TRANSPORT AND TELECOMMUNICATION INSTITUTE

Alexander V. BEREZHNOY

INVESTIGATION OF THE TRAFFIC FLOW MODELS MANAGING PARAMETERS INFLUENCE ON THE EFFICIENCY OF THE URBAN TRAFFIC CONTROL

SUMMARY OF THE PROMOTION WORK

RIGA – 2008

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Scientific supervisors:
Dr.habil.sc.ing., Professor
Igor V. Kabashkin
Dr.sc.math., Professor
Sharif E. Guseynov

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OFFICIAL OPPONENTS:
Dr.habil.sc.ing., Professor Valery A. Kutev
Director of Electronics department
Transport and Telecommunication Institute, Latvia
Dr.habil.sc.ing., Professor Yuri N.Shunin
Vice-rector, Information systems management institute (ISMA), Latvia
Dr.sc.ing. N.I.Kobasko
Director of technology and R&D, "IQ Technologies Inc.", Ohio, USA

The defence of the thesis will be delivered in 07.10.2008 at 16:00 o'clock at the special Promotion Council of Transport and Telecommunication Institute on award of a doctor's degree to the address: 1, Lomonosova Street, conf. hall 4-130, Riga, Latvia, tel. (+371) 67100594, fax: (+371) 67100535.

CONFIRMATION

I confirm that I developed the promotion work that is presented to the promotion council of Transport and Telecommunication Institute to obtain the scientific degree of Doctor of Science in Engineering. The promotion work has not ever been presented to any other promotional council to obtain the scientific degree.

July 04, 2008 Alexander V. Berezhnoy

The promotion work is written in English, contains an introduction, 3 chapters, conclusions, 65 figures, 220 formulas, 12 tables, 256 pages in total. Bibliography contains 122 sources.

ABSTRACT

The promotion work "Investigation of the Traffic Flow Models Managing Parameters Influence on the Efficiency of the Urban Traffic Control" by Alexander V. Berezhnoy to obtain the scientific degree "Doctor of Science in Engineering in Telematics and Logistics" has been worked out. Scientific supervisors of the work are Dr.habil.sc.ing., Professor Igor V. Kabashkin, Dr.sc.math., Professor Sharif E. Guseynov.

The work is performed to investigate the mathematical models describing the motion process of vehicular traffic. Construction of mathematical models for determination of the characteristic of traffic flow density is carried out in the work. The unequivocal finding of traffic flow density in the considered domains of urban transport system allows predicting the further utilization condition of a street-road network and in appropriate way to change the controlled parameters of corresponding transport management and control systems. As a result of managing influences implementation there arises an opportunity of purposeful rational change of the density of traffic flows on the controlled road sections that substantially promotes the traffic congestions problem solution and raises a degree of controllability of transport system.

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1. ACTUALITY OF THE PROBLEM

In the majority of the world's cities, megapolises and agglomerations there exists a global and extremely complicated situation with the overloading of an urban street-road network nowadays. The permanent increase of vehicles number leads to road congestions, and consequently results in the increase of road accidents quantity; leads to the economic losses expressed in infringement of public transport schedules, travel time expenses for operative transport motion, breakdown of delivery timeliness both for the goods and passenger traffic in logistics chains; affects the ecological situation with environmental contamination, noise pollution, landscape infringements etc. All the listed problems of the urban transport environment could be expressed in total expenses increase, connected with the maintenance and operation costs for vehicles and elements of a road-transport infrastructure.

As very actual and sharp costing problems are most often stressed out the insufficient throughput of a street-road network, which is not keeping up with intensive growth of traffic volumes; the limited degree of controllability of the vehicular traffic flows, influenced, mainly, by the static nature and "localness" of the majority of transport regulation ways especially in conditions of high dynamics of road situation changes; unpredictability of driving process participants behaviour, owing to the fact that transport system should be regarded as techno-social system.

In order to successfully overcome the specified problems there is a whole set of the decisions, which are based on various methods of management of the road-transport conditions. For finding out an effective strategy of management of the traffic flows it is necessary to consider a wide spectrum of traffic flow characteristics, influence dependencies for both external and internal factors and its dynamic properties. It is practically impossible without a preliminary estimation of possible consequences of some planned changes in topology of a transport network, changes of a route map of the city, introduction or removal of any organization of the traffic flows as concerns the road traffic signs, and the traffic lights or the information pointers, and so forth. The decision of the mentioned issues class is interfaced to known complexities of carrying out the full-nature tests and experiments that in its turn is connected with exclusively high economic expenses and directly touches upon the human life safety area. Therefore, even at a preliminary stage, it is already traditionally involved the mathematical apparatus of modelling and simulation of vehicular traffic processes for forecasting results of introduction and the analysis of the offered construction cases. The scientific basis of modelling of the vehicular traffic motion process is the mathematical theory of traffic flows.

The main classical question of the traffic flows theory, which is of essential interest to modern applied sphere of traffic management, is studying of the existing interrelation amongst the general characteristics of a traffic flow. If to be more precise, the most significant is the question of exact and unequivocal determination of the general characteristics of a traffic flow and in particular, the traffic flow density. For this purpose various mathematical models describing behaviour of a traffic flow in various conditions are applied. Such unequivocal determination of the density characteristic of a traffic flow allows increasing traffic management efficiency, due to improvement of quality of traffic lights regulation modes.

The given dissertation work is devoted to research of the density characteristic of a traffic flow as a function depending on spatial coordinates and on time. There are widely known many different hydro-, gas-, electro-dynamic models, models of statistical physics and the other model-analogues allowing determining the density of traffic flows depending on traffic situation. In the most cases the mentioned types of models investigate the one-dimensional traffic flows at the one-lane roads. In the given research there are investigated both one-dimensional and 2-D mathematical models aimed for multilane roads. Vehicular traffic models of hydro-, gas-, electro-dynamics origin as well as statistical physics models are constructed on the basis of fundamental laws of the corresponding scientific areas. During the adaptation process of given models to the traffic flows motion process description there should be established a relation between original physical laws and dependencies, which describe the traffic flow generated by transport system. As a rule, the mentioned adaptation requires taking into consideration some suggestions and assumptions on the existence of functional dependence amongst the general characteristics of traffic flow, for instance, the functional dependence between velocity and density of traffic flow in the Witham's and Greenshields-Greenberg's models that lead to some contradictions, like the negative values of velocity and/or traffic density. Therefore, construction of mathematical models of macroscopic and/or microscopic nature as well as probabilistic models "from zero" taking into account only transport systems behaviour laws becomes essentially topical, i.e. the above mentioned assumptions arising during adaptation process, which may lead to various contradictions are absent in models produced "from zero". In the present research the construction "from zero" of mathematical models for traffic flow density characteristic going directly from the nature of vehicular traffic flow is carried out.

2. PURPOSE AND OBJECTIVES OF THE RESEARCH

The purpose of the promotion work is to unequivocally determine the characteristic of density of vehicular traffic flow in any point of multilane road at any time moment in order to support the decision-making at forecasting the given road section utilization condition and at selection of time parameters of corresponding traffic lights regulation adaptive operating cycles.

The unequivocal finding of density of the traffic flow in the required road section at any time moment will allow carrying out more effective resource-saving traffic flows management. Besides, the unequivocal determination of density of the traffic flow at the considered transport system domains will allow to change the transport system controlled parameters in such a way that vehicles distribution in the single road section, urban district or a city scale, i.e. densities of the traffic flows at the road network bottlenecks should remain within the allowed limits for the peak hours in order to avoid various negative consequences for transport system, such as impact of traffic congestions.

To realize the brought forward purpose there are primarily to be set and solved the following problems:

1. Carrying out the review of the most well-known existing models of the traffic flow, revealing their scope of application, analysing its major lacks and restrictions. Performing a substantiation of expediency for development of traffic flow density mathematical models starting "from zero" for the cases of simplified – one-dimensional traffic flow and realistic – 2-D traffic flow.
2. Construction of mathematical models for the traffic flow density characteristic determination and the analysis of application scope, revealing of the major lacks and restrictions of designed mathematical models.
3. Analytical solution of the stated mathematical problem formulations for traffic flow density determination and obtained solutions investigation in terms of possible technical-engineering implementation.
4. Carrying out of the numerical experiments and check on adequacy of constructed vehicular traffic flow mathematical models based on example of real data and simulation results.

