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MIHAILS SAVRASOVS

JAUNAS PIEEJAS IZSTRĀDE TRANSPORTA PLŪSMU MODELĒŠANAI UN ANALĪZEI MEZOSKOPISKĀ LĪMENĪ

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Zinātniskais vadītājs
Dr. habil. sc. ing., profesors
Jurijs Tolujevs

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MIHAILS SAVRASOVS

DEVELOPMENT OF NEW APPROACH FOR SIMULATION AND ANALYSIS OF TRAFFIC FLOWS ON MESOSCOPIC LEVEL

DOCTORAL THESIS

to obtain the scientific degree Doctor of Science in Engineering

Scientific area "Transport"

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Dr. habil. sc. ing., Professor

Jurijs Tolujevs

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ИНСТИТУТ ТРАНСПОРТА И СВЯЗИ

САВРАСОВ Михаил Михайлович

РАЗРАБОТКА НОВОГО ПОДХОДА ДЛЯ МОДЕЛИРОВАНИЯ И АНАЛИЗА ТРАНСПОРТНЫХ ПОТОКОВ НА МЕЗОСКОПИЧЕСКОМ УРОВНЕ

ДИССЕРТАЦИОННАЯ РАБОТА

на соискание учёной степени доктора инженерных наук

Научная область “Транспорт”
направление “Транспорт и логистика”

Научный руководитель
Dr. habil. sc. ing., профессор
Юрий Толуев

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ANOTĀCIJA

Mihaila Savrasova promocijas darbs „Jaunas pieejas izstrāde transporta plūsmu modelēšanai un analīzei mezoskopiskā līmenī”. Darba zinātniskais vadītājs habilitētais inženierzinātņu doktors, profesors Jurijs Tolujevs.

Darbā ir aprakstīti mezoskopiskā modeļa transporta plūsmu izstrādes rezultāti, izpēte un pielietošana, kas ir realizēts uz „Discrete Rate” imitācijas modelēšanas pieejas bāzes.

Izvēlētas tēmas aktualitāte ir saistīta ar nepārtrauktu transporta līdzekļu skaita pieaugumu uz ceļiem un, līdz ar to – ekonomiskie zaudējumi, ekoloģiskās situācijas pasliktināšana pilsētās un nelaimes gadījumu pieaugums.

Viena no šīs problēmas atrisināšanas iespējam ir tā sauktās ilgtspējīgas transporta sistēmas izveidošana. Starp ilgtspējīgas transporta sistēmas izveidošanas instrumentiem ir imitācijas modelēšana. Imitācijas modelēšana šajā gadījumā ir instruments, kurš ļauj optimizēt esošo transporta sistēmu un padarīt jaunu transporta sistēmas elementu ieviešanu par pamatotu.

Darbā detalizēti ir apskatīta imitācijas modelēšanas loma transporta satiksmes sfērā un ir identificēti tālāki šā instrumenta attīstības posmi transporta plūsmu analīzē. Turklāt darbā ir veikts esošo transporta plūsmu analīzes instrumentu apskats un detalizēti izskatīti imitācijas modelēšanas instrumenti: mikroskopiskie modeļi, mezoskopiskie modeļi un makroskopiskie modeļi.

Detalizēti ir izpētīti šobrīd eksistējošie mezoskopiskie modeļi, pie tam ir atzīmēts to galvenais trūkums – realizācija slēgtā programmatūrā, kas apgrūtina to izmantošanu praksē. Jauna izstrādāta mezoskopiska modeļa pamatā ir „Discrete Rate” imitācijas modelēšanas pieeja, kura šobrīd tiek izmantota materiālu plūsmu modelēšanai loģistikas sfērā. Darbā sīki ir aprakstīti pieejas pamatprincipi un ir parādīta tā atšķirība no zināmām klasiskām pieejam. Uz eksistējošas koncepcijas bāzes ir piedāvāti un formulēti divi modeļi: nenoslogotam un noslogotam tīklam. Ir aprakstīti izstrādāto modeļu izmantošanas piemēri dažādiem transporta sistēmas objektiem. Uz piemēru bāzes ir sīki aprakstīts modeļa realizācijas algoritms, realizēta modeļa validācija un noteikti reāli parametri, nepieciešamie dotajām modeļa tipam.

ABSTRACT

Doctoral thesis – Mihails Savrasovs. “Development of new approach for simulation and analysis of traffic flows on mesoscopic level”. The scientific supervisor of the thesis is Dr.habil.sc.ing., professor Jurijs Tolujevs.

The work presents the results of the development, investigation and application of a new type of mesoscopic model, which is based on simulation approach called “Discrete Rate”. The actuality of the theme is underlined by a continuing growth of vehicle number on roads. This leads us to economic losses, deterioration of ecology, and an increasing incident rate on transport. One of the directions of solving this problem is a development of the so-called sustainable transport systems. Among tools for a sustainable transport system development, simulation could be emphasized. In this case, simulation is a tool that allows us to do an optimisation of existing transport system and to make the introduction of new elements into the system more reasonable.

In this work the role of simulation in transportation sphere is overviewed in detail. At the same time, the trends of further development of the tool for traffic analysis are emphasized. Different state-of-art traffic analysis tools have been completed, focusing on simulation models of the following types: microscopic, mesoscopic and macroscopic. A detailed overview of the existing mesoscopic models has been performed and the main disadvantage is emphasized – realization in closed software, which makes it difficult to use them in practice. The new type of mesoscopic model has been developed, which is based on simulation approach called “Discrete Rate”, which is currently used for material flow simulation in the sphere of logistics. In this work, the concept of the approach is described in detail, focusing on the difference between the “Discrete Rate” approach and the classical ones. Based on the existing concepts, two kinds of models are proposed and formulated: the model for an uncongested network and the one for a congested network. A few examples of the model application of to different kind of transport systems are given. Based on the examples, operation algorithm of the proposed models is demonstrated and the validations of model completed, as well as realistic input parameters of model are defined.

АННОТАЦИЯ

Диссертационная работа Михаила Саврасова «Разработка нового подхода для моделирования и анализа транспортных потоков на мезоскопическом уровне». Научный руководитель хабилитированный доктор инженерных наук, профессор Юрий Иванович Толуев.

В настоящей работе представлены результаты разработки, исследования и применения мезоскопической модели транспортных потоков, реализованной на базе подхода имитационного моделирования „Discrete Rate". Актуальность выбранной тематики подчёркивается непрерывным ростом числа транспортных средств на дорогах, и как следствие - экономические потери, ухудшение экологической ситуации в городах и рост числа несчастных случаев. Одним из способов решение этих проблем является построение так называемой устойчивой транспортной системы. Среди инструментов, которые могут быть использованы при построении устойчивой транспортной системы, можно выделить имитационное моделирование. Имитационное моделирование в данном случае является инструментом, который позволяет оптимизировать существующую транспортную систему и сделать введение новых элементов транспортной системы более обоснованным. В работе подробно рассмотрена роль имитационного моделирования в сфере транспортного сообщения и выделены дальнейшие пути развития этого инструмента анализа транспортных потоков. Приведён обзор существующих инструментов анализа транспортных потоков и более подробно рассмотрены инструменты имитационного моделирования: микроскопические модели, мезоскопические модели и макроскопические модели. Детально исследованы существующие на сегодняшний момент мезоскопические модели, а так же выделен их основной недостаток – это реализации в закрытом программном обеспечении, что затрудняет их использование на практике. Разработанная новая мезоскопическая модель базируется на подходе имитационного моделирования „Discrete Rate", который на сегодняшний день используется для моделирования материальных потоков в логистической сфере. В работе подробно описан основной принцип подхода и показано его отличие от известных классических подходов. На базе существующей концепции предложены и сформулированы два вида моделей: для незагруженной и загруженной сети. Приведены примеры использование разработанных моделей для различных объектов транспортной системы. На базе примеров подробно описан алгоритм реализации модели, проведена валидация модели и определены реалистичные параметры, необходимые для данного вида моделей.

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CONTENTS

ABBREVIATIONS

AB – Agent Based

AICC – Autonomous Intelligent Cruise Control

AMS – Anisotropic Mesoscopic Simulation

APTS – Advanced Public Transportation System

AR – Acceptance Region

ATIS – Advanced Traveller Information System

ATMS – Advance Traffic Management Systems

AVCSS – Advanced Vehicle Control and Safety System

CA – Cellular Automata

CAD – Computer –Aided Design

CONTRAM – CONtinuous TRaffic Assignment Model

CVO – Commercial Vehicle Operations

DES – Discrete Event Simulation

DR – Discrete Rate

DRTRM – Discrete Rate Traffic Reference Model

DSS – Decision Support System

DynaMIT – Dynamic Network Assignment for the Management of Information to Travellers

ESS – Estimated Sum of Square

FHWA – Federal Highway Administration

GIS – Geographical Information Systems

GPSS – General Purpose Simulation System

GUI – Graphical User Interface

HCM – Highway Capacity Manual

HGV – Heavy Goods Vehicle

ICU – Intersection Capacity Utilisation

ITS – Intelligent Transportations System

LOS – Level of Service

OD – Origin – Destination

P&R – Park and Ride

PCU – Passenger Car Unit

RSS – Residual Sum of Squire

SC – Supply Chain

SD – System Dynamics

SIR – Speed Influence Region

SSE – Sum of Squared Errors

TDM – Travel Demand Management

VBA – Visual Basic for Applications

VDF – Volume Delay Function

VR – Virtual Reality

INTRODUCTION

Actuality of the problem

Science in 1886 when the first engine of vehicle was patented the number of vehicles on roads was growing dramatically [1]. Growth of the number of vehicles is almost linear for both EU [2] and Latvia [3]. This led us to the problem how to build sustainable transport systems. The term sustainable transport system has lot definitions [4], but the main idea is that sustainable transport system should cover 3 aspects of transport systems [5]. They are: effectiveness, security and ecological questions [2]. Here we face a problem of building a sustainable transport network [6]. The analysis of current situations shows that all of these 3 aspects are not fully covered by modern transport systems. The situation is dramatic. Cities are filled with vehicles, thus bringing about congestions, which in turn have resulted in economical losses. At the same time, congestions increase the use of petrol, and emission volumes are growing. Growth of emissions impacts the health of inhabitants. The development of a sustainable transport network is a complex problem that should cover all aspects of transport systems - from permanent survey of traffic volumes and households to development of new technologies in such areas like petroleum production and vehicles engines modernization. A series of tools that could be used for the development of new generation of transport systems could be mentioned here (the tools directly connected with transport systems):

- Use of ITS (Intelligent Transportation Systems);
- Use of P&R (Park and Ride);
- Optimisation of existing transport infrastructure;
- Implementation of new transport infrastructure elements based on rigorous analysis of development consequences;
- Development of intermodality;
- Intensive use of district roads;
- Use of thoughtful parking policy;
- Cargo traffic limitation in city centre;
- Use of sound tax policy,

etc.

Here it must be noted that two of the most important tools are as follows: optimisation of the existing transport infrastructure and rigorous and comprehensive analysis of the impact rendered by a new infrastructure element upon another part of the transport infrastructure.

These two are the most widely used tools for a sustainable transport system development. Normally these tasks could be solved by using different tools of traffic flow analysis.
Among the existing tools for traffic analysis, simulation stands alone as a powerful mathematical tool. The use of traffic simulation tools could give a good and highly accurate output data for decision-making. Unfortunately, simulation models used for today are known as microscopic models; as regards macroscopic models, they are not as effective as they have their own disadvantages. This fact has motivated us to extend the class of mesoscopic models which should avoid disadvantages inherent in microscopic and macroscopic models. The use of mesoscopic models instead of micro and macromodels could save time spent on simulation model development and produce the output result with an insignificant error at the same time. A mesoscopic traffic model could serve as a good tool for transport planners, since it could increase the efficiency of their works. Obviously it will be a good support tool for the creation of a sustainable transport system.

Degree of the theme study

The history of the traffic flow simulation began in 1955 by D.L.Gerlough's issuing his thesis: "Simulation of freeway traffic on a general-purpose discrete variable computer" at the University of California, Los Angeles. From that time on, a number of scientists and practitioners made efforts toward the development of traffic simulation theory. The following could be mentioned with regard to the microscopic simulation of traffic flows: R.Wiedemann in 1974 has published a description of 74 Wiedemann car-following models in his article "Simulation des Straßenverkehrsflusses", P.G.Gipps in 1970 developed a Gipps' car-following model; some other works by R.E.Chandler, D.C. Gazis, G.Lee, J.C.Bender, R.E.Fenton etc. could me mentioned as well. The research of these scientists helped to create a basis for traffic flow simulation software. In the meantime, a development of macroscopic models was started. Macroscopic traffic models take into consideration cumulative traffic stream characteristics (speed, flow, and density) and the interrelationships between them. Here, a number of scientists could be mentioned referring to the macroscopic simulation: M.J.Lighthill, G.B.Whitham, P.I.Richards, C.F.Daganzo, S.K.Godunov etc.

The disadvantages of these simulation approaches motivate one to develop another class of traffic simulation models which are called mesoscopic. In 1989, D.R. Leonard proposed a model called CONTRAM. The model assumes grouping of vehicles into pockets and routing them through the network. Another scientist, M. Ben-Akiva, proposed a mesoscopic model

DYNAMIT in 1996. This model grouped vehicles by cells, which define such properties like speed etc. A bit earlier in 1994, R. Jayakrishnan proposed a queue-server approach application for traffic flow simulation. This idea became popular, and the model FASTLANE was presented by C.Gawron in 1998 and, subsequently, the model DTASQ – by M. Mahut (in 2001). Application of cellular automata for traffic flow simulation was made by B.W. Bush by development of TRANSIMS. The last widely known paper dedicated to mesoscopic model was published in 2004 as doctoral dissertation by Wilco Burghout in Royal Institute of Technology (Stockholm, Sweden).

The analysis of the above-mentioned sources enables one to draw the following conclusions on practical application of mesoscopic models, motivating the author to undertake a study in this area:

- mesoscopic traffic flow simulation is a very perspective type of simulation in terms of lack of any high requirements to input data and a low time of model development and experimentation, etc;
- the term "mesoscopic traffic model" still does not have any official definition (different scientists interpret it differently); this allows one to use different approaches to model development, but leads to confusion within scientific and transport planners' community at the same time;
- mainly mesoscopic models are realized in proprietary software used by government institutions, or are developed without practical approbation. This leads to scarcity of mesoscopic simulation tools on the market.

As a base for the development of a new type of mesoscopic traffic model, a Discrete Rate (DR) simulation approach is used. The concept proper of DR approach is based on event planning using piecewise linear aggregates, which were described by Buslenko in 1973. The DR approach was presented on Winter Simulation Conference by D.Krahl in 2009, who implemented this approach in ExtendSim software. In the meantime, prof. J.Tolujevs in his research demonstrated concepts and examples of DR application in logistics area.

Object and subject of the research

The object of the research is traffic flow. The subject of research is simulation-based models of the traffic flow.

Objectives and tasks of the research

The main objective of the promotional work is the following: Development, investigation and application of a new type of model in the class of mesoscopic traffic models.

As regards the stated divisions and objectives of the present research, the following tasks are considered:

1. To consider problems of traffic system simulation modelling.
2. To conduct the analysis of:
   a) the currently used traffic system simulation tools;
   b) the existing simulation approaches and detailed mesoscopic traffic models.
3. To offer and develop the concept of the new type of traffic mesoscopic model.
4. To develop the models with synthetic and real data, providing for numerical examples of the offered mesoscopic approach.
5. To test the validity of the new type of traffic mesoscopic model by different examples.
6. To demonstrate the advantages and disadvantages of the offered model.
7. To provide scientifically-grounded recommendations on model use.

Research hypotheses

The study puts forward the following hypotheses:

1. The Discrete Rate approach could be used for traffic flow simulation on mesoscopic level of detail.
2. The results obtained through application of the proposed model are adequate for further decision-making

Methodology and methods of research

The promotional work research is based on: system theory, methods of statistical data analysis, modern theory of simulation; analysis of scientific literature, scientific papers, press releases, EU Commission’s reports, simulation guidelines, internal software manuals, and the Internet sources. The computer-based support for necessary investigations was: Microsoft Office Excel 2010, Statistica 8 package, PTV VISION VISSIM microscopic simulation software, and simulation software ExtendSim 8.

Scientific novelty of the research

The novelty of this research consists of:

1. The development of a new approach for traffic flow simulation based mainly on DR approach and called by author as DRTRM (discrete rate traffic reference model), which could be used by transport planners as a fast and exact tool in the course of decision support process.
2. The DRTRM methodology development for traffic simulation in research and practical applications.
3. The developed DRTRM methodology for traffic simulation can be used in scientific studies and practical work.
4. DRTRM validation results allowing one to accept the approach as credible.

Practical value of research

1. Development of methodology and algorithms for DRTRM usage in transport application.
2. Justification of the advantages of the proposed new mesoscopic traffic simulation model as against microscopic models, based on numerical examples.
3. Practical approbation of DRTRM use for specific Case Studies:
   a) The Project "Development of the Model of Intelligent Transport System of Europe-Asia Multimodal Corridor for Optimisation of Latvia-Belarus International Logistics Chain (TRANSLAB)" announced by Ministry of Education and Science of the Republic of Latvia (2007-2009).
   b) Latvian National Research Programme "Local Resouces Long-Term Utilization. New Products and Technologies (NatRes)". Project Nr. 4. „Development of the Long-Term Programme of the Latvian Transport System Harmonization" (LATRANS).
4. The obtained results became the basis for part of lectures delivered on the course: "Transport Modelling" of program 43525 "Transport Commercial Operations" and "Simulation modelling in transport and logistics" of program 4534507 "Transport and Logistics" of Transport and Telecommunication Institute.

Author' publication

The main results of this investigation are published in 18 articles [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18], [19], [20], [21], [22] and presented in 9 scientific conferences held in Latvia, Germany, Spain, Poland and Croatia, where the author has presented reports according to the subject of the promotional work and connected with it. The list of scientific publications is given in the Appendix 1.

Structure of the promotional work

Introduction. The object and the subject of research are defined on the basis of timeliness; a review of scientific literature has been carried out with respect to the issues related to the formulated goal and tasks of the research.

Chapter 1. Review of traffic analysis tools. The types of traffic analysis tools are examined and classified, the role of simulation in transportation area is analysed, and trends in simulation-based traffic analysis tools are summarized. The review and analysis of different models classified as mesoscopic, are presented. These are: CONTRAM, DYNAMIT, DYNASMART, FASTLANE, DTASQ, MEZZO, AMS and Cellular Automata.

Chapter 2. Comparative analysis of simulation approaches. The place of DR approach among well-known classical simulation approaches as discrete event simulation, system dynamics, and agent-based simulation is emphasized. The comparison of these approaches is done on the general level and in detail.

Chapter 3. Development of the concept of a mesoscopic discrete rate traffic simulation model. This chapter presents the DRTRM mesoscopic model for traffic simulation, treated as an extension of DR approach. Two alternatives of the model are formulated: the one for congested and the other for uncongested network. Mathematical formalisation of both is performed.

Chapter 4. Comparing the developed approach to microsimulation. Case studies. This chapter demonstrates the use of DRTRM approach for simulation of a single, traffic light-controlled crossroad, as well as simulation of two interconnected crossroads. Microsoft Excel and VBA are used to code both models. Simulation is based on synthetic data to show the idea of application of DRTRM approach for such a kind of simulation. Some results of simulation are presented as well; a kernel of the simulation model is shown in tabular form. The main idea of these examples is to show the simulation algorithm in details.

Chapter 5. Mesoscopic simulation of an urban transport corridor. The last chapter demonstrates an example of simulation of the urban transport corridor using DRTRM approach. The urban transport corridor consists of 11 crossroads and most of them are controlled by traffic lights. The simulation is based on real input data obtained in the course of the traffic flow study project implemented in Riga City Centre (2011). Within the framework of the project, the microscopic model was created, allowing one to test mesoscopic model output data against the already validated microscopic model. The mesoscopic model was created in ExtendSim simulation software. Special attention in this chapter is paid to determination of the form of the crossroad passing function used in the proposed traffic model.

Conclusions. Summary of the work, as well as main findings and conclusions, are presented.

The list of author's publications, VBA code of models, different supporting tables and figures are to be found in **Appendices.**

1. REVIEW OF TRAFFIC ANALYSIS TOOLS

This chapter gives a deep overview of the role of simulation in the transportation sphere by describing the current state of application. Also special attention is paid to the future development trends in simulation. Classification of a simulation-based model will be done, focusing on the currently existing mesoscopic models.

1.1 Current state

As soon as computers had become powerful, simulation started to play a vital role in system analysis and experimentation. Simulation came to the sphere of transport very late. In general, the development in traffic simulation since the early 1950-ies and 1960-ies has been very intensive [23]. This fact is connected with a few reasons: the 1st one is connected with lack of any dedicated tools for simulation; the 2nd one is associated with absence of theoretical background and lack of knowledge of fundamental processes taking place in transport systems; finally, the 3rd one is, to my mind, the most important in terms of the small volume of road traffic, which does not cause any problem for transport infrastructure, economics, and human health.

However, the situation has changed dramatically. Cities are packed with vehicles, and this fact has given impetus to performing profound researches in the area of transportation. First, fundamental processes connecting flows in transport systems have been studied. The knowledge of these processes motivates one to develop specific simulation software aimed directly at traffic flow simulation. Thus, experts have received a very powerful tool for transport system analysis, and now it is treated as one of the most powerful traffic analysis tools. At the moment, there is a number of simulation software packages widely used. Some of them have become standard tools in some countries, e.g., VISSIM and VISUM for Germany [24], AIMSUN for Spain [25], PARAMICS for the United Kingdom [26].

Some groups of institutions using these tools could be mentioned, such as: universities and scientific research institutions using simulation software for studying processes in transport systems; consulting companies applying software for commercial consultancy in the area of transportation; governmental agencies mostly using simulation models already developed by someone. Some groups of end users could also be mentioned, as scientists, transport planners, employees in traffic control centres, etc.

The area of application of traffic simulation software is very large and could be divided into the following major subareas:

- ITS – intelligent transportation systems;
- DSS – decision support system;
- Strategic, tactical and operational development of the transport system.

Let's go through these areas to show the practical application of simulation software. The first one is ITS.

ITS

The concepts of ITS include the use of modern information technologies to control and monitor the state of transport systems. The ITS concepts are very closely connected with telematics [27]. There is a number of examples when simulation is used as a part of ITS system [28]. The brightest example is the application of simulation in the Berlin Traffic Control Centre. Here, simulation is used as a forecasting tool for obtaining short-term predictions. The concept of the ITS system used in the Berlin Traffic Control Centre could be seen on Figure 1.1.

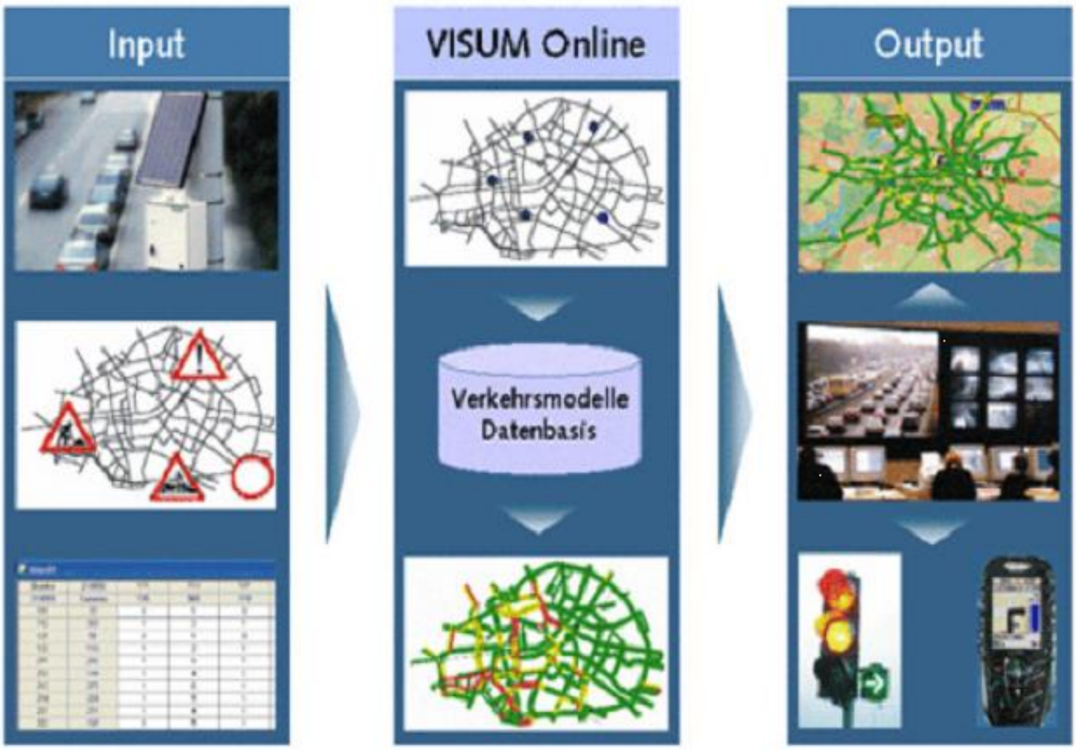


Figure 1.1. **ITS concept in Berlin traffic control centre [29]**

The main component of ITS demonstrated on Figure 1.1 is a traffic simulation model based on input data, to make a short-term prediction of the level of congestion on roads. The following information is used as the input data: the current information on traffic flow characteristics in different parts of the transport network, obtained by traffic data collection tools; information about limitations existing within the transport network - like maintenance and renovations; information about inhabitants' trips coded in the OD (origin-destination) matrix with regard to different time of day, time of week, etc. The traffic simulation software is fed with this information to produce output data - e.g., in the graphical form. The green colour shows normal level of congestion of the road, the yellow one signalises about potential problems, and the red one attracts one's attention to the already congested parts of the transport systems. This output data could be used to adjust traffic light working modes, to put notification messages into the information boards for the drivers, and send information about congestions to navigators of the end users, so the navigators could change the driving route according to the current state of the transport network.

DSS

The use of DSS is a very popular direction used for decision support. The same tendency is also observed in transport area. A number of examples concerning the ideas of using DSS in transportation sphere could be found. For example, in [30] the authors propose a DSS framework for the investment planning of an advanced traffic signal control system. The proposed DSS framework is presented on Figure 1.2.

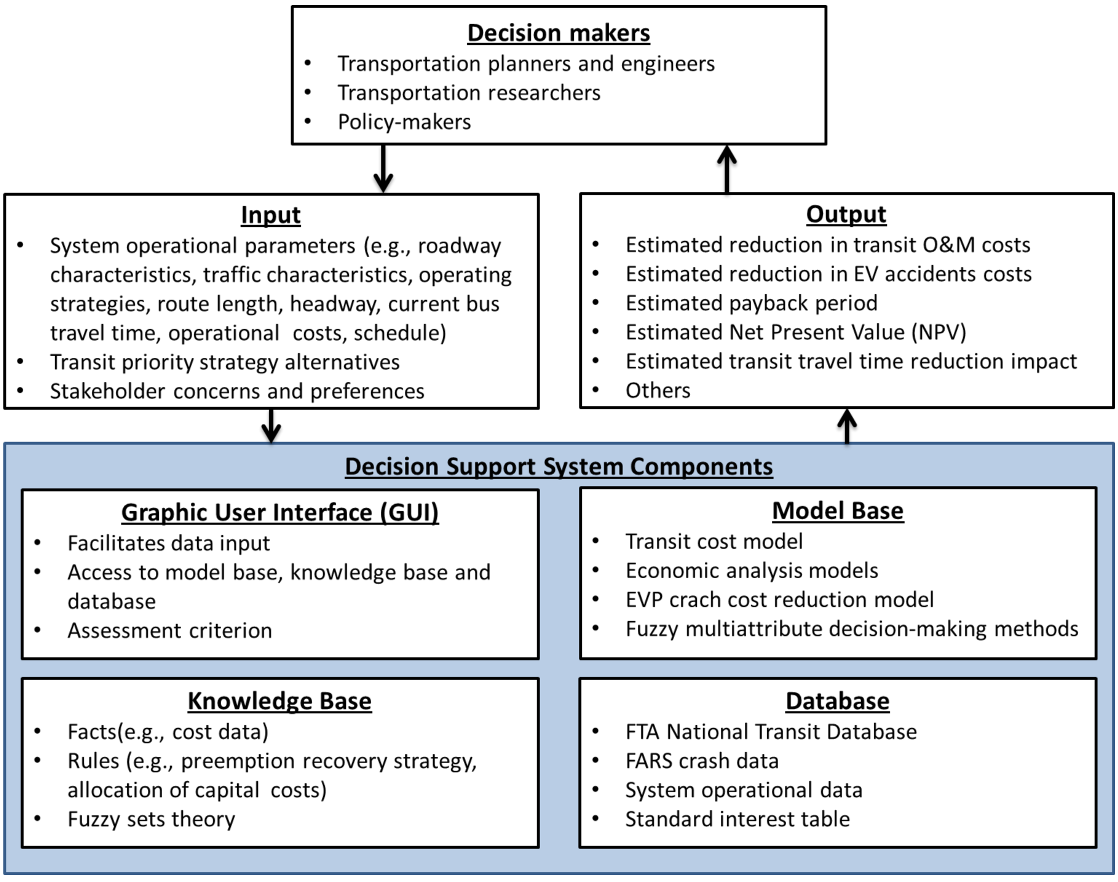


Figure 1.2. **DSS framework for traffic signal control system investment planning [30]**

The necessary part of DSS is a base of models as it could be seen from DSS framework picture. In this particular example, a different kind of models is proposed for using, including analytical and simulation models. At the same time, the developers of this framework emphasize the importance of having user interface, knowledge base, and database of one's own. The idea behind DSS has been pointed not to take decision automatically, but to furnish the expert or a group of experts with all available information on the required problem, and to recommend different kinds of solutions for the problem. That is why the work of DSS could be described in the following way: the user of DSS must enter input data to DSS using GUI, the DSS uses knowledge base, model base and the data available in the database to provide an output consisting of different indices. The DSS user analyzes the output results and estimates their consistency; and using this output is a decision of the problem.

Strategic, tactical and operational development of the transport system

In most cases, traffic flow simulation software has been used separately from such complex systems as ITS and DSS for solving a particular problem. Of course, in this case software by itself could be treated as DSS, but in most cases - as it was mentioned before - DSS is something more complex than just a single software application. As it was mentioned before, separately software could be applied on different levels: strategic, tactical, and operational. There are a number of examples in literature, which show the application of simulation. For example, the use of simulation models at national level could be presented as the following schema (see Figure 1.3). It should be mentioned that the proposed schema is a real example of a model application at national level for Germany to solve strategic questions of transport system development.

Here it should be emphasized that the above-mentioned schema could be used at the tactical level as well, provided local organizations are involved. The two main players according to this schema are the national Government/local municipalities and transport organizations. Transport organizations do the main analysis based on the usage of traffic analysis tools (transport plans and programs in this case) provide output data to national government/local municipalities. National government/local municipalities use the output data when working out the future area development strategy. Furthermore, it should be mentioned that input data (country/city development plan, transport system development plan, statistical data, etc.) for the analysis are provided by national government/local municipalities [31].

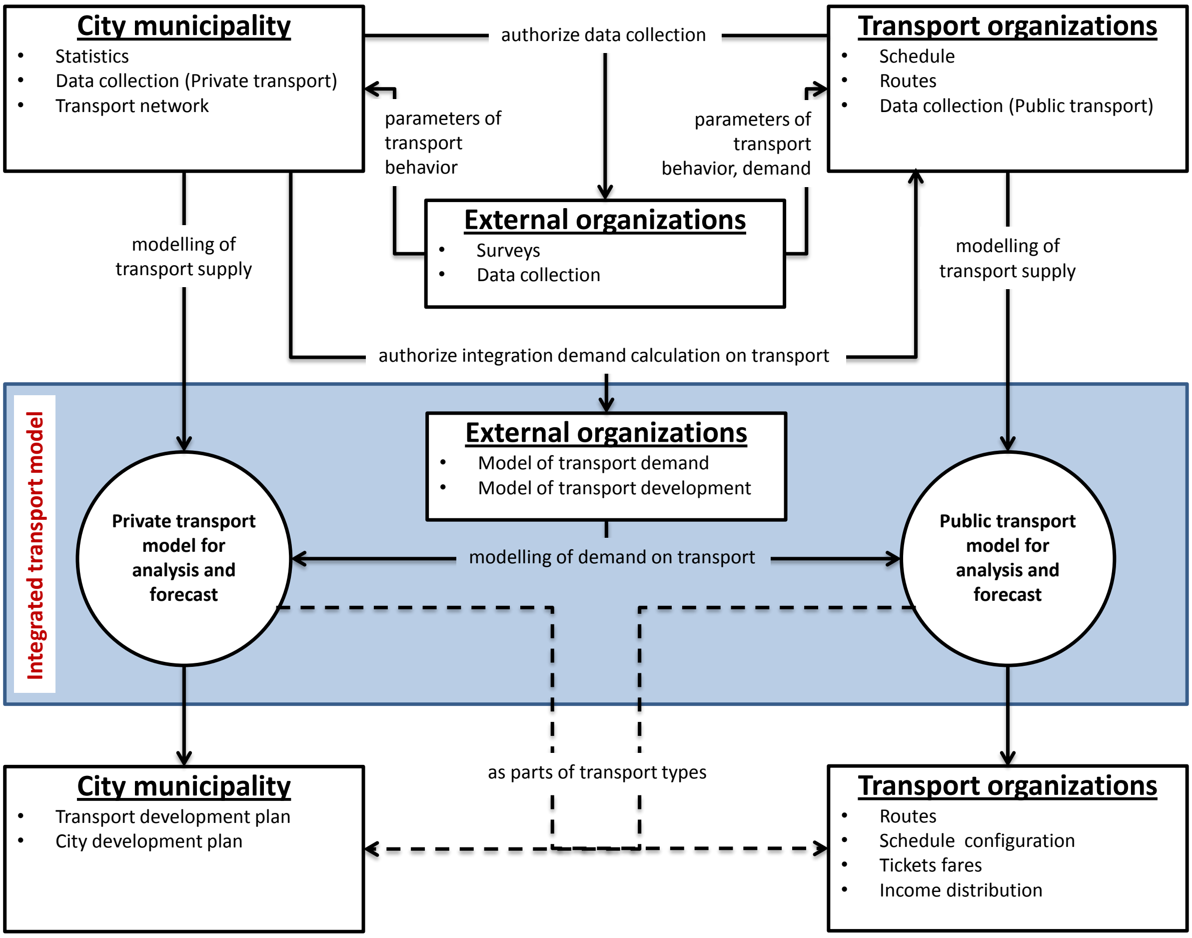


Figure 1.3. **The schema of transport model use on national level [32]**

The general process of transportation analysis is presented on Figure 1.4. An overview of transportation analysis process, along with its various evaluation contexts and the types of traffic analysis tools typically used in each context, is shown on Figure 1.4. Typically, transportation analysis needs some results from the policies and objectives of state/regional/local transportation plan. A transportation improvement (project) goes through several phases, including planning, project development, design, implementation and post-implementation operational assessment and modification. Each of these phases requires different methodologies and tools as it is shown on Figure 1.4. A project's early planning stage normally involves a sketch planning or travel demand modelling techniques. These methodologies help agencies screen different transportation improvements, resulting in the selection of a few candidate transportation improvements. Later stages (such as project development or post-implementation modifications) normally involve application of some more rigorous and detailed techniques, such as traffic simulation and/or optimisation [31].