3. RESEARCH FOCUS

In the dissertational work the well-known methods of modelling used at investigation of vehicular traffic are considered and it is carried out the review of some major existing models describing the vehicular driving process. At present, there have been already investigated a lot of macroscopic and microscopic models of a traffic flow, particularly – one-dimensional. The most popular of them are macroscopic models of Greenberg, Greenshields, Ligthill-Whitham, Prigogine, Daganzo, Payne, Burger, Weidlich-Hilliges, Cremer, Kühne, Papageorgiou, Kerner-Konhauser, etc.; microscopic models of Reuschel, Newell, Cremer-Ludwig, Pipes, Herman-Rothery, Gazis, Chandler, Gipps, Chow, Helbing, Kometani-Sasaki, Kühne-Kroen, Kühne-Rödiger, Fox-Lehmann, Helbing-Tilch, Nagel-Schreckenberg, May-Keller, Chang-Lai, Tomer, Nakayama, Nakanishi, Manstetten, Hoefs, Bleile, Klauck, etc. Some of these models have deterministic nature: knowing the initial state of the flow completely defines its coordinates at any consequent moment of time. It is significant that amongst the listed models there are also the models of non-deterministic nature.

At the heart of deterministic models there is a suggestion of functional dependence existence amongst certain mathematical parameters of traffic flow, for instance, functional dependence between the velocity of vehicles and distances between them in the traffic flow; functional dependence between the velocity of traffic flow and its density, etc. It is significant that requirement of the existence of functional dependence can lead to some contradictions. For example, an intuitively correct presumption of functional dependence between the velocity and density of flow can lead to negative values of density or velocity. It makes to presume the existence of equilibrium flow state, i.e., it is necessary to have a requirement that average velocity of the flow at any moment of time would correspond to the equilibrium value at the given vehicles density. The mentioned assumptions oblige the investigator to restrict the range of application of the constructed mathematical model, by restraining consideration of traffic flow process only in road sections without crossings, etc.

Therefore, in parallel with the deterministic models, there appears the necessity of constructing the non-deterministic models of traffic flows and elaboration of methods for solving them. There are the well-known basic features of traffic flow such as uncertainty, finiteness, distance dependence on time and other, which, being ignored, do not allow us to consider the built of mathematical models as complete and real enough, to use them as traffic motion models.

In the present work the non-deterministic models are considered, in which traffic flow is presented as flow of particles in the investigated environment with the allowed motion in both directions as forward and as well opposite. By analogy with physical phenomenon, for description of vehicular traffic can be used mathematical tools of statistic physics, where random displacements of particles affected by random moments are permitted. In non-deterministic models it is possible only with certain probability to predict motion of each separate vehicle. However, if there is a large amount of vehicles, moving "accidentally" and "independently" of each other, the behaviour of traffic system in whole can be predicted quite well. In the dissertation work exactly this fact is taken into account at the studies of traffic flows.

The analysis of works in the considered area as a whole has shown that do not exist universal models for today, which are equally well-suited for the description of traffic motion process in various situations and are taking into account the large variety of influencing factors. Moreover, in some existing general models besides the known restrictions, some contradictions mentioned above are revealed, which reason of occurrence is the insufficient degree of adaptation while borrowing of the laws acting in other scientific areas. Therefore, for the further perfection of existing models, in the given investigation the construction of mathematical traffic flows models proceeding directly from the nature of traffic is carried out firstly, and only subsequently the further development of models occurs in a direction of designing and the analysis of a 2-D case of traffic motion.

4. METHODOLOGY AND METHODS OF THE RESEARCH

In order to investigate the objectives in the dissertation the mathematical analysis methods, PDE theory methods, approximation methods, inverse coefficient problems theory methods as well as numerical methods are used. Besides, by means of applying statistical physics laws and traffic flows mathematical theory laws, mathematical models describing driving process in a stream on multilane road on spatial variables are deduced for one-dimensional and 2-D cases. The constructed models describe both discrete and continuous vehicular traffic. As a representation apparatus used in discrete models for the quantity of vehicles in the specified fixed point of a road section at the given moment of time as a difference of vehicles amount between the neighbouring nodes of discrete existentional grid. At construction of traffic flows continuous models methods of differential equations in partial derivatives theory are applied.

In the dissertation the following methods of the mathematical analysis and mathematical physics are used:

- methods of limiting transitions in functional and numerical sequences;
- methods of comparison of indefinitely big and small sequences;
- a method of functions decomposition in Taylor's series;
- elements of the vector and tensor analysis theory;
- methods of the non-uniform differential equations reduction to homogeneous;
- methods of polynomial and splines interpolation;
- methods of the coefficient inverse problems theory of mathematical physics;
- methods of the stability analysis both for mathematical statements and for the received solutions of differential equations;
- proofs of existence and uniqueness of the considered problems;
- Tikhonov's steady methods for the solution of ill-posed problems.

In the dissertation work the scientific and educational literature in the mentioned areas, thematic materials of periodicals, proceedings of the international conferences and so forth were used.

For the constructed models adequacy check the numerical experiments are carried out. For its realization the statistics on urban traffic flows has been gathered and processed, the software has been developed, calculations in software package MathCAD have been carried out and simulation in the VISIM environment has been performed.

5. SCIENTIFIC INNOVATION

The new scientific results obtained in the dissertation work are the following:

1. The discrete and continuous non-deterministic traffic flow models for determination of the traffic density characteristic in cases of one-dimensional and 2-D traffic flows are constructed.
2. The integral managing parameter – sensitivity coefficient, which determination allows performing an estimation of efficiency of the traffic management, is deduced and interpreted.
3. Requirements to the traffic flow management are revealed, allowing providing such mode of vehicular traffic motion at which on an investigated road section the best stream throughput mode is selected, providing an exclusion of occurrence of the traffic congestions.
4. Engineering-technical requirements to quantity and allocation of measuring systems sensors for traffic flow registration are formulated. Invariance of requirements to a control system in relation to measuring systems gauges allocation for gathering primary measuring information is proved.
5. The technique for determination and forecasting of quantitative changes of the traffic flow density characteristic in any arbitrary chosen point of an investigated road section is proposed.

6. PRACTICAL VALUE AND REALIZATION

In the promotion work the technique for the traffic density characteristic determination in the given section of multilane road at the arbitrary time moment is proposed. Finding of traffic density in the required point of the given road section at the arbitrary time moment allows to realize the adaptive management concept of the traffic flows. For this purpose in the research work the scientific approach to the traffic lights regulation time parameters selection for the known function of a traffic flow density is offered.

Besides, the unequivocal determination of the traffic flow density at the considered transport system domains will allow to change the transport system controlled parameters in such a way that vehicles distribution in the single road section, urban district or a city scale, i.e. density of the traffic flows at the road network bottlenecks should remain within the allowed limits for the peak hours in order to avoid various negative consequences for transport system, such as impact of traffic congestions.

Practical recommendations on selection of the primary data gathering form for the traffic flow as well as on the choice of measuring systems stationary sensors are resulted in the work. The condition according to which it is sufficient to get benefit of using the only one additional sensor for determination of density of the traffic flow, besides located at the beginning and at the end of considered road site is formulated and proved for as much as long interval of road.

It is offered to use the sensitivity coefficient present in the constructed models, which can be considered as the integrated characteristic of a vehicular traffic flow, as a key evaluation parameter of the efficiency analysis of transport system dynamic characteristics at the traffic organization processes.

Results of the researches conducted in the given work can be used while solving questions of the operative traffic organization in the city scale. The constructed models can also be applied for the purpose of urban forward planning, at development of new inhabitable areas, as well as to construction of new roads and highways and the analysis of possible traffic utilization in residential areas of cities.