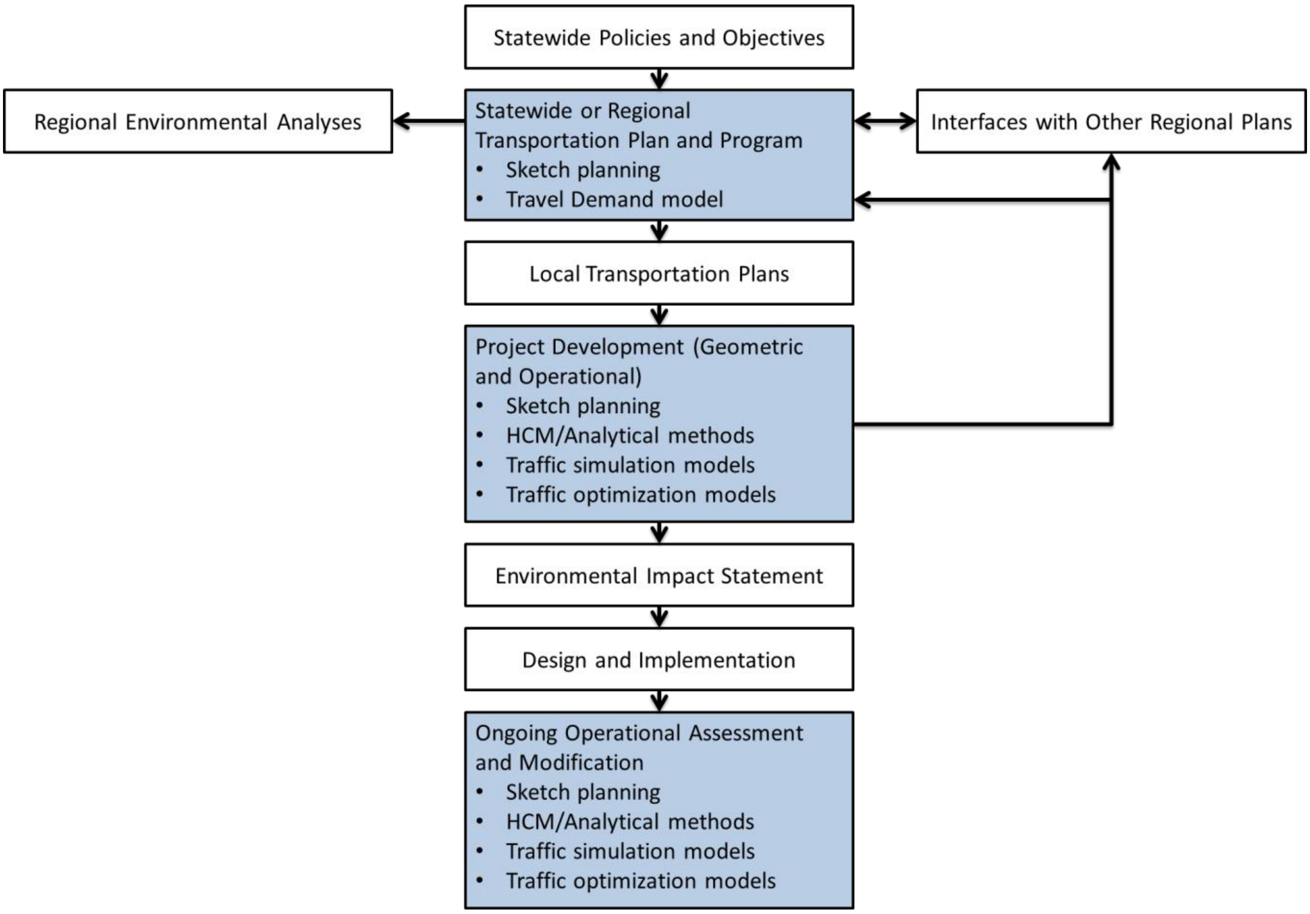


Figure 1.4. **Overview of transportation analysis process [31]**

Traffic analysis tools are designed to assist transportation professionals in evaluating the strategies that best address the transportation needs of their jurisdiction. Especially traffic analysis tools (including simulation ones) can help practitioners [31] to:

- **Improve the decision-making process.** Traffic analysis tools (including simulation of course) allow practitioners to reach the better planning/engineering solutions of complex transportation problems. Traffic analysis tools are used to evaluate: the impact of the deployment of traffic management and other strategies; they help to define priorities among different projects and compare potential improvements or alternatives [31].
- **Evaluate and prioritise planning/operational alternatives.** This typically consists of comparing "no build" scenario with some alternatives, which have different types of future improvements. The impacts are presented as performance indices and are evaluated as the difference between "no-build" and alternative scenarios. The evaluated data can be used to select the best alternative or prioritise improvements, increasing the probability of having a successful implementation [31].

- **Improve design and evaluation time and cost.** Traffic analysis tools (including simulation) are relatively less costly as compared to: pilot studies; field experiments; full implementation costs; etc. These tools can be used to evaluate different deployment combinations or other complex scenarios within a relatively short time [31].
- **Reduce disruption of traffic.** Traffic management control strategies come in many forms and options, and tools provide a way to cheaply estimate the effect prior to full development of management strategy. They may be used to initially test new concepts of transportation management system without the inconvenience typical to a field experiment [31].
- **Present/market strategies to the public/stakeholders.** Most of professional traffic analysis tools have a wide range of graphical and animation opportunities which could be used to demonstrate "what if" scenarios to the public, local authorities, etc [31].
- **Operate and manage the existing roadway capacity.** Some tools provide an opportunity of solving optimisation tasks. This brings about new possibilities of finding and recommending the best solution to maximize performance of the traffic systems [31].
- **Monitor performance**. Tools can also be used to evaluate and monitor the performance of existing transportation facilities. At the moment, many companies have come to the idea of linking monitoring systems directly to the traffic analysis tools for a more straightforward and real-time analysing process [31]. For example, PTV VISION product Traffic Counts can be linked to the PTV VISION VISUM simulation software [24].

On Figure 1.4 the places of traffic flow analysis tools (including simulation) are presented by the blue rectangles application in transportation analysis process.

1.2 Trends in traffic simulation

Simulation as a traffic analysis tool is currently an everyday tool for practitioners and researches in all fields of the profession. Some of the development trends will be shortly discussed below. The modern trends in traffic simulation area could be grouped in the following directions, which will be described in detail:

- **Application of new knowledge from fundamental traffic flow analysis area.**

Scientists do fundamental researches in transport systems, providing a new knowledge about processes inherent in the transport systems. This information is extremely important to update the existing models and to develop new types of models. A good example could be the introduction to the public of a new version of HCM (Highway Capacity Manual) 2010 [33] and a new manual ICU (Intersection Capacity Manual) 2003 [34], making experts look at the internal processes of transport systems from a different angle. Another good example is fundamental traffic analysis gaining a better insight into the fundamental speed-flow-density relationship [35].

- **Use of GIS and CAD systems.**

Another trend is more precise descriptions of physical roads and street environment, especially in local applications, like in simulation. In both cases, the use of graphical user interfaces and integration of GIS (Geographical Information Systems) and CAD (Computer-Aided Design) systems are a feasible approach. At the moment, developers are only approaching the use of GIS systems in their applications. PTV VISION products have a number of scripts that can provide communication of simulation software with GIS systems (Google Maps) to obtain information about public transport roots and locate the graphical representation of the simulated crossroads. The main idea of these scripts is to simplify model development and to reduce model implementation time. In the long run, a traffic simulation model will be united with GIS system as they will jointly use a single database for storing information about transport infrastructure [36].

- **Application of parallel and cloud computing.**

Simulation objects become increasingly larger. It requires a lot of computation to complete the simulation. The current situation is the following – most of the software is programmed to use all cores of a processor. Of course, it gives some performance, but still simulation in future will require usage of parallel computing to obtain results. However, at the moment only a number of professional software can boast with the ability of doing parallel computations. The American TRANSIMS development work is an example of a network approach. The simulation of the traffic system of the whole city is based on massive use of parallel computing [37], [38] featuring an advantage that becomes ever more common to modern applications.

- **Integration of new general simulation and programming principles.**

In addition to parallel computing, the modern programming and simulation methods have their effect on the traffic flow simulation. Object-oriented programming has been found very suitable in the description of a great number of parallel interactions in traffic [39]. Object-oriented programming opened a window to the agent-based simulation, which is currently becoming a very powerful simulation approach capable of modelling individual characteristics and behaviour of the object. With respect to the traffic flow simulation, it opens a variety of development directions, like submicroscopic simulation of the traffic flows, taking into account a more detailed description of vehicle characteristics (like type of fuels used, fuel consumption, etc.) and a more detailed description of the driver's behaviour. A good example of the application of agent-based simulation is traffic library of the AnyLogic software [40].

- **Development of open environment.**

Another way of looking at the need for system-level simulation is the development of open environment, where several analysis tools can be used interactively to solve the problems, with each of them being most suitable. As an example of this, we can mention the FHWA TRAF-program family and the FHWA (Federal Highway Administration) Traffic Management Laboratory, whose primary goal is the development of a distributed, real-time test bed to simulate traffic conditions for ATMS (Advance Traffic Management Systems) [41]. Another example is giving PTV VISION developers a possibility to create an environment for simulation where macro- and micro-simulation could be used depending on a specific task, but there is still a lot of work to be done in this direction [24].

- **Simulation of control systems as a part of traffic.**

Simulation of control systems as a part of traffic operations is also becoming more and more important in the era of ITS. Comprehensive research performed in the field of transport and telematics has given one the possibility of implementing ITS systems into the actual environment. It means that for adequate traffic flow simulation, one should take into account the availability of such systems, as the ITS interacts with the traffic. More and more simulation systems will be embedded into control systems in the future to anticipate the state of the traffic flow and the effect of an alternative control measure. On the other hand, developers of simulation systems try to integrate some parts of control systems behaviour by implementing the method and algorithms in their simulation software. As an example, such implementations of signal control algorithms as SCOOT, SCATS, etc. in PTV VISION VISSIM could be mentioned [24].

- **Use of virtual reality systems.**

Virtual reality systems and programming tools are used extensively today, especially in simulation, where the driver's reaction and behaviour must be analysed in greater detail. Therefore, traffic safety-related simulation will possibly be an area that greatly benefits from VR (Virtual Reality) technology. Of course there is no reason why VR tools could not be used in a more traditional way - for demonstration of results to decision-makers and the public [42]. For example, the laboratory of virtual reality of Magdeburg Frauhofer Institute could be mentioned in this context. But still the bulk of the currently existing software could produce 2D and 3D animation (in microscopic simulation tool). Macroscopic deals only with 2D, but there are some papers and ideas as to the way of how that could be changed [43].

- **Development of new types of models.**

Scientists are developing new types of models, which, after testing, could be integrated into the current simulation software or even lead to new software development. A good example is the use of fuzzy logic theory. In traffic flow simulation, rule-based approaches like HUTSIM and TRANSIMS are becoming more and more common. In this kind of a framework, the use of fuzzy logic to describe the human perception can easily be used, and there are several applications of fuzzy car-following models available [44], [45], [46]. Integration of fuzzy logic will lead to more accurate details of driver's behaviour in microscopic simulation and the same for macroscopic simulation [47]. Another example is introduction of a new assignment algorithm LUCE into PTV VISION VISUM software [48]. Another direction of the research is development of the so-called mesoscopic traffic simulation models and hybrid transport models [49], [50], [51].

1.3 Review of the state-of-the-art of the traffic analysis tools

Traffic analysis tools are a collective term used to describe a variety of software-based analytical procedures and methodologies, and supporting different aspects of traffic and transportation analyses. To date, numerous traffic analysis methodologies and tools have been developed by public agencies, research organizations, and consultants. Traffic analysis tools can be grouped into the following categories [31]:

- **Sketch-planning tools:** Sketch-planning methodologies and tools produce general order-of-magnitude estimates of travel demand and traffic operations in response to transportation improvement. They enable one to evaluate specific projects or alternatives without conducting an in-depth engineering analysis. Such technologies are primarily used to prepare preliminary budgets and proposals, and are not considered to be a substitute for the detailed engineering analysis needed later in the project implementation process. Sketch-planning approaches are typically the simplest and the cheapest of the traffic analysis techniques. Sketch-planning tools perform some or all of the functions of other types of analytical tools, using simplified analyses techniques and highly aggregated data. However, sketch-planning techniques are usually limited in scope, analytical robustness, and presentation capabilities [31]. The following examples of sketch-planning tools could be mentioned: HDM (Highway Design and Management), SMITE (Spreadsheet Model for Induced Travel Estimation), TransDec (Transportation Decision) etc.

- **Travel demand models:** Travel demand models have specific analytical capabilities, such as the forecast of travel demand and the consideration of choice of destination, mode, time-of-day of the travel, and route, and the representation of traffic flow in the highway network. These are mathematical models that forecast future travel demand based on current conditions, and make future projections of household and employment characteristics. Travel demand models have been originally developed to determine the benefits and the impact of major improvements of highway traffic in metropolitan areas. However, they are not designed to evaluate travel management strategies such as intelligent transportations systems (ITS)/operational strategies. Travel demand models have only limited capabilities of accurate estimation of changes in operational characteristics (such as speed, delay, and queuing) resulting from implementation of ITS/operational strategies. These inadequacies generally occur because of poor representation of the dynamic nature of traffic in travel demand models [31]. The following examples of travel demand models could be mentioned: VISUM, TransCAD, TRANSIMS, EMME/2, CUBE/TRIPS etc.

- **Analytical/deterministic tools (HCM-based).** Most analytical/deterministic tools implement the procedures of the Highway Capacity Manual (HCM). These tools quickly predict density, speed, delay, and queuing with respect to a variety of transportation facilities, and are validated with data fields, laboratory test beds, or small-scale experiments. Analytical\deterministic tools are good at analysing the performance of isolated or small-scale transportation facilities; however, they are limited in their ability to analyse network or system effect [31]. The following examples of analytical/deterministic tools could be mentioned: FREWAY (Freeway Delay Calculation Program); ARTPLAN (Arterial Planning); CATS (Computer-Aided Transportation Software); ARCADY (Assessment of Roundabout Capacity and Delay), ICU (Intersection Capacity Utilization), TGAP (Traffic Gap Analysis Program) etc.

- **Traffic signal optimisation tools** are primarily designed to develop optimal signal-phasing and timing plans for isolated, arterial streets, or signal networks. This may include capacity calculations; optimizations of cycle length splits including left turns; and coordination/offset plans. Some optimisation tools can also be used for optimising ramp metering rates for freeway ramp control [31]. The following examples of traffic signal optimisation tools could be mentioned: Synchro; TRANSYT-7F; PROGO; TEAPAC/SIGNAL2000, etc.

- **Macroscopic simulation models: Macroscopic simulation models:** Macroscopic simulation models are based on the deterministic relationships of the flow, speed, and density of the traffic stream. The simulation in a macroscopic model takes place on a section-by-section basis rather than by tracking individual vehicles. Macroscopic models have considerably fewer demanding computer requirements than microscopic models. However, they are unable to analyse transportation improvements in as much detail as microscopic models do [31]. The following examples of macroscopic simulation models could be mentioned: BTS (Bottleneck Traffic Simulator); VISUM; EMME/2; SATURN; NETCELL, etc.

- **Mesoscopic simulation models:** Mesoscopic simulation models combine the properties of both microscopic (discussed below) and macroscopic simulation models. As in microscopic models, the mesoscopic models' unit of traffic flow is an individual vehicle. Their movement, however, follows the approach of the macroscopic models and is governed by the average speed on the travel link. Mesoscopic model travel simulation takes place on an aggregate level and does not consider any dynamic speed/volume relationships. Mesoscopic models as such provide less fidelity than micro-simulation tools, but they are superior to the typical planning analysis techniques [31]. The following examples of macroscopic simulation models could be mentioned: CONTRAM (Continuous Traffic Assignment Model), DYNAMIT-P; MesoTS, etc.

- **Microscopic simulation models:** Microscopic models simulate the movement of individual vehicles based on car-following and lane-changing theories. Typically, vehicles enter a transportation network using a statistical distribution of arrivals (a stochastic process) and are tracked through the network over small time intervals (e.g., 1 second or a fraction of a second). Typically, upon entry, each vehicle is assigned a destination, a vehicle type, and a driver type. Computer time and storage requirements for microscopic models are large, usually limiting the network size and the number of simulation runs that can be completed [31]. The following examples of macroscopic simulation models could be mentioned: AIMSUN2; CORSIM; VISSIM; SimTraffic; PARAMICS; AUTOBAHN, etc.

As this research focuses mainly on simulation tools, only traffic analysis tool based on simulation will be taken into account hereinafter.

Transport models could be classified according to a wide range of criteria. Some of them could be mentioned here:

- by solution: analytical vs. simulation;
- by process representation: dynamic vs. static;
- by variables scale: continuous vs. discrete;
- by process: stochastic vs. deterministic;
- by time-changing approach: time-stepped vs. event-based, etc.

But the most popular classification, which is widely used, is based on model detail level [52]. At the moment, 4 levels of the transport models are distinguished. They are as follows:

- Sub-microscopic models.
- Microscopic models.
- Mesoscopic models.
- Macroscopic models.

Both microscopic and macroscopic models are treated as classical ones, and, generally, there is hardly any trouble at all to define the meaning of these models. The examples of microscopic and macroscopic models could be found in [53].

1.3.1 Submicroscopic models

Similar to microscopic simulation models, sub-microscopic simulation models describe the characteristics of individual vehicles in the traffic stream. However, apart from a detailed description of driving behaviour, also vehicle control behaviour patterns (e.g. gear change, AICC operation, etc.) in response to the prevailing surrounding conditions are modelled in detail. Moreover, the functioning of specific parts (sub-units) of the vehicle is described. Figure 1.5 shows the fundamental principles of sub-microscopic traffic models. The input parameters of sub-microscopic models are mostly like those used in microscopic models plus some additional characteristics describing properties of vehicles and specific properties of the drivers; among a large set of such parameters, a few could be mentioned: fuel type used by vehicle, engine type installed in vehicle, use of navigation systems, type of gear used in vehicle, and so on. Most of these additional parameters are used to estimate specific output data, e.g., influence of traffic flow upon the ecological situation.

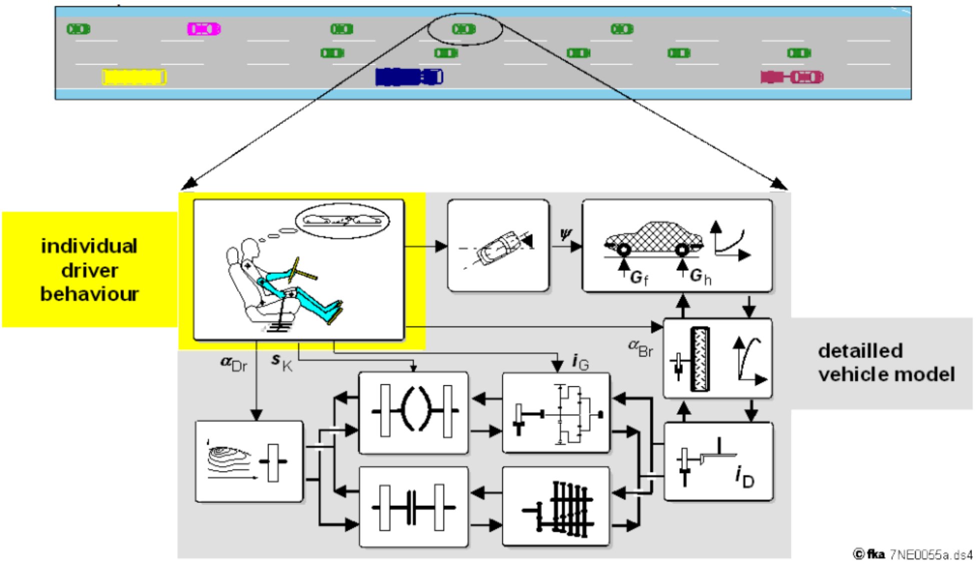


Figure 1.5. **Sub-microscopic traffic model concept [54]**

1.3.2 Microscopic models

According to [55], the micro-models are characterized by a detailed description of the traffic flow and the infrastructure. The modeling objects for this level are crossroads, groups of crossroads, bridges, flyovers, traffic roundabouts, etc. The main application is the decision-making on the tactical level [56], [57]. The tasks, which could be solved by using such models, could be enumerated as follows:

- choice and search for the optimal traffic light cycle;
- optimal traffic organizations;
- check-up of architectural according to requirements;
- estimation of different characteristics (queue length, LOS (level of service), delay times, etc).

Certainly, to be able to solve the above-stated problems, a simulation model should use the data. The general input and output data for microscopic simulation could be seen in Table 1.1.

Table 1.1.

Input and output data for microscopic models

Input data	Output data
• Number of lanes • Lanes width • Road location high level • Allowed speed • Transport flow structure • Flow intensity or OD (origin-destination) matrix • Priority rules • Traffic lights parameters • Acceleration function • Deceleration function	• Queue length (m) • Delay times (s) • Level of service (LOS) • Travel times (s) • Average speed (m/s) • Maximal speed (m/s) • Minimal speed (m/s) • Animation

As could be seen from the Table 1.1 there are a lot of data required to develop the microscopic model. Normally this data could be obtained from transport surveys aimed at collecting information on the traffic flow intensity. Animation could be treated as an important result of such a type of simulation. In general, animation could help one in process of model validation and analysis of results. An example of the animation from the microscopic model could be seen on Figure 1.6.

Figure 1.6. **Example of animation from microscopic transport model [14]**

However, such a detailed modeling brings about a lot of disadvantages typical to the microscopic simulation. They could be characterized as follows:

- Large resource requirements (time, money, staff) to be met in process of model development;
- a batch of runs should be completed to get reliable results;
- a big number of input data;
- the need to disaggregate input data;
- model parameters calibration;
- the model's high sensitiveness to errors in input data;
- The need of calibration.

These disadvantages usually make microscopic simulation projects time- consuming. This fact puts a very strict limitation on application of such a kind of simulation.

1.3.3 Mesoscopic models

The mesoscopic models are not so widely used. It is connected with the fact that different scientists interpret the term "mesoscopic modeling" in different ways. Some researchers suggest that the mesoscopic modeling should integrate characteristics of both microscopic and macroscopic levels [51]. Moving deeper, the following definition could be formulated: "Mesoscopic models combine the properties of both microscopic and macroscopic simulation models. These models simulate individual vehicles, but describe their activities and interactions based on the aggregate (macroscopic) relationships" [31]. Another definition sounds like "Mesoscopic models of traffic flows imply the estimation of the macroscopic indicators on the microscopic level".

1.3.4 Macroscopic models

The traffic flow on the macroscopic level is presented in general, and is associated with a fluid flow (hydrodynamic model) or with gases (gas-dynamic model) [52]. The transport infrastructure is presented with a low level of detail: crossroads as nodes, streets as links connecting the nodes. The objects of modelling are districts of the city, the city proper, regions of the country, the country proper, etc. [58]. The output data is presented with the average values in the aggregated form and could be used for decision-making at the strategic level. The tasks which could be solved on the macroscopic level, could be enumerated as follows:

- roads loading level over the entire modelling object;
- traffic intensity over all of the modelling objects;
- public transport route and schedule optimization;
- traffic distribution forecast in the network;
- system bottleneck research;
- estimation of public transport route effectiveness.

Typically, the data required to build macroscopic models has aggregated characteristics. Obviously, due to aggregated characteristics of the input data we should not expect high precision of the output results. But low precession of the output data is compensated by the possible size of modelling objects. All of the general input and output data are presented in Table 1.2.

Table 1.2.

Input/output data for macroscopic models

Input data	Output data
• Transport infrastructure • Capacity of the links • Free flow speed for each link • Allowed turns for each node • Transport systems • Transport zones • OD matrix • Volume delay function for each type of link	• Free flow travel time from one transport zone to another • Travel time from one transport zone to another in a loaded network • Average speed of travel for each couple of zones • Loading level for each link • Intensity for each link • Public transport effectiveness indices • Graphical representation of the results

The below-stated data has been received mainly from different types of transport surveys (including mobility survey, household survey etc.). These surveys are oriented to collect information about people mobility, attraction points, travel goals, travel transport system, and statistical information for each transport zone. This information is necessary for developing the origin destination matrix, which describes travels between transport zones. In general, the results are aggregated for modelling horizon and are static. It means that we could not observe any dynamics of the modelled system. The results are presented in the graphical and tabular form.

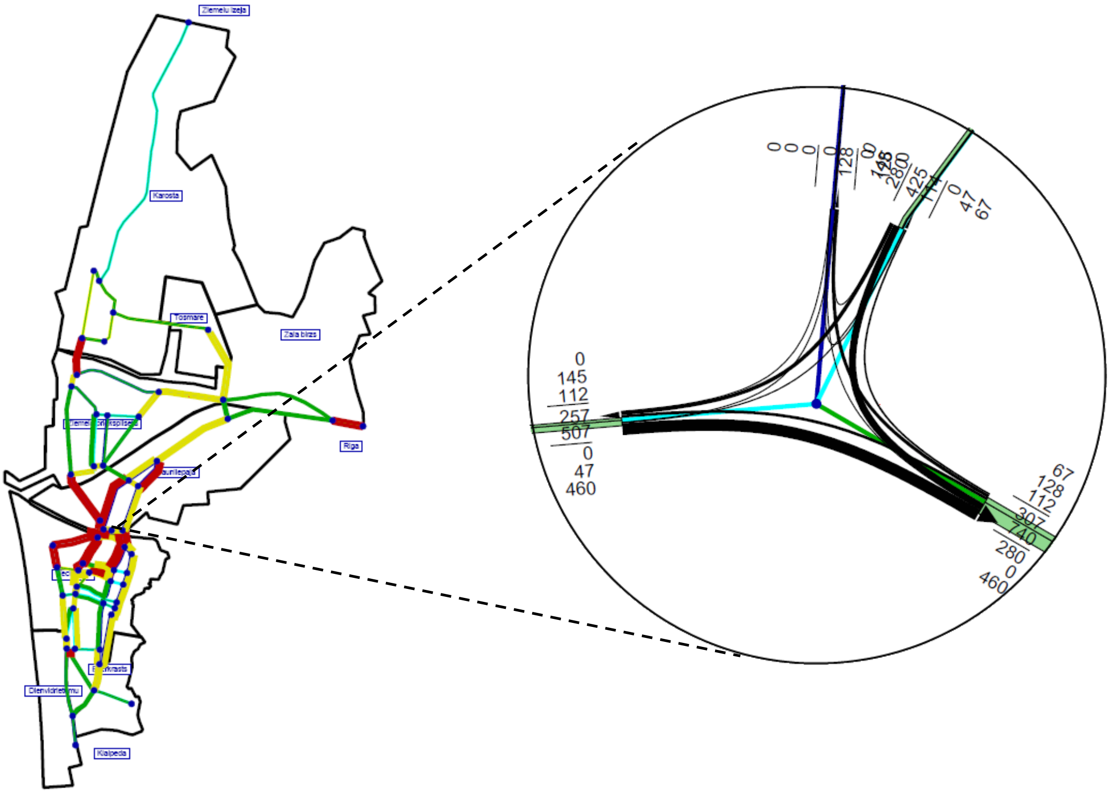


Figure 1.7**. An example of macroscopic modelling result [20]**

Figure 1.7 demonstrates one of the possible outputs obtained from the macroscopic model. The figure shows the loading level of different roads of the city, highlighted in colour. The red means that the loading level exceeds 120%, the yellow shows that the loading level is between 80% and 120%, and the green represents information that the loading level is less than 80% [20]. Some more examples of data representation at macroscopic level could be found in [59].

The output results could be used effectively by a transport engineer as the decision-supporting information. But normally simulation on the macro-level has some limitations which complicate its use:

- output results have general characteristics;
- the results are static;
- an additional research (survey) should be done to determine input data.

1.4. Application of the term "mesoscopic" in traffic flow simulation theory

In literature, the definition and concepts of mesoscopic traffic flow models vary widely, and some of the models have distinct modelling conceptions. In this work only simulation-oriented models have been taken into account.

1.4.1 CONTRAM

CONTRAM was the first-ever developed model presenting the aspects of mesoscopic modelling. The name of the model itself stands for the "CONtinuous TRaffic Assignment Model" [43]. Its structure is based on the link-node network, where the link behaviour of a batch of vehicles is determined by the free-flow speed on that link, or a speed/flow relation. At the moment, the speed/flow relation is also known as the volume delay function. The size of pocket is the same as a rule, and could be given by user or automatically calculated using the following formula [60]:

$$P = k\frac{\sum_{i=1}^{n} Q_i}{\sum_{i=1}^{n} \sqrt{Q_i}},$$

where

- Q_i – the total volume of vehicles loaded on that O-D movement within each time slice;
- n – the number of time slices with data;
- k – user-supplied scaling factor.

Additionally, the links have saturation flow limits. The nodes represent crossroads in the model; that is why the additional delay value is calculated. This value is connected with the signal timing plans, average give-way delays, etc. Basically, this information is estimated from real observations. Also the throughput of each node is known; a queue could exceed the available link space. The link space is determined by the link length and the number of lanes for this link. If the link is filled then it cannot accept any new vehicle batches. A characteristic feature of CONTRAM is that the model is designed iteratively and progressively within iterated assigned proportions of OD (Origin-Destination) flows, until one or more of a set of "equilibrium conditions" are met [60], [61]. All the aspects of the model could be represented graphically on Figure 1.8.

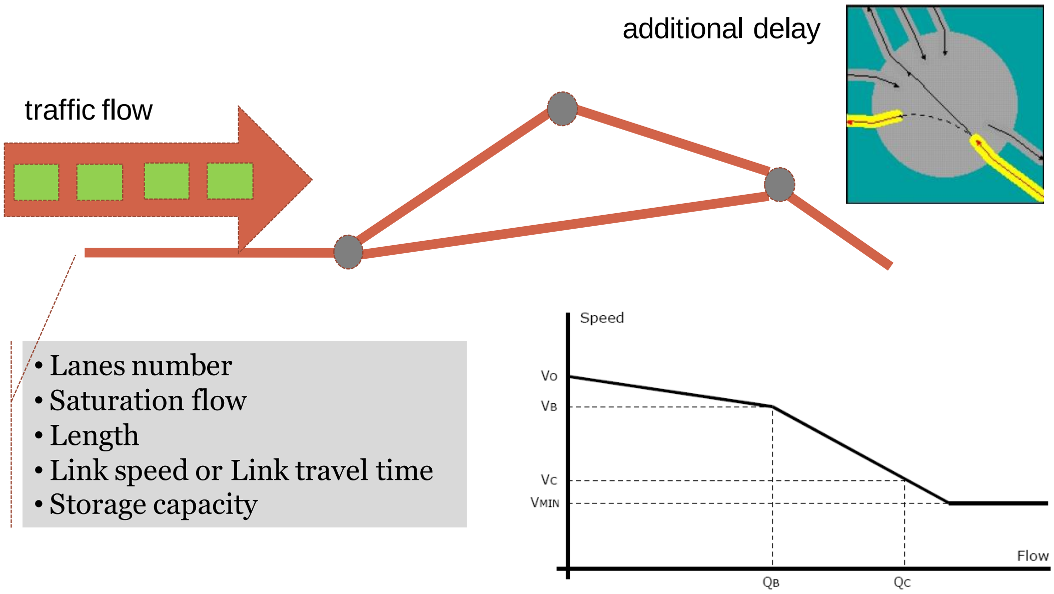


Figure 1.8. **CONTRAM model concepts**

The basic input data for the model could be mentioned:

- saturation Flow (pcu/h);
- link length (m);
- green time (%);
- free flow time (s);
- free flow speed (km/h);
- speed/flow curve number;
- number of lanes;
- storage capacity (pcus);
- link number;
- OD matrix.

The output data of the CONTRAM model could be drawn:

- total flow (veh);
- average flow (veh/h);
- speed (km/h);
- queue (veh);
- delay (sec);
- capacity (pcu/h);
- PCU ratio;
- generalized cost;
- congestion index;
- vehicles stopping (%);
- capacity reduction (%);
- etc.

Figure 1.9 demonstrates an example of a traffic simulation model based on CONTRAM.

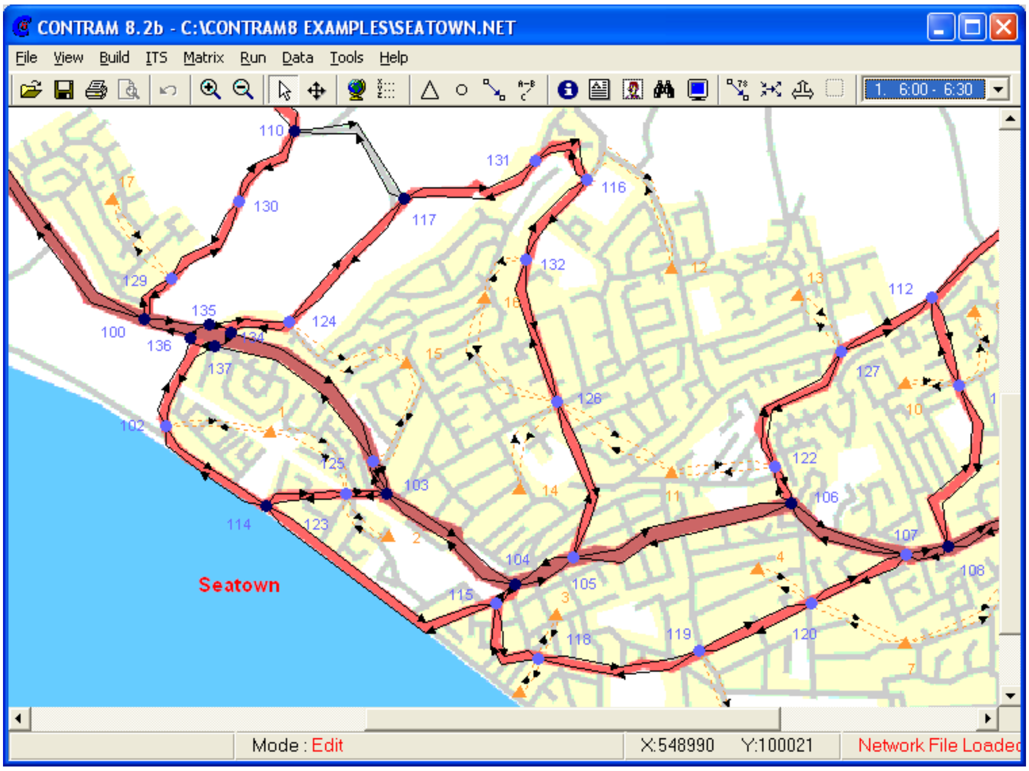


Figure 1.9. **Simulation using CONTRAM model [60]**

1.4.2 DynaMIT

DynaMIT (**Dy**namic **N**etwork **A**ssignment for the **M**anagement of **I**nformation to **T**ravellers) is a real time dynamic traffic assignment system that provides traffic predictions and travel guidance [62], [63]. This system has been developed since 1990's at MIT, with the aim of being able to simulate and predict the effects of real-time traffic information provided to the drivers [64]. It consists of two main engines [65]:

- The supply simulator, which simulates movements of the vehicles over the network;
- The demand simulator, which simulates the time-varying Origin-Destination (OD) demand flows, disaggregates route choices, etc.

These two modules interact continuously with each other, and with the traveler information that is provided. The modelling of the road network is performed as per link, per segment, and per lane. Like the previous model, the model examined represents the network as a set of links and nodes. Links and nodes are treated as static elements of the network. The dynamic components are designed to capture aspects of traffic dynamics. The dynamic components of the network are continuously updated. Each link is divided into segments that capture variation of traffic conditions along the link. While most of the segments are defined in advance, some additional segments of the network can be dynamically created to capture the presence of incidents. Each segment has a capacity constraint at its downstream and depending on the nature of the segment; this capacity constraint can appear due to the static physical characteristics of the road or the dynamic occurrence of an incident. Each segment has a moving part and a queuing part. The moving part represents the portion of the segment where vehicles can move at some speed. The queuing part represents vehicles that are queued up [66]. The above-mentioned logic could be demonstrated by using Figure 1.10. Queuing model . Traffic dynamics is captured by two major models: a deterministic queuing model and a speed model. As a matter of fact, the queuing model is a family of models. Each specific queue status (formation, dissipation, blockage, etc.) is captured by a different model. As an example, the position *q(t)* of a given vehicle joining a dissipating queue at time *t* is given by:

$$q(t) = q(0) + l(ct\text{-}m),$$

where

- $q(0)$ – the position of the end of the queue at time 0;
- l – the average length of vehicles;
- c – the output capacity (i.e. the dissipation rate);
- m – the number of vehicles moving between the considered vehicle and the end of the queue at time 0.