7. WORK APPROBATION

Results of the dissertation were reported and discussed at the following conferences and seminars:

- International conference "Nordic-Baltic Transport Research Conference" (Riga, 2000);
- TransBaltica 2002 – VII International Conference "Transport. Communications. Logistics" (Riga, 2002);
- RelStat'03 – "Reliability and Statistics in Transportation and Communication" (Riga, 2003);
- International conference "Lithuania without Science – Lithuania without Future" (Vilnius, 2004);
- ICRAT 2004 – 1st International Conference on Research in Air Transportation (Zilina, 2004);
- RelStat'05 – "Reliability and Statistics in Transportation and Communication" (Riga, 2005);
- Scientific-practical and education-methodical conference "Science and technology – step into the future" (Riga, 2006);
- INFRADAY'07 – 6th International Conference on Applied Infrastructure Research "Sustainability and Reliability of European Infrastructure: Investment, Innovation, and Regulation" (Berlin, 2007);
- RelStat'07 – "Reliability and Statistics in Transportation and Communication" (Riga, 2007);

- Extended seminar of electronics department "Investigation of vehicular traffic management models with assistance of ITS systems and its making use for increase of urban transport system efficiency " (Riga, 2007);
- First International Conference on Soft Computing Technologies in Economy (Baku, 2007);
- Interdepartmental seminar of computer networks department and mathematical methods and modelling department "On some problems of traffic flow density determination and on traffic management issues" (Riga, 2007);
- Research and Academic Conference "Research and Technology – Step to the Future". (Riga, 2008).
- MBITS'08 – International conference "Modelling of Business, Industrial and Transport Systems" (Riga, 2008).

Research materials on the dissertation topic are used during listed below projects implementation process that are realized according to Riga city council and financed from European funds as well as from the source of Latvian scientific council grants:

- European COST Action 355 "Changing behaviour towards a more sustainable transport demand" (2004-2008);
- Intelektuālā rajona tīkla un tā realizācijas pilotprojekta funkcionēšanas koncepcijas un nacionālā modeļa izstrāde uz Daugavpils Akadēmiskā parka bāzes, 2006.-2009. LZP sadarbības projekts Nr. 06.0027 (2006-2009);
- Development of the model of Europe-Asia multimodal corridor intelligent transport system for optimization of Latvia-Belarus international logistics chain (TransLaB). Ministry of Education and Science of Latvia (2007-2009).
- Development of the stable analytical and numerical methods for solving of inverse and direct problems of deterministic and probabilistic systems. Ministry of Education and Science of Latvia Nr. IZM/08 (2008);
- Transporta sastrēgumu monitoringa metodoloģijas izstrāde plūsmu uzlabošanai pilsētā. Rīgas dome. (2007);
- Transporta intelektuālo sistēmu attīstība Latvijā. LZP sadarbības projekts Nr. 02.0001. (2001-2005);
- Innovative Vocational Education and Training in Transport Area (IVETTA). EU Leonardo da Vinci Programme Nr. 2004 – 1818 / 001-001 LE2 74SUB. (2004-2005);

- Latvijas Transporta sistēmas optimizācija. LZP sadarbības projekts Nr. 02.0001. (1999-2001).

8. PUBLICATIONS

Research results on dissertation subject are published in 22 scientific works incl. 16 articles are published in various scientific magazines and periodicals and 6 theses of scientific reports are published in proceedings of the international conferences. Problems considered in publications have been covered at the international scientific conferences, which have taken place in Latvia, Lithuania, Azerbaijan, Slovakia and Germany.

9. STRUCTURE OF THE THESIS

The dissertation consists of the introduction, three chapters, conclusions, bibliography and 2 appendices. The total amount of the dissertation (without appendices) contains 256 pages, 65 figures, 12 tables. The list of the literature includes 122 names of information sources.

In the first chapter "Urban Transport System Condition Analysis Methods Review" the review of the system of urban transports general problems, in general and vehicular traffic, in particular is carried out. Classification of existing approaches to solving the urban traffic motion problems is offered. The analysis is carried out and comparative characteristics of the existing mathematical deterministic and non-deterministic macroscopic models are resulted.

The second chapter "Construction and Analysis of Non-Deterministic One-Dimensional Traffic Flow Model on the Multilane Road" is purposed to development of own discrete and continuous mathematical models for density of traffic flow in the simplified one-dimensional case "from zero". Scope of models application, advantages, shortcomings and restrictions of the constructed discrete and continuous models are considered. The probabilistic interpretation of the introduced concepts, the received formulas and statements is given in this chapter.

In the third chapter "Construction and Analysis of Non-Deterministic 2-D Traffic Flow Model on The Multilane Road" are constructed discrete and continuous mathematical models for the determination of traffic flows 2-D scalar density. The main assumptions are analysed, shortcomings and restrictions are revealed and probabilistic interpretation of the constructed models is given. Adequacy verification of the constructed model is resulted, numerical experiment is realized and examples showing the practical making use of the obtained theoretical results are given.

10. DESCRIPTION OF THE MAIN RESULTS OF THE RESEARCH

In the first chapter the express analysis of the major existing problems of the urban transport system is carried out. As very topical and sharp costing problems are most often stressed the traffic congestion and street-road network overload problems, driving safety problems and traffic ecological impact problems. The review of the transport system general components is carried out. According to that the transport system will consist of a street-road network, road infrastructure objects, traffic flows and technical management and control systems, as central object of research of the given dissertation is considered the traffic flow. Its general characteristics, properties and features are determined in dissertation, and also the factors influencing generation of traffic flows are considered.

Classification of existing approaches to the solution of problems of traffic motion in the city scales (Fig. 1) is offered.

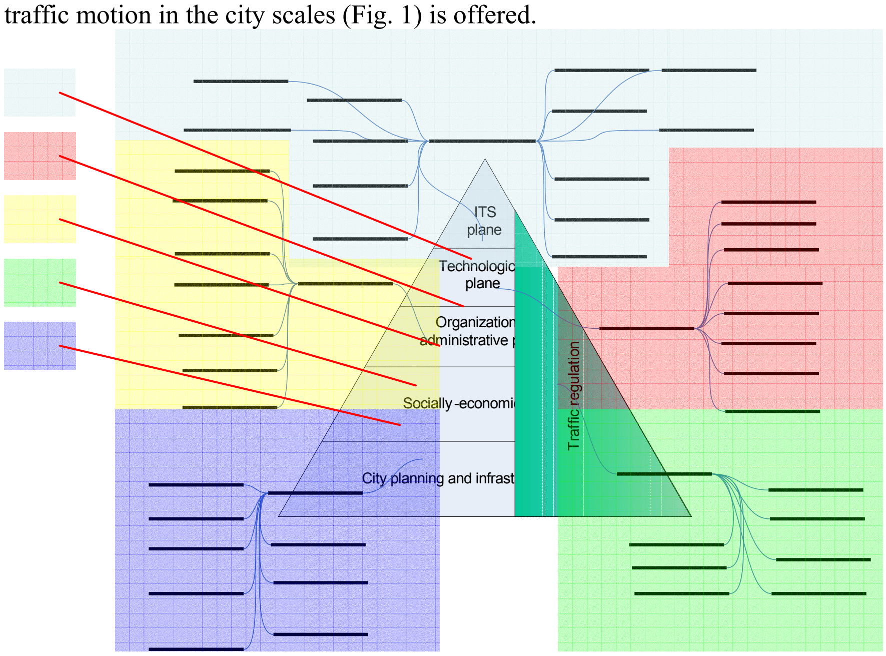


Fig. 1. *Classification of the traffic management and control methods*

According to the proposed classification, all problems considered in dissertation work are solved within the framework of technological methods and management and intelligent transport systems classes.

The comparative analysis of the European projects aimed at introduction of modern technological methods of urban traffic management is carried out in the work. The architecture of the automated control systems, technical methods of management and control for traffic flows, as well as ways of information gathering, which is necessary for realization of managing influences are considered.

Approaches to an evaluation of traffic flows management efficiency in the city scale are analysed. Examples of measured urban transport system parameters from position of traffic efficiency evaluation are as follows: travel time for regulated highways; vehicles average velocity; average delay values for vehicles at crossroads; number of stops at the red traffic lights signal; number of road accidents, including road catastrophes; pollution degree for the city soil, water and air; noise pollution level; land use and landscape infringements and others.

Efficiency of the traffic automated control system, in general case, represents a functional, depending on a set of services, which can be realized within the framework of the given system:

$$E = f\left(E_{S_i}\right),$$

where E is traffic management automated road system (TMARS) efficiency, S_i is the i^{th} service realized within the framework of TMARS system.

Efficiency of i^{th} service, on an example of traffic flows automated control service, depends on such properties of system, as reliability and quality of decision-making, etc., in its turn adhered to the system internal architecture.

$$E_{S_{ACK}} = \prod_{i=1}^{N} e_i\left(a_j\right) \to \max \,\middle|\, TCO \to \min,$$

where e_i is i^{th} system property, depending on system construction architecture a_j, and the TCO is total cost of ownership index, $j = 1, \ldots, M$.

The concept of TMARS efficiency as a control toolset develops not only of an estimation of a parameters-indicators degree of change in view of solved problems, but also of an estimation of profitability of system on spent resources (time, labour, material, land use, finances, information, power and so forth) and estimation of a centralized management system reliability degree (Fig. 2).

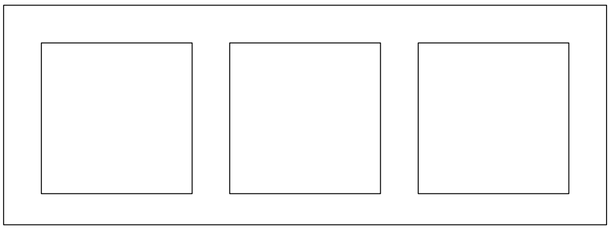


Fig. 2. *System efficiency concept as management and control means*

Evaluation of TMARS efficiency can be carried out on the basis of comparison of external results before the management system introduction with the results of the previous road situation.

In a general case, given class systems can be evaluated by means of applying the parameter of losses that looks like as follows:

$$C = W_0 - W\ ,$$

where W is a parameter of average losses, which are taking place at use of centralized management and control automated system; W_0 is a parameter of average losses when management of city traffic is carried out without intervention of TMARS system. Thus, the estimation of TMARS system efficiency can also be expressed through parameters of losses as follows:

$$E = 1 - \frac{W}{W_0} = -\frac{C}{W_0}.$$

As an evaluation criteria used in the research work for the managing influences introduction in the transport system the changes in density of the traffic flow on the investigated road section are considered.

In the dissertation the known modelling methods for motorized driving process investigation are reviewed. Some major existing models describing the process of traffic flow motion are listed and the systematisation of mathematical models of the traffic flow is performed. Comparative analysis of the proceeded models application area is made; the main advantages, shortcomings and restrictions of the models are revealed.

It is noted that to all considered models is peculiar the presence of existence of the requirement of mutually unequivocal functional dependence. However, in some cases the similar assumption leads to some contradictions. For example, intuitively true assumption of functional dependence between velocity and density of a traffic flow can lead to negative values of flow density or traffic velocity that compels to set the assumption of existence of an equilibrium condition for a stream. That's why there are made demands regarding conformity of an average traffic flow velocity during each moment of time to its equilibrium value at the given traffic density. The specified assumptions considerably restrict a scope of mathematical models, being limited to consideration of traffic flows motion process only on roads sections without crossings, etc. Therefore it is shown that alongside with the deterministic models there is a need of construction of models of the non-deterministic traffic flows, and development of methods of its solution. In the dissertation major properties of a traffic flow (like uncertainty, finiteness, dependence of distance on time, and so forth) are in details described and its ignoring does not allow considering the constructed mathematical models as high-grade and real enough to be accepted as models of vehicular traffic motion.

In the second chapter, because of the mentioned reasons the concept of traffic flow "restoration" (self-restoration) has been introduced, meaning the inversely proportional dependence of vehicles quantity at any fixed point of the considered road section on the average arithmetic value of vehicles in the neighbouring points. On the basis of the introduced traffic flow "restoration" concept, it has been constructed the discrete non-deterministic model, which under the presence of some initial data allows to determine the demanded vehicles quantity in the traffic flow in any point of investigated road section at any moment of time:

$$U(m,n+1) \approx \frac{U(m-1,n)+U(m+1,n)}{2},$$

where a $U(m,n)$ function $(m = 0, \pm 1, \pm 2, ...;\ n = 0,1,2,...)$, giving the vehicles quantity at time moment $n \cdot \Delta t$ at point with coordinates $m \cdot \Delta x$. The analogue of the obtained recurrent expression for the traffic flux level characteristic has been constructed.

Features, properties, restrictions and application scope of the constructed discrete model have been analysed. It is shown that the proposed discrete model loses its adequacy near by the dynamic boundaries of a traffic flow, and in internal points takes into account bi-directional traffic motion of a one-dimensional stream with the presence of driving overtaking possibilities.

In the promotion work there are formulated mathematical conditions only under which satisfaction the transition from discrete to continuous model becomes possible. As a result of performing limiting transitions at $\Delta x \to 0$, $\Delta t \to 0$, the continuous model for traffic flow density determination is obtained:

$$\frac{\partial \rho(x,t)}{\partial t} = a^2(t) \cdot \frac{\partial^2 \rho(x,t)}{\partial x^2}, \ 0 < x < l, \ 0 < t \le T,$$

where

$$a^2 = \lim_{\substack{\Delta x \to 0 \\ \Delta t \to 0}} \left(\frac{\Delta x}{\sqrt{2 \cdot \Delta t}} \right)^2 > 0$$ is a sensitivity coefficient.

The deduced continuous model allows determining unequivocally required density of a traffic flow for the given initial and boundary conditions on density:

initial condition

$$\rho(x,t)\big|_{t=0} = h(x), \ 0 \le x \le l,$$

boundary conditions of the first type (Dirichlet conditions)

$$\rho(x,t)\big|_{x=0} = 0, \ 0 \le t \le T,$$

$$\rho(x,t)\big|_{x=l} = 0,\ 0 \le t \le T,$$

consistency conditions

$$\begin{cases} h(0+0) = h(l-0) = 0, \\ h''(0+0) = h''(l-0) = 0, \end{cases}$$

and requirement

$$\begin{cases} a^2(t) \in C[0,T], \\ h(x) \in C^4[0,l]. \end{cases}$$

The received continuous model completely coincides by its form with mass equation, known from the theories of heat conductivity, diffusion and the theory of a filtration. It is mathematically proven that the sensitivity coefficient, which is present in continuous model is known as mass transfer coefficient in the mass equation, but is not a constant, is unknown and is a subject to determination for the case of vehicular traffic modelling. Given coefficient in the general case depends on spatial and time arguments, traffic flux and velocity.

Solution of the direct problem, which is rooted in traffic flow density determination, can be found in the form of Green's function of the considered problem at the priori known sensitivity coefficient and given initial, boundary and corresponding consistency conditions.

The question of sensitivity coefficient existence and its unequivocal determination, which is not a priori given in the traffic flows theory, has been studied. For this purpose an inverse problem of sensitivity coefficient unequivocal determination has been formulated.

$$\frac{\partial \rho(x,t)}{\partial t} = a^2(t) \cdot \frac{\partial^2 \rho(x,t)}{\partial x^2},\ 0 < x < l,\ 0 < t \le T$$

initial condition

$$\rho(x,t)\big|_{t=0} = h(x),\ 0 \le x \le l,$$

boundary condition of the first type (Dirichlet conditions)

$$\rho(x,t)\big|_{x=0} = 0,\ 0 \le t \le T,$$

$$\rho(x,t)\big|_{x=l} = 0,\ 0 \le t \le T,$$

consistency conditions

$$\begin{cases} h(0+0) = h(l-0) = 0, \\ h''(0+0) = h''(l-0) = 0, \end{cases}$$

requirement

$$\begin{cases} a^2(t) \in C[0,T], \\ h(x) \in C^4[0,l]. \end{cases}$$

and additional condition

$$\rho(x,t)\big|_{x=\tilde{x}} = \tilde{\rho}(t),\ 0 \le t \le T\ ,$$

where $\tilde{\rho}(t)$ is a known function and $\tilde{x} \in (0,l)$ is a set point, in which it is expedient to allocate telemetric measurement systems information collection sensors for monitoring and registration of density of the traffic flow.