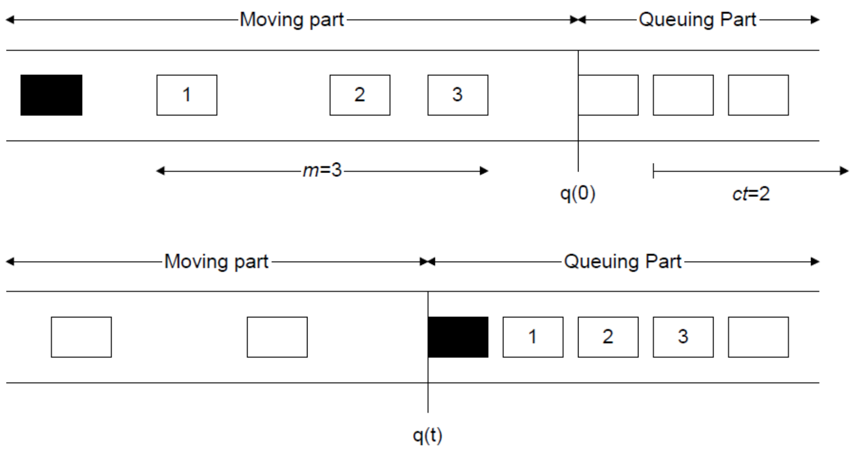


Figure 1.10. **Queuing model [66]**

The speed model is based on the following assumptions. For a specific moving part of a segment, two speeds are computed. The speed at the upstream end of a segment (v_u) is a function of the average density on the moving part of the segment. The speed at the downstream end (v_d) is the speed at the upstream end of the next segment. An acceleration/deceleration zone of length δ is the speed at the upstream end of the next segment. An acceleration and deceleration zone of some length is defined at the end of the moving part. Before that zone, each vehicle is moving at a constant speed [66]. Within the zone, the speed of vehicles varies linearly as a function of the position, as illustrated on Figure 1.11.

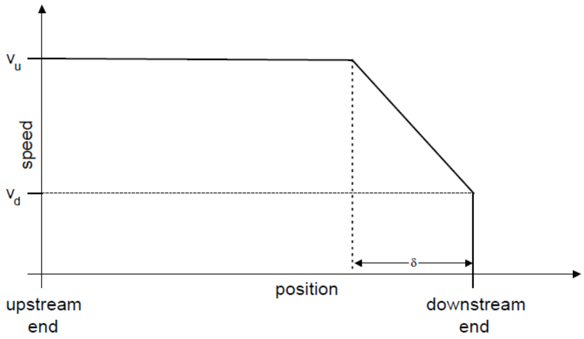


Figure 1.11. **Speed model [64]**

Based on the model description we can conclude that input data for DynaMIT model is in general the same as for CONTRAM model. The possible output data could be split into two parts. The first one could be called primary data:

- flow;
- speed;
- density;
- queue length;
- spillbacks.

This data is calculated for each segment of the road. The second part of the data is calculated based on the primary data and could include the following information:

- total savings in travel delays;
- costs;
- revenues;
- air pollution;
- safety;
- etc. [63].

1.4.3 DYNASMART, FASTLANE, DTASQ

Another idea is based on the queue-server approach [67], [68], [69] where the roadway is modelled as a queuing and a running part. This approach represents a road as a queuing and running carriageway. Lanes are usually not modeled individually. Vehicles are modelled individually, and have their individual speed. At the same time, their behavior is not modelled in detail. Vehicles run on the carriageway of the road with a speed which is defined by using macroscopic speed-density function. On reaching the downstream end, a queue server transfers vehicles to the next road segment. The advantage of this approach is connected with simulation of vehicles individually and with easy calibration of macroscopic speed/density relationships. The capacity of the servers (at nodes) is a matter of saturation flows and their variance (estimated or calculated). In case of signal controllers, simulation servers are replaced by state-changing gates (open/closed state) according to the respective states of the signal control (green/yellow/red). The adaptive signal control is more difficult for the model since the positions of the vehicles on the link are not known, and therefore it is difficult to know when exactly they pass the detectors connected to the signal control [51]. The concepts of the models are presented on Figure 1.12.

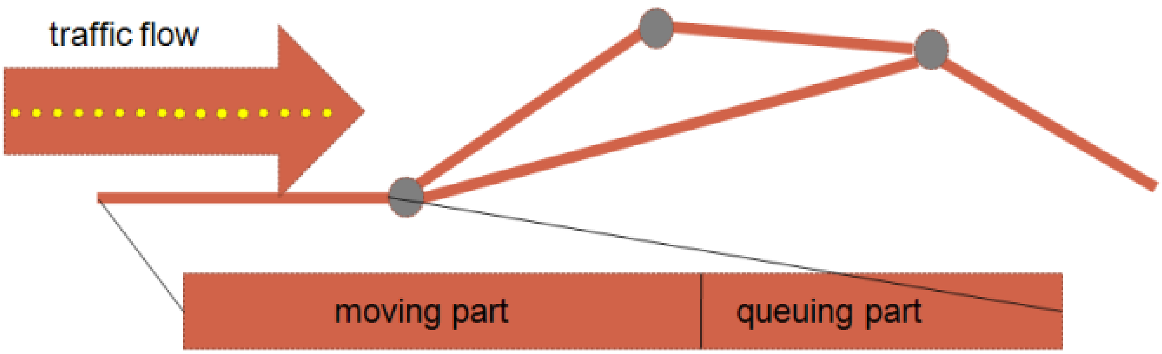


Figure 1.12. **Moving and queuing part**

The output data could be drawn as follows:

- intensities per link;
- average speed per link;
- average travel time for each origin-destination point;
- average delays per directions;
- routes.

Here it should be noted that the great advantage of these models is that vehicles are modelled individually; this gives us information about the routes of travel. It is not available in the previously described models, because in those models vehicles have been grouped in an unspecified manner.

1.4.4 AMS

AMS stands for anisotropic mesoscopic simulation. The AMS model is developed based on two intuitive concepts and traffic characteristics [70]:

- at any time, a vehicle's prevailing speed is influenced only by the vehicles in front of it, including those that are in the same or adjacent lanes;
- the influence of traffic downstream upon a vehicle decreases with the increasing distance.

These two characteristics define "anisotropic" property of the traffic flow and provide the guiding principle for AMS model. The influence of traffic on a single vehicle is captured in what is known as a Speed Influence Region (SIR), as shown on Figure 1.13. The SIR for vehicle *i* is defined as vehicle *i*'s immediate downstream roadway section in which the stimulus significantly influences vehicle *i*'s speed response.

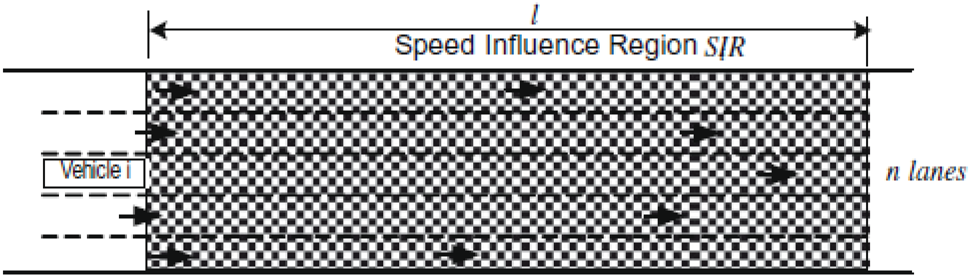


Figure 1.13. **Speed Influence Region concepts demonstration [70]**

The AMS work principle is based in the following calculations (an example is given for one vehicle):

- At the beginning of each simulation interval, for each vehicle *i*, the prevailing speed of vehicle *i* is determined by equation, which is the non-increasing speed-density relationship function $v_i^t = f(k_i^{t-1})$.
- The AMS model evaluates (at every simulation interval) the vehicle's speed based on the SIR density within the previous simulation interval. The SIR density is calculated based on equation $k_i^{t-1} = min\left[k_{queue}, \frac{N_i^{t-1}}{nl}\right]$ if the SIR spans a homogeneous highway segment; otherwise, equation $k_i^{t-1} = min\left[k_{jam}, \frac{N_i^{t-1}}{mx_i^{t-1}+n(l-x_i^{t-1})}\right]$ is used.

  The following notation for the above-stated formula is used:

i – subscript denoting vehicle;

t – subscript denoting a simulation interval;

l – SIR length;

x_i^{t-1}– distance between vehicle *i* and the upstream edge of the end of lane or the location of bottleneck point within SIR at clock tick *t-1;*

m – number of lanes for the SIR area designed with x_i^{t-1};

v_i^{t-1}– prevailing speed of vehicle *i* during simulation interval *t;*

k_i^{t-1}– density of the SIR with respect to vehicle *i*;

N_i^{t-1}– number of vehicles present in SIR for vehicle *i*, excluding vehicle *i;*

k_{jam} – *queue density*, $f\left(k_{jam}\right) = 0$

During AMS simulation, each vehicle maintains its own prevailing speed and SIR at the beginning of a simulation interval. Therefore, travelling distances of individual vehicles are likely to differ, even though the vehicles are on the same link. The AMS model proper lies very close to microscopic models based on the car-following concept. The major difference between AMS-based model and the car-following concept-based one lies in using SIR, which is a macroscopic measure of all vehicles present in SIR region - instead of an inter-vehicle measure between the target and the leading vehicle(s). The input parameters of AMS model are very close to the inputs of microscopic model, because the only way of calculating speed is changed. So the same picture is also observed with output parameters - they are the same as in microscopic model. The use of this AMS model could decrease a model execution time and save calculation performance of the computer. This feature allows one to build more spatially dispersed models as compared to microscopic simulation.

1.4.5 MEZZO

The traffic system in Mezzo [51] is described by a graph with two objects: nodes and links. The nodes represent the point of traffic flow intersection and the origin and destination of traffic. The links are the road between nodes. Links are unidirectional, so if one has to simulate a bidirectional road, two links should be used. Lanes are not presented in the model separately. Usually nodes have a number of incoming and outgoing links.

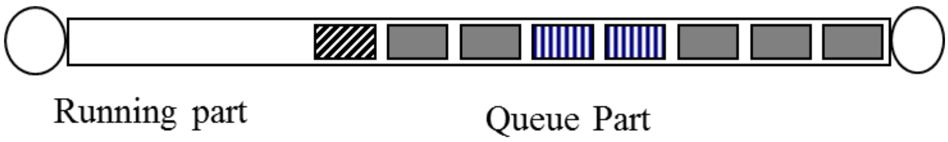


Figure 1.14. **Running and queuing part in Mezzo [51]**

As it can be seen on Figure 1.14, links are divided into the running part and the queue part. The running part is a segment of road with the ability of free movement for vehicles. The queuing part represents the segment of road occupied by vehicles standing in queue. This means that the boundary between the running part and queue part is dynamic, and usually varies over time, depending on variations in the inflow and outflow. The running part contains all the moving vehicles. The speed of vehicles within the running part is calculated as a function of density in the running part $V = f(density)$. The function is defined by the following way [51]:

$$V(k) = \begin{Bmatrix} V_{free}, k < k_{min} \\ V_{min} + \left(V_{free} - V_{min}\right)\left(1 - \left(\frac{k - k_{min}}{k_{max} - k_{min}}\right)^{a}\right)^{b}, k \in [k_{min}, k_{max}] \\ V_{min}, k > k_{max} \end{Bmatrix},$$

where

$V(k)$ – speed assigned to the vehicle;

k – the current density on the running part of the link;

V_{min} – minimum speed;

V_{free} – free flow speed;

k_{min} – minimum density;

k_{max} – maximum density;

a, b – model parameters.

The queue part is defined to contain - at time t - those vehicles that have the earliest exit time smaller than t. In other words, all the vehicles should have left the link, if it were not for some delay caused at the downstream node. So as it was said before, the expected exit time could be calculated as $t_{expected} = t_{current} + (link\ length/speed)$. At any time $t_{current}$ all vehicles with $t_{expected} < t_{current}$ are on the running part; all vehicles with $t_{expected} \geq t_{current}$ are on the queue part.

At the downstream end of the link, the node connects it to other links. Each connection between the incoming and the outgoing link is called the turning movement (even the straight going movements) [51]. See Figure 1.15 for an example.

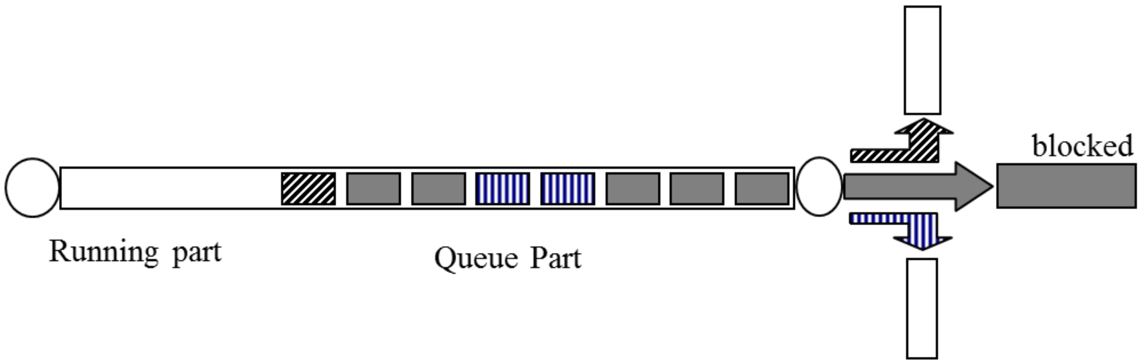


Figure 1.15. **Turning movements in Mezzo [51]**

Queue part contains all vehicles that should have left the link; to simulate the turning movement, a stochastic queue-server for each turning movement is used. The capacity of queue-server is assumed to vary according to a (truncated) normal distribution, which is a common way to represent time headways of traffic at high flows [71].

The possible output data could be described here:

- average speed;
- average density;

- average income flow;
- average outcome flow;
- average queue length;
- travel time;
- travel routes.

1.4.6 Cellular Automata

Furthermore, some authors [51] believe that cellular automata (CA) could be assigned to mesoscopic traffic modelling. Notwithstanding the fact that another authors classify cellular automata models as microscopic ones, the idea that cellular automata belong to mesoscopic simulation could be true. As it was defined in the previous chapter, a microscopic model is characterized by a detailed traffic flow and transport infrastructure modeling. Should this definition be accepted, cellular automata could be assigned to mesoscopic models since transport infrastructure and vehicle interaction process is not described in detail. The main idea of using cellular automata in traffic simulation could be described in the following way. The road by itself is divided into homogeneous cells [72]. Usually the size of a cell is defined by the size of PCU. Each cell has two possible states: being occupied by a vehicle or not being occupied by a vehicle. The set of vehicle behaviour rules is minimalistic (most notably, the Nagel-Schreckenberg rules [73]), which determine for each time step the number of cells traversed by a vehicle. The Figure 1.16 shows the representation of the model based on cellular automata.

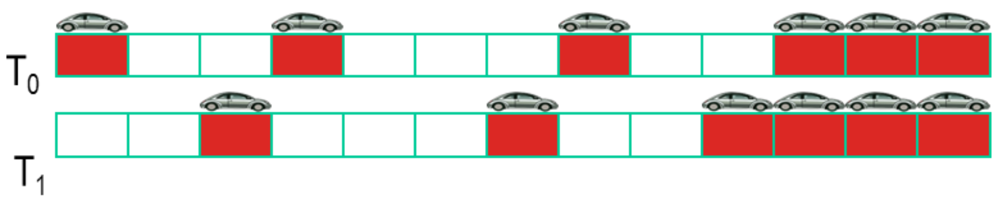


Figure 1.16. **Typical configuration of the road in CA based simulation [73]**

As it was stated above, vehicle behavior is simplified and is subject to the rules which could be mentioned here [57]:

- **Acceleration:** if the velocity v of a vehicle is lower than v_{max} and if the distance to the next car ahead is larger than v+1, the speed is advanced by one [*v*→ *v*+*1*].
- **Slowing down (due to other cars):** if a vehicle at site *i* seems the next vehicle at site *i*+*j* (with j≤ v), it reduces its speed to *j-1 [v→ j-1]*.

- **Randomization:** with probability *p*, the velocity of each vehicle (if greater than zero) is decreased by one [*v*⟶ *v-1*].
- **Car motion:** each vehicle is advanced v sites.

Through the steps one to four, very general properties of single-lane traffic are modelled on the basis of integer valued, probabilistic automation rules. But this simple model already shows nontrivial and realistic behavior. There are a number of examples of research of this model and its practical application. The Figure 1.17 demonstrates an example of traffic simulation model based on CA (cellular automata).

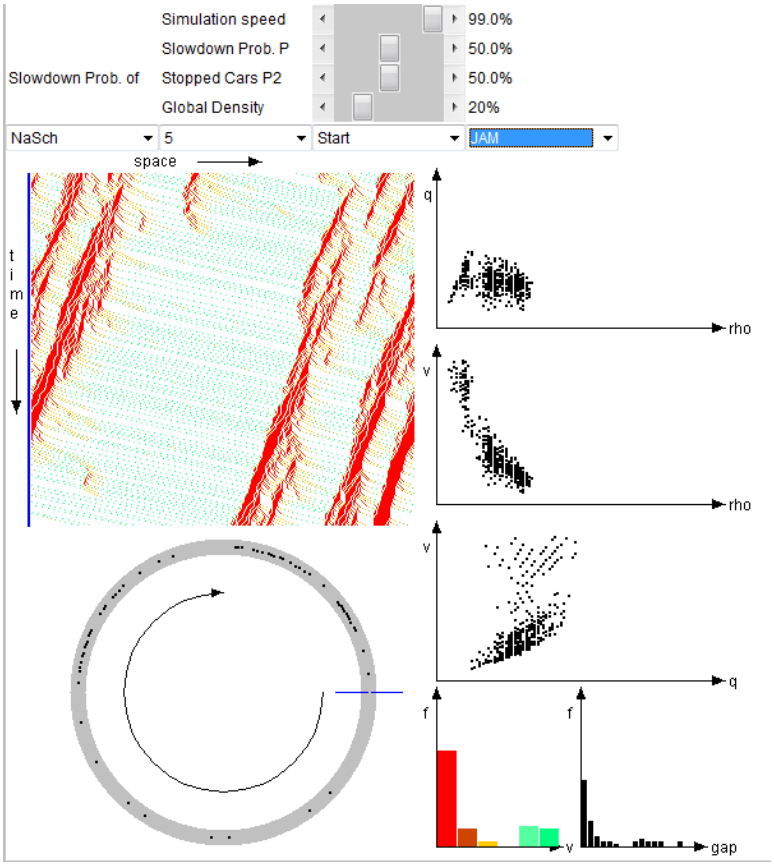


Figure 1.17. **A traffic model based on CA [74]**

1.5 Summary

- The development of traffic simulation tools was due to the problems, which started to occur in transport systems, with the main of them being: congestions, growing number of accidents, economical losses. But prior to the development, a lot of work had been done on studying processes inherent in transport systems, data processing operations had been performed, and just after that, a different kind of tools for traffic analysis was developed.

- The application areas of simulation are very large. They include: integration of simulation systems into ITS as a short-time or medium-time forecast tool; adding different types of simulation models to DSS to generate more reasonable solutions and to take the optimal decision of the problem, and - last but not least - simulation is applied as a tool for strategic, tactical, and operational development of the transport systems.
- Nowadays a simulation has become a powerful traffic analysis tool. It is used by a wide range of specialists - from researchers to politicians. The primary role of the traffic analysis tools (including simulation) is the following: to improve the decision-making process; to evaluate and prioritise planning/operational alternatives; to improve design and evaluation time and cost; reduce disruption of traffic; to present/market strategies to the public/stakeholders; to operate and manage the existing roadway capacity; to monitor performance.
- The current trends in simulation as a tool of traffic analysis are connected with the following directions: application of new knowledge from the fundamental traffic flow analysis area; use of GIS and CAD systems; application of parallel and cloud computing; integration of new general principles of simulation and programming; development of open environment; simulation of control systems as a part of traffic; use of virtual reality systems; development of new types of models.
- “Traffic analysis tools” is a collective term used to describe a variety of software-based analytical procedures and methodologies that support different aspects of traffic and transportation analysis. The traffic flow simulation software is a part of traffic analysis tools.
- Different types of traffic analysis tools could be classified by purpose into the following groups: sketch–planning tools; travel demand models; analytical/deterministic tools (HCM-based); traffic signal optimisation tools; macroscopic simulation models; mesoscopic simulation models; microscopic simulation models (including sub-microscopic models).
- The disadvantages of traffic simulation at the microscopic and macroscopic levels have already been presented. In general, for microscopic model the disadvantages are connected with the lack of resources necessary to develop, validate, calibrate and

experiment with microscopic models. The output data refinement could be mentioned as an advantage. As regards a macroscopic model, it requires less input data for developing, but the output results are presented in the aggregated form and usually could be used only at the strategic level of decision support.

- Mesoscopic models fill the gap between the microscopic and macroscopic models, by gaining advantages of these models.
- The major part of scientists agrees that mesoscopic models represent individual vehicles, but describe their activities and interactions based on aggregate relationships. However, different notation could be used.
- A number of mesoscopic models were overviewed in this chapter of the promotional work. They are CONTRAM, DynaMIT, DYNASMART, FASTLANE, DTASQ, AMS, MEZZO and Cellular Automata. The above-mentioned models were described conceptually. The input and output parameters were drawn out for each model.
- In general, input parameters for all the models are the same. Almost all models (CONTRAM, DynaMIT, DYNASMART, FASTLANE, DTASQ, MEZZO) use link-node notation to describe a transport network. The transport network in AMS could be defined in different ways. The author of AMS model points out that link-node notation, cell-based notation and continuous space notation could be used to present a transport infrastructure. The CA use cell-based notation and it is only one way to present transport infrastructure in these models.
- The output data for the models, in general, are the same, but some differences could be noted. The models DYNASMART, FASTLANE, DTASQ, MEZZO, AMS and Cellular Automata could output travel route information, because they model vehicles individually, not grouping them as in CONTRAM and DynaMIT.
- As the input and output data for all models are mostly the same, the main difference between the models appears in traffic modelling. The way of traffic modelling influences exactness of final results.
- The main problem of all these models is that they are realized in closed software and are not available for public or still described conceptually without any software.

2. COMPARATIVE ANALYSIS OF SIMULATION APPROACHES

The simulation is a powerful tool for analysis and investigation of various systems. We could mark universality of this tool as the main advantage of simulation. The universality means the ability to model different types of the systems. Simulation branch is presented by 4 approaches, they are as follows: agent-based simulation, system dynamics, discrete event and discrete rate simulation.

2.1 Discrete event simulation

The discrete event simulation (DES) is the paradigm which is realized in the major part of software tools (for example, GPSS (General Purpose Simulation System), Extend, Arena, Witness, Automod, FlexSim, em-Plant, etc.). This approach is known since 1960-ies, when it was described by Geoffrey Gordon. A discrete event simulation model is defined by three attributes [75].

- stochastic – at least some of the system-state variables are random;
- dynamic – the time evaluation of the system-state variables is important;
- discrete event – significant changes in the system state variables are associated with the event that occurs at discrete time instance only.

Moreover, the concept of the approach could be described using Figure 2.1.

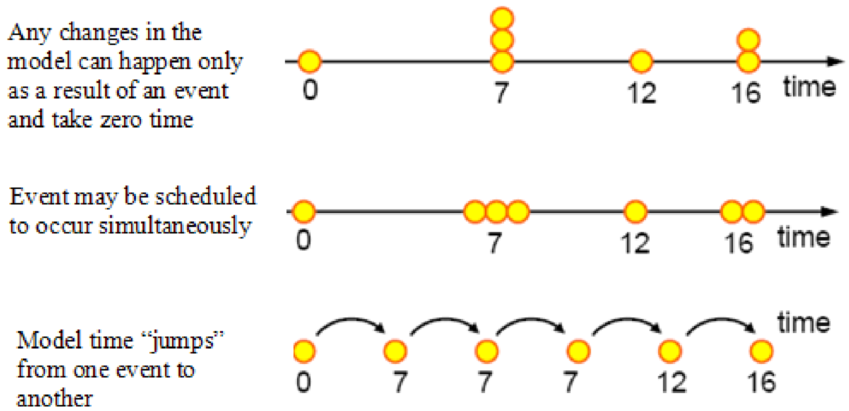


Figure 2.1. **Concept of discrete event approach [76]**

The model of the implemented system using this approach could be presented as a sequence of blocks (see Figure 2.2), which performs different operations: arrival, delay, resource use, split, combine, etc. These operations are being performed on the entities which could be presented as a flow. Usually these entities are called transacts. Transacts can have different type of attributes, which could be used to describe different processing logic. It should be noted that transact do not have their own behavior; their behavior is subject on the process described, that is why it is mentioned in some literature sources that it is a process centric approach. It means that the main element in this approach is a process.

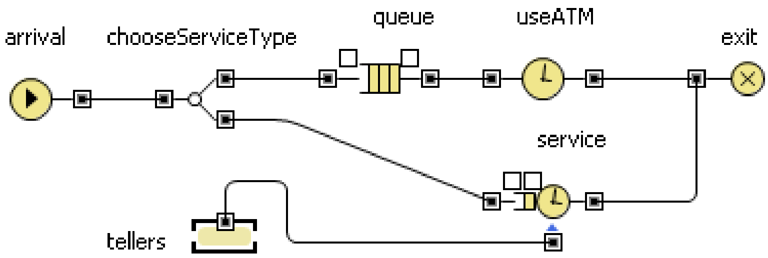


Figure 2.2. **Model realized by means of discrete event approach in AnyLogic [76]**

2.2 Agent-based simulation

The second paradigm is not better known today, but its popularity is growing. This paradigm is the agent-based (AB) modelling. The main concepts of the paradigm appeared only in 1990-ies. At present, various definitions exist in literature; from the practical point of view, it can be defined as essentially-decentralized, individually-centric (as opposed to system level) approach to the model design. When designing an agent-based model, the modeller identifies the active entities - the agents (they can be people, companies, projects, assets, vehicles, cities, animals, ships, products, etc.), defines their behaviour (drivers', reactions, memory, states, etc.), places them into a certain environment, perhaps, establishes connections and runs the simulation [77]. At present, there are only few software products available on the market; they are: RePast, Swarm, ASCAPE, NetLogo and AnyLogic. Graphically, the conception of the agent- based approach could be presented on Figure 2.3.

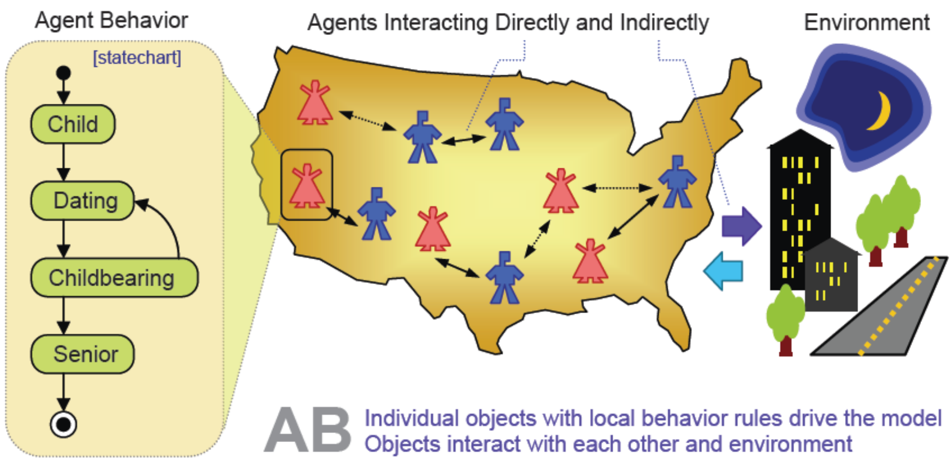


Figure 2.3. **Agent-based model architecture [78]**

The model of the system consists of the description of agents' behavior and characterization of the environment where agents are "living". The agents' behavior is mainly described by the finite state automata (also called state chart) as presented on Figure 2.3. The number of finite state automata is not limited, so each state chart could present different aspects of agent's existence. For example, one state chart could describe different living phases of the agent (for example, a child, young man, senior, etc.); the second one could describe educational phases (for example, kindergarten, school, university, etc.) and so on [79]. The state in state charts could be changed with time, and here we have a variety of possibilities, why it could be changed. For example, the following reasons could be mentioned: by probability, by time, by influence of another agent, by influence of environment, and so on. The number of agents in the system is not a fixed value, and it could be changed during simulation by removing agents from the system (e.g., in case of a person's death) and by "injecting" agents into the system at the same time (birth of a person).

2.3 System Dynamics

This approach was developed by Jay Forrester in 1950-ies. Firstly, Forrester applied a developed approach for analyzing industrial systems. According to [80], system dynamics (SD) deals with time-dependent behavior of the managed systems with the aim of describing the system and, by using qualitative and quantitative models, gaining an understanding of how the information feedback governs the system behavior; moreover, the aim was to design robust information feedback structures and develop control policies through simulation and optimization. In fact, system dynamics is a set of differential equations. That is why the system dynamics does not deal with a particular object, but rather with aggregated sets of objects flowing from one stock to another with an intensity which is defined with flow variables. Currently there is a lot of software dealing with system dynamic approach, and we can enumerate the following which was the most popular in due time: VenSim, PowerSim, iThink and AnyLogic. Basically, the difference between simulation software exists only in the model components visualization, user's interface, and reporting module. The main elements of the system dynamics model, which are presented in all of the software, are the following: stocks, flow variables, and variables. An example of the dynamic system model is presented on Figure 2.4.

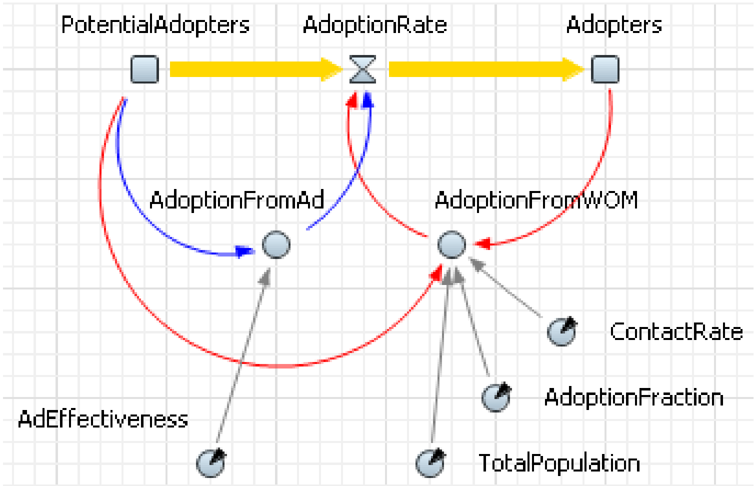


Figure 2.4. **Realized model using system dynamics approach in AnyLogic [81]**

In general, the results of the modelling are presented as graphs of the processes (see Figure 2.5). System dynamic models are very useful for experiments of the type "what if".

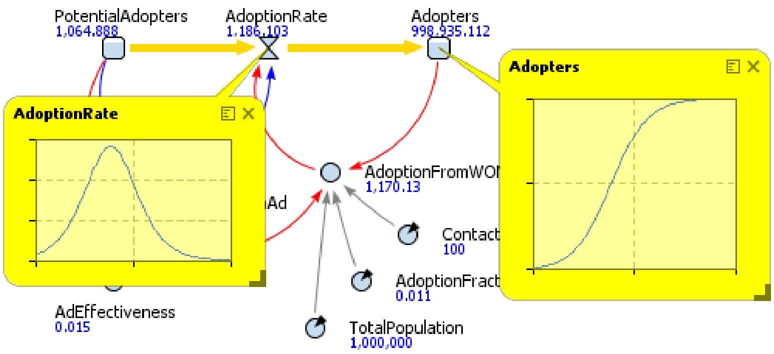


Figure 2.5. **Result of modelling using system dynamics approach [81]**

2.4 Hybrid simulation

This approach is not the classical one, but it has been widely used over the last few years. The main idea of this approach is to combine different approaches in one model. Simulation of different parts of the system using different approaches imparts flexibility to the model development process and reduces the time needed for the development.[78]. There exist no rigorous rules as to what kinds of approaches should be combined, it is up to developer. The only limitation in this case is a feature of the software used. The most well-known software, which allows building hybrid models, is AnyLogic. The Figure 2.8 demonstrates the main idea of hybrid simulation. The first part of the picture shows the combining of agent-based simulation and system dynamic approach. The partners of SC (Supply Chain) could be described as agents; the process of production is described by system dynamic approach.

The second part of the picture shows the combination of the system dynamic approach and agent-based simulation. By agents, population is simulated; using system dynamic approach is presented by the behaviour of environment. The third example shows the combination of discrete event simulation and agent-based modelling. The agents present the customers and workers, the process of customer service is described by discrete event simulation.

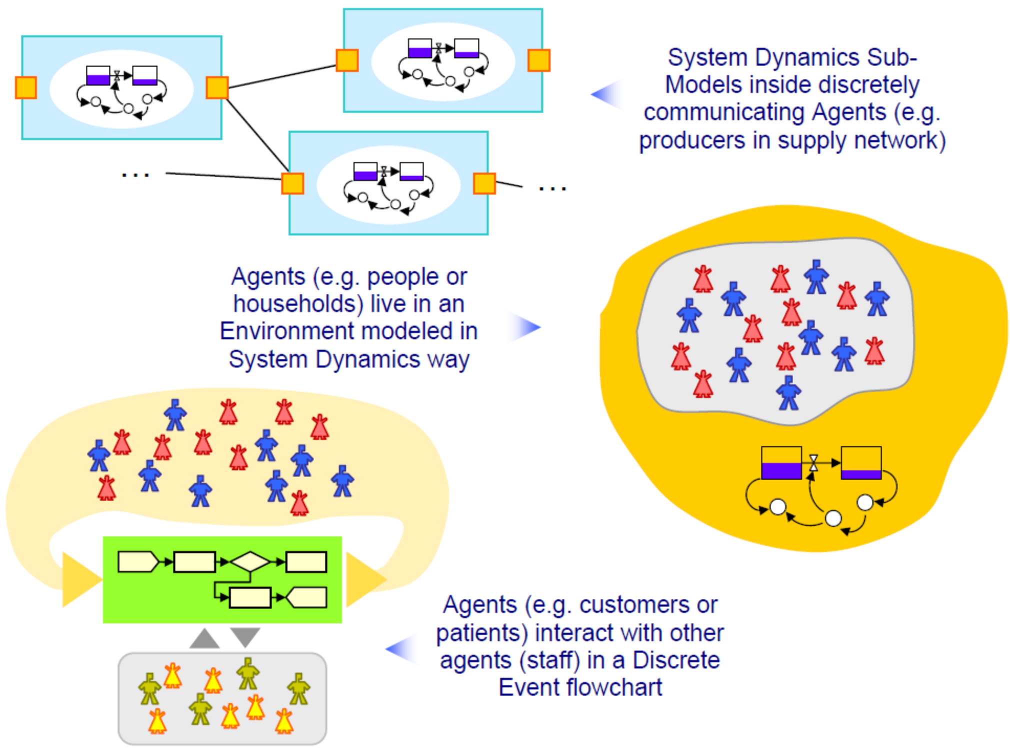


Figure 2.6. **Combining different simulation approaches [78]**

2.5 Discrete rate simulation

The philosophy behind this approach can be described with the phrase "event planning for continuous processes" [82]. The representation of individual flow objects that reproduce persons, job orders, goods, etc. is dispensed with. The only employed members that are used in the model represent the respective quantities of objects or materials and can be modified with mathematical formula in every step of the discrete simulation time. This type of mesoscopic modelling and simulation is the method helping one to complete planning tasks in production. Furthermore, the results of simulation at mesoscopic level can be introduced as graphs of the processes, which are very useful in practice. The concept of simulation on mesoscopic level specifies the development of the principally new class of models.

Mesoscopic modelling shows only discrete changes of the corresponding continuous flows. It means that flow intensity *λ(t)* stays unchangeable in each interval of time between flow changes. Function *λ(t)* could be called a slice constant function. Figure 2.7 presents diagrams of income and outcome flow for a simple store. The last graph presents contents of the store for the given input and output flows. Since *λ(t)* is a slice constant function, the graph of the store contents could be only a piecewise-linear function. The main advantages of such process representation in mesoscopic modelling are probably forecasting (planning, calculating) moments of time, then contents of store and cumulative value of flow reach the given values.

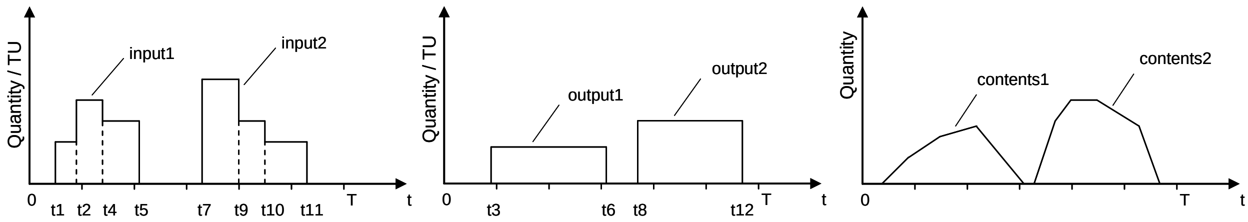


Figure 2.7. **Process presentations in discrete rate approach [83]**

So dual properties are characteristic of the mesoscopic model:

1. its flow processes characterized by intensity λ(t) (as in case of model of a continuous type);
2. for processes in store and for cumulative values of flows the future events (as in case of models of discrete events) could be planned.

The single continuous fragments of the flow, that will be called the batches of product, could be treated as objects. A peculiar feature of mesoscopic model is the possibility to control the path of any batch of product during its movement through the model structure. The main components used in mesoscopic model could be mentioned here: source, funnel, and transport element. The source element is required to inject flow inside the system; the main parameter of this component is intensity. However, it should be noted that intensity in this case is not just a constant value, but could vary depending on time. The next and the most important element is a funnel. Funnels could be divided into two classes: single channel and multi-channel funnels. Such a division allows one to simulate multi product systems. The idea of a funnel is presented on Figure 2.8.

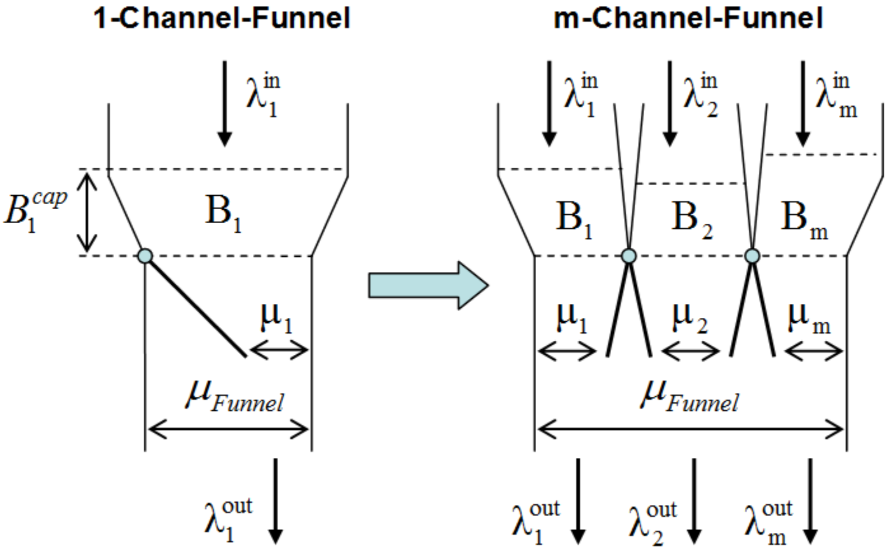


Figure 2.8. **Single channel funnel and multi-channel funnel [83]**

First, let us describe the notation used on Figure 2.8. The notation is equal for both single-channel and multi-channel funnel.

λ_i^{in} – input flow for the channel *i;*

B_i^{cap} – the capacity for the channel *i;*

B_{funnel}^{cap} – the capacity of the funnel;

B_i – the current level of product for the channel *i;*

μ_{i} – the processing rate for channel *i*;

μ_{funnel} – the processing rate for funnel;

λ_i^{out}- the output flow from channel *i*.

The following restrictions are defined:

$$B_i \leq B_i^{cap}, \ \ \sum B_i^{cap} \leq B_{funnel}^{cap},$$

$$\lambda_i^{out} \leq \mu_i, \quad \sum \mu_i \leq \mu_{funnel}.$$

The output flow could be calculated using the following equation

$$\lambda_i^{out} = \begin{cases} 0, & \lambda_i^{in} = 0 \ and \ B_i = 0 \\ \lambda_i^{in}, & \lambda_i^{in} > 0 \ and \ \lambda_i^{in} \leq \mu_i \ and \ B_i = 0 \\ \mu_i, & B_i > 0 \end{cases}$$

The funnel level of product in the channel could be obtained by using the following equation:

$$B_i\left(t + \Delta t_j\right) = B_i(t) + \left(\lambda_i^{in}(t) - \lambda_i^{out}(t)\right) \cdot \Delta t_j$$

It should be noted that parameters like λ_i^{in} and μ_{i} can be changed every $t_j = t_{j-1} + \Delta t_j$. It means that these parameters are time dependent and could be rewritten in the following way: $\lambda_i^{in}(t)$ and $\mu_{\mathrm{i}}(t)$.

The transporting element is used to model a movement of product. The function of this element is to delay the movement of product from one point to another.

The step Δt value could be fixed or variable. The type of step Δt, which should be used in the model depends on the type of system simulated and the processes inside the simulated systems. Most important in both cases is a scheduling of events. Figure 2.9 presents an example of event scheduling for continuous processes in a funnel.

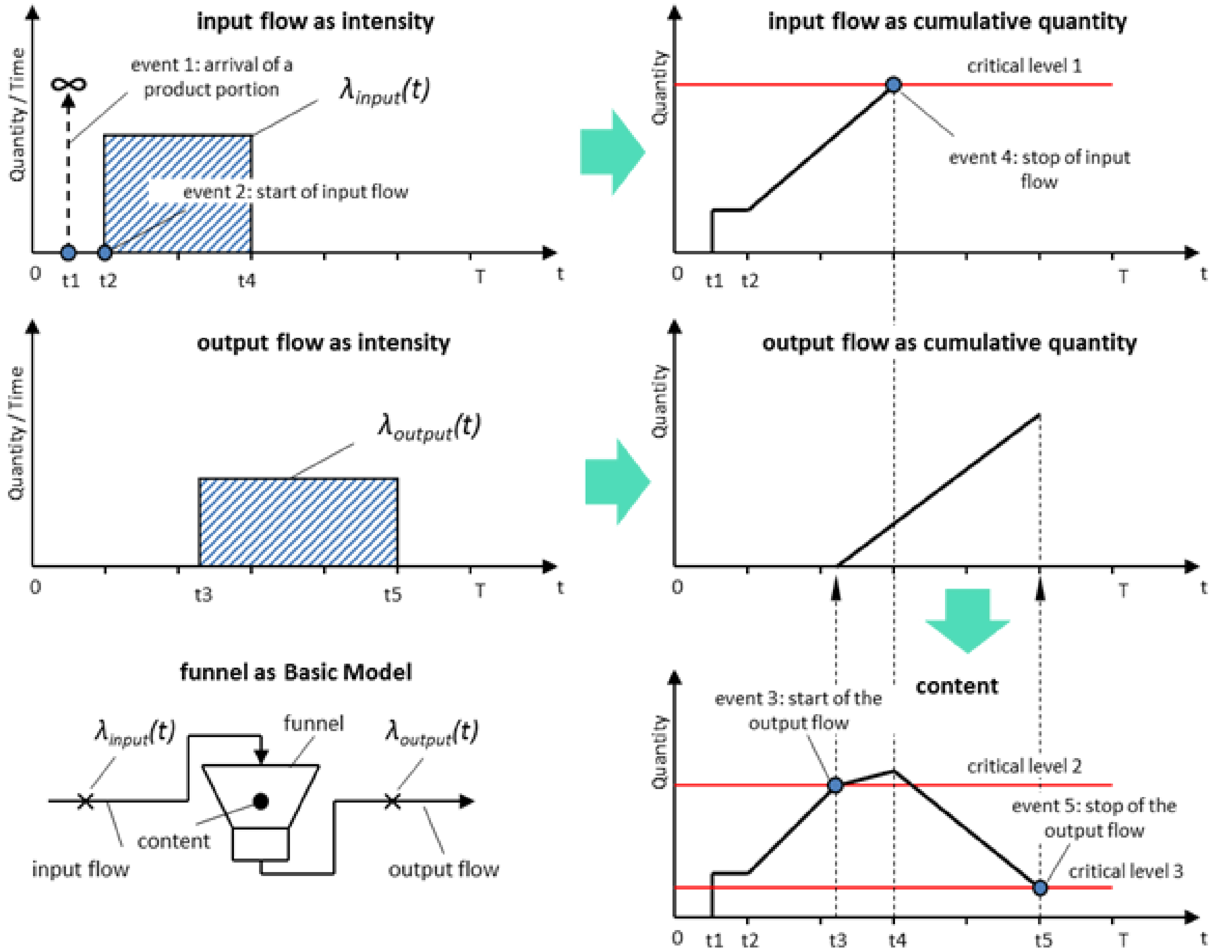


Figure 2.9. **Event scheduling for continuous processes [84]**

The first two events (event 1 and event 2) are assumed to be independent of the system's current state. Specific conditions are specified for all other events [84]:

- Event 3: The funnel's output flow can only start when its stock has reached the critical level 2.

- Event 4: The funnel's input flow is terminated when the entire quantity of products in the flow has reached the critical level 1.
- Event 5: The funnel's output flow is terminated when its stock has reached the critical level 3.

Times t3, t4 and t5 can be calculated precisely and entered into the simulators' chain of future events. Ultimately, only five events must be processed within the variable time interval Δt. In terms of performance, the advantage of this type of time advance over a fixed time step Δt is obvious.

As it has been mentioned above, two possible types of a time step could be selected for mesoscopic modeling. They are modelled by fixed time step and variable time step. The use of variable time step gives more flexibility to the model and, as a result, a more exact output result at the end, on the one hand; on the other hand, the fixed time step is simpler in realization. The Figure 2.10 shows the general difference in both cases: the application of fixed time step and the use of variable time step.

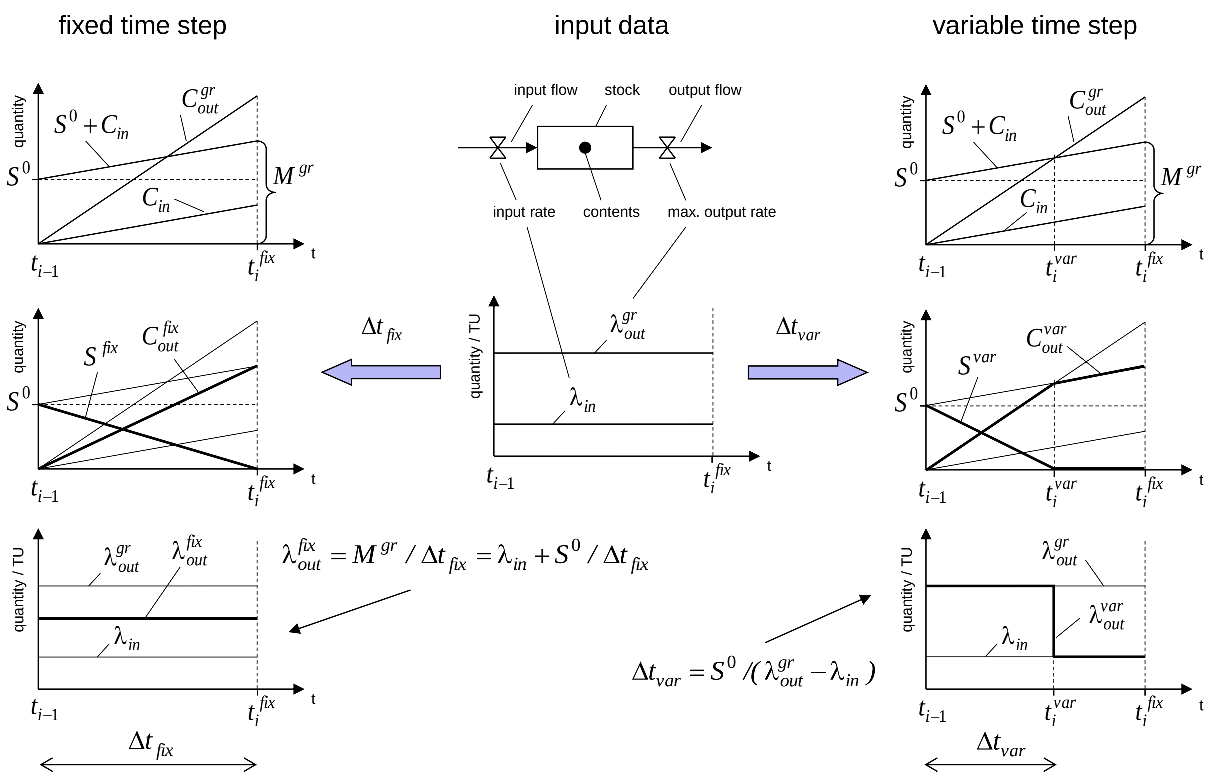


Figure 2.10. **Mesoscopic modelling by fixed and variable time step [84]**

The main feature of mesoscopic modelling is the point that all intensities of the flows $\lambda(t)$ in each step Δt of the model time t stay permanent. At the same time, a step value Δt could be given a fixed value or should be calculated for each new step. Further this situation will be described, based on simple stock (see Figure 2.10).

It is expected that the current time of the process is t_{i-1}. The following values are given:

- the level of stock $S(t_{i-1})$ in time t_{i-1}, equal to S^0;
- fixed time step Δt_{fix};
- permanent intensity of the input flow λ_{in};
- maximum value of the intensity of the output flow λ_{out}^{gr}.

In case of fixed time step, the next moment of time is determined using the following formula:

$$t_i^{fix} = t_{i-1} + \Delta t_{fix} \tag{2.1}$$

If output flow of stock is not taken into account, the level of stock for time t_i^{fix} will be equal to:

$$S\left(t_i^{fix}\right) = S^0 + C_{in}\left(t_i^{fix}\right) = S^0 + \lambda_{in}\Delta t_{fix} = M^{gr} \tag{2.2}$$

If the maximum value of intensity of the output flow λ_{out}^{gr} is so small, that the following condition is true

$$C_{out}^{gr}\left(t_i^{fix}\right) = \lambda_{out}^{gr}\Delta t_{fix} \leq M^{gr} \tag{2.3}$$

the real intensity of the output flow λ_{out}^{fix} during all the interval Δt_{fix} will be equal to λ_{out}^{gr}. The case when condition 4.3 is not true will be described below. This means, that level of stock could be decreased to 0, until the time moment t_i^{fix}. Intensity of the output flow λ_{out}^{fix} in this case cannot be λ_{out}^{gr} within the time interval Δt_{fix}, because it will lead to negative values of the level of stock within time t_i^{fix}.

Figure 2.10 shows the two cases as follows:

- next moment of time for the process t_i^{fix} is determined by fixed time step Δt_{fix};
- next time step of the process Δt_{var} is determined by moment of time t_i^{var}, for which an event will occur (the level of stock will reach 0).

For fixed step Δt_{fix}, any changes in flows within the time interval could not be reflected. It is supposed that at the end of time interval Δt_{fix} cumulative volume of output flow will be M^{gr}. This means that, fixed intensity λ_{out}^{fix} could be determined by the following formula:

$$\lambda_{out}^{fix}\Delta t_{fix} = M^{gr} \tag{2.4}$$

It follows from equations 2.3 and 2.4 that:

$$\lambda_{out}^{fix} = \frac{M^{gr}}{\Delta t_{fix}} = \frac{\left(S^0 + \lambda_{in}\Delta t_{fix}\right)}{\Delta t_{fix}} = \lambda_{in} + S^0/\Delta t_{fix} \tag{2.5}$$

Permanent intensity λ_{out}^{fix} also means that level of stock $S^{fix}(t)$ during interval Δt_{fix}, will be evenly (linearly) reduced to 0. The actual moment of time t_i^{var}, when stock will be empty cannot be determined in this case.

On the other hand, modelling with variable time step Δt_{var}, is based on preliminary calculations of moments of time t_i^{var}, then the determined conditions are true. In this example, these conditions are as follows:

$$S^0 + \lambda_{in}\Delta t_{var} = \lambda_{out}^{gr}\Delta t_{var} \tag{2.6}$$

From 2.6 it follows:

$$\Delta t_{var} = S^0/\left(\lambda_{out}^{gr} - \lambda_{in}\right) \tag{2.7}$$

and

$$\mathrm{t}_i^{var} = t_{i-1} + \Delta t_{var} \tag{2.8}$$

Thus, using modelling with variable time step, two time intervals (t_{i-1}, t_i^{var}) and $\left(t_i^{var}, t_i^{fix}\right)$ have been constructed. During the first interval real intensity λ_{out}^{var} is equal to maximum value of λ_{out}^{gr}, but during the second one, it equals to λ_{in}. The level of stock is reduced to 0 at the end of the first interval, and stays subsequently at this level within the whole interval $\left(t_i^{var}, t_i^{fix}\right)$.

This example shows the difference between modeling presentation with the fixed and the variable time step in mesoscopic simulation.

On Figure 2.11 an example of mesoscopic model is presented. Instead of stores, it presents the so-called multichannel funnels. Because of the parallel channels, batches of products in the same funnel could be divided at the same time. Funnel channels (see funnels stock1 and stock2 on Figure 2.11) have numbers, which are the numbers of parallel flows of products (see products Pr1 and Pr2 on Figure 2.11). The conformity between the product batches created and processed in the model and the numbers of parallel flows is given in the frame of conceptual model.

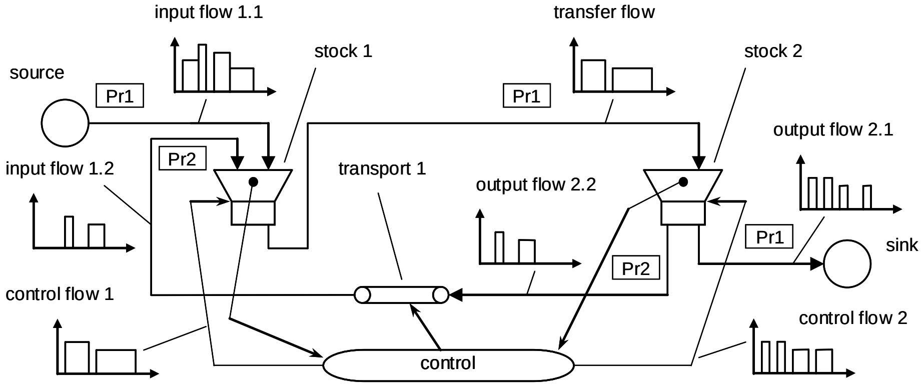
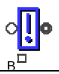
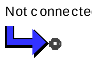
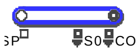
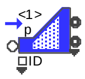
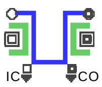
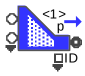
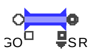
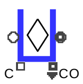
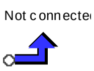


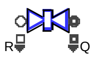
Figure 2.11. **Example of discrete rate model structure [83]**

The discrete rate (DR) which is based on proposed mesoscopic is realized in ExtendSim [85] software starting from version 7. It fully realizes the above-described concepts with some additional elements, which are required for a more flexible model development [86]. The blocks of the discrete rate library of ExtendSim software and their function are presented in Table 2.1.

Table 2.1.

Discrete rate library blocks [87]

Block	Block name	Description
	Bias	Prioritizes the flow going through it
	Catch Flow	This block catches flow sent by Throw Blocks or Diverge blocks
	Change unit	Changes the flow unit of measurement
	Convey Flow	Delays the movement of flow from one point to another
	Diverge	Distributes the input flow to two or more outputs
	Interchange	The Interchange block represents a tank, or holding area, where the flow can interact with items generated by discrete event blocks
	Merge	Merges flows from multiple inputs into one output
	Sensor	Reports the potential upstream supply rate and potential downstream demand rate
	Tank	Acts as a source, intermediate storage, or sink. As a residence-type block, the Tank has the capacity of holding definite amounts of flow as time advances
	Throw flow	This block sends the flow to be received by Catch Flow block

Block	Block name	Description
	Valve	Controls, monitors, and transfers the flow

2.6 Comparison of the simulation approaches

All the above-described simulation approaches could be divided into groups. The division could be based on detail level. Discrete event simulation and agent- based simulation are mainly used at microscopic level. System dynamics approach could be applied at macroscopic level. Finally, the discrete rate approach deals with mesoscopic level. The Figure 2.12 below shows the relations between levels from the point of view of input and output data.

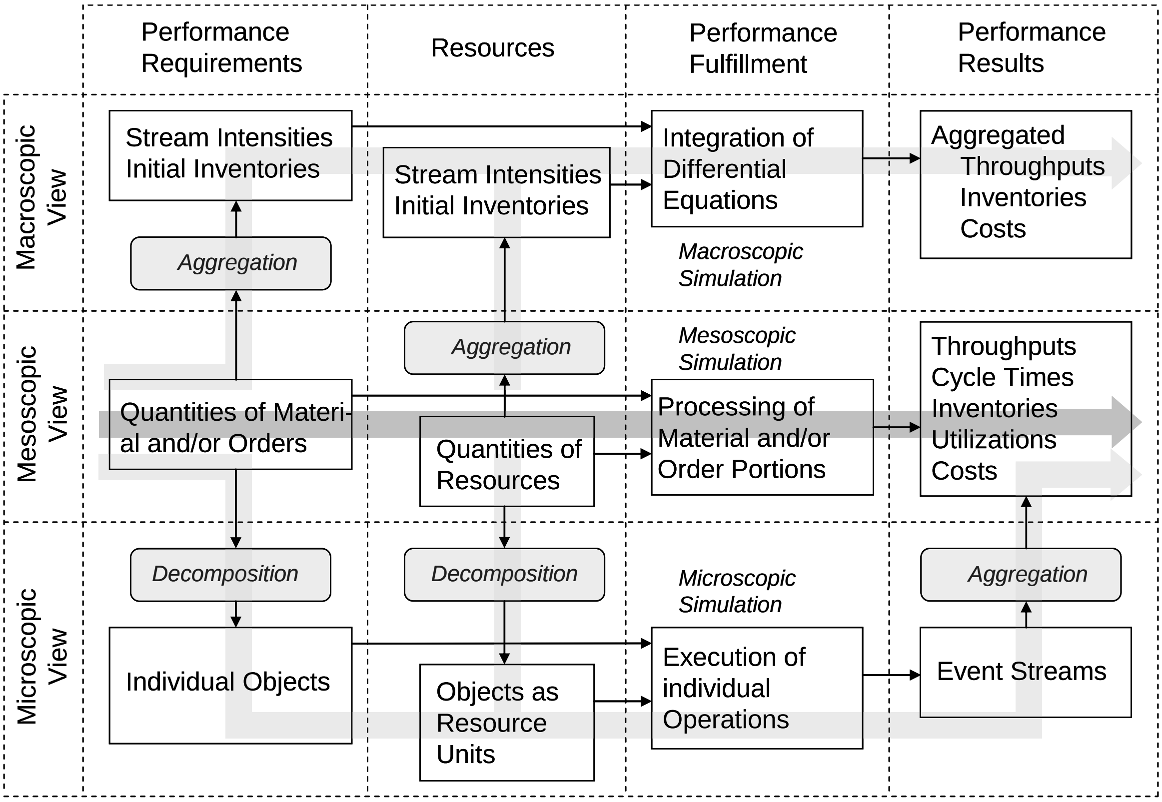


Figure 2.12. **Relationship between levels [84]**

It could be seen from Figure 2.12 that application of macroscopic approaches is often connected with such operation as data aggregation; that is why the final results are frequently not so exact. Application of microscopic approaches is connected with decomposition operation. Moreover, constructing and experimenting with the model is a time-consuming process. At the same time, simulation efforts are growing while abstraction level is lowering. This could be demonstrated by using Figure 2.13. It means that each group (micro, meso, and macro) have disadvantages in their application. Nevertheless, it should be noted that certain advantages of application of each group of approaches still exist.

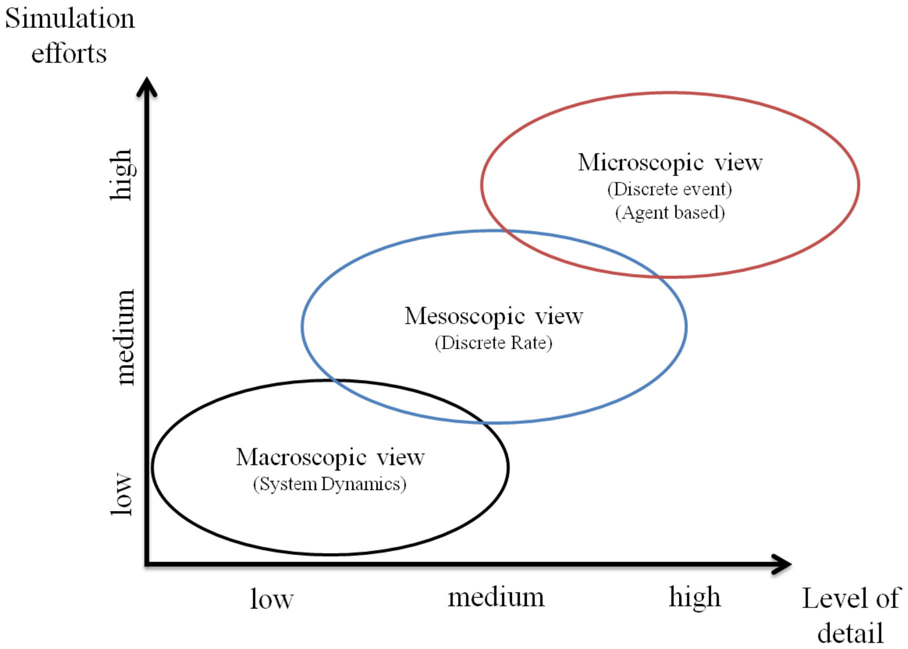


Figure 2.13. **Simulation efforts [84]**

The following table (see Table 2.2) could be constructed to show advantages and disadvantages of each group of approaches.

Table 2.2.

Advantages and disadvantages of micro, meso and macro

View	Advantages	Disadvantages
Microscopic	• ***Graphical representation*** of the model could be developed – all the modern simulation software allows us to create the graphical representation of the model. Such representation could be used for two purposes: for model validation and as a poster child of the systems functioning. Many problems and bottlenecks of the system could be detected quickly because of the graphical representation. • Microscopic simulation allows constructing models with a very ***high level of detail.*** This possibility allows one to build very detailed and precise models and do a very profound research of the systems.	• ***High requirements to resources*** (staff, computers, money, time etc.) – large resources should be involved during model creation. First of all, special software for simulation should be acquired. The second is a model construction; validation and experimentation are very time-consuming processes. The third is the high level of hardware requirements. • ***High level of developer's subjective*** system presentation, i.e. the models are developed by people and they could bring in their subjective understanding of the system. To minimize this effect, a validation should be done. • ***A set of runs*** for each experiment should be performed to get the result – because of a stochastic environment of the investigated systems a set of runs should be executed for each experiment. After that, the results should be aggregated and the data analyses should be done using statistical packages.

View	Advantages	Disadvantages
Mesoscopic	• ***Models could be created faster*** than models of microscopic levels • There is ***no need to do a lot of runs*** for an experiment • ***The result will be more precise*** than the results from macro level, because the system algorithm is modelled in high details. • **Simulation proper could be completed faster** as compared to the microscopic simulation; that is why it could be used at tactical and operational level of decision-making	• ***Theoretical background*** has not been implemented fully as yet. • There are ***only a few of dedicated software packages*** which introduce discrete rate approach. • The ***graphical possibilities*** are limited; results are mainly presented as graphs.
Macroscopic	• ***Complex systems*** – very complex systems could be modeled and both qualitative and quantitative factors could be taken into account • ***Graphs of process*** – there is no animation at this level, but still it is possible to obtain graphs of processes which help one to gain understanding of the entire system.	• ***Simulation representation*** – results are presented as the graphs of processes. Animation is not available at this level. Therefore, the simulation process is not as representative as the micro-level simulation. • ***Results*** – results of simulation should not be treated as exact values, because of the object aggregation and system simplification. • ***No flexibility*** – there no possibility of changing algorithm of the system during simulation time; therefore, sometimes it could cause problems and the system will not be similar to real-world system. • **Additional statistical methods**, like regression analysis and time series, should be used to find functional dependence between factors. • **A deep research of the modelled system** should be completed to find out dependencies between factors. This fact leads to the complexity of model realization.

A more profound comparison between different simulation approaches could result in the construction of Table 2.3 below. This Table 2.3 demonstrates a detailed comparison between all the approaches except hybrid simulation, because it is not the classical one and as it has been mentioned before, just uses a combination of classical simulation approaches.

Table 2.3.

Detailed comparison of the simulation approaches [88]

Factor	System Dynamics	Discrete event	Discrete rate	Agent- based
What is modelled	Values that flow through the model.	Distinct entities ("items" or "things").	Bulk flows of homogeneous stuff. Or, flows of other	Distinct active entities, called agents.

Factor	System Dynamics	Discrete event	Discrete rate	Agent- based
			distinct entities where sorting or separating is not necessary.	
What causes a change in state	A time change	An event	An event	An event and time change
Time steps	Interval between time steps is constant. Model recalculations are sequential and time-dependent.	Interval between events is dependent on when events occur. Model only recalculates when events occur.	Interval between events is dependent on the event occurrence time. Model only recalculates when events occur.	In some models, time step is a constant and could be defined as a very small value. In some models, events define the time intervals.
Characteristics of what is modelled	Track characteristics in a database or assuming the flow being homogeneous.	Using attributes, unique characteristics are assigned to items, which can then be tracked throughout the model.	Track characteristics in a database or assuming the flow as being homogeneous.	Using attributes, unique characteristics and behaviour are assigned to items
Ordering	FIFO	Items can move in FIFO, LIFO, Priority, time-delayed, or customized order.	FIFO	Could be given customized order if necessary
Routing	Values need to be explicitly routed by being turned off at one branch and turned on at the other (values can go to multiple places at the same time).	By default, items are automatically routed to the first available branch (items can only be in at one place at a time).	Flow is routed based on constraint rates and rules that are defined in the model (flow can be divided into multiple branches).	The routing is done based on agent behaviour
Statistical detail	General statistics about the system: amount, efficiency, etc.	In addition to general statistics, each item can be individually tracked: count, utilization, cycle time.	In addition to general statistics - effective rates, cumulative amount.	Information about each agent. Average characteristics with respect to all agents or groups of agents
Typical uses	Scientific (biology, chemistry, physics), engineering (electronics, control systems), finance and economics, System Dynamics.	Manufacturing, service industries, business operations, networks, systems engineering.	Manufacturing of powders, fluids, and high- speed, high-volume processes. Chemical processes, ATM transactions. Supply chains.	Social processes, biology, traffic flows

2.7 Summary

- The main goal of this chapter was to analyse the difference between classical simulation approaches existing at the moment, and to show the place of discrete rate simulation approach among them. To reach this goal, a concept of each simulation approach has been described in detail, focusing on the discrete rate approach.

- Furthermore, the comparison of the existing approaches from different points of view has been completed, showing the place of discrete rate approach. At the same time, the table of advantages and disadvantages of each group of approaches is presented to emphasize the fact that discrete rate approach is just an additional tool, which could be used if necessary. This approach does not substitute the existing approaches, but gives a larger degree of freedom to analysts in their researches.
- The main advantage of discrete rate approach is less intensive simulation efforts as compared to the microscopic groups of approaches, and more precise results as compared to the macroscopic approach at the same time.
- The analysis of typical use of approaches shows that discrete rate approach is mainly oriented to the logistics sphere dealing with material flows. This makes it necessary to use this approach for traffic flow simulation, as traffic flows also could be called a material flow. The sole example of application of mesoscopic approach in this direction could be found in [89]. Of course, an additional upgrade of discrete rate concepts is required to be able to simulate traffic flows as the traffic flow simulation has the specifics of its own.

3. DEVELOPMENT OF THE CONCEPT OF A MESOSCOPIC DISCRETE RATE TRAFFIC SIMULATION MODEL

As it has been mentioned above, mesoscopic concepts at the moment are mainly used for simulation of logistic systems. The goal of this chapter is to demonstrate a transformation of the approach described in Chapter 2 for traffic flow simulation. Traffic flow simulation is a very specific area of application of simulation, because of the difference between processes inherent in traffic and those inherent in logistic systems. In this chapter, two mathematical models have been observed: the first one for uncongested network, the second - for congested network. Most generally, the model for congested network could be treated, as it takes into account possible queues in transport systems.

3.1 Concept of DRTRM

The concept of DRTRM is based on simulation approach called discrete rate. Two different models could be proposed: one for uncongested network and the other for congested network. It should be noted that the last model could be also used for uncongested network too, but the development time in this case will be higher, while the output results are almost the same as those obtained from the model of uncongested network. Let's consider the first model for uncongested network.

3.1.1 Uncongested network

Let's consider a network with two signalized crossroads linked together by the road. The Figure 3.1 presents an example of such a network with notations used here to describe mathematical model for traffic flow simulation. As it could be seen from Figure 3.1, a number of input flows from different directions marked by $\lambda_1, \lambda_{2,}\lambda_3, \lambda_4$. exist in the transport system. At the same time, intensities of the output flows marked by $\beta_1, \beta_{2,}\beta_3, \beta_4$ exist. As compared to the logistics systems (traffic flows), a very complex interaction between flows could be observed in this case. This interaction is determined by splitting input flows by different directions; simultaneously, some control mechanism should be applied to prioritize flows. To show the flows' interaction, Figure 3.1 could be represented with a higher detail level by using Figure 3.2, which explains interaction between flows.