The theorem defining sufficient conditions needed for unequivocal determination of the required sensitivity coefficient is formulated and proved.

Theorem 1. *Let us suppose that the following two conditions are added to the above formulated inverse problem:*

A. $\forall x \in (0,l)\ \ h''(x) > 0$ *or* $h''(x) < 0$,

B. $0 < a^2(t) \in C^1[0,T]$.

Then the coefficient $a^2(t)$ *in the formulated inverse problem can be determined uniquely.*

Satisfaction of these sufficient conditions narrows an application scope for the continuous model for finding out the traffic flow density. However it is shown that performance of the theorem conditions allows to predict a situation of traffic jam occurrence on investigated road section and thus to control the least "possible" density of traffic flow.

The non-linear equation of the first type relevant to unknown sensitivity coefficient is obtained:

$$Aa^2(t) = \tilde{\rho}(t),$$

where

$$Aa^2(t) \overset{def}{\equiv} \sum_{n=1}^{\infty} h_n \cdot \sin\frac{\pi \cdot n \cdot \tilde{x}}{l} \cdot e^{-\frac{\pi^2 \cdot n^2}{l^2} \cdot \int_0^t a^2(\tau)d\tau}.$$

It is proved that the received equation is unstable, and consequently, the necessity of making use of the regularizing methods providing the stability of its solution, for example, a method of Tikhonov’s regularization is shown. It is described the regularizing algorithm based on a Tikhonov’s regularization method, allowing to find the approximate stable solution of investigated inverse problem.

The analysis of the constructed discrete and deduced continuous models for a traffic flow density determination in a one-dimensional case has been carried out. It is proved that the revealed restrictions of discrete model remain as well for continuous model after performing the limiting transition: near by dynamic boundaries of a traffic flow the continuous model loses its adequacy.

Probabilistic interpretation of the constructed models is given according to which the sensitivity coefficient can be treated as half of dispersion of a total coordinates increment of the vehicle at independent "jumps" across the nodes of discrete grid for a time unit; the traffic density characteristic is interpreted as density of a mathematical mean of "vehicles weight"; in case of consideration of single vehicle motion, function of density can be understood as function of probability distribution for the location of the given "single" vehicle.

Recommendations for practical use of sufficient conditions for sensitivity coefficient determination are formulated as well as complexities arising during this application are investigated.

1. Condition $\rho(x,t)\big|_{t=0} = h(x), 0 \le x \le l$ means that for initial time moment it is required to receive the distribution of traffic flow density on the investigated road section $[0,l]$.
2. Condition $\rho(x,t)\big|_{x=0} = \varphi_1(t), 0 \le t \le T$ in terms of practical realization requires to obtain a density distribution function in the small vicinity to the right of the beginning of examined road section, i.e. in the segment $[0,+\varepsilon], \varepsilon << l$ for the fixed time interval $[0,T]$. As ε variable it is taken the value equal to 1,5-2 average vehicle length taking into account the minimum permissible distance between vehicles in the flow for the measured average traffic velocity.
3. Condition $\rho(x,t)\big|_{x=l} = \varphi_2(t), 0 \le t \le T$ means that for the fixed time interval $[0,T]$ it is necessary to obtain the traffic density in the left end vicinity of the considered road section, i.e. in segment $[l-\varepsilon,l], \varepsilon << l$.
4. Condition $\rho(x,t)\big|_{x=\tilde{x}} = \tilde{\rho}(t), 0 \le t \le T; \forall\tilde{x} \in (0,l)$ for the practical realization means that it is needed to find out traffic density in the small vicinity of the arbitrary chosen internal point of investigated road section for the time period $[0,T]$.

Simultaneous performance of the listed parameters measurements at the fixed values T and l for a investigated road section allows to obtain all necessary initial data for the solution of the formulated coefficient inverse problem.

The comparative analysis of interpolation methods, which are necessary for transformation of the initial discrete data into continuous functions, has been carried out, since it is necessary that the initial data have been set as continuous and continuously-differentiated functions for the making use of continuous model. It is shown that for the solution of a given problem it is expedient to use only cubic interpolation by spline-functions.

The numerical experiments are conducted as in order to investigate the solutions of obtained models at the various initial and boundary conditions, as for practical application relevant to some sections of street-road network of the Riga city.

The software for realization of the offered discrete model of a one-dimensional vehicular traffic motion is developed. Values of non-stationary sensitivity coefficient and required vehicles quantity in a one-dimensional traffic flow for various values of initial and boundary conditions are received.

Taking the example of various initial data (various initial distributions of a traffic flow at all distance of considered road section) for the set road section there have been calculated quantities of vehicles in the subsequent moments of time. The structure for performing the numerical experiments is shown in the table 1.

Table 1.

Numerical experiments structure for the discrete traffic flow model

		One-dimensional discrete model	
		Artificial initial conditions	Adapted initial conditions
Closed system		+	+
Open system	without sources	+	+
	having sources	-	-

As the initial vehicles quantity distribution along the considered road section there were taken mathematical functions as representatives of various degree functions and the data, which were received as experimental vehicle distributions across the examined road section and transferred to final value (approximated) on a numerical OX axis.

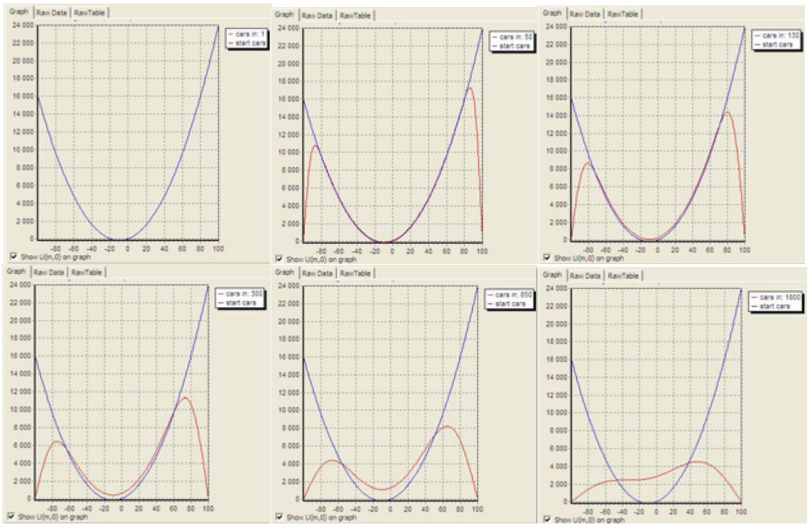


Fig. 3. *Vehicles number square function* $U(m,0) = 2 \cdot m^2 + 40 \cdot m + 5$ *time evolution across the road section*

As the initial data for the approximation, there have been taken vehicles observation results from a file of a traffic flow video record at Raņķa dambis street in the morning peak hours.

Fig. 4. *Vehicles initial distribution function across the Raņķa dambis street road section*

In all the conducted numerical experiments it has been noted that change of vehicles amount on a road section in a certain time interval results in alignment of traffic flow density to some value corresponding to a difference between the amount of vehicles driving on considered road section and vehicles leaving the given site on its both boundaries. Such a behaviour nature of traffic flow is explained by the model existent property of a traffic flow restoration.

Thus, any distribution of vehicles quantity at the road section will tend to a linear form, and velocity of alignment process will be caused by the road geometrical characteristics as well as by the traffic flow density functions on the borders of the given site.

There were also performed numerical experiments for the continuous model, where have been considered examples for the closed transport system (with homogenous boundary conditions) and for the open transport system (for the non-homogenous boundary conditions).

Table 2.

Numerical experiments structure for the continuous model

		One-dimensional continuous model	
		Artificial initial conditions	Adapted initial conditions
Closed system		+	+
Open system	without sources	+	+
	having sources	-	-

By means of using standard software package MathCAD the continuous model of a one-dimensional traffic flow for finding out of traffic density is realized. The verification of the offered mathematical models in order to check its adequacy has been performed.