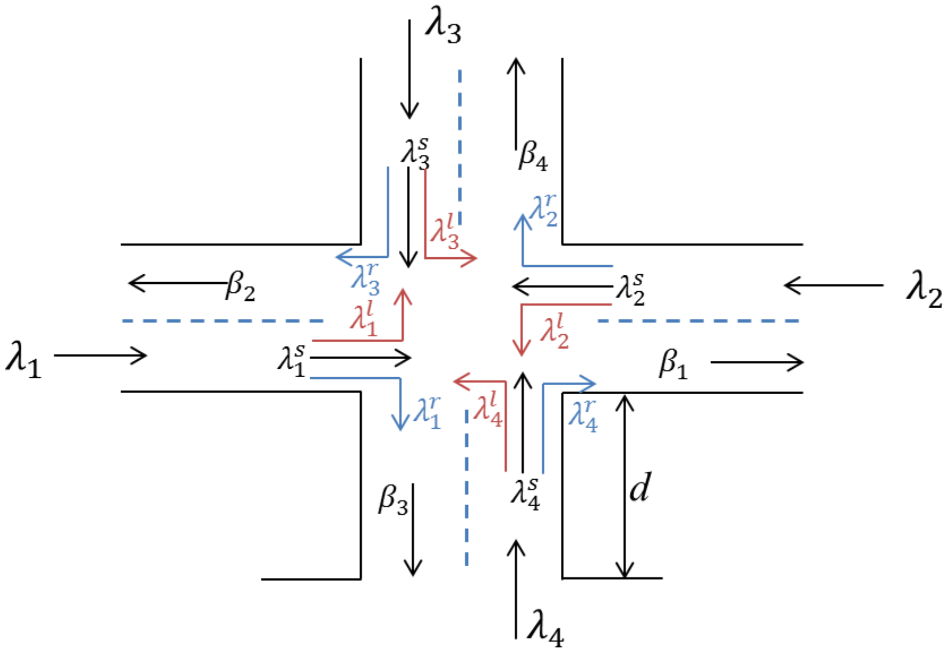


Figure 3.1. **Example of transport network**

The transport network shown on Figure 3.1 could be also presented according to the discrete rate notation with funnels and transporting element as shown on Figure 3.2. As it could be seen from Figure 3.2, the transport network is described using 5 funnels and 1 transporting element.

The next description of the notation used on Figure 3.2 could be given:

λ_i – input intensity of the flow from direction *i=1..5 (*used units: PCU per time unit);

β_i- output flow value from direction *i=1..5 (*used units: PCU per time unit);

$\lambda_i^l, \lambda_i^s, \lambda_i^r$ - intensity of the flow for the turns (*l-left; s-straight; r-right*) from direction *i=1..5* (used units: PCU per time unit);

b_i^l, b_i^s, b_i^r- queue length for the turns (*l-left; s-straight; r-right*) from direction *i=1..5* (used units: PCU);

$\mu_i^l, \mu_i^s, \mu_i^r$-the capacity for the turns (*l-left; s-straight; r-right*) from direction *i=1..5* (used units: PCU per time unit);

$\beta_i^l, \beta_i^s, \beta_i^r$ – output flow rate for the turns (*l-left; s-straight; r-right*) from direction *i=1..5* (used units: PCU per time unit);

B_5^{cap}- the maximum value of queue length for direction 5 (used units: PCU);

The following equations could be written, which define intensity of the flow by direction and by turns:

$$\begin{cases} \lambda_i^r(t) = \lambda_i(t)p_i^r \\ \lambda_i^s(t) = \lambda_i(t)p_i^s \\ \lambda_i^l(t) = \lambda_i(t)p_i^l \\ p_i^r + p_i^s + p_i^l = 1 \end{cases} \tag{3.1}$$

where

p_i^r, p_i^s, p_i^l – a probability of turns (*l-left; s-straight; r-right*) from direction *i=1..5;*

t – current time.

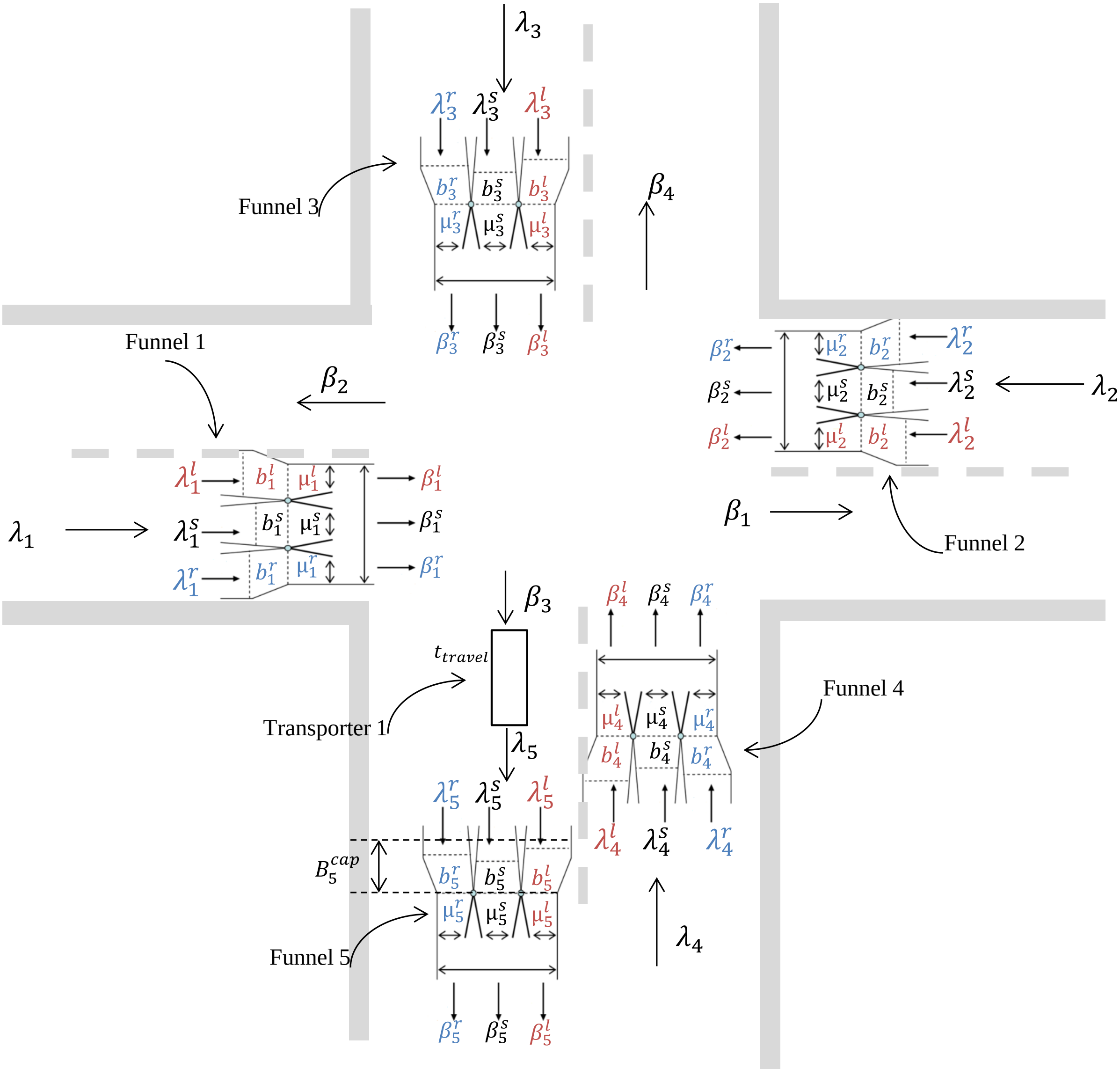


Figure 3.2. **Transport network in discrete rate notation**

The Equation 3.2 defines the output intensity for each direction calculated as a sum of intensities from each turn.

$$\begin{cases} \beta_1(t) = \beta_1^s(t) + \beta_4^r(t) + \beta_3^l(t) \\ \beta_2(t) = \beta_2^s(t) + \beta_3^r(t) + \beta_4^l(t) \\ \beta_3(t) = \beta_3^s(t) + \beta_1^r(t) + \beta_2^l(t) \\ \beta_4(t) = \beta_4^s(t) + \beta_1^r(t) + \beta_2^l(t) \\ \beta_5(t) = \beta_5^s(t) + \beta_5^r(t) + \beta_5^l(t) \end{cases} \tag{3.2}$$

The Equation 3.3 is used to determine throughput capacity for the turns *r-right* and *s-straight.* As it could be seen the calculation for turns *l-left* is not performed as left turns should interact with straight direction, and normally straight direction has priority.

$$\begin{cases} \mu_i^r(t) = f_i^r(\Delta t(t)) \\ \mu_i^s(t) = f_i^s(\Delta t(t)) \end{cases} \tag{3.3}$$

where

$\Delta t(t)$– time step in time *t* (in this demonstration - equal to the duration of green phase (t_{green} for simplification) is equal for all directions and all crossroads;

$f_i^n()$ – function (called passing function), which determines throughput capacity from direction *i=1..5* and turn $n \in (l, s, r)$.

The form of the above-mentioned function could be empirically found in the course of observations of real traffic; this task is solved in Chapter 5. In Equations 3.4 and 3.5 below, the output intensity for straight and right turns is calculated, taking into account the input intensity from turn, queue volume, and throughput capacity.

$$\beta_{i\in(1,2,4,5)}^s(t) = \begin{cases} 0,\ \lambda_i^s(t) = 0\ and\ b_i^s(t) = 0 \\ \lambda_i^s(t), \lambda_i^s(t) > 0\ and\ \lambda_i^s(t) \le \mu_i^s\ and\ b_i^s(t) = 0 \\ \mu_i^s, b_i^s(t) > 0 \end{cases} \tag{3.4}$$

$$\beta_{i\in(2,3,4,5)}^r(t) = \begin{cases} 0,\ \lambda_i^r(t) = 0\ and\ b_i^r(t) = 0 \\ \lambda_i^r(t), \lambda_i^r(t) > 0\ and\ \lambda_i^r(t) \le \mu_i^r\ and\ b_i^r(t) = 0 \\ \mu_i^r, b_i^r(t) > 0 \end{cases} \tag{3.5}$$

Based on calculated output intensity for straight and right turns, the throughput capacity for left turns could be calculated (see Equation 3.6). To calculate throughput capacity, it is necessary to define the time left for vehicles to complete the left turn. This time is calculated as a difference between t_{green} and the time used by straight flow to pass all vehicles. To calculate the time needed to pass the crossroad, the inverse of passing function is used. To simulate the possibility of crossroad passing during the yellow phase of traffic light, additional capacity h is applied.

$$\begin{cases} \mu_1^l = f_1^l\left(t_{green} - f_2^{s-1}\left(\beta_2^s(t)\right)\right) + h \\ \mu_2^l = f_2^l\left(t_{green} - f_1^{s-1}\left(\beta_1^s(t)\right)\right) + h \\ \mu_3^l = f_3^l\left(t_{green} - f_4^{s-1}\left(\beta_4^s(t)\right)\right) + h \\ \mu_5^l = f_5^l(t_{green}) + h \end{cases} \tag{3.6}$$

where

h – additional capacity, explained by crossing of the crossroad during yellow color of traffic light;

$f_i^{n-1}()$ – inverse passing function which determines the time required for crossroad passing.

As soon as throughput capacity for the left turns is defined, the output flow intensity for the left turns could be calculated by using Equation 3.7. As it was mentioned before, this calculation is performed taking into account the intensity of flow for turns, throughput capacity, and queue volume.

$$\beta_{i\in(1,3,5)}^l(t) = \begin{cases} 0,\ \lambda_i^l(t) = 0\ and\ b_i^l(t) = 0 \\ \lambda_i^l(t), \lambda_i^l(t) > 0\ and\ \lambda_i^l(t) \le \mu_i^l\ and\ b_i^l(t) = 0 \\ \mu_i^l, b_i^l(t) > 0 \end{cases} \tag{3.7}$$

According to the conceptual model (see Figure 3.2), we must take into account the available free capacity of the road between crossroads. It could be calculated using Equation 3.8, as the difference between the full capacity and the capacity already used.

$$\Delta b_5(t) = B_5^{cap} - b_5^r(t)\ - b_5^s(t)\ \ - b_5^l(t) \tag{3.8}$$

By calculating the available free capacity, the output flow could be calculated by using Equation 3.9 and 3.10. As β_1^r and β_3^s do not interact (because of traffic light phases), we should not care about the priority of calculating Equation 3.9 and 3.8.

$$\beta_1^r(t) =$$

$$= \begin{cases} 0,\ \lambda_1^r(t) = 0\ and\ b_1^r(t) = 0 \\ \lambda_1^r(t), \lambda_1^r(t) > 0\ and\ \lambda_1^r(t) \le \mu_1^r\ and\ b_1^r(t) = 0\ and\ \lambda_1^r(t) \le \Delta b_5(t) \\ \mu_1^r, b_1^r(t) = 0,\ \mu_1^r \le \Delta b_5 \\ \Delta b_5\ , b_1^r(t) = 0 \end{cases} \tag{3.9}$$

$$\beta_3^s(t) =$$

$$= \begin{cases} 0,\ \lambda_3^s(t) = 0\ and\ b_3^s(t) = 0 \\ \lambda_3^s(t),\ \lambda_3^s(t) > 0\ and\ \lambda_3^s(t) \le \mu_3^s\ and\ b_3^s(t) = 0\ and\ \lambda_3^s(t) \le \Delta b_5(t) \\ \mu_3^s, b_3^s(t) = 0, \mu_3^s \le \Delta b_5 \\ \Delta b_5\ , b_3^s(t) = 0 \end{cases} \tag{3.10}$$

But, it should be taken into account that the right β_1^r output flow has to interact with the left β_2^lflow. As priority is always given to the right turn, the intensity β_1^r has been calculated earlier (see Equation 3.9). That is why it is only now that we could calculate throughput capacity for the left turn from direction 4. The calculation could be performed by using Equation 3.11. This equation takes into account the time left to complete the passing after all vehicles have passed the crossroad from the right turn of direction 1, plus additional capacity$h$.

$$\mu_2^l = f_2^l\left(t_{green} - f_1^{r^{-1}}\left(\beta_1^r(t)\right)\right) + h \tag{3.11}$$

Equation 3.12 is used to complete the calculation of the output intensities, by calculating the last - β_2^l.

$$\beta_2^l(t) =$$

$$= \begin{cases} 0,\ \lambda_2^l(t) = 0 \ and\ b_2^l(t) = 0 \\ \lambda_2^l(t),\ \lambda_2^l(t) > 0 \ and\ \lambda_2^l(t) \le \mu_2^l \ and\ b_1^r(t) = 0 \ and\ \lambda_2^l(t) \le \Delta b_5(t) \\ \mu_2^l, b_2^l(t) = 0, \mu_2^l \le \Delta b_5 \\ \Delta b_5\ , b_2^l(t) = 0 \end{cases} \tag{3.12}$$

3.1.2 Congested network

The second model proposed for a congested network simulation is based in general on the previously described model, but it takes into account the length of the road between crossroads, queue growing, and travel time between crossroads. To make these options available, the model for uncongested network should be updated in the following way (see Figure 3.3).

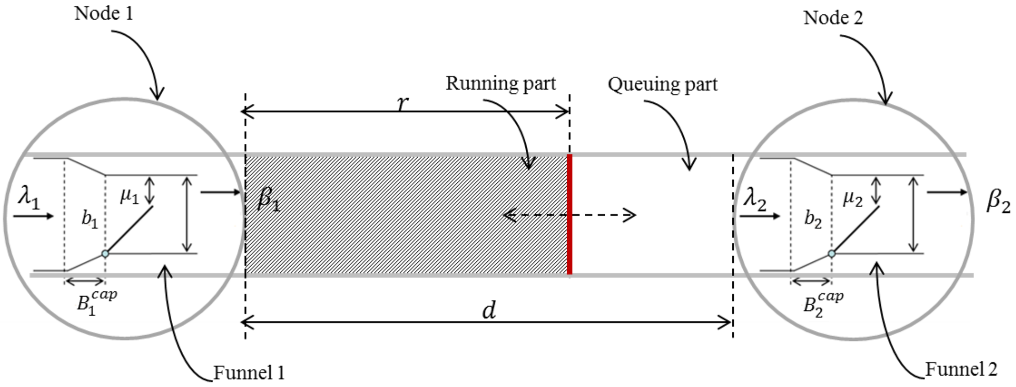


Figure 3.3. **Model for congested network**

On Figure 3.3 above, two crossroads connected by the road are presented. We could describe the notations as follows:

d – length of the road between crossroads (measured in PCU);

r – length of the running part of the road (measured in PCU);

d-r – length of the queuing part of the road (measured in PCU);

λ_1- intensity of income flow for funnel 1 (measured in PCU per time unit);

λ_2- intensity of income flow for funnel 2 (measured in PCU per time unit);

$B_1^{cap} = \infty$ - maximum length of the queue in the 1st funnel (measured in PCU);

$B_2^{cap} = d$ - maximum length of the queue in the 2nd funnel (measured in PCU);

b_1 – length of queue in the 1st funnel (measured in PCU);

b_2 – length of queue in the 2nd funnel (measured in PCU);

μ_1 – capacity for the 1st funnel (measured in PCU per time unit);

μ_2 – capacity for the 1st funnel (measured in PCU per time unit);

β_1- outflow intensity for the 1st funnel (measured in PCU per time unit);

β_2- outflow intensity for the 1st funnel (measured in PCU per time unit).

There is no need to describe the way how traffic flow is processed through the funnels, as it is described in the previous subchapter; that is why more attention should be paid to the mechanism, which allows modelling the congested network. The main problem of the previously described model was that it disregarded the fact that the queue of the vehicles can grow and occupy a part of the road. Therefore, the following mechanism is proposed in this mnodel. The road between crossroads (let's call it "link") is divided in two parts: running part and queuing part. The division of roads into two parts is dynamic, it means that during simulation the running part could be increased or decreased depending on the queuing part. The length of the queuing part is equal to the length of the queue of the 2nd funnel. This could be written in the following way:

$$r(t) = d - b_2(t) \tag{3.13}$$

The running part of the link is used for the movements. A movement in this model is defined by the travelling time, which could be calculated by using one of the volume delay functions (VDF), e.g., the following one [90]:

$$\begin{gathered} \begin{cases} t_{cur} = t_0(1 + a \cdot sat^b), if\ sat < sat_{crit} \\ t_{cur} = t_0(1 + a \cdot sat^b) + (q - q_{max})d, if\ sat \geq sat_{crit} \end{cases} \\ sat = \frac{q}{q_{max}c} \\ sat_{crit} = 1 \end{gathered} \tag{3.14}$$

where

t_0 – free flow travel time (measured in time units);

q – traffic quantity within the running part (measured in PCU);

$q_{max} = r$ – capacity (measured in PCU);

t_{cur}- travel time in loaded network (measured in time units);

a, b, c, d – parameters of the VDF.

The free flow travel time t_0 could be defined by the following well-known equation:

$$t_0 = \frac{r}{v}$$

where

v – speed on link;

3.3 Area of application

The area of application of mesoscopic model should be pointed out. For the description of the area of application, the tables provided in [31] will be used and updated according to the described concepts of mesoscopic DRTRM. According to [31], different layers of application areas could be used: analytical context, study area/geographic scope, facility type, management strategy and application. The different areas in each category could be listed here with a short description according to [31]:

- **Analytical context:**
  - **Planning:** This phase includes short- or long-term studies or other nation-wide, regional, or local transportation plans (e.g., master plans, congestion management plans, ITS strategic plant, etc.);
  - **Design:** This phase includes approved and funded projects being subject to analysis of alternatives or preliminary design to determine the best option for implementation. This phase also includes the analysis of roadway features needed to operate at a described level of service;
  - **Operations/Constructions:** These projects share many similar characteristics with design projects, but are performed to determine the best approach for optimizing or evaluating the existing systems.
- **Study area/Geographic scope:**
  - **Isolated location:** limited study, such as a single intersection or interchange;
  - **Segment**: linear or small-grid roadway network;

- **Corridor/Small network**: expanded study area that typically includes one major corridor with one or two parallel arterials and cross-streets connecting them - typically less than 520 square kilometres;
- **Region**: Citywide or countrywide study area involving all freeway corridors and major arterials, typically 520 square kilometres or larger.

- **Facility type:**
  - **Isolated intersections:** single crossing point, between two or more roadway facilities;
  - **Roundabouts:** non-signalised intersection with a circulatory roadway around a central island with all entering vehicles yielding to circulating traffic;
  - **Arterial:** a signalized street that preliminary serves through traffic that secondarily provides access to abutting properties;
  - **Highway:** a high-speed roadway connecting major areas or arterials, with little or no traffic signal interruption;
  - **Freeway:** a multilane, divided highway with a minimum of two lanes for the exclusive use of traffic in each direction and the full control of access without traffic interruption;
  - **HOV Lanes:** an exclusive highway or street lane for vehicles with a defined minimum number of occupants;
  - **HOV Bypass Lane:** exclusive on-ramp lane for vehicles with a defined minimum number of occupants;
  - **Ramp: a** short segment of roadway, connecting two roadway facilities;
  - **Auxibility lane:** additional lane of a freeway to connect an on-ramp and off-ramp;
  - **Reversible lane:** a roadway lane that changes directions during different hour of the day;
  - **Truck lane:** designated lane for commercial vehicles, but not for public transit vehicles;
  - **Bus lanes:** a highway or street lane reserved for public transport;
  - **Toll Plaza:** facility where payment transactions for the use of roadway take place;
  - **Light-Rail lane:** Electric-powered railway system operating single cars or short trains.

- **Management strategy and application:**
    - **Freeway management:** it controls, guides, and warms traffic in order to improve the flow of people and goods on limited-access facilities;
    - **Arterial intersections:** includes intersection or arterial operations, such as geometric improvements, parking adjustment, and signal timing for individual intersections.
    - **Arterial management:** applies state and local planning, capital and regulatory and management tools to enhance and/or preserve the transportation functions of the arterial roadway through the use of surveillance devices, advanced signal algorithms, and coordination;
    - **Incident management:** manages unexpected incidents so as to minimize their impact on transportation facilities;
    - **Emergency management: it** represents public safety and other agency systems supporting coordinated emergency response.
    - **Work zones:** uses traffic control devices and traveler information to maximize the availability of roadway facilities during construction or maintenance;
    - **Special events:** manages planned events to minimize the impact on transport facilities' availability for the public;
    - **Advanced public transportation systems (APTS):** applies advanced technologies to the operations, maintenance, customer information, planning and management functions for transit agencies;
    - **Advanced traveler information system (ATIS):** ranges from simply furnishing multimodal travelers with a fixed transit schedule information, including real-time traffic conditions and other information, to selecting support mode and route ;
    - **Electronic payment system:** allows travelers to pay transportation service by electronic means;
    - **Rail grade crossing monitors:** manages traffic at highway-rail intersections;
    - **Commercial vehicle operations (CVO):** performs advanced functions that support commercial vehicles operations;
    - **Advanced vehicle control and safety system (AVCSS):** includes vehicle systems such as vehicle and drivers' safety monitoring, intersection warming etc.;

- **Weather management:** includes automated collection of weather condition data and use of that data to provide road condition information;
- **Travel demand management (TDM):** TDM strategies are designed to maximize person throughput or influence the need for or time of a traveler. TDM includes employer trip reduction programs, construction of park-and-ride lots and some alternative work schedules.

The results of analysis of application area for the proposed mesoscopic model DRTRM could be observed in appendices 2 - 4. Moreover, these appendixes show not only the application area, but also a comparison between application areas for different traffic analysis tools. The analysis of application areas gives the following conclusion, which further could be used for defining possibilities of application of DRTRM:

- From the point of view of analytical context, DRTRM could be used without any limitation for design and operations/construction, but with an additional featuring for planning.
- Analysis of the study area/geographic scope gives the following results: DRTRM could be used without limitation for the following objects, such as: isolated locations, segments and corridors, and small networks. For the region, an additional development of the model is necessary.
- According to the types of facilities, there are a number of facilities which could be modeled using the proposed models, in particular: isolated intersections, Roundabouts, Arterial, Highway, Freeway, Ramps. All other facilities are subject to additional development of the model.
- According to the management strategies and application developed, the model could support without limitations: Freeway management, Arterial intersections, and Work zones. All others require additional development of the model or even cannot be realized by this tool at all.

3.4 Summary

- The goal of this chapter is to describe the main concept of the proposed discrete rate mesoscopic traffic simulation model. Two variants of the models are proposed: for uncongested and for congested network. It should be emphasized that the last model could also be used for uncongested network, but the development time will be higher in this case, while the output results will be almost the same as those obtained from the model of uncongested network. Both of the described models are based on discrete rate, and both of them assume that vehicles are grouped into the batches of different length dependent on income intensity and *Δt.* The *Δt* is not a constant value, but is defined according to the traffic light cycles; in case of absence of traffic lights in the simulated network, the value of *Δt* could be defined as a constant value by the user.

- Further advantages and disadvantages of the proposed model are defined. The disadvantages of this approach are as follows: animation is not available; lack of simulation software; model not proved by time; it is not capable of tracing individual vehicles; it is geographically and time- dependent; not able to reproduce different types of vehicles; detailed drivers' behaviour could not be reproduced. These disadvantages should be taken into account in process of model selection for research or for a project. At the same time, a number of advantages have been described: a lower development time; more accurate output data; no replication needed; a higher simulation speed; software being cheaper than professional tools. These advantages could be good reasons for using a proposed traffic simulation approach. In general, almost all of these points refer to the reduction of model development time, which sometimes is necessary in case of time shortage.
- For the proposed model, the area of application has been described in detail, taking into account analytical context, study area/geographical scope, facility type, management strategy and application. As the basis of this description, a FHWA-proposed alternative has been used. The reference to analytical context shows that the proposed model could be used in two cases: design and operations/constructions. In both of them, mesoscopic model could be used without limitations. Further reference to the study area and geographical scope has been done. The following items are covered by mesoscopic model: isolated locations; segments, the Corridor, and a small network. For the region, the proposed items partly cover functionality because of the complexity of the modelling object. The following facility types are covered by the model: isolated intersection; roundabouts; arterial; highway; freeway and ramp. Finally, mesoscopic models could be used in the following management strategies: freeway management; arterial intersection; management of arterials and work zones. For the item, special events could be used partly because of the high data aggregation level.

4. COMPARING THE DEVELOPED APPROACH WITH MICROSIMULATION. CASE-STUDIES

This chapter demonstrates an application of DRTRM, described in the previous chapter, for simulation of traffic flows in two transport nodes. The first simulation object is a crossroad with traffic light control, while the second one is two crossroads connected into a network. For demonstration purposes, both models are based on artificial input data. The main goal of this chapter is to show an example of realization of the model using DRTRM and to complete a validation of the constructed model.

4.1 Simulation of crossroad

The first example shows the use of the proposed DRTRM for simulation of traffic flows on one symmetric crossroad. Artificial data is used for demonstration purposes in this example.

4.1.1 Modelling and source data description

The modeling object is a symmetric crossroad with traffic light control as on Figure 4.1. Transport flows are created in sources 1 to 4. They can pass the crossroad by using the following directions: right (*r*), straight (s) and left (l). Two opposite sources have their green phase simultaneously. During this time, the two remaining sources have their red phase. That means that they do not interfere with each other. The modelling task is to estimate the dynamics of all 12 queues and the crossroad capacity utilization. Transport flows are given and traffic light phases will be determined.

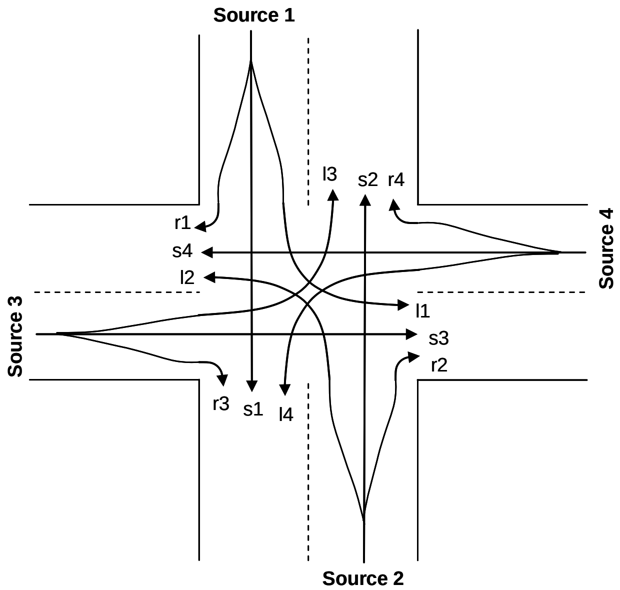


Figure 4.1. **Conceptual model of the crossroad**

To determine the number of vehicles (*q*), the mesoscopic approach uses the queue length of vehicles waiting at a crossroad. This concept is also used for describing the number of created vehicles and vehicles passing a crossroad. If the number of vehicles in the incoming flow is known, the queue length of vehicles can be easily estimated using empirical data. The flows in the model will be described in terms of meter/minutes (m/min).

The simulation in this case study is based on synthetic data. Random values are used for the description of all 12 incoming flows. Table 4.1 shows the numerical parameters of stationary incoming flows, which are used in the example presented here. The number of vehicles for each traffic light cycle is generated in compliance with the given distribution laws. In this example, the cycle duration is 50 seconds (20s +5s+20s+5s).

Table 4.1.

Parameters of the incoming vehicle flows

Incoming flow	Distribution law	Mean (m/min)	Left border (m/min)	Right border (m/min)	Crossroad passing
r1	uniform	20	15	25	0,6
s1	uniform	65	40	90	0,7
l1	uniform	10	5	15	0,6
r2	uniform	20	15	25	0,6
s2	uniform	65	40	90	0,8
l2	uniform	10	5	15	0,6
r3	uniform	15	10	20	0,6
s3	uniform	55	40	70	1,2
l3	uniform	10	5	15	0,6
r4	uniform	20	15	25	0,6
s4	uniform	65	40	90	0,8
l4	uniform	10	5	15	0,6

An empirical function (see Figure 4.2) is used to create a more realistic process of crossroad passing. This function has been estimated through direct observation of real crossroad passing processes. This function can be used for all directions. The function is used in the model as a direct and an inverse function. Figure 4.2 shows that within the first 25.5 seconds of a traffic light cycle a vehicle flow with the length $q1$=122 m can pass the crossroad and within the first 32.5 seconds a flow of $q2$=185 m.

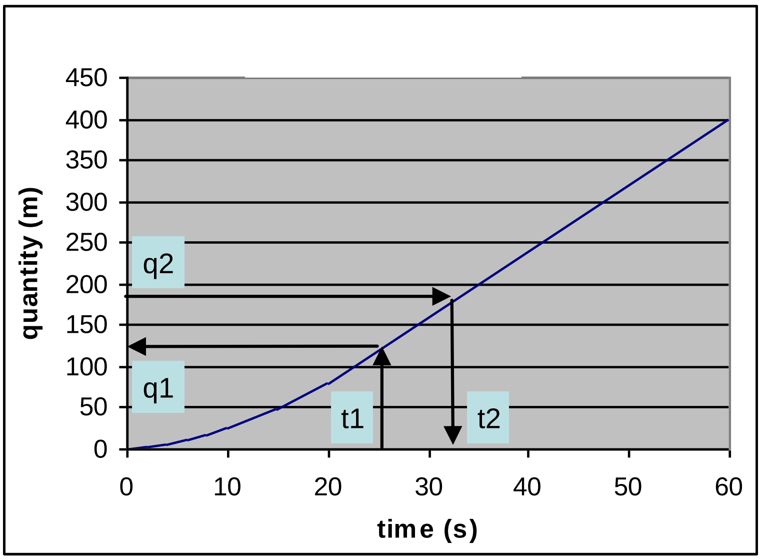


Figure 4.2. **Dynamics of vehicle flow during crossroad passing**

4.1.2 Principles of a mesoscopic model for a crossroad

The principal structure of the crossroad model is presented on Figure 4.3. The model consists of four autonomic components, because no direct relationships exist between the traffic flows of the four directions. Each component has three parallel channels. The channels generate, delay, and release traffic flows. The delay and the release are realized by a multichannel funnel described above. The content of each channel of the funnel is numerically equal to the length of the queue. The control component of the model (Flow Control) defines the quantity of vehicles, which can pass the crossroad within each traffic light cycle, for the different directions.

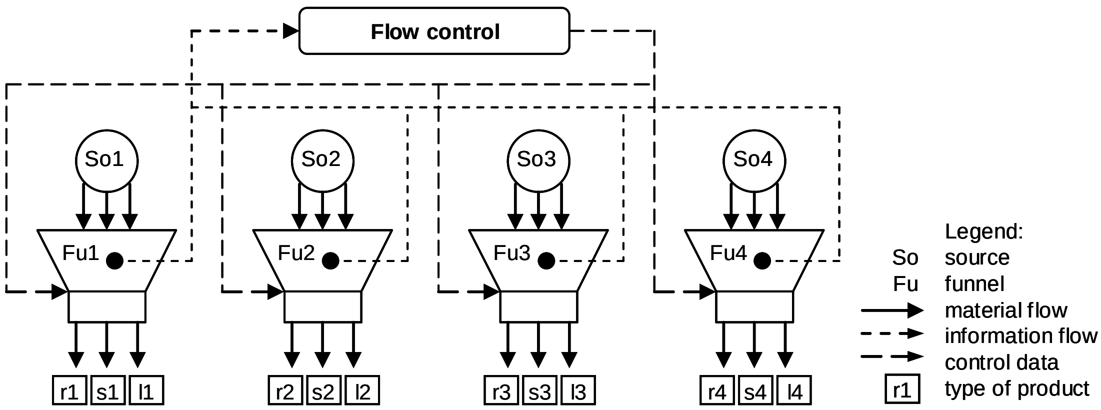


Figure 4.3. **Principal structure of the mesoscopic model**

The traffic light cycle length is used as the discrete time step Δt (in this example the cycle length is 50 seconds). In each step Δt, the data shown in Table 4.2 is calculated and updated. The table is an example of a standard realization of an internal data structure in a mesoscopic model. By itself, the model is realized in Microsoft Excel 2003 using Visual Basic for Application (VBA). Selection of a development tool depends on the possibility of using internal spread-sheets, internal programming languages, and internal visualization tools as graphs. The Table 4.2 is a main source of information in this model. The Table 4.2 processing algorithm is written in VBA (see Appendix 6) and called by user's command (pushing on button.). The general scheme of processing algorithm could be presented using Figure 4.4 and Figure 4.5. Figure 4.4 shows the general view of the algorithm. In the step 1, data initialization should be completed. This step is important to load input data for the model. Input data consists of simulation starting conditions (like existing queues) and definition of necessary parameters (like the number of cycles to simulate). The clearing of all working table is also completed in this step.

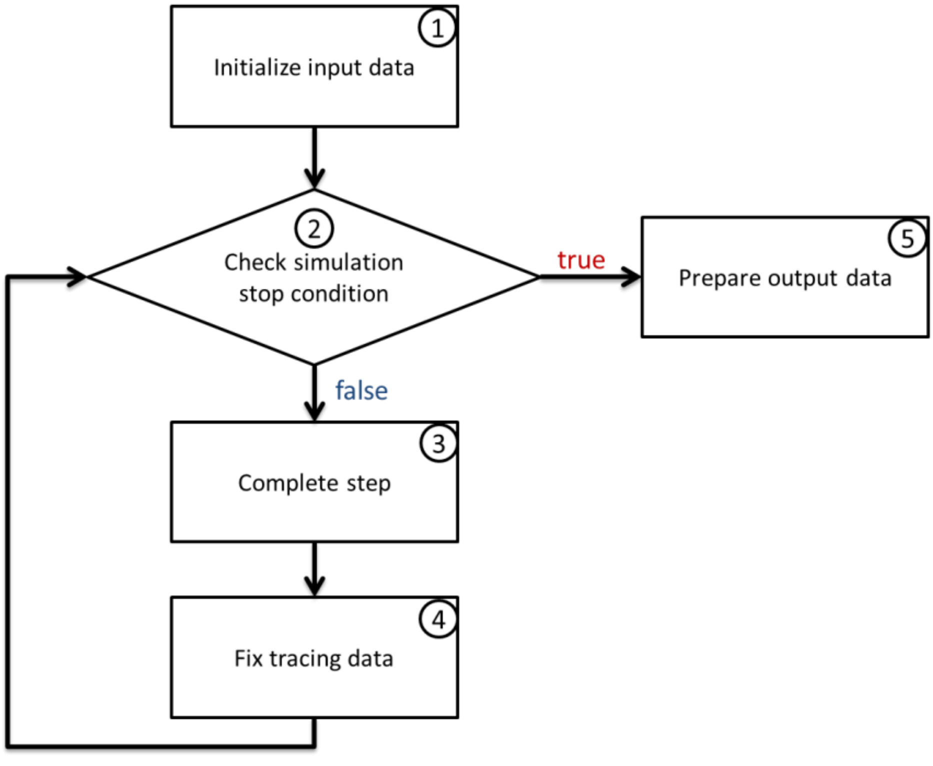


Figure 4.4. **General simulation procedure**

The step number 2 is important to check if it is necessary to stop the simulation. To comply with this decision, the number of simulated traffic light cycles is compared with that given by the user. If the stopping condition is true, the step number 5 is executed.