Studying the continuous model solutions behaviour nature for a set of typical cases, as changeable parameters functions of initial conditions, boundary conditions and non-stationary sensitivity coefficient were considered. The majority of experiments are carried out at observance of the following assumptions:

1. about identical number of lanes in each of directions;
2. regarding the traffic flows symmetry moving in opposite directions;
3. the sensitivity coefficient is calculated only once and remains constant during all experiment.

For carrying out of numerical calculations while studying the traffic flow one-dimensional continuous model behaviour nature, there were taken 6 various functions of initial distribution of traffic flow density, corresponding to initial conditions of a considered problem aimed on demonstration of some properties of researched model.

Table 3.

The direct problem initial condition functions

1. $p0wave(x) := \left(\sin\left(\frac{\pi \cdot x}{1}\right) + \frac{\cos(x \cdot 3)^2}{1} \right)$	4. $p0scw(x) := \begin{vmatrix} f \leftarrow \left[\sin\left(\frac{\pi \cdot x}{1}\right) \cdot \left|\cos\left(\frac{\pi \cdot x}{1}\right)\right| + \left|\frac{(\cos(x \cdot 4))^2}{1} \right. \right. \\ f \leftarrow 0 \ \text{if} \ f < 0 \end{vmatrix}$

2. $p0sin(x) := \sin\left(\frac{\pi \cdot x}{1}\right) \cdot 1$	5. $p0sc(x) := \sin\left(\frac{\pi \cdot x}{1}\right) \cdot \left\lvert \cos\left(\frac{\pi \cdot x}{1}\right) \right\rvert$
3. $p0wave2(x) := \begin{array}{\|l} f \leftarrow \left(\left(\sin\left(\frac{\pi \cdot x}{1}\right)\right)^2 \cdot \cos \right. \\ f \leftarrow 0 \ \text{if} \ f < 0 \end{array}$	6. $p0asimectic(x) := \sin\left(\frac{\pi \cdot x}{1}\right) \cdot \lvert \cos(x) \rvert$

For each of the listed functions there have been received density distribution functions. The required solution has been calculated at the various sensitivity coefficient a^2 values. In order to investigate the influence of sensitivity coefficient on the traffic flow density function behaviour, calculations for the following values of sensitivity coefficient a^2 are performed: 0, 1, 2.5, 5, 40.

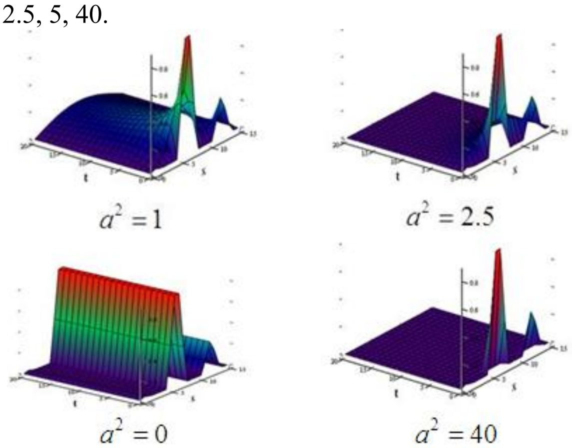


Fig. 5. *Traffic flow density surface for the various sensitivity coefficient values*

Apparently from the function of density surfaces, the general influences of sensitivity coefficient will consist in a smoothing effect, and, than more is the value of sensitivity coefficient, the more sharply there is performed a transition to boundary function values. And finally during some period of time initial distribution of a traffic flow density passes to the distribution of the linear form, with the values equal to values of boundary conditions in the beginning and at the end of the investigated road section. The received calculation results are summarized in Appendix 1.

There has been also investigated the given problem solution on the basis of the adapted data received as a result of traffic flows measurements at selected sections of a street-road system of the Riga city.

In order to approbate a technique and check the adequacy of the constructed models in work there has been carried out the video observation of various sites of a street-road system of Riga city. For convenience of carrying out of counting the vehicles amount procedure and performing the model adequacy check, on the basis of the video observation data the simulation model has been applied (Fig. 6).

Fig. 6. *Traffic flow video observation frames and modeling simulation results for the Raņķa dambis street*

Applying cubic spline-interpolation, there has been received final approximation functions for the initial and boundary conditions.

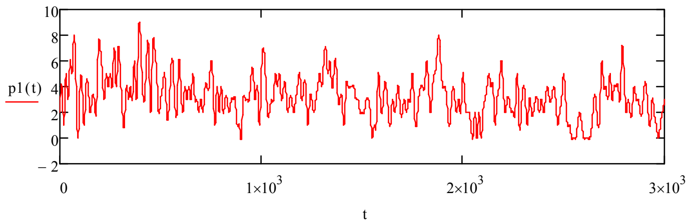


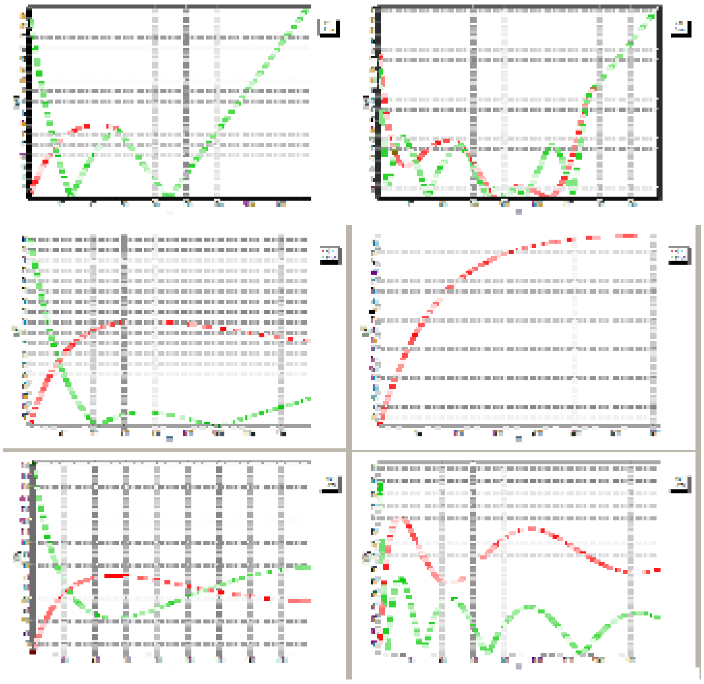
Fig. 7. *Graphical representation of boundary condition interpolation function at the beginning of investigated road section of Raņķa dambis street during the whole period of observation time*

Study of the sensitivity coefficient character change possesses significant practical value. Since the situation on roads of a city street-road network changes in time under the influence of huge amount of factors, alongside with road conditions change dynamics there is present a change of a sensitivity coefficient parameter. From the scientific point of view a special importance represents research of a problem of sensitivity coefficient influence on the traffic flow density characteristic. Besides from the practical sense the knowledge of sensitivity coefficient enables to carry out a comparative evaluation of road sites in terms of dynamic factors influence and to perform traffic flows management.

Sensitivity coefficient function unequivocal determination, depending on time is a result of the inverse problem solution. In order to find the sensitivity coefficient the solution of transcendent equation is carried out. The solution is found for the closed system, i.e. under zero boundary conditions. The purpose of conducting the experiment is revealing the nature of received functions of the inverse problem solution depending on changeable parameters of model. As changeable parameters there are proceeded the function of initial conditions, function of an additional condition, the distance of considered road section and observation time period.

The following functions of initial conditions – a constant and convex function have been chosen for the experiments. The additional condition is submitted as linear $f(t)=T-t$, intensively $f(t)=\left[\exp\left(x\cdot\frac{1}{0.2\cdot T}\right)\right]^{-1}\cdot L$ and gradually $f(t)=\frac{\left(1-x^2\right)}{T}+T$ decreasing functions. Using the iteration approaches the criterion of the solution reception was considered the least integrated value of errors on all steps of calculation.

Along with the change of a road section distance, the form dependencies remains constant and it happens only the change of a time scale. As an example there are shown the results of 6 experiments that are grouped by functions of initial conditions are shown. In a column a) solutions for constant value of initial conditions function are located, in a column b) there are placed solutions for convex function of initial conditions. In this case the purpose of demonstration of influence degree in changing of initial conditions on the inverse problem solution function is pursued.



a) b)

Fig. 8. *Time dependence of the sensitivity coefficient function for the experiments: a) 6 (upper), 33 (middle), 24 (lower); b) 15 (upper), 42 (middle), 51 (lower).*

As a numerical experiments conduction result there were obtained traffic flow density functions at the considered road section for the different values of sensitivity coefficient at the different time moments.

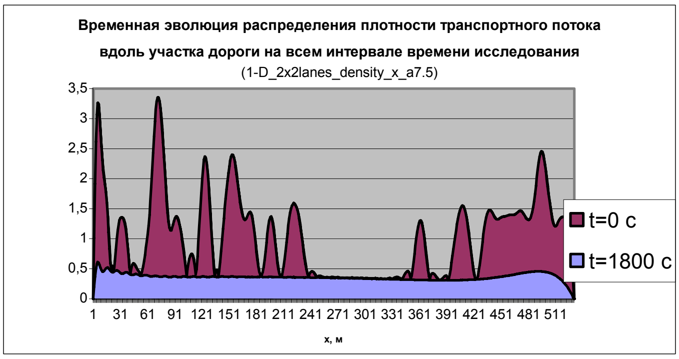


Fig. 9. *Calculated traffic flow density distribution across the total distance of Raņķa dambis street road section in various time moments for the fixed value of sensitivity coefficient 7,5*

The study has shown that the least significant factor is regarded to be the changeable parameter of observation time duration. It has been noticed that for small distances of examined road intervals the behaviour of solution function possesses a high degree of non-stationarity. Thus, investigation of small length road sections is undesirable owing to small informativity of received results. Results of non-stationary sensitivity coefficient functions evaluations at the given values of other changeable parameters, according to the experiments plan are resulted in the Appendix 2.

In the third chapter, analogically to one-dimensional case, the concept of "restoration" (self-restoration) of the traffic flow lying in the basis of construction a discrete model has been introduced, meaning the assumption when the vehicles quantity at any point becomes less than average arithmetic value in the neighbouring points (for each point of domain there are equally four next points, with the exception of for edge points of "dissipated" traffic flow) this quantity increases, and vice-versa.

Following the reasoning of one-dimensional case, on the basis of the introduced traffic flow "restoration" property, the discrete model for determination of vehicles quantity in any point of the investigated road section at any time moment for a case of a 2-D traffic flow is constructed.

$$U\left(m_1, m_2; n+1\right) \approx \frac{U\left(m_1-1, m_2; n\right)+U\left(m_1+1, m_2; n\right)}{4}+$$
$$+\frac{U\left(m_1, m_2-1; n\right)+U\left(m_1, m_2+1; n\right)}{4}.$$

Given expression means that in the subsequent moment of time at any point of considered road section will come equally $\frac{1}{4}$ share of all vehicles located in the previous time instant at the neighbouring "horizontally to the left" point, equally $\frac{1}{4}$ share of all vehicles located in the previous moment of time at the neighbouring "horizontally to the right" point, equally $\frac{1}{4}$ share of all vehicles located at the neighbouring "vertically upper" point (i.e. ahead) and equally $\frac{1}{4}$ share of all vehicles located at the neighbouring "vertically lower" point (Fig. 10).

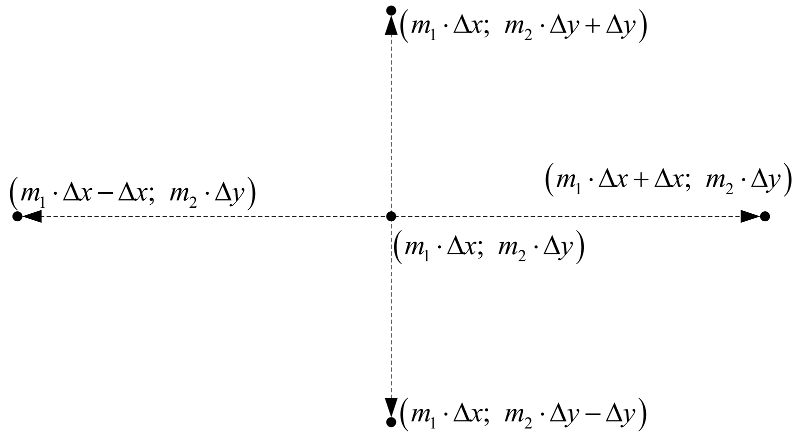


Fig. 10. *Possible vehicle locations at discrete grid*

Conditions are formulated under which performance the transition from discrete to continuous model for determination of density of the vehicular traffic flow becomes possible. In two different ways, by performing limiting transition at $\Delta x \to 0,\ \Delta y \to 0,\ \Delta t \to 0$ there has been deduced continuous model for density of the traffic flow determination in case of 2-D traffic.

$$\frac{\partial \rho(x,y,t)}{\partial t} = a^2 \cdot \left[\frac{\partial^2 (x,y,t)}{\partial x^2} + \frac{\partial^2 (x,y,t)}{\partial y^2} \right],$$

where

$a^2 = \lim\limits_{\substack{\Delta x \to 0 \\ \Delta y \to 0 \\ \Delta t \to 0}} \dfrac{\frac{1}{2} \cdot (\Delta x)^2 + \frac{1}{2} \cdot (\Delta y)^2}{4 \cdot \Delta t} > 0$ is a sensitivity coefficient of the continuous 2-D model.

There has been formulated the direct problem for the unequivocal determination of density of the traffic flow for the various boundary conditions types. Below is given the statement of the direct problem for the boundary Dirichlet conditions.

$$\frac{\partial \rho(x,y,t)}{\partial t} = a^2(t) \cdot \left[\frac{\partial^2 \rho(x,y,t)}{\partial x^2} + \frac{\partial^2 \rho(x,y,t)}{\partial y^2} \right],$$

$$0 < x < l_1,\ 0 < y < l_2,\ 0 < t \le T$$

initial condition

$\rho(x, y, t)\big|_{t=0} = \rho_0(x, y),\ 0 \le x \le l_1,\ 0 \le y \le l_2$,

boundary conditions of the 1st type

$\rho(x, y, t)\big|_{x=0} = \rho_1(y, t),\ 0 \le y \le l_2,\ 0 \le t \le T$,

$\rho(x, y, t)\big|_{y=0} = \rho_2(x, t),\ 0 \le x \le l_1,\ 0 \le t \le T$,

$\rho(x, y, t)\big|_{x=l_1} = \rho_3(y, t),\ 0 \le y \le l_2,\ 0 \le t \le T$,

$\rho(x, y, t)\big|_{y=l_2} = \rho_4(x, t),\ 0 \le x \le l_1,\ 0 \le t \le T$,

and corresponding consistency conditions.