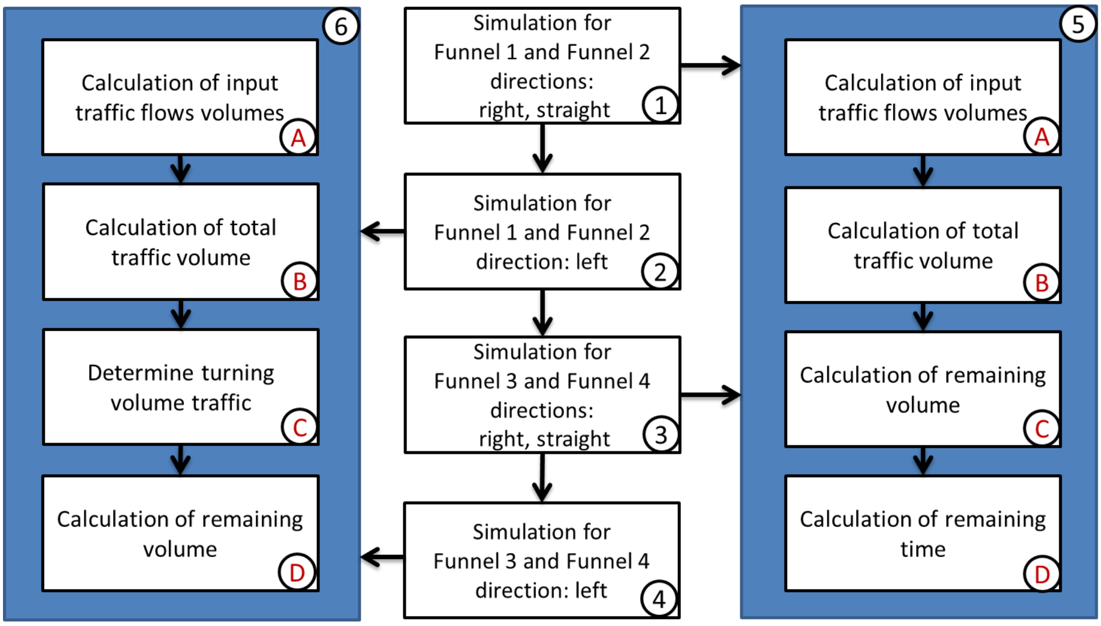


Figure 4.5. **Step simulation procedure**

The step number 5 prepares simulation output data and builds the graphs. If stop condition is false, the 3rd step is executed. The 2nd step does the main simulation and is described in detail on Figure 4.5 The 4th step fixes the data after each step and copies the model output data into the resulting table, which will be used subsequently by the procedure 5.

The detail description of stepwise procedure is presented on Figure 4.5. The procedure starts with step 1, which includes calculation for funnel 1 and funnel 2, but only for right and straight directions. In this step the sub-procedure (number) 5 should be executed. The sub-procedure execution involves the following steps: calculation of input traffic volume (5A) according to the law provided in Table 4.1; calculation of total traffic volume (5B) wishing to pass through (the current input traffic volume + the one remaining from previous cycle); calculation of the remaining volume (5C) for this cycle (calculation is based on using exemplary chart to determine the volume of traffic passing through) and calculation of the remaining time (5D) which could be used to complete the left turn.

Only then computation for the right and the straight direction is completed, and the procedure goes over to the 2nd step. The 2nd step involves the calculation for the left turn for funnel 1 and funnel 2. Here sub-procedure (number) 6 is also used. In particular, the sub procedure has the following steps: calculation of input volume (6A) for the left turn, according to the Table 4.2; calculation of total traffic volume (6B), which wishes to pass through (current input traffic volume + the one remaining from the previous cycle); determination of turning traffic volume (6C), which is based on calculation of the remaining time and the guaranteed passed flow and, finally, calculation of the remaining traffic volume (6D). The same computations as those described above are done with respect to the steps number 3 and 4, which use the same sub-procedures.

Further, let's describe the passing traffic flow to get a detailed understanding of what is happening in sub procedure 5 and 6.

The calculation of data for *r*- and *s*-type flows is trivial. The value of the "phase capacity" is determined with the exemplary chart (see Figure 4.6). If the accumulated length of the waiting flow and the incoming vehicles is smaller than the "phase capacity", the "duration of pass flow" can be calculated with the exemplary chart. Line 1 on Figure 4.6 shows the process of vehicles' entering during a traffic light cycle. Curve 2 presents the process of all vehicles passing the crossroad before the green phase ends. Then t_{left} begins the interval for the left turn. The data calculation for direction *l* (left turn) is done after the data calculation for straight-on flow *s*. If the condition $t_{left}=0$ applies during a cycle (curve 3 on Figure 6.4), it is assumed that the defined minimal flow of vehicles can complete the left turn. Such case is presented for flow *l2* in Table 4.2. It is assumed that a vehicle flow of 8 meters can pass the crossroad during yellow light.

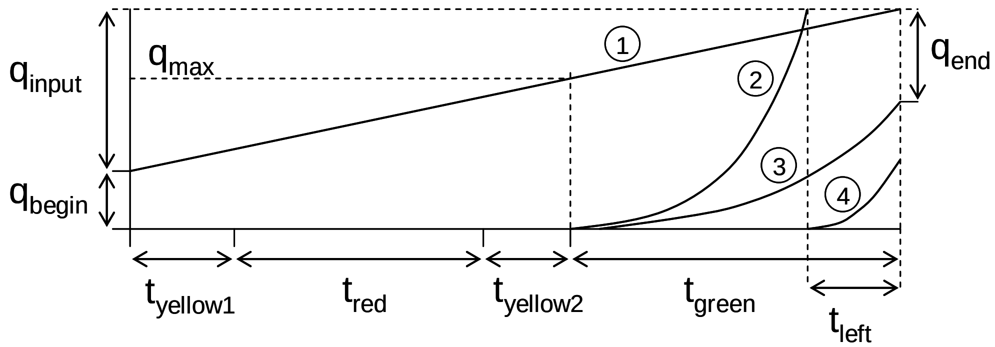


Figure 4.6. **Schema of process for one traffic light cycle**

For $t_{left}>0$ the "phase capacity" (curve 4 on Figure 4.6) can be calculated using the exemplary chart for direction *l*. Then the value of passed vehicles can be calculated, taking into account driving during yellow light. Table 4.2 shows that the flow *s2* has passed the crossroad in 17.31s, which results in $t_{left}=2.69$ s for flow *l2***.** Initial values for the queues of all *12* directions can be defined before the model execution begins.

Table 4.2.

Kernel of the crossroad mesoscopic model

Cycle number:	20 of 20			Time (s):	1000			
	Remaining from previous cycle (m)	Arrival per cycle (m)	Wish to drive through (m)	Duration of green phase (s)	Phase capacity (m)	Passed through volume (m)	Duration of pass flow (s)	Remaining on current cycle (m)
Funnel 1								
right (r1)	0.00	18.85	18.85	20	48.00	18.85	11.40	0.00
straight (s1)	33.78	39.70	73.48	20	56.00	56.00	20.00	17.48
left (l1)	7.20	8.16	15.36	20	48.00	11.02	2.69	4.33
total	40.98	66.71	107.69		152.00	85.87		21.81
Funnel 2								
right (r2)	0.00	20.37	20.37	20	48.00	20.37	11.94	0.00
straight (s2)	0.00	50.22	50.22	20	64.00	50.22	17.31	0.00
left (l2)	3.65	11.57	15.22	20	48.00	8.00	0.00	7.22
total	3.65	82.16	85.81		160.00	78.59		7.22
Funnel 3								
right (r3)	0.00	12.71	12.71	20	48.00	12.71	8.91	0.00
straight (s3)	0.00	40.64	40.64	20	96.00	40.64	11.93	0.00
left (l3)	0.00	5.40	5.40	20	48.00	5.40	2.59	0.00
total	0.00	58.75	58.75		192.00	58.75		0.00
Funnel 4								
right (r4)	0.00	17.06	17.06	20	48.00	17.06	10.75	0.00
straight (s4)	11.74	39.00	50.74	20	64.00	50.74	17.41	0.00
left (l4)	0.00	12.10	12.10	20	48.00	12.10	8.07	0.00
total	11.74	68.16	79.89		160.00	79.89		0.00

Model execution can be done in two modes: full execution and step-by-step execution, which allows one to perform monitoring of data changes in every traffic light cycle. The Table 4.2 is a result of full execution.

4.1.3 Data output and interpretation of simulation results

Necessary data are copied from Table 4.2 into the process trace spreadsheet (step 4 in general simulation procedure) during mesoscopic model execution. Diagrams of incoming flows (column “arrivals per cycle”) and outgoing flows (column “passed-through volume”) can be presented in differential and integral (cumulative) forms for all components of the model. The trace spreadsheet also contains the funnel content data (column “remaining on current cycle”). Table 4.3 presents the outcomes of modeling 20 cycles. The frames “input (sum)” and “output (sum)” show the total length values of funnel entrance and funnel exit. The data within the frame “queue (maximum)” present the maximal lengths of the queues during model execution. The queue lengths are changing during a traffic light cycle. The column “remaining on current cycle” in Table 4.2 does not present the maximal values of the queues but the minimum values because the values are written in the trace file at the end of every green phase.

Table 4.3.

Output data of the mesoscopic model of a crossroad

Cycle number:	20 of 20		Time (s):	1000							
Funnel 1											
input (sum)				output (sum)				queue (maximum)			
right	straight	left	total	right	straight	left	total	right	straight	left	total
330.9	1115.9	187.5	1634.3	335.9	1108.4	188.2	1632.5	12.6	69.3	18.1	96.3
Funnel 2											
input (sum)				output (sum)				queue (maximum)			
right	straight	left	total	right	straight	left	total	right	straight	left	total
352.4	1117.0	161.6	1630.9	357.4	1127.0	159.3	1643.7	15.4	50.4	15.2	72.1
Funnel 3											
input (sum)				output (sum)				queue (maximum)			
right	straight	left	total	right	straight	left	total	right	straight	left	total
275.1	886.8	159.9	1321.8	280.1	896.8	164.9	1341.8	10.9	34.5	12.8	54.9
Funnel 4											
input (sum)				output (sum)				queue (maximum)			
right	straight	left	total	right	straight	left	total	right	straight	left	total
345.4	1173.1	186.2	1704.8	350.4	1183.1	191.2	1724.8	16.8	54.6	15.1	86.4

The method used for estimating the maximal queue length for each traffic light q_{max} can be explained by using Figure 4.6. It is assumed that the number of incoming vehicles grows linearly (straight line 1). It is also assumed that the queue length does not grow anymore when the green phase starts. This assumption can be verified through real process surveys. If the queue end responses to the beginning of the queue motion during a green phase start, the value of q_{max} approaches to $q_{begin}+ q_{input}$ (see Figure 4.6). Figure 4.7 presents the value of q_{max} for 20 cycles of traffic light. The incoming queues are aggregated for each direction. Table 4.3 presents the maximum value of a queue in the column “total”.

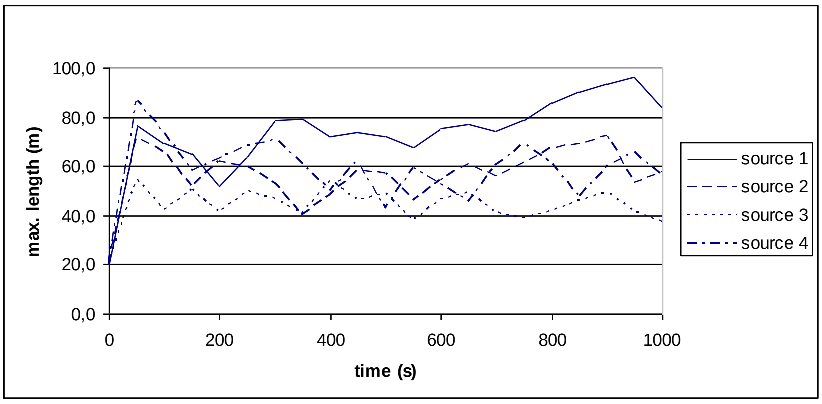


Figure 4.7. **Dynamics of queues of incoming flows**

The source data for modelling (see Table 4.1) has been chosen in such a way, that flow *s1* is slower (coefficient 0.7) then the other straight-on flows. Figure 4.7 shows that the queue for source 1 has a growing tendency. The queue dynamics for source 2 and 4 are similar since the parameters of all the flows are the same. The queue for source 3 is the shortest queue since flow *s3* has the smallest flow intensity (55 m/min on average) and the biggest passing speed (coefficient value is 1.2).

4.1.4 Summary

- This part of the chapter demonstrates a case study of application of discrete rate approach for traffic flow simulation in the signalized crossroad. For demonstration purposes, the synthetic input data has been used for the model.
- The model has been coded using Microsoft Excel possibilities, as it provides ready spreadsheets, programming language, charts and computing options. The coded algorithm in this chapter has been described.
- The main tasks are: to develop model according to the discrete rate approach principle, estimate the dynamics of queues, and evaluate intersection capacity utilization. All of these tasks have been solved successfully. Queues dynamics is presented in both views as a trace table of the model and as a chart for graphical representation of the obtained results. Intersection capacity utilization has been computed based on the model output data and is 51%. It should be noted that this value shows the average utilization of the crossroad. At the same time, capacity utilization could be calculated by direction and by funnel. In real situation it is also important to provide both of the above-mentioned values to do a profound analysis of the research object.
- Another task which will be solved in the next model is - to make a validation of the developed discrete rate.

4.2 Simulation model of the two connected crossroads

This case study shows the possibility of DRTRM in modeling two crossroads connected and controlled by traffic light. As the previous example, this one is also based on artificial data for demonstration purposes.

4.2.1 Modelling and source data description

The modelling object is a fragment of a transport network. The conceptual model is presented on Figure 4.8. The fragment consists of two symmetric, traffic-light signalized crossroads. The crossroads are connected with the road, which is a part of the model. The flow of the vehicles enters the network from 6 zones, which are enumerated by numbers 1, 2, 3, 5, 6 and 8. Each income flow is divided between three moving directions: right (*r*), straight (*s*) and left (*l*). The geometry of the crossroads is constructed in such a way, that vehicles entering the network from one zone and belonging to one direction *r*, *s* and *l*, can reach and pass the crossroad independently from other directions. Only the vehicles turning left (flow *l*) depend on *s* duration of the flow which passes crossroad straight in the counter lane, during green phase of traffic light. Because of the modelled object's topology, the total entering flow of the left crossroad (see Figure 4.8), which enters from zone 4, is equal to sum of flows *r5*, *s8*, and *l6*. In the same way, entering flow for the right crossroad is calculated as sums of flows *r2*, *s3*, and *l1*. The modelling task is to estimate the dynamics of all 24 queues, calculate the crossroads capacity utilization, make experiment with the model and validate a developed model with the microscopic model. The data used in this example is synthetic. Transport flows are given and traffic lights phases will be determined.

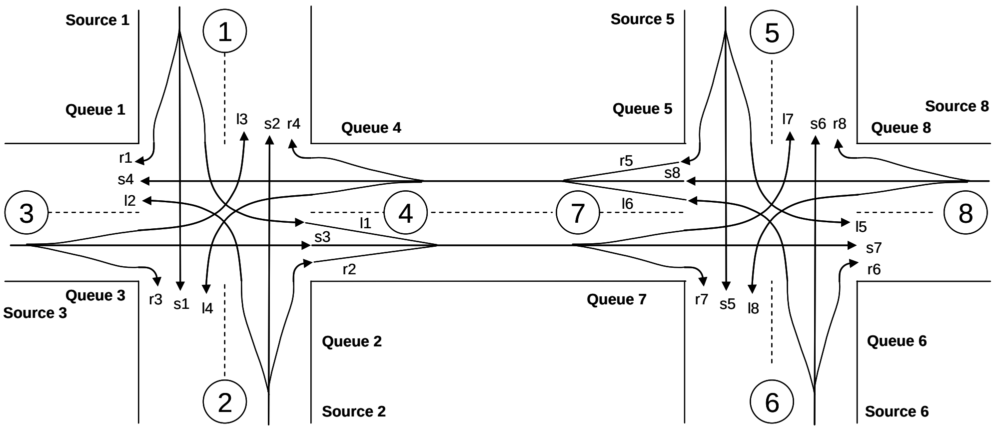


Figure 4.8. **Conceptual model of the transport network fragment**

The mesoscopic approach uses the queue length of vehicles waiting at a crossroad to determine the quantity of vehicles (*q*). This concept is also used for describing the number of created vehicles and vehicles passing a crossroad. If the number of vehicles in the incoming flow is known - for example, 10, then the queue length of vehicles can be easily estimated using empirical data. The flows in the model will be described in terms of meter/minutes (m/min).

Table 4.4 shows the numerical parameters of stationary incoming flows, which are used in the example presented here. The parameters for all 6 stationary flows are the same. The number of vehicles for each traffic light cycle is generated in compliance with the given distribution laws, taking into account cycle duration. In this example, the cycle duration for the first (left) crossroad is 60 seconds (25s+5s+25s+5s). For the second (right) crossroad, two variants of cycles are implemented for the research. They are: the first variant is 70 seconds (30s+5s+30s+5s), the second one is 90 seconds (40s+5s+40s+5s) (see Figure 4.9).

1st Crossroad (left)					Cycle time
	25 s	5 s	25 s	5 s	60 s
2nd Crossroad (right)					Cycle time
1st variant	30 s	5 s	30 s	5 s	70 s
2nd Crossroad (right)					Cycle time
2**st variant**	40 s	5 s	40 s	5 s	90 s

Figure 4.9. **Traffic lights parameters**

During the flows, generating distribution between directions (*r, s, l*) is done according to the proportion *r/s/l=0,25/0,6/0,15*.

Table 4.4.

Parameters of the incoming vehicle flows

Incoming flow	Distribution law	Flow intensity mean value (m/min)	Crossroad passing
r	Uniform	20	0,6
s	Uniform	65	0,8
l	Uniform	10	0,6

The model takes into account the length of the road, connecting zones 4 and 7. So the maximal total amount of the vehicles for directions 7→4 (Queue 4) or 4→7 (Queue 7) should be defined. In this example, the value 130m is used as the highest-level limit for both directions. The queue length for internal flows (see Figure 4.8) is not limited.

To maintain a high plausibility of the crossroad-passing picture, the empirical function (see Figure 4.10) is used, which could be estimated during direct observation of the passing process, with respect to each direction of crossroad passing. It is assumed that the function is preserved for all directions, and its numerical values could be obtained based on exemplary chart, by way of multiplying the value of function by the coefficient presented in Table 4.4 in the column "Crossroad passing". The function is used in the model as a direct and as an inverse function. Figure 4.10 shows that during the first *t1*=25,5s of a traffic light cycle a vehicle flow with the length *q1*=122m can pass the crossroad and the flow with the length *q2*=185m will need a *t2*=32,5s to pass the crossroad.

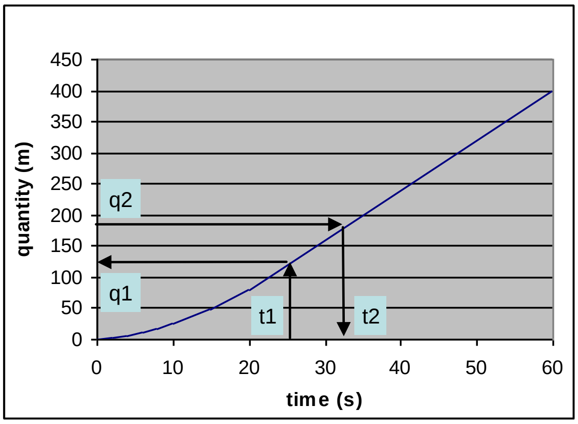


Figure 4.10. **Dynamics of vehicle flow during crossroad passing**

4.2.2 Principles of a mesoscopic model for two crossroads

The principal structure of two crossroads model is presented on Figure 4.11. The model consists of eight fragments. Links between them are defined according to vehicle flows (see Figure 4.8). Each fragment includes three parallel channels, which generate, delay, and pass flows of vehicles through the crossroad. The last two functionalities are realized using "multichannel funnel", which has been described previously. The content of each channel of the funnel is numerically equal to the length of the queue. The control component of the model (Flow Control) defines the quantity of vehicles, which can pass the crossroad within each traffic light cycle for the different directions.

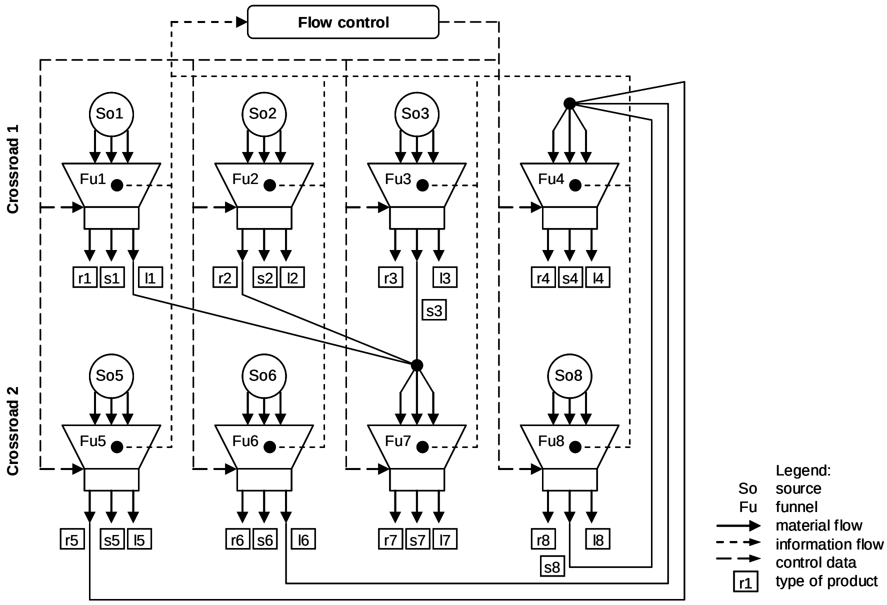


Figure 4.11. **Principal structure of the mesoscopic model for two crossroads**

The kernel of the mesoscopic model of one or several crossroads is presented in the Table 7.2. The event-planning algorithm determines the next pair of funnels, for which the green phase is finishing. The model time is “jumping” to the moment then this event will happen. For these funnels, new values of variables listed in table heads are calculated. Calculation is performed from left to right. The data calculation for direction *l* (left turn) is done after the data calculation for straight-on flow s is performed. After processing, the event-planning algorithm chooses the next pair of funnels, for which the green phase is finishing. For the event-planning and processing algorithm, the variable *t* is defined for each crossroad: variable *t1* for the event time of the first (left) crossroad, and variable *t2* for the event time of second (right) crossroad. We define the time interval Δt as the corresponding duration of half-cycle of the traffic light “yellow + green”. For example, for crossroad the first value is defined $\Delta t1$=30s, but for the second modelling has been done with two variants $\Delta t2$=35s and $\Delta t2$=45s.

Table 4.5.

Kernel of the two crossroads mesoscopic model

	Remaining from previous cycle (m)	Arrival per cycle (m)	Wish to drive through (m)	Duration of green phase (s)	Phase capacity (m)	Passed through volume (m)	Duration of pass flow (s)	Remaining on current cycle (m)
Crossroad 1			**Cycle number: 16**			**Time (s): 990**		
Funnel 1								
right (r1)	0,00	17,42	17,42	25	72,00	17,42	10,88	0,00
straight (s1)	0,00	44,09	44,09	25	96,00	44,09	16,11	0,00
left (l1)	31,52	7,76	39,28	25	72,00	17,10	7,05	22,18
total	31,52	69,26	100,78		240,00	78,60		22,18
Funnel 2								
right (r2)	0,00	17,21	17,21	25	72,00	17,21	10,80	0,00
straight (s2)	0,00	53,49	53,49	25	96,00	53,49	17,95	0,00
left (l2)	4,23	7,75	11,98	25	72,00	11,98	8,89	0,00
total	4,23	78,44	82,67		240,00	82,67		0,00
Funnel 3								
right (r3)	0,00	16,22	16,22	25	72,00	16,22	10,44	0,00
straight (s3)	30,85	56,37	87,22	25	96,00	87,18	23,62	0,04
left (l3)	0,00	9,45	9,45	25	72,00	9,45	3,64	0,00
total	30,85	82,04	112,89		240,00	112,85		0,04
Funnel 4								
right (r4)	0,00	30,28	30,28	25	72,00	30,28	15,39	0,00
straight (s4)	0,00	72,68	72,68	25	96,00	72,68	21,36	0,00
left (l4)	8,87	18,17	27,04	25	72,00	9,65	1,38	17,38
total	8,87	121,13	130,00		240,00	112,62		17,38
Crossroad 2			**Cycle number: 14**			**Time (s): 1015**		
Funnel 5								
right (r5)	18,86	26,11	44,97	30	96,00	26,18	14,05	18,79
straight (s5)	0,00	84,75	84,75	30	128,00	84,75	23,24	0,00
left (l5)	2,36	13,91	16,27	30	96,00	16,27	11,37	0,00
total	21,22	124,78	146,00		320,00	127,21		18,79
Funnel 6								
right (r6)	0,00	19,39	19,39	30	96,00	19,39	11,59	0,00
straight (s6)	0,00	56,99	56,99	30	128,00	56,99	18,63	0,00
left (l6)	29,17	9,32	38,50	30	96,00	0,00	6,76	38,50
total	29,17	85,70	114,87		320,00	76,38		38,50
Funnel 7								
right (r7)	0,00	28,97	28,97	30	96,00	28,97	15,04	0,00
straight (s7)	0,00	69,52	69,52	30	128,00	69,52	20,86	0,00
left (l7)	14,14	17,38	31,52	30	96,00	16,09	6,49	15,43
total	14,14	115,86	130,00		320,00	114,57		15,43
Funnel 8								
right (r8)	0,00	27,11	27,11	30	96,00	27,11	14,39	0,00
straight (s8)	0,00	86,43	86,43	30	128,00	86,43	23,51	0,00
left (l8)	0,00	12,44	12,44	30	96,00	12,44	9,14	0,00
total	0,00	125,98	125,98		320,00	125,98		0,00

The calculation algorithms of variables presented in Table 4.5 for flows of *r*, *s* and *l* types are described in detail in the previous chapter. Here it should be mentioned that the defined minimal flow of vehicles is assumed to be capable of completing the left turn, even if the passing flow exists during all the green phase of the traffic light. Before model execution, it is possible to establish an initial condition of the queues. The value of the maximal modelling time is then defined. The model is developed using Microsoft Excel 2003 [91].The model execution could be done entirely or on a stepwise basis, and it will give one a possibility to control value of the variables in each step Δt. Moreover, when the full mode is available, the modelling result will be obtained.

4.2.3 Data output and interpretation of simulation results

Necessary data are copied from Table 4.5 into the process trace spreadsheet during mesoscopic model execution. Diagrams of incoming flows (column "arrival per cycle") and outgoing flows (column "passed-through volume") can be presented in differential and integral (cumulative) forms for all components of the model. The trace spreadsheet also contains the contents of the funnels (column "remaining on current cycle").

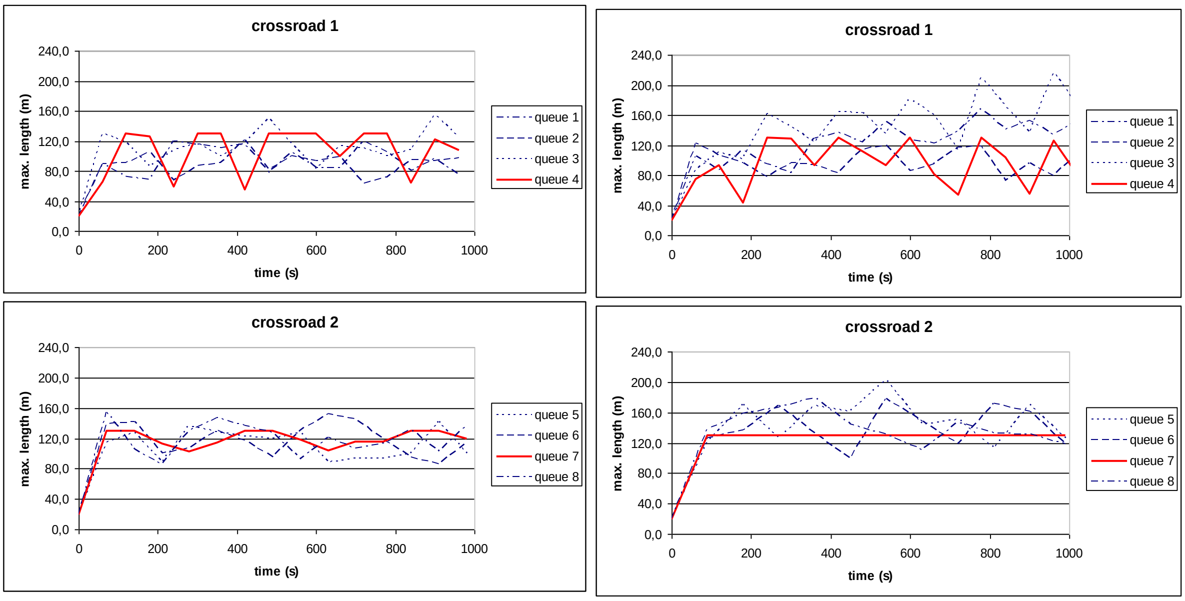
Table 4.6.

Output data of the mesoscopic model of a crossroad

Crossroad 1	**Cycle number: 16**			**Time (s): 990**							
Funnel 1											
input (sum)				output (sum)				queue (maximum)			
right	straight	left	total	right	straight	left	total	right	straight	left	total
329,5	997,5	163,5	1490,4	334,5	1007,5	168,5	1510,4	24,6	89,2	15,0	119,1
Funnel 2											
input (sum)				output (sum)				queue (maximum)			
right	straight	left	total	right	straight	left	total	right	straight	left	total
308,9	1028,1	164,1	1501,1	313,9	1038,1	169,1	1521,1	22,8	86,0	13,1	120,6
Funnel 3											
input (sum)				output (sum)				queue (maximum)			
right	straight	left	total	right	straight	left	total	right	straight	left	total
309,0	1125,9	177,1	1612,0	314,0	1135,9	182,1	1632,0	25,0	126,5	18,2	155,3
Funnel 4											
input (sum)				output (sum)				queue (maximum)			
right	straight	left	total	right	straight	left	total	right	straight	left	total
385,1	924,2	231,1	1540,4	390,1	934,2	219,9	1544,3	32,5	78,0	39,7	130,0
Crossroad 2	**Cycle number: 14**			**Time (s): 1015**							
Funnel 5											
input (sum)				output (sum)				queue (maximum)			
right	straight	left	total	right	straight	left	total	right	straight	left	total
341,3	1027,9	173,3	1542,5	346,3	1037,9	178,3	1562,5	31,6	101,2	19,8	141,1
Funnel 6											
input (sum)				output (sum)				queue (maximum)			
right	straight	left	total	right	straight	left	total	right	straight	left	total
343,3	1075,1	161,5	1579,9	348,3	1085,1	166,5	1599,9	28,9	99,5	34,3	151,7
Funnel 7											
input (sum)				output (sum)				queue (maximum)			
right	straight	left	total	right	straight	left	total	right	straight	left	total
404,6	971,0	242,8	1618,3	409,6	981,0	244,1	1634,6	32,5	78,0	27,7	130,0
Funnel 8											
input (sum)				output (sum)				queue (maximum)			
right	straight	left	total	right	straight	left	total	right	straight	left	total
334,7	1111,7	178,1	1624,4	339,7	1121,7	183,1	1644,4	29,0	105,4	22,3	154,5

In Table 4.6 the final data is presented which has been obtained during model execution with the model run time equal to 1000s. Within the frames “input (sum)” and “output (sum)”, the length of the flow fixed on the corresponding funnel entrance and exit is presented. The frame “queue (maximum)” gives the maximum value of the queue during simulation.

On Figure 4.12 the results of two experiments with different traffic light cycle durations are presented. For each direction, only total length of the queue is given; the maximal value of the queue could be obtained from Table 4.6 in column “queue (maximum)-total”. As the input flows are stochastic, the process presented on Figure 4.12 is stochastic, too.



a) traffic light cycle crossroad 1: 60s (25+5+25+5)
traffic light cycle crossroad 2: 70s (30+5+30+5)

b) traffic light cycle crossroad 1: 60s (25+5+25+5)
traffic light cycle crossroad 2: 90s (40+5+40+5)

Figure 4.12. **Dynamics of queues for eight directions of vehicles income**

During visual analysis of the post-run outputs, the qualitative conclusion could be made for each queue - mostly for Queue 4 and Queue 7.

A certain increase on crossroad 2 during the traffic light cycle could be seen from Figure 4.12:

- For each “red” phase of traffic light on crossroad 2, Queue 7 (the fragment of road 4→7) is filled to the maximum value 130m.
- Queues: Queue 1 and Queue 3 have a tendency to grow, the capacity of the crossroad 1 in directions 1→4 and 3→4 is not enough to pass the flows because of a large amount of the vehicles in road 4→7.
- Queues: Queue 2 and Queue 3 have a tendency to stable state.

4.2.4 Mesoscopic model validation

To validate the constructed mesoscopic model, we are using simulation on microscopic level. According to the conceptual model (see Figure 4.8) a microscopic model of the crossroads has been constructed. The model is developed in professional simulation software PTV VISION VISSIM [92]. This product is widely used for microscopic simulation of traffic. On Figure 4.13, a screenshot from model animation is presented. To simplify validation process, the type of all input flows has been defined as deterministic. The quantitative characteristics of the developed microscopic model could be mentioned here: number of links and connectors – 66; number of vehicle inputs – 18; number of routes – 24; number of conflict areas – 8; number of signal groups – 2; number of traffic lights – 24. For data collection, 24 data collection points have been used, which are configured so as to collect information about queues.

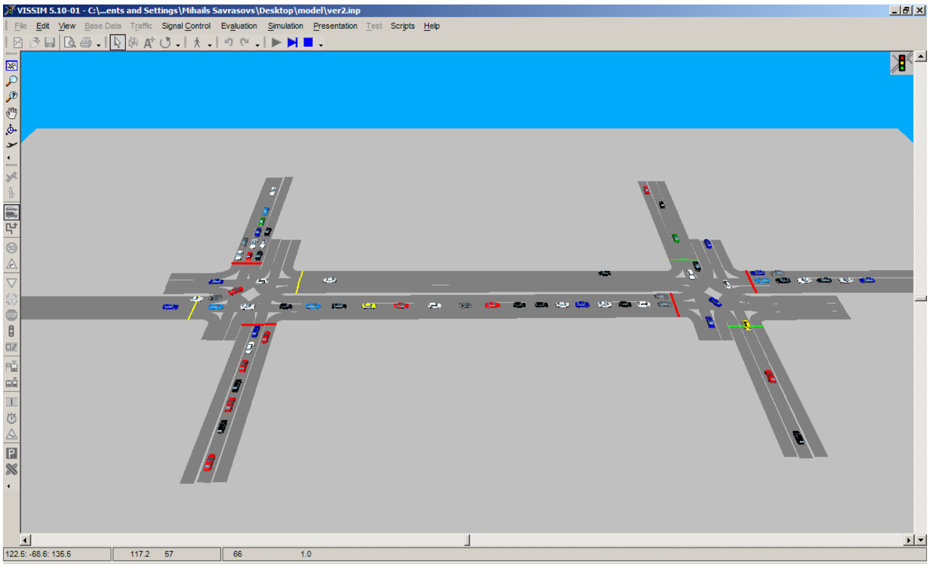


Figure 4.13. **Crossroads microscopic model realized in VISSIM**

For model validation, we use qualitative and quantitative approaches [93]. So validation of the mesoscopic model has been done at two levels. The first level consists of the comparison between qualitative conclusions from mesoscopic model and microscopic animation.

4.2.4.1 Animation-based validation

According to the observed animation of the simulation process in microscopic model, it could be concluded that mesoscopic model on qualitative level represents microscopic level. Such conclusions have been made based on the following points:

- According to animation, road 4→7 is filled by vehicles all the time, which can also be seen on Figure 4.13.
- Queue 1 is growing during simulation. Figure 4.13 demonstrates that vehicles from direction 1→4 could not get to the road, because road 4→7 is already filled.
- During simulation, animation demonstrated that there are some problems in direction 3→4, caused by the road 4→7.
- Queues: Queue 2 and Queue 4, according to animation video, have a stable state, which means that the roads are filled during the “red” phase of traffic light, while almost all of the vehicles pass the crossroad during the “green” phase.

Thus, according to qualitative analysis, all 3 conclusions which have been made according to mesoscopic models are also approved by microscopic model. The next step is qualitative comparison of the two models.

4.2.4.2 Queue dynamics comparison

As a comparison parameter, the queues length is used. The comparison has been done for all queues, but here only the most interesting results are presented. On Figure 4.14, results for Queue 1, Queue 4, Queue 6 and Queue 7 are shown.

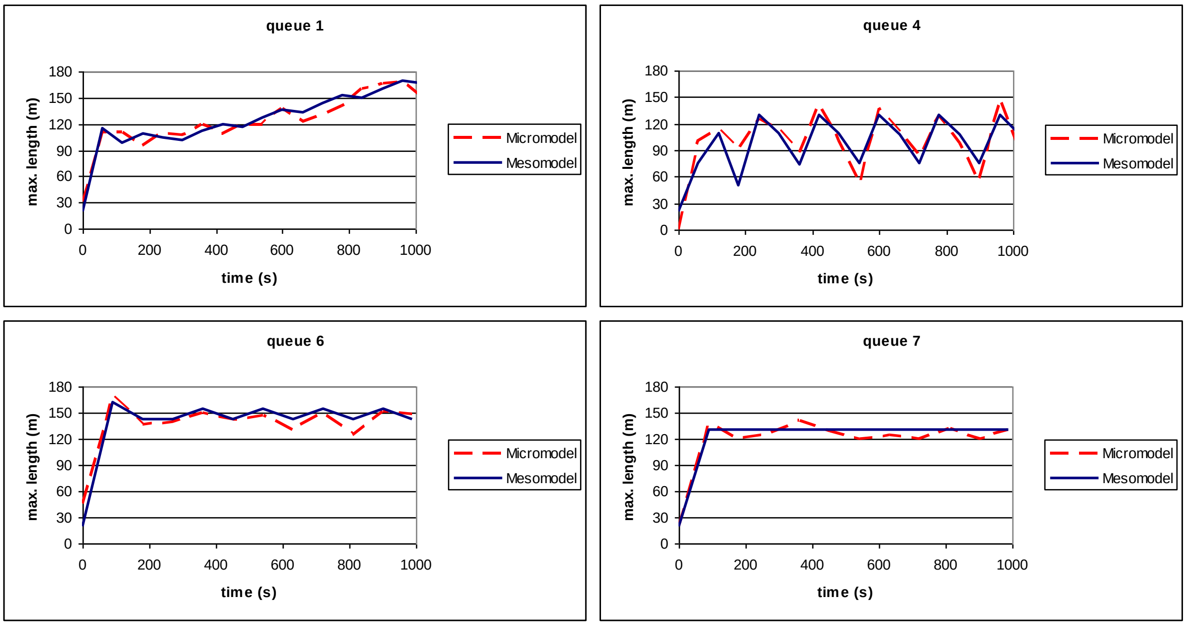


Figure 4.14. **Maximum length comparison for queues**

According to the graphs on Figure 4.14, mesoscopic model successfully presents the queue accumulation process. Of course, there are some differences in results, but fluctuation in the microscopic model could be explained by random numbers, which are used to define the length of vehicles, while in mesoscopic model all vehicles have the same size. Also the comparison of the results could be done using Box-Whisker plots[94].

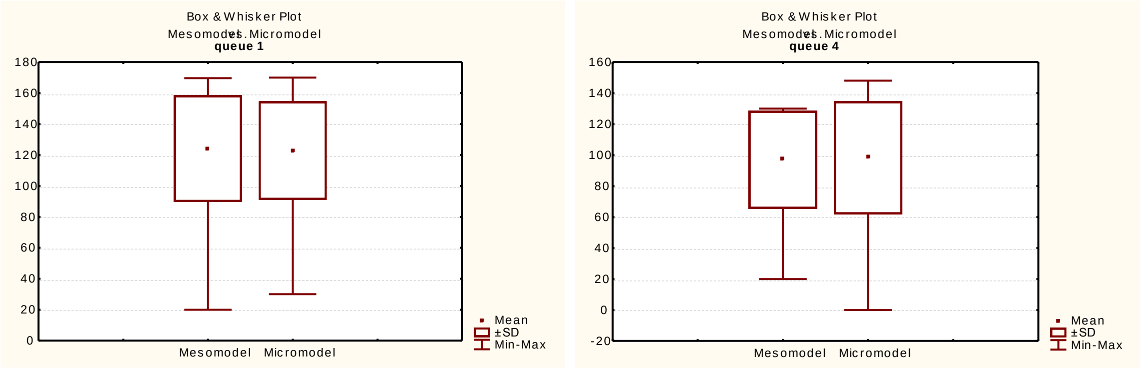


Figure 4.15. **Box-Whisker plot for Queues 1 and 4**

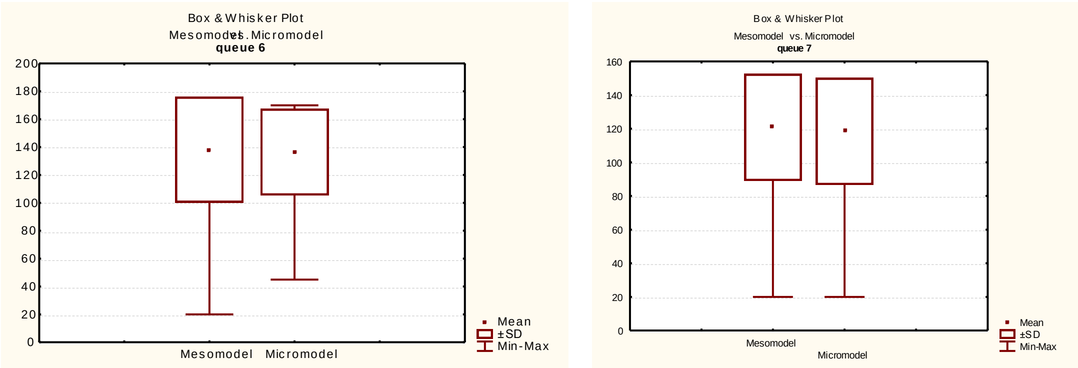


Figure 4.16. **Box-Whisker plot for Queues 6 and 7**

As it could be seen on Figure 4.15 and Figure 4.16, queues mean length, and length standard deviations convince us that output data from both models are homogeneous. To test a hypothesis about outputs homogeneity, the following tests have been used: Student's and Mann-Whitney criteria [95]. But before, as Student's test assumes normal distribution of data, test on normal distribution should be completed. To test data on normal distribution, Person's chi-square test is performed [96], [97]. The results of the testing are presented in the table below (see Table 4.7). The analysis of the Table 7.4 shows that for cells marked by green colour the hypothesis about normal distribution law could not be rejected with the $n=50$ (*n*- power of sample) and $\alpha=1\%$. The cells marked with the pink colour show that hypothesis about normal distribution law could not be accepted with the $n=50$ (*n*- power of sample) and $\alpha=1\%$. For the Queue 7 the computation has not been done, because a variance of the sample is equal to zero. It is explained by constant value of the output result for mesoscopic model.

The constant result is explained by maximum possible value of queue 130m (the queue between two crossroads).

Table 4.7.

Testing results on normal distribution

Data set	Micromodel	Mesomodel
Queue 1	χ^2=5.61; df=5;p=0.34 NRA: (0;15.08)	χ^2=10.89; df=4;p=0.03 NRA: (0;13.27)
Queue 2	χ^2=3.08; df=5;p=0.54 NRA: (0;15.08)	χ^2=5.62.89; df=5;p=0.34 NRA: (0;15.08)
Queue 3	χ^2=18.72; df=3;p=0.00031 NRA: (0;11.34)	χ^2=9.44; df=3;p=0.023 NRA: (0;11.34)
Queue 4	χ^2=9.86; df=5;p=0.07 NRA: (0;15.08)	χ^2=6.43; df=3;p=0.09 NRA: (0;11.34)
Queue 5	χ^2=1.79; df=3;p=0.62 NRA: (0;11.34)	χ^2=5.13; df=4;p=0.27 NRA: (0;13.27)
Queue 6	χ^2=12.64; df=3;p=0.005 NRA: (0;11.34)	χ^2=4.81; df=3;p=0.19 NRA: (0;11.34)
Queue 7	χ^2=18.34; df=4;p=0.00105 NRA: (0;13.27)	*Variance equal to 0*
Queue 8	χ^2=6.98; df=5;p=0.22 NRA: (0;15.08)	χ^2=9.76; df=4;p=0.045 NRA: (0;13.27)

After the test on normality has been completed, the test for homogeneity could be performed in turn, with Student's t and Mann-Whitney u test.

4.2.4.3 Test for homogeneity

The Student's t test will be used and Mann-Whitney u test will be performed [96]. As some of the data is not normally distributed the t test will not be performed for Queue 7, Queue 6 and Queue 3, but still u test will be completed. The results of the testing hypothesis on homogeneity are presented in Table 4.8.

Table 4.8.

Tests results on homogeneity

Data set	t test	u test
Queue 1	t=0.12;df=98;p=0.89 NRA: (-2.63;+2.63)	u=1226.5;Z=0.16;p=0.87 NRA: (-2.58;+2.58)
Queue 2	t=0.30;df=98;p=0.76 NRA: (-2.63;+2.63)	u=1159.5;Z=0.63;p=0.53 NRA: (-2.58;+2.58)
Queue 3	-	u=1230.5;Z=0.13;p=0.89 NRA: (-2.58;+2.58)
Queue 4	t=1.14;df=98;p=0.25 NRA: (-2.63;+2.63)	u=1207.5;Z=0.29;p=0.76 NRA: (-2.58;+2.58)

Data set	t test	u test
Queue 5	*t=0.47;df=98;p=0.60* *NRA: (-2.63;+2.63)*	*u=1182.5;Z=0.47;p=0.64* *NRA: (-2.58;+2.58)*
Queue 6	-	*u=1204;Z=0.32;p=0.75* *NRA: (-2.58;+2.58)*
Queue 7	-	*u=1175;Z=-0.52;p=0.60* *NRA: (-2.58;+2.58)*
Queue 8	*t=1.29;df=98;p=0.20* *NRA: (-2.63;+2.63)*	*u=1074;Z=1.21;p=0.22* *NRA: (-2.58;+2.58)*

The application of Student's t test and Mann-Whitney u test (see Table 4.8) shows that hypothesis about homogeneity could not be rejected with $n_1=50$, $n_2=50$ and $\alpha=1\%$.
Moreover, a more advanced validity test could be applied when testing validity of the developed model.

4.2.4.4 Confidence interval test

The first approach is based on confidence interval. Let's assume that we have n real data $X_1, X_2, \dots X_n$ and the m data from simulation model $Y_1, Y_2, \dots Y_m$. For simplification, m is equal to n. In this case, a mean value for X and Y could be noted in the following form $\mu_x = E(X_j)$ and $\mu_y = E(Y_j)$. The task is reduced to building a confidence interval for the mean values difference $\xi = \mu_x - \mu_y$. Further, one can decide on the validity of the model by simple checking if the confidence interval has a zero value inside; should the confidence interval include zero, it will imply that the model is valid, otherwise the model is not valid. The confidence interval could be calculated using the following formula [98]:

$$\left[\bar{Z}(n) \pm t_{n-1,1-\alpha/2}\sqrt{D[\bar{Z}(n)]}\right],$$

where

n - number of observations;

Z_j - difference $Z_j=X_j- Y_j$

$\bar{Z}(n)$ – mean value;

$D[\bar{Z}(n)]$ – variance.

The mean value could be calculated using the following formula:

$$\bar{Z}(n) = \frac{\sum_{j=1}^{n} Z_j}{n}$$

The value of variance could be calculated using the following formula:

$$D[\bar{Z}(n)] = \frac{\sum_{j=1}^{n}\left[Z_j - \bar{Z}(n)\right]^2}{n(n-1)}$$

Table 4.9.

Confidence interval calculation (*α=1%*)

Data set	Confidence interval
Queue 1	*[-1.943;5.146]*
Queue 2	*[-3.157;4.950]*
Queue 3	*[-6.548;8.697]*
Queue 4	*[-1.014;11.711]*
Queue 5	*[-2.219;6.992]*
Queue 6	*[-6.03;8.719]*
Queue 7	*[-1.941;0.701]*
Queue 8	*[-1.534;1.239]*

As it could be seen from Table 4.9, all confidence intervals include zero, it means that there is no significant mean difference between values obtained from microscopic and mesoscopic models. The calculation of the confidence interval is done with n=50 and *α=1%.*

4.2.4.5 NAIVE test

The first one is a NAIVE test. Let's assume we have a number of real data *x* and the same number of outputs from simulation model *y*. Based on that data, a regression line $\hat{y} = \beta_0 + \beta_1 x$. must be computed. The task of validation is reduced to the task of testing the following hypothesis: $H_0: \beta_0 = 0 \; and \; \beta_1 = 1$, if hypothesis is not rejected, the model could be treated as valid [99]. To test the hypothesis, F-statistic is used in the following form [100]:

$$F_{2,n-2} = [(n-2)/2]\left[SSE_{reduced} - SSE_{full}\right]/SSE_{full}$$

As it could be seen from the above-stated formula, the sum of squared errors (SSE) without and with hypothesis must be calculated; this corresponds to the full and reduced regression model respectively:

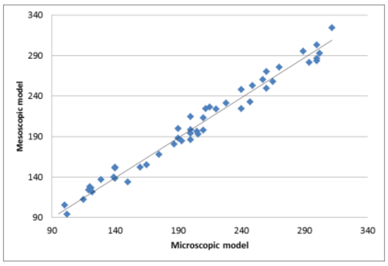
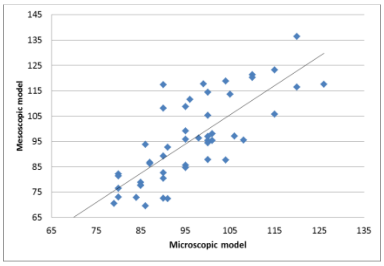
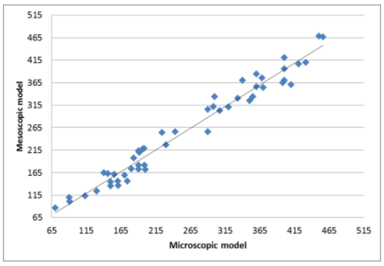
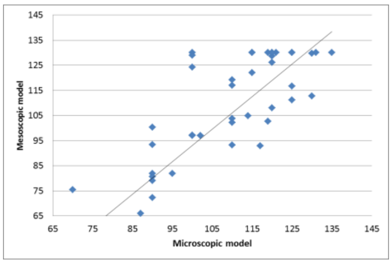
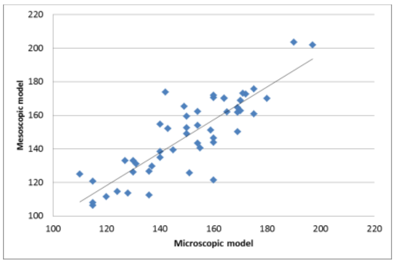
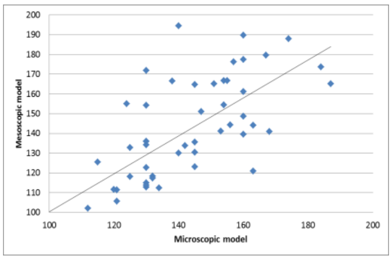
$$SSE_{full} = \sum_{i=1}^{n}(y_i - \hat{y}_i)^2$$

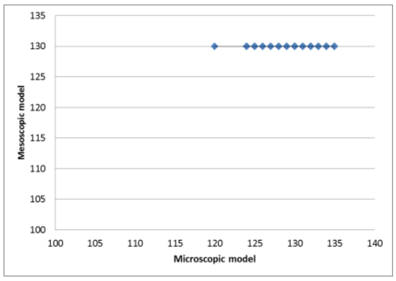
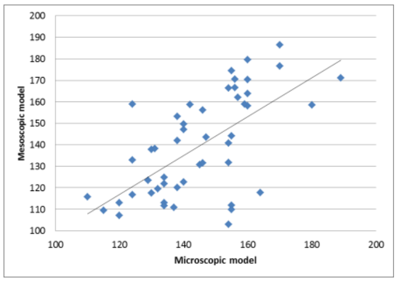
$$SSE_{reduced} = \sum_{i=1}^{n}(y_i - x_i)^2$$

The application of this test for data gives the results, presented below in Table 4.10.

Table 4.10.

Validation results by NAIVE test

Data set	General model Characteristics	F_{naive}	Graphical representation
Queue 1	$R^2=0.978$ Adj. $R^2=0.978$ $SEE=9.39$ $F(1;48)=2199.502$ $RSS=4240$ $ESS=194303.6$ $y=0.517892+0.989189x$	$SSE_{full}=4295$ $SSE_{reduced}=4446$ $F_{naive}(2;48)=0.8487$ *NRA: (0;5.07)*	
Queue 2	$R^2=0.627$ Adj. $R^2=0.619$ $SEE=10.63$ $F(1;48)=80.84$ $RSS=5427.82$ $ESS=9141.46$ $y=-15.3282+1.1506x$	$SSE_{full}=5486$ $SSE_{reduced}=5688$ $F_{naive}(2;48)=0.8869$ *NRA: (0;5.07)*	
Queue 3	$R^2=0.96$ Adj. $R^2=0.965$ $SEE=19.9$ $F(1;48)=1378.362$ $RSS=19178.7$ $ESS=550732.8$ $y=8.255250+0.963953x$	$SSE_{full}=19192$ $SSE_{reduced}=20026$ $F_{naive}(2;48)=1.0432$ *NRA: (0;5.07)*	
Queue 4	$R^2=0.674$ Adj. $R^2=0.667$ $SEE=16.159$ $F(1;48)=99.37$ $RSS=12533.86$ $ESS=25948.72$ $y=-36.6014+1.2971x$	$SSE_{full}=12575$ $SSE_{reduced}=15345$ $F_{naive}(2;48)=5.2870$ *NRA: (0;5.07)*	
Queue 5	$R^2=0.7253$ Adj. $R^2=0.719$ $SEE=12.354$ $F(1;48)=126.776$ $RSS=7326.76$ $ESS=19351.32$ $y=0.312457+0.982520x$	$SSE_{full}=7284$ $SSE_{reduced}=7576$ $F_{naive}(2;48)=0.9625$ *AR: (0;5.07)*	
Queue 6	$R^2=0.464$ Adj. $R^2=0.453$ $SEE=19.7254$ $F(1;48)=41.69$ $RSS=18676.57$ $ESS=16223.93$ $y=3.812868+0.963554x$	$SSE_{full}=18665$ $SSE_{reduced}=18783$ $F_{naive}(2;48)=0.1520$ *NRA: (0;5.07)*	

Data set	General model Characteristics	F_{naive}	Graphical representation
Queue 7	$R^2=0.001$ $Adj.\ R^2=-0.019$ $SEE=0.00199$ $F(1;48)=0.064$ $RSS=0.000192$ $ESS=0.000000256$ $y=130.0023-0.0000207x$	$SSE_{full}=0.000007$ $SSE_{reduced}=619$ $F_{naive}(2;48)\rightarrow\infty$ $NRA: (0;5.07)$	
Queue 8	$R^2=0.411$ $Adj.\ R^2=0.399$ $SEE=18.48$ $F(1;48)=33.614$ $RSS=16406.57$ $ESS=11489.43$ $y=8.843561+0.901485x$	$SSE_{full}=16551$ $SSE_{reduced}=18154$ $F_{naive}(2;48)=2.32$ $NRA: (0;5.07)$	

The results presented in Table 4.10 show that, with respect to some data sets, the NAIVE test shows that null hypothesis should be rejected. It means that the data from mesoscopic and from microscopic models differs. The cells marked with pink highlight the specific data sets with respect to which the hypothesis has been rejected. Also it must be noted that for data set Queue 7, a regression model construction has been problematic, as data from mesoscopic model do have variance equal to zero; for this problem solution, a data from mesoscopic model are a bit blurred, but still a NAIVE test reports that the hypothesis should be rejected. The test has been performed with $n_1=50$, $n_2=50$ and $\alpha=1\%$. Also it should be noted that the quality of the regression model is important in this case. That's why the green colour in the table shows a significant regression model, the pink one - insignificant regression models. The decision has been made based on R^2 and $Adj.\ R^2$ and F, i.e. only 5 of 8 data sets passed the validity test.

4.2.4.6 NOVEL test

The second is Novel test, which has been described by Kleijnen, Bettonvil, Groenendal [100]. Let's assume that we have real data x and output data from simulation model y. The n differences must be calculated $d_i = x_i - y_i$, it is assumed that the number of data from real system and simulation model is the same ($n=m$). Further, the n sums must be calculated $q_i = x_i + y_i$. On calculation of differences and sums, the regression line should be calculated

$d = \gamma_0 + \gamma_1 q$ and null hypothesis should be formulated in the following way: $H_0: \gamma_0 = 0 \; and \; \gamma_1 = 0$. In case if hypothesis is not rejected, the model could be treated as valid. To test the hypothesis, F-statistic is used in the following form [93]:

$$F_{2,n-2} = [(n-2)/2]\left[SSE_{reduced} - SSE_{full}\right]/SSE_{full}$$

As it could be seen from the formula above, the sum of squared errors (SSE) without and with the hypothesis must be calculated; this corresponds to the full and reduced regression model respectively:

$$SSE_{full} = \sum_{i=1}^{n} \left(d_i - \hat{d}_i\right)^2$$

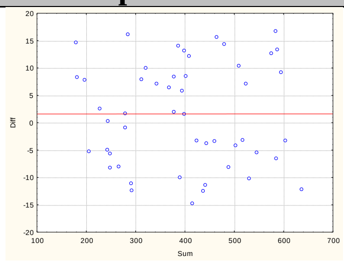
with $\hat{d}_i = c_0 + c_1 q_i$ where c_0 and c_1 are the ordinary least square estimators of γ_0 and γ_1. The reduced regression model implies $\hat{d}_i = 0$, so

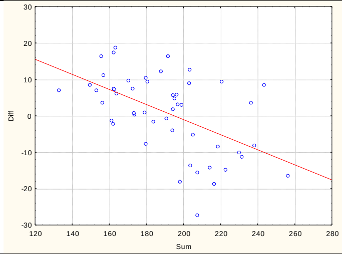
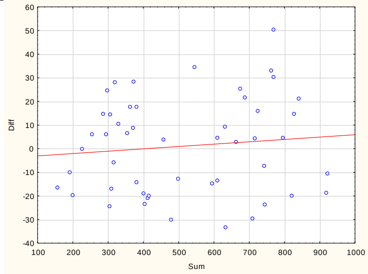
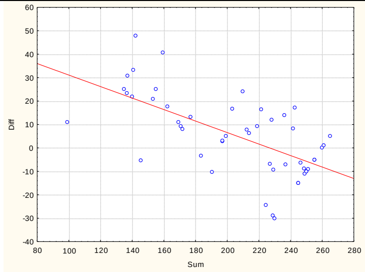
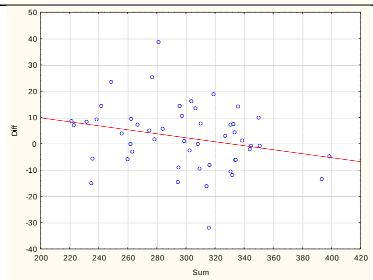
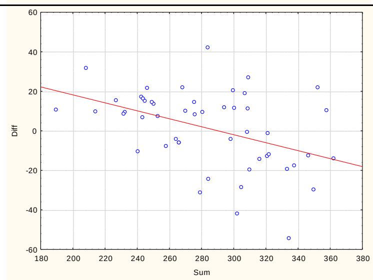
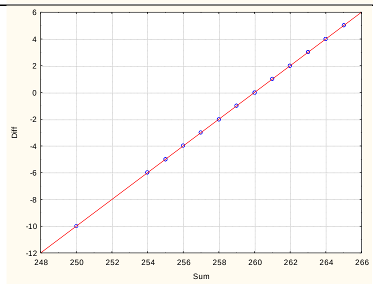
$$SSE_{reduced} = \sum_{i=1}^{n} d_i^2$$

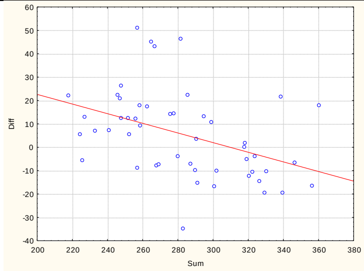
The application of this test for data gives the following results presented below (see Table 4.11). The test has been performed with $n_1=50$, $n_2=50$ and $\alpha=1\%$. First of all, it should be noted that for data set Queue 7 the test is not performed, as no unexplained variance has been found; all the other data sets are tested. Data sets are marked with green where the hypothesis could not be rejected, while pink colour means that the hypothesis could not be accepted. Mostly if the hypothesis is not rejected, the regression model by itself is valid. That's why there is no information about regression models for this test. Only in three cases could the hypothesis not be rejected.

Table 4.11.

Validation results by NOVEL test

Data set	Regression model *d=f(q)*	F_{novel}	Graphical representation
Queue 1	$d=1.589617+0.00003q$	$SSE_{full}=4318$ $SSE_{reduced}=4446$ $F_{novel}(2;48)=0.7126$ NRA: (0;5.07)	

Data set	Regression model $d=f(q)$	F_{novel}	Graphical representation
Queue 2	$d=40.38334+0.20710q$	$SSE_{full}=4040$ $SSE_{reduced}=5688$ $F_{novel}(2;48)=9.7960$ *NRA: (0;5.07)*	
Queue 3	$d=-4.05860+0.00995q$	$SSE_{full}=19740$ $SSE_{reduced}=20026$ $F_{novel}(2;48)=0.3474$ *NRA: (0;5.07)*	
Queue 4	$d=55.6184-0.2453q$	$SSE_{full}=8271$ $SSE_{reduced}=15345$ $F_{novel}(2;48)=20.5251$ *NRA: (0;5.07)*	
Queue 5	$d=25.03369-0.07579q$	$SSE_{full}=6797$ $SSE_{reduced}=7576$ $F_{novel}(2;48)=2.7481$ *NRA: (0;5.07)*	
Queue 6	$d=58.51204-0.20157q$	$SSE_{full}=15207$ $SSE_{reduced}=18783$ $F_{novel}(2;48)=5.6436$ *NRA: (0;5.07)*	
Queue 7	Calculation cannot Be performed. No unexplained variance.	Calculation cannot be performed. No unexplained variance.	

Data set	Regression model $d=f(q)$	F_{novel}	Graphical representation
Queue 8	$d=63.85626-0.20621q$	$SSE_{full}=13793$ $SSE_{reduced}=18154$ $F_{novel}(2;48)=7.5879$ *NRA: (0;5.07)*	

4.2.5 Summary

- This chapter demonstrates a case study of the application of discrete rate approach for traffic flow simulation in the two connected signalized crossroads. For demonstration purposes, synthetic input data has been used for the model.
- The model is coded by using Microsoft Excel possibilities as it provides ready-made spreadsheets, programming language, charts, and computing options.
- The main tasks are as follows: to develop model according to the discrete rate approach principles; estimate queues dynamics; make an experiment of the model to select the best control of the second crossroad, and to make a profound validation of the created model. As the model uses synthetic data, a corresponding microscopic model has been created in PTV VISION VISSIM software. All of these tasks are solved successfully.
- To make a validation of the created mesoscopic model, different kinds of tests have been performed, including, in particular: qualitative validation based on microscopic model animation; test for homogeneity using Student's t criteria and Mann-Whitney u test; a confidence interval test; NAIVE and NOVEL test. Also it should be noted that some of these tests require a normal distribution of the output data. That is, before validation all data sets have been tested on normal distribution using Pearson's chi-square test.
- A test on normal distribution shows that almost all data sets have been distributed by normal distribution low. For Queue 3 and Queue 6, the test of data from microscopic model has failed, while with respect to mesoscopic model, a hypothesis about normal distribution low could not be rejected. For data set Queue 7, the hypothesis has been rejected for both microscopic and mesoscopic data. The mesoscopic data test could not even be completed because variance of the data is equal to 0.

- The application of Student's t criteria and Mann-Whitney u test for testing data on homogeneity has shown good results, which means that data characteristics do not differ significantly in general. The confidence interval test also has shown the same results for all data sets.
- The applied NAIVE test doesn't give a picture as sound as that for data set Queue 4 and Queue 7 - the test fails. Also it should be noted that for some data sets like Queue 6 and Queue 7 the constructed regression models have a very low R^2 and *Adj.* R^2 values, but still according to F the model is significant.
- The NOVEL test shows that it fails for data sets Queue 2, Queue 4, Queue 6 and Queue 8. For the Queue 7 it has not been possible to complete the test as the mesoscopic data do not have any variance. Thus, out of 8 datasets, 4 have failed by application of NOVEL test. It should be noted that for data sets Queue 6 and Queue 2 the value of F_{novel} is located very close to the AR, so maybe increase of sample will lead to the positive result.
- Consequently, by aggregating all the results of the test performed, the following table could be constructed to summarize the results (see Table 4.12 below.) According to the final conclusion, the model could be treated as valid and credible.

Table 4.12.

Validation results summary

Data set	Qualitative validation (animation)	Test for homogeneity	Confidence interval test	NAIVE test	NOVEL test
Queue 1	Valid	Valid	Valid	Valid	Valid
Queue 2	Valid	Valid	Valid	Valid	Not valid
Queue 3	Valid	Valid	Valid	Valid	Valid
Queue 4	Valid	Valid	Valid	Not valid	Not valid
Queue 5	Valid	Valid	Valid	Valid	Valid
Queue 6	Valid	Valid	Valid	Valid	Not valid
Queue 7	Valid	Valid	Valid	Not valid	Not valid
Queue 8	Valid	Valid	Valid	Valid	Not valid

5. MESOSCOPIC SIMULATION OF AN URBAN TRANSPORT CORRIDOR

This chapter demonstrates an example of application of the proposed DRTRM model for traffic flow simulation on the real object with real data, taking into account traffic lights located in the simulated area.

5.1 Modelling object and input data description

The object of research is the urban transport corridor located in Riga city. This transport corridor connects the centre of the city and residential districts inside and outside the city.

5.1.1 General input data

The modelled part of the transport corridor consists of 10 crossroads. The length of researched part of the transport corridor is approximately 1500 meters. The schema of transport corridor is presented on Figure 5.1.

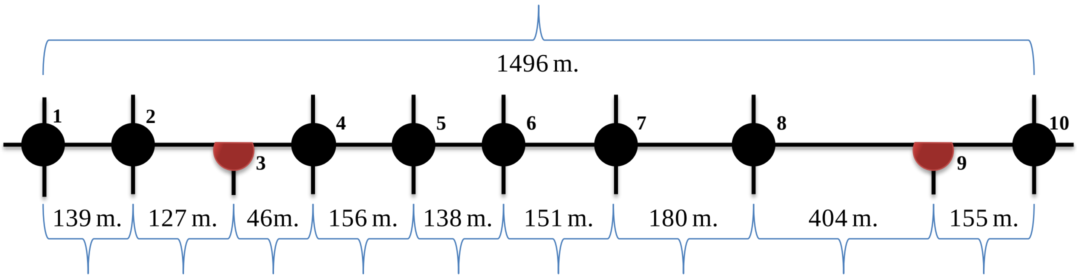


Figure 5.1. **Schema of Transport Corridor**

To simplify reference to crossroads of the transport corridor, all the crossroads are numbered. It should also be noted that the direction from city centre goes from left to right. Most of the crossroads are controlled by traffic lights, except crossroads 3 and 9. The traffic lights are organized so as to produce a green wave. The data about traffic lights is obtained from city council, and shows the duration of cycle and the duration of each light. An example of traffic light operation data with respect to the 1st crossroad is presented on Figure 5.2.

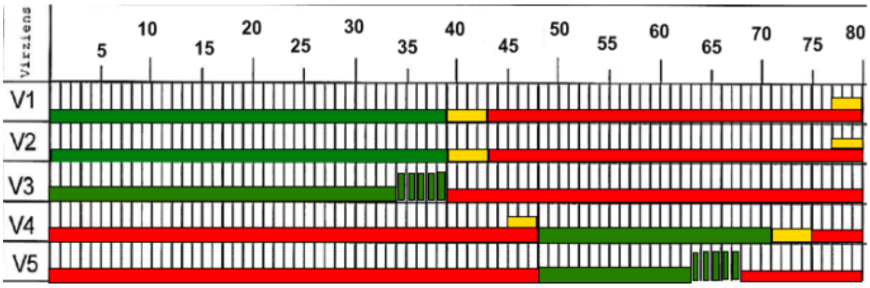


Figure 5.2. **Example of Traffic Light Data [101]**

The volumes of traffic are obtained during traffic counts. The traffic counts have been estimated during morning peak hours from 8:00-9:00 a.m. During traffic counts, a number of vehicle types have been differentiated: bicycles, motorcycles, passenger vehicles, light cargo trucks, HGV, buses, single trolleybuses, and twin trolleybuses. The mesoscopic approach in the current formulation does not take into account any types of vehicles; that is why all the collected data are aggregated into PCU (passenger car unit). The conversion has been done according to the following table (see **Error! Not a valid bookmark self-reference.**) showing multiplication coefficients for each type of a vehicle.

Table 5.1

Multiplication Coefficients [101]

Vehicle types	Coefficients
Bicycles	0.3
Motorcycles	0.5
Passenger vehicles	1
Light cargo trucks	1.5
HGV	2
Busses	2.5
Single trolleybuses	3
Twin trolleybuses	4

Finally, all traffic volumes are presented in PCU. The schema showing an example of calculated volumes, can be observed on Figure 5.3 (the numbers within the rectangle identify the crossroad).