The solution of the stated problem for the required 2-D traffic flow scalar density determination can be found in the following form

$$\rho(x, y, t) = \int_0^{l_1} d\xi \int_0^{l_2} G(x, y, \xi, \eta; t, \tau)\Big|_{\tau=0} \cdot \rho_0(\xi, \eta)\, d\eta +$$

$$+\int_0^t a^2(\tau)\, d\tau \int_0^{l_2} \left.\frac{\partial G(x, y, \xi, \eta; t, \tau)}{\partial \xi}\right|_{\xi=0+0} \cdot \rho_1(\eta, \tau)\, d\eta +$$

$$+\int_0^t a^2(\tau)\, d\tau \int_0^{l_1} \left.\frac{\partial G(x, y, \xi, \eta; t, \tau)}{\partial \eta}\right|_{\eta=0+0} \cdot \rho_2(\xi, \tau)\, d\xi -$$

$$-\int_0^t a^2(\tau)\, d\tau \int_0^{l_2} \left.\frac{\partial G(x, y, \xi, \eta; t, \tau)}{\partial \xi}\right|_{\xi=l_1-0} \cdot \rho_3(\eta, \tau)\, d\eta -$$

$$-\int_0^t a^2(\tau)\, d\tau \int_0^{l_1} \left.\frac{\partial G(x, y, \xi, \eta; t, \tau)}{\partial \eta}\right|_{\eta=l_2-0} \cdot \rho_4(\xi, \tau)\, d\xi,$$

where

$G(x, y, \xi, \eta; t, \tau)$ is a Green's function for the stated problem that could be written in the following form

$$G(x, y, \xi, \eta; t, \tau) \overset{def}{\equiv} \frac{4}{l_1 \cdot l_2} \cdot \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \left\{ \sin\frac{\pi \cdot n \cdot x}{l_1} \cdot \sin\frac{\pi \cdot n \cdot \xi}{l_1} \times \right.$$

$$\left. \times \sin\frac{\pi \cdot m \cdot y}{l_2} \cdot \sin\frac{\pi \cdot m \cdot \eta}{l_2} \cdot e^{-\pi^2 \cdot \left\{ \left(\frac{n}{l_1}\right)^2 + \left(\frac{m}{l_2}\right)^2 \right\} \cdot \int_0^{t-\tau} a^2(\tau_1)\, d\tau_1} \right\}.$$

In the given work there has been made a statement and are given solutions for the different boundary conditions – Dirichlet, Newton's, Neumann's and mixed boundary conditions of the 1st and 2nd types. The given boundary conditions are regarded to be the entire set of conditions for problems investigation in statistical physics. Relevant to the flows of traffic various boundary conditions mean different information gathering forms gained from the measuring systems on the boundaries of the road section. For the majority of listed boundary conditions there exists an applied interpretation on the types of measured data. For the Newton's boundary conditions such treatment is not obvious, therefore the solution for the corresponding problem is shown for the completeness of investigated boundary problems.

It is shown, that despite of external similarity of the received continuous model to the mass equation, in the obtained model there are available some essential differences, namely while the mass coefficient is considered as a known constant, the vehicular traffic flow sensitivity coefficient is unknown and is a subject to determination. Moreover, the given coefficient is a function depending on spatial and time arguments, and also depends on velocity and flux level of a traffic flow.

Additional conditions of quantitative and qualitative nature are formulated at which performance of the problem of unequivocal determination of the required sensitivity coefficient for a 2-D traffic flow has the unique solution.

It is proved that the initial inverse problem of required sensitivity coefficient determination at the formulated additional requirements is reduced to the solution of the equation of the first type with the non-linear operator. It is shown, that the received problem is an unstable problem and therefore, its solution does not possess the stability to even rather small perturbances of the initial data.

It is shown that if the requirement of function smoothness from an additional condition will be ignored, the non-linear problem of required sensitivity coefficient determination can have no solution at all, i.e. the given requirement is a necessary condition of solution existence for considered inverse problem. It is considered an alternative regularizing algorithm, which is distinct from considered in chapter 2 Tikhonov's regularizing algorithm and allowing in the obtained non-linear problem unequivocally and with required accuracy to determine the required sensitivity coefficient.

It is proved that the formulated additional condition of qualitative nature allows to use the only one additional gauge of measuring systems for gathering primary information in convenient, both from technical, and from economical points of view, and located in arbitrary internal point of investigated road section. Hence, within the framework of considered mathematical model, disappears a necessity for installation of additional devices of traffic flow density registration as any additional measuring data on density are superfluous in unequivocal determination of sensitivity coefficient and cannot positively influence a control problem of density of the traffic flow.

The general shortcomings of the offered discrete and continuous models are revealed. In particular, it is shown that mass equation widely used in transport systems, borrowed from statistical physics, at the edges of "dissipated" traffic flow ceases to be adequate model for determination of traffic flow density. Probabilistic interpretation for the sensitivity coefficient is given, which is regarded to be the important controllable parameter of the traffic flow. Besides, the probabilistic treatment for a limiting transition from the offered discrete model to continuous model is given. It is shown, that the 2-D scalar density of a traffic flow from the probabilistic interpretation viewpoint can be treated as a density of mathematical mean of "vehicles weight", and in a case, when "casual motion" of the single automobile is considered (i.e. the flow, as such is absent) 2-D scalar function means the probability density, i.e. it can be interpreted as a function of probability distribution for the location of the given "single" vehicle.

Performing the numerical experiments in case of 2-D non-deterministic model there has been considered the similar problem of collected data interpolation from the discrete tabulated form. Relevant to the 2-D traffic model there has been offered a vehicular flow density determination methodics allowing to use the initial data from satellite or video surveillance for the traffic flows management and control by means of intelligent traffic lights application.

Unfortunately, owing to the ultimately long duration of carrying out the numerical calculations in computing environments (construction of a surface for the function of non-stationary density for constant value of sensitivity coefficient having quite a large step of discreteness on a powerful modern computer takes the calculation time over one month) have not allowed to make high-grade 2-D model numerical calculations and to carry out the adequacy verification of the constructed 2-D model in the numerical way for today.

CONCLUSIONS

1. At the presented dissertation work there is proposed a scientific approach to urban traffic flows management consisting in unequivocal determination of density characteristic of the traffic flow as a function depending on spatial coordinates and time, in any point on the arbitrary selected section of multilane road, at any subsequent moment of time, relevant to the initial investigation instant and based on the initial data set, received from primary means of information gathering, according to requirements of initial, boundary and additional conditions suggested in the developed mathematical model. Given approach allows perfecting and increasing the traffic management, and controlling efficiency as well to reduce the congestion level on a street-

road network in a direct way, and provides achieving the purpose of the presented dissertation work.

2. The discrete non-deterministic model is constructed. It describes traffic flow mathematically and allows determining the required vehicles quantity in a stream in any section of investigated road site at any time moment for the simplified case of one-dimensional motion and for realistic case of 2-D motion process.

3. The continuous model of the vehicular traffic allowing to determine unequivocally a traffic flow density under set initial and boundary conditions for one-dimensional and 2-D cases is obtained. The continuous model received for a one-dimensional case completely coincides by its form with the mass equation, known from the theories of heat conductivity, diffusion, as well as from the theory of filtration.

4. It is mathematically proven that included in the continuous model sensitivity coefficient, which is regarded to be the key integral characteristic of traffic flow and interpreted as mass transfer coefficient in the mass equation, is non-constant, is unknown and in general case is dependent on the spatial and time coordinates, traffic flux and traffic velocity and is subject to determination.

5. Additional conditions of quantitative and qualitative nature are formulated in the dissertation work at which performance the problem of unequivocal determination of the required sensitivity coefficient for a 2-D traffic flow has the unique solution. For this purpose the theorem of uniqueness is formulated, which conditions performance allows to predict a situation of traffic jam occurrence on investigated road section and thus to control the least "possible" density of traffic flow.

6. Features, properties, restrictions and application area of the proposed analytical models are analysed and probabilistic interpretation of the constructed models is given. Recommendations for practical use of sufficient conditions for sensitivity coefficient determination are formulated as well as complexities arising during its application are investigated.

7. It is proved, that the formulated additional condition of qualitative nature allows to use the only one sensor of measuring systems for gathering primary information in convenient, both as from technical, as well from economic points of view, and located in arbitrary internal point of investigated road section. Hence, within the framework of

considered mathematical models, there disappears a necessity for installation of additional devices of traffic flow density registration as obtained theoretical results show that any additional measuring data on density is superfluous in unequivocal determination of sensitivity coefficient and cannot make any influence to the solution of a traffic flow density management and control problem.

8. The software for realization of the proposed discrete model of a one-dimensional vehicular traffic motion is developed. The developed software tool allows receiving the required vehicles quantity in a one-dimensional traffic flow for various values of initial and boundary conditions in any point of coordinate grid at any discrete time moment. By means of using standard software package MathCAD the continuous model of a one-dimensional traffic flow for finding out of traffic density is realized. The verification of the offered mathematical models in order to check its adequacy is performed.

9. The practical approbation of obtained theoretical results for the 2-D traffic flow scalar density determination in case of non-stationary sensitivity coefficient on specific road sections of a street-road system of Riga city is performed.

CO-AUTHOR IN PUBLICATIONS

Articles

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