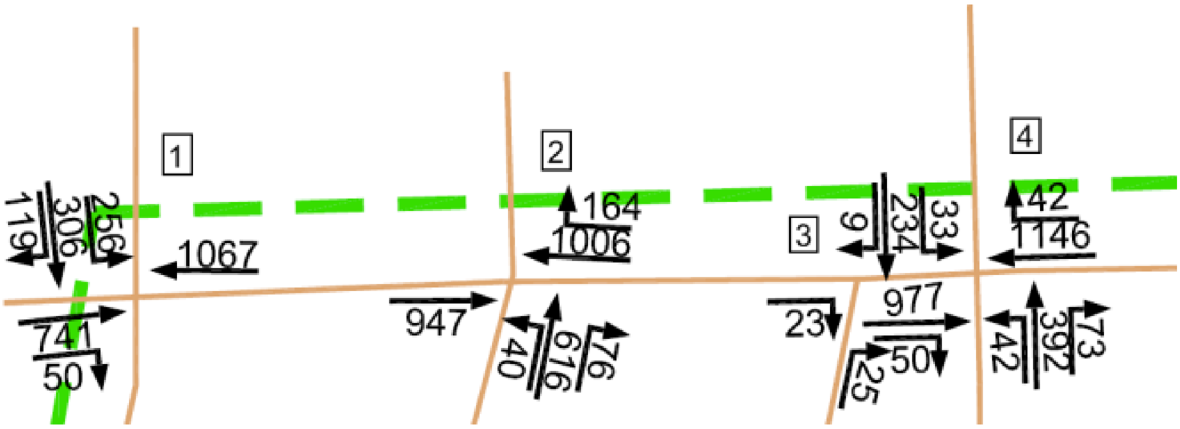


Figure 5.3. **An Example of Volumes of Traffic in PCU [101]**

Here it should be noted that the defined volumes are used as input ones, and for calculation of probability of a further movement. Each crossroad is described using a schema of the crossroad and the allowed movements. An example of such description can be seen on Figure 5.4.

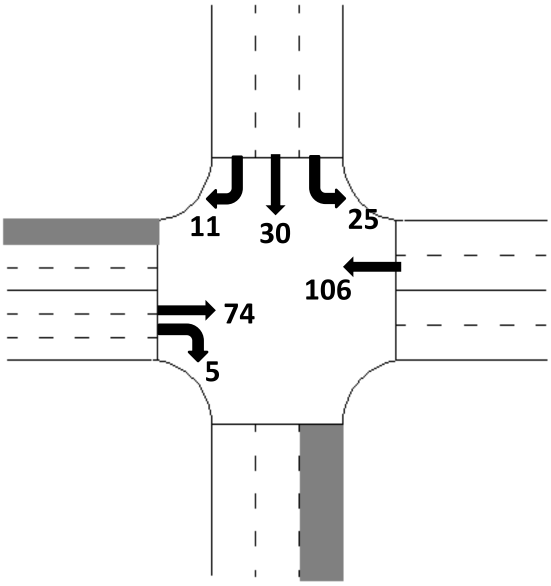


Figure 5.4. **An example of crossroad schema**

Finally, according to the following general information about the transport corridor:

- The allowed speed across the transport corridor is 50 km/h;
- The transport corridor has two lanes per each direction;
- all public transport stops are made outside the main roads and are located in pockets;
- traffic lights are managed for creating a green wave across the entire transport corridor.

The main task is to model the described transport corridor and define the level of service (LOS) for each crossroad, using simulation results.

Table 5.2.

Level of Service [102]

Level of Service	Average delay time (s)
A	≤10
B	> 10 – 20
C	> 20 – 35
D	>35 – 55
E	> 55 - 80
F	>80

It is necessary to underline at this point that LOS will be calculated based on average delay value on crossroad. The following table demonstrates the LOS levels according to HCM 2000 standard [102].

In general, the first 3 levels (A, B, C) can be treated as normal delay time on crossroad. The D level warns about possible problems. And finally, the last 2 are the worst levels, attesting to the existence of the problem of crossing this crossroad.

As the presented mesoscopic approach is quite new and not widely used, validation of data of simulation results is required. For the model output data validation, the defined LOS levels will be used by applying the microscopic model constructed in PTV VISION VISSIM software. Microscopic simulation results can be observed in Table 5.3.

Table 5.3.

Microscopic Model Output Data [101]

Crossroad Number	LOS	Average delay time (s)
1	B	14.5
2	B	13.8
3	A	1.6
4	B	17.3
5	B	18.1
6	B	11.2
7	C	20.6
8	C	31.2
9	A	2.1
10	D	41.5

As it can be seen from Table 5.3, the transport corridor is not too congested. Most of the LOS for crossroads are A and B. Only 3 crossroads have level C and D. So, the same results are expected from mesoscopic model. It should be noted that the development within the framework of the project-based microscopic model is validated against real situation.

5.1.2 Passing function form determination

As it has been discussed before, the modelling object is a real object; therefore, to obtain a valid simulation model, some realistic parameters must be included into the model. Most of these parameters could be obtained directly (e.g., traffic lights cycles, distance between crossroads), some of them could be collected (e.g., traffic counts), but some of them require an additional research. One of these parameters has a form of the passing function, which in general has been described in Chapter 3 (e.g., see subchapter 3.1.1). In Chapter 4, a theoretical form of this function has been used. But the task of determination of the form of realistic function is still topical. To obtain the form of the function, a regression analysis has been used. The data for analysis is obtained from video records made during peak hours in Riga at two crossroads (Brivibas street – Elizabetes street and Dzelzavas-Lielvardes streets). More than 7 hours of video have been analysed. The analysis is performed manually with additional programming help, which activates a 5-second timer to stop video. The data has been collected with the interval of 5 seconds. The number of vehicles is measured in PCU (transition to PCU is done according to The volumes of traffic are obtained during traffic counts. The traffic counts have been estimated during morning peak hours from 8:00-9:00 a.m. During traffic counts, a number of vehicle types have been differentiated: bicycles, motorcycles, passenger vehicles, light cargo trucks, HGV, buses, single trolleybuses, and twin trolleybuses. The mesoscopic approach in the current formulation does not take into account any types of vehicles; that is why all the collected data are aggregated into PCU (passenger car unit). The conversion has been done according to the following table (see **Error! Not a valid bookmark self-reference.**) showing multiplication coefficients for each type of a vehicle.
Table 5.1. More than 50 data sets (400 observations) in total are obtained and analysed. The data sets are presented on Figure 5.5.

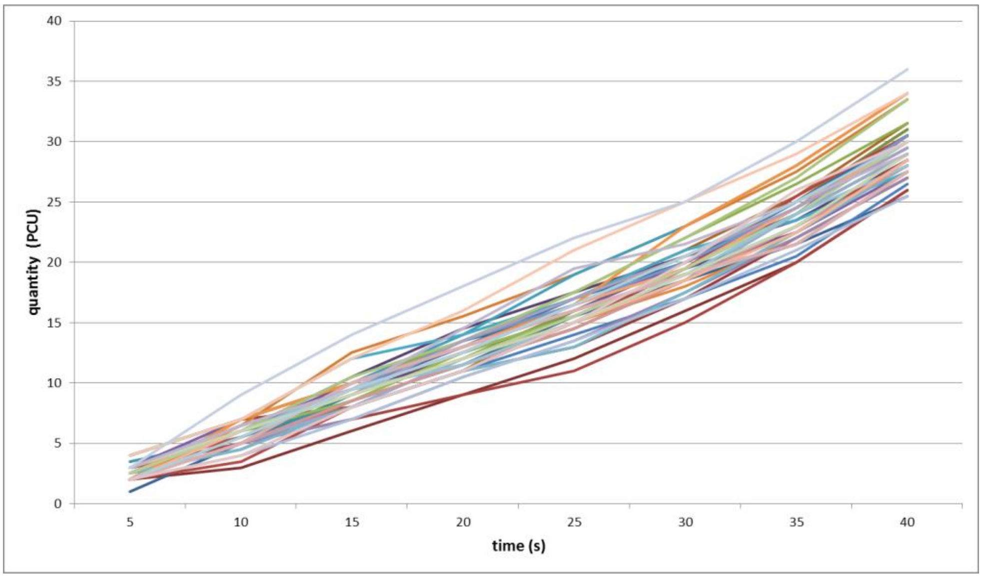
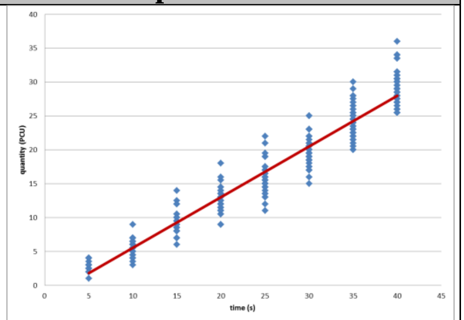
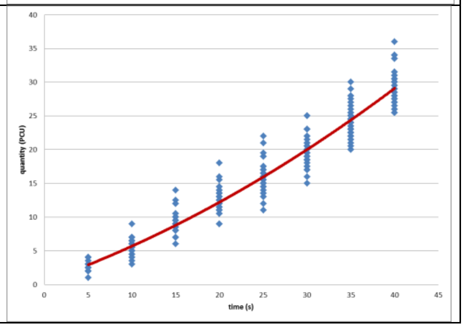


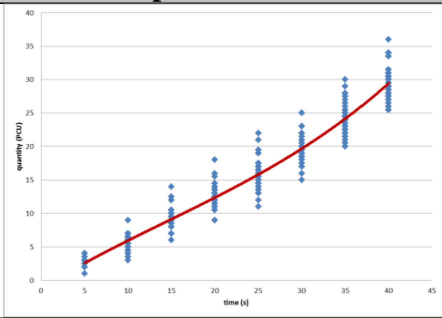
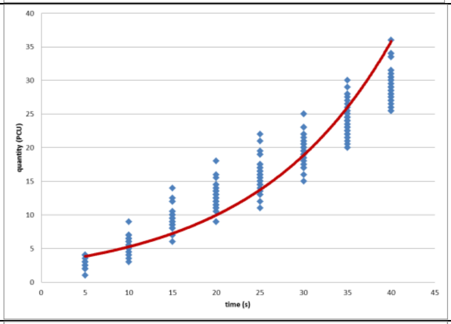
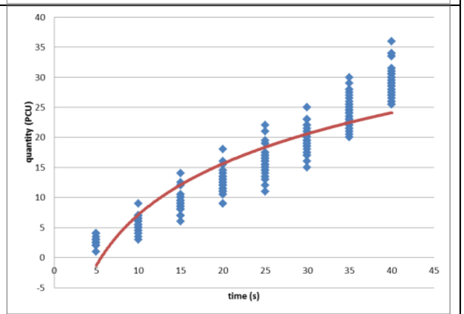
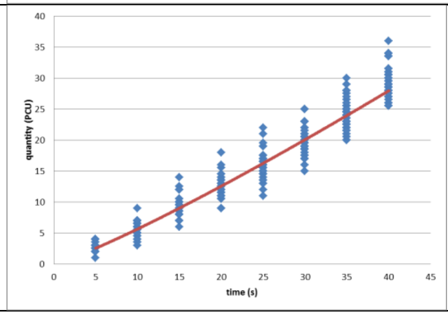
Figure 5.5. **Graphical representation of datasets**

Different regression models constructed on the basis of the presented data sets can be seen in Table 5.4, accompanied with graphical representation and regression models characteristics.

Table 5.4.

Regression models

#	**Regression Model**	**Model characteristics**	**Graphical representation**
1.	$q=-1.93714+0.74793*t$	$R^2=0.9533$ $Adj.\ R^2=0.9531$ $SEE=1.90$ $F(1;398)=8124.59$ $RSS=1438.67$ $ESS=29368.35$	
2.	$q=0.4584+0.46*t+0.0064*t^2$	$R^2=0.9603$ $Adj.\ R^2=0.96$ $SEE=1.756$ $F(2;397)=4795.83$ $RSS=1224.43$ $ESS=29582.59$	

#	Regression Model	Model characteristics	Graphical representation
3.	$q=-1.1479+0.794*t-0.01113*t^2+0.00026*t^3$	$R^2=0.9613$ $Adj.\ R^2=0.961$ $SEE=1.736$ $F(3;396)=3276.21$ $RSS=1193.16$ $ESS=29613.86$	
4.	$q=2.7824*e^{0.0639*t}$	$R^2=0.8928$ $Adj.\ R^2=0.8925$ $SEE=0.2542$ $F(3;396)=3314.6$ $RSS=25.72$ $ESS=214.20$	
5.	$q=-20.971+12.22*ln(t)$	$R^2=0.839$ $Adj.\ R^2=0.839$ $SEE=3.5243$ $F(1;398)=2082.31$ $RSS=4943.41$ $ESS=25863.61$	
6.	$q=0.3966*t^{1.1534}$	$R^2=0.9602$ $Adj.\ R^2=0.96$ $SEE=0.155$ $F(1;398)=9595.81$ $RSS=9.55$ $ESS=230.37$	

As it could be seen from Table 5.4, all the proposed models could be treated as significant according to the high values of R^2, $Adj.\ R^2$ and F criteria. The highest values of R^2, $Adj.\ R^2$ are observed for the model number 3. At the same time, the lowest value of SEE is fixed for the 6^{th} model. To select the one best describing the passing process, a forecast for the further time interval could be made for all the proposed models. Figure 5.6 shows a forecast given by all 6 models. The 4^{th} model gives a very optimistic forecast and that is why it could not be treated as realistic model of the passing process. On the other hand, the 5^{th} model gives very pessimistic results and could not be taken into account. The 1^{st} and 6^{th} models have almost the same behaviour on the observed forecasting horizon, but the value of R^2 and *Adj.* R^2 is smaller than that in models 2 and 3. The 3rd model have a higher R^2 and *Adj.* R^2 among the 2nd and the 3rd model, therefore it could be selected as a valid model for crossroad passing process.

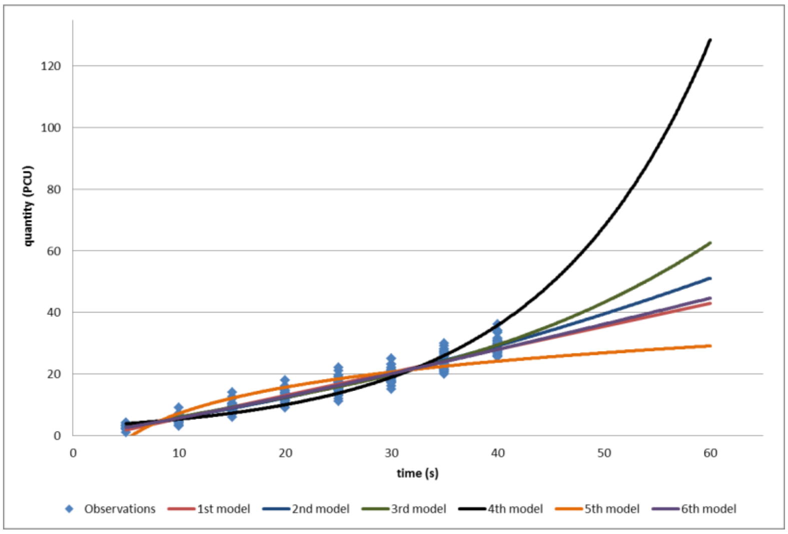


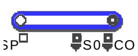
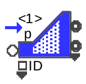
Figure 5.6. **Forecast with the models**

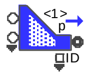
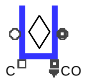
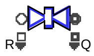
5.2 ExtendSim Rate library blocks in the frame of transport modelling

As it has been mentioned before, creation of models with the coding in Microsoft Excel is a time-consuming procedure and requires additional knowledge from the researcher. On the other hand, the code complexity is growing exponentially with the growth of the simulation object. That is why the use of ExtendSim simulation software proposed as rate library of this application fully realizes the logic of discrete rate approach. The table below shows the role of library block in transport modelling (see Table 5.5). It should be noted that not all blocks of the library are useful in transport modelling; out of 11 blocks mentioned in Chapter 3, only six blocks are proposed for transport modelling. Of course, additional ExtendSim blocks could be involved to collect and visualise output data.

Table 5.6.

Main role of block in transport model

Block	Block name	Role in transport model
	Convey flow	Could be used to simulate a movement between two geographical points (e.g., between two crossroads).
	Diverge	Could be used to simulate a splitting of the transport flow by different directions (e.g., on crossroads turning left, turning right, and moving forward).

	Merge	Could be used to merge traffic flows together.
	Sensor	Could be used as the main source of information for controlling flows and the flow interaction.
	Tank	Could be used as a source and sink. Also could be used to represent capacity of the road.
	Valve	Controls, monitors, and transfers traffic flow.

5.3 Mesoscopic model development

A mesoscopic model of the urban transport corridor has been developed in ExtendSim software using a discrete rate library, which is described in Chapter 3. As it has been mentioned above, not all library blocks are used, but only some of them.

It is possible to create hierarchical structure of model in ExtendSim application. That is why to simplify a process of model development the custom library blocks have been created: those called “node_sig” and “node”. These blocks present crossroads with different management strategy:

- Signalised crossroads (node_sig) – crossroads controlled by traffic light with a definite cycle length. This group includes 8 crossroads (crossroads numbers: 1, 2, 4, 5, 6, 7, 8 and 10).
- Non-signalised crossroads (node) – crossroads not controlled by traffic light. This group includes only 2 crossroads (crossroads numbers: 3 and 9).

The general high-level part of the model of transport corridor is presented on Figure 5.7.

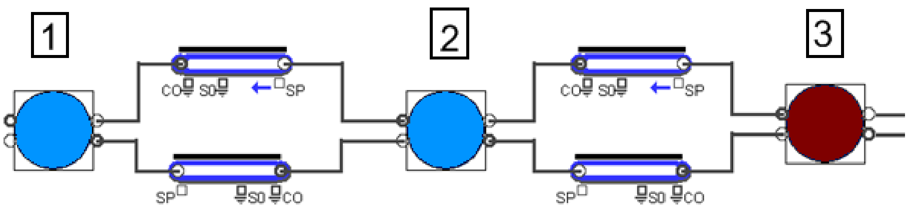


Figure 5.7. **Example of Constructed General High-level Model**

Custom library blocks are presented as circles. The blue colour circles (light ones) represent signalised crossroads, while the red colour (dark circles) - non-signalised crossroads.

An example of the internal structure of the block “node” is presented below (see Figure 5.8).

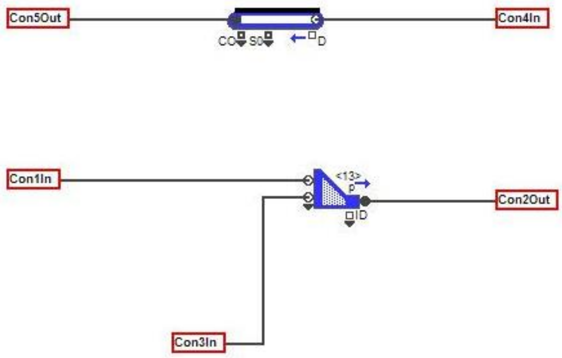


Figure 5.8. **Internal Structure of the Block “node”**

As it can be seen from Figure 5.8, the internal structure of the block is very simple. The traffic flow moving to the city centre does not face any problem; that’s why the input of the block is directly connected to its output via “convey flow” block with a very small delay time equal to 0.01. The flows going from the centre and from the bottom input must be merged; this is done by using “merge” block from standard-rate library. Flows have different priorities. The higher priority is assigned to the flow moving from city centre; the lower priority is assigned to the flow which goes from the bottom.

The structure of the “node_sig” block is much more complicated as compared to “node” block. It is due to the complexity of the crossroads and to traffic lights existing at the crossroads. The traffic light operation is modelled by using “Lookup Table” from Value library block. The input parameter of the block is a current time in model, while the output of the block is presented by three outputs: r (right), s (straight), l (left). The output values are determined according to a predefined table. Possible values of outputs are defined as 0 and 1. “0” means that movement in the respective direction is forbidden, “1” means that the movement is allowed. Each leg of the crossroad has its own control block “Lookup Table” except that the legs are controlled by a single traffic light program. The Figure 5.2 shows the traffic light data for the first crossroad. The Figure 5.9 demonstrates the controlled legs of the crossroad.

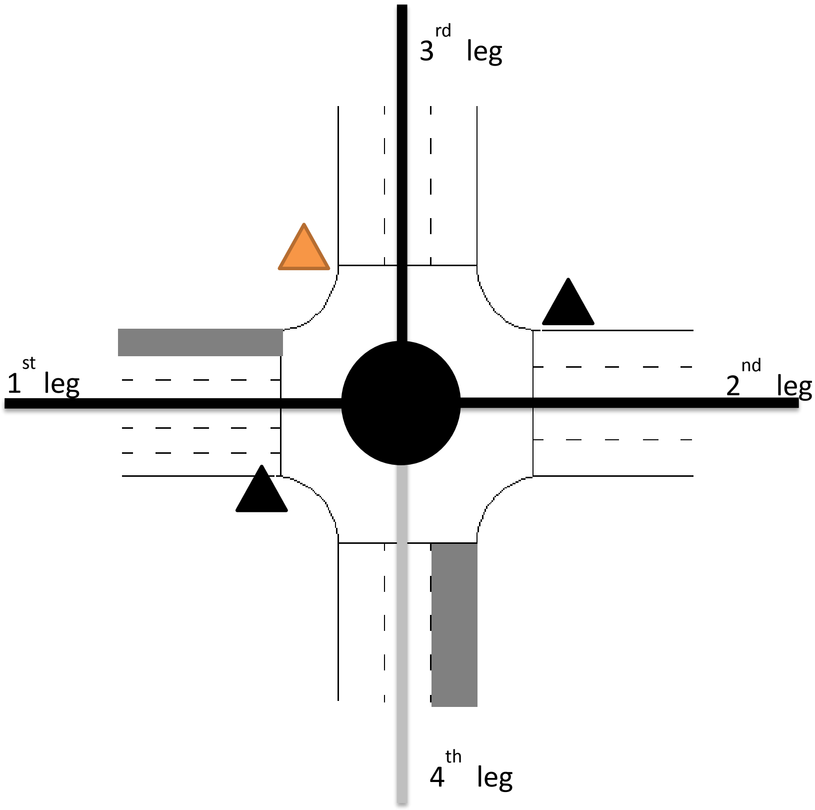


Figure 5.9. **Signal Heads of the Traffic Light**

As it can be seen from Figure 5.9, legs 1 and 2 have the same program, while leg 3 has a separate program; that is why 2 “Lookup Table” blocks have been used to define the control over this crossroad. Table 5.7 and Table 5.8 demonstrate the content of the two Lookup Tables used to model traffic light control on this crossroad.

Table 5.7.

Example of Lookup Table for Leg 1 and 2

Record ID	Time	left	straight	Right
1	0	1	1	1
2	43	0	0	0
3	80	1	1	1

Table 5.8.

Example of Lookup Table for Leg 3

Record ID	Time	left	straight	Right
1	0	0	0	0
2	47	1	1	1
3	80	0	0	0

It should be noted that some directions mentioned in Table 5.7 and Table 5.8 are not allowed for movement (e.g., see Figure 5.4), but still have 1 in this direction; it does not affect the model validity because the traffic flow distribution is not controlled by lookup tables. The parameter of Lookup Table "Repeat table every:" should be adjusted to traffic light cycle length which in this case is 80 seconds.

Each movement on the crossroad is presented by a fixed sequence of the Rate library blocks. Figure 5.10 shows the sequence of the blocks for the 1st crossroad.

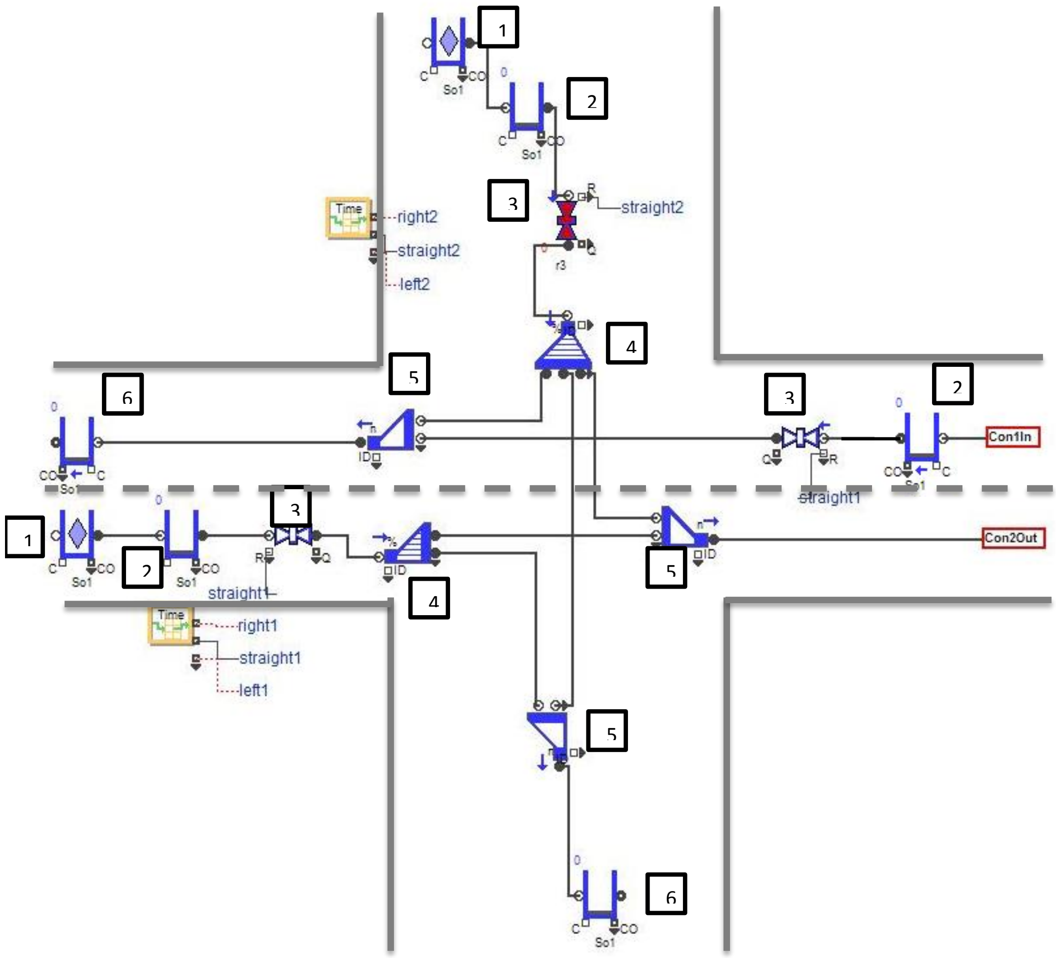


Figure 5.10. **Example of Internal Structure of the Block "node_sig" for the 1st crossroad**

The internal structure of the block "node_sig" for the 1st crossroad consists of 14 main and 2 additional blocks. Additional blocks are required to simulate traffic light work. The main blocks define the behaviour of the traffic flow. The description of functionality of each block used in this structure can be found below:

- Blocks marked by 1 are required to generate input flow. Rate library block “Tank” is used.
- Blocks marked by 2 are used to accumulate traffic flow before crossroad passing. Rate library block “Tank” is used in capacity mode.
- Blocks marked by 3 control passing of the crossroad by traffic flow with the given intensity. Rate library block “Valve” is used. Here it should be noted that these blocks are controlled by using additional “Lookup table” blocks.
- Blocks marked by 4 are used to split traffic flow by different directions. Rate library block “Diverge” is used in proportional mode.
- Blocks marked by 5 are used to combine traffic flow from different directions. Rate library block “Merge” is used in neutral mode.
- Blocks marked by 6 are used to store a traffic flow which leaves the system. Rate library block “Tank” is used in capacity mode.

Finally, the constructed model of the urban transport corridor has approximately 132 blocks, among them 16 are additional blocks used to simulate traffic light work. It stands to reason that, to provide for output data collection and processing, some additional blocks from different ExtendSim libraries have been used.

5.4 Comparison of results and analysis of simulation output data

The constructed mesoscopic model of urban transport corridor has been used to simulate traffic movement across the corridor in morning peak hour from 8:00-9:00. The constructed model requires validation, because a new approach to traffic simulation is used. The microscopic model, developed earlier and validated on real survey data, is used for validation. The validation of the mesoscopic model has been done by using two different types of output data, available at microscopic and mesoscopic level: observed volumes of traffic and delay time one crossroads (LOS). The required data has been obtained from microscopic model by executing 50 runs. The aggregated data are processed by calculating the mean value. Comparison of the simulated volumes of traffic shows the average difference of 20% between the results simulated by microscopic and mesoscopic models. Comparison between the delay times is proposed in Table 5.9.

Table 5.9.

Delay and LOS Comparison

Crossroad Number	Microscopic model		Mesoscopic Model	
	LOS	Average delay time, s	LOS	Average delay time, s
1	B	14.5	B	18.6
2	B	13.8	B	17.5
3	A	1.6	A	1.2
4	B	17.3	B	17.6
5	B	18.1	C	21.6
6	B	11.2	B	14.3
7	C	20.6	C	30.8
8	C	31.2	D	45.5
9	A	2.1	A	1.2
10	D	41.5	E	55.6

As it can be seen, LOS for crossroads is generally matched. It should be noted of course that delay time for all the crossroads has a difference, and sometimes this difference does not influence LOS, except for the LOS for crossroads number 10, 8 and 5. For crossroad number 10 the difference is equal to 14 seconds. For 8 and 5, the difference is not so significant.

Generally, the behaviour of traffic flow in the mesoscopic model is very close to the real situation existing in this urban transport corridor. The difference of the output data can be explained by higher level of abstraction of the mesoscopic model. At the same time, the speed of development of mesoscopic model is 3 times less. In general, a big difference in the development time is connected with simplification of the network representation at mesoscopic level. At the same time, the difference between output data is not significant. This is the main advantage of the mesoscopic approach, expressed by a significant decrease of development time and expectable precessions of the output results. More detailed results of comparison between the development issues at microscopic and mesoscopic level could be seen in Table 5.9. Table 5.9 is constructed by aggregating expert estimations of time consumption needed to implement the main elements of the model.

Table 5.9.

Development and experimenting time for micro- and mesoscopic models

Development and experimenting issue (min)	**Microscopic model**	**Mesoscopic model**
Transport network implementation	175	60
Implementation of traffic lights	60	30
Conflict areas and priority rules implementation	115	60
Movement routes implementation	30	30
Traffic flow implementation	30	60
Time spent on experimentation	350	10
Total implementation and experimentation time	**760**	**250**

At the same time, it should be pointed out that the developed model can be applied only to uncongested network. If the network is congested – a higher error in the output results will be expected. It is connected with the fact that the queue growth does not influence running distance between crossroads. In this model, the running distances between crossroads are constants.

5.5 Advantages and disadvantages of DRTRM

This section deals with the advantages and disadvantages of the proposed concept of a mesoscopic traffic model. We would state the disadvantages first:

- **Animation not available:** for the proposed model, animation will not be available as traffic is grouped in batches. This point will lead to the difficulty of the results, making it difficult to present the model to the wide public. At the same time, the possibility of using animation as a qualitative validation will be partly lost as well. The only possible graphical output from the model is graphs, which could demonstrate the dynamic of some models output data - e.g., queue dynamics. Partly these data could be used for qualitative validation.
- **Lack of simulation software:** at the moment, there are two possibilities of how the proposed model could be used. The first one is a realization of the model using universal programming languages, which give high development flexibility, but at the

same time it will make developer code secondary functionality of the model - like data collection, data procession, report generation, and so on. Another possibility is to use the software already available. But the list of the software is very short; just one position is ExtendSim, which has a discrete-rate library. And it should be pointed out that this library mainly is focused on logistics area; consequently, some additional traffic modelling features are required, like traffic light control, data collection blocks, and so on.

- **Model not proved by time:** the proposed model is a new one and it has not been mastered profoundly – unlike the existing microscopic and macroscopic models of traffic. A profound and long-term research of the model should be done to find the situation that the model is unable to recreate real traffic behaviour. Studies of this type are performed by different experts in different countries and in different situations. And it is the only way how the model could be researched deeply.
- **Not able to trace individual vehicles:** the proposed model is based on combining vehicles into flow batches; that is why it is not possible to trace an individual vehicle route via the network and, therefore, a number of output data could be obtained only in general form, like average vehicle travelling time, average vehicle travelling speed, and so on.
- **Geographically and time dependent:** this point means that some of the model parameters are country- and time-dependent. It poses the necessary task of calibrating parameters according to the location of the real object and the time selected for the simulation. This is an additional work that should be completed to obtain a valid model. As an example of such parameters, the following could be mentioned: coefficients for obtaining traffic in PCU; crossroad passing function, VDF, and so on.
- **Not able to reproduce different types of vehicles:** the presented model aggregates different types of vehicles into a homogenous flow which is measured in PCU. It means that different types of vehicles cannot be used in the model, except if they are travelling on the selected lane. This disadvantage will lead to degradation of accuracy of output data.
- **Detailed drivers' behaviour could not be reproduced:** the model operates with a flow; that is why detailed drivers' behaviour is not taken into account. This could lead to the problem that the model will not be valid as compared to the real system. Input

parameters, which describe the behaviour of the vehicles, are very general and sometimes could not reflect the real behaviour.

The next is the advantages of the proposed model:

- **Lower development time:** the transport model development experience shows that the development of a mesoscopic model takes less time as compared to that of a microscopic model. This is mostly connected with the fact that a microscopic model claims for a detailed description of the transport infrastructure. Description of traffic infrastructure includes the following: geometry of the road; road width; road gradient; number of lanes; location of traffic lights, and so on. For mesoscopic simulation, definition of most of these parameters is not necessary; that reduces the model development time. The following estimation could be done: it is three times faster to develop a mesoscopic model than an equal microscopic model. This fact is a big advantage allowing us to obtain output data faster and to make a quicker decision.
- **Output data more precise:** the obtained output data are more precise as compared to the macroscopic simulation. This fact is connected with a high abstraction level of macroscopic model. For instance, normally macroscopic models do not use information about traffic lights directly, only inform of special VDF functions for turns. As mesoscopic model describes these processes, output data are more adequate to the real data.
- **No replication needed:** normally the proposed model does not require determining stochastic factors, as they have some level of aggregation. This leads to the good point of saving time needed to complete replications as in microscopic models. Usually minimum of 10 replications are done for microscopic model to obtain realistic output data. Another point that goes in this direction is due to the following - for mesoscopic model no replication data processing is required. This point also leads to reduction of modelling time.
- **Higher simulation speed:** as compared to the microscopic model, which is mostly based on permanent time change with some small interval, the proposed model uses the event-based time changing. This allows one to reduce a time spent on simulation. Empirically it could be concluded that simulation of the model created using mesoscopic approach could take up to ten times less than simulation of the equal microscopic model.

- **Software cheaper than professional tools:** as it was mentioned above, currently only one simulation tool could be used to make a mesoscopic model with DRTRM - i.e., ExtendSim. The ExtendSim software is the so-called universal simulation tool. It means that ExtendSim could be used for simulation of any kind of a system. Usually the price for universal simulation software is few times cheaper than a price for specialized professional software for transport model development. As an example, the price of base license for the PTV VISION VISSIM (microscopic simulation tool) is approximately 10000 Euro, while the price for the base version of ExtendSim with discrete rate library is only 2000 Euro.

5.6 Summary

- This chapter shows the example of application of discrete rate approach for traffic flow simulation in a real urban transport corridor located in Riga city. As a real object is simulated, the real input data must be entered to the model. The main source of input data is a project held by Transport and Telecommunication Institute in 2010-2011.
- Moreover, the research of passing function has been completed within the framework of this chapter. By application of regression analysis for the data obtained from video recording of passing process on two crossroads, this task has been completed. In the aggregate, more than 7 hours of videos have been analysed. The number of observations used for further regression analysis is 400. Six different regression models have been constructed and subsequently tested for suitability. It should be pointed out that all of the constructed models are significant according to the R^2, *Adj.* R^2 and F criteria. But a profound analysis shows that the best model is a polynomial model of the third order, since it has the highest R^2, *Adj.* R^2 and F criteria and it is able to reproduce limitation behaviour of the model. Further analysis of this problem should include the following points: a limitation point search, based on simulation model, as it is difficult to obtain it from a real traffic observation; research of the influence of HGV and public transport upon the passing function form.
- The model has been constructed in the ExtendSim 8.0 simulation software, which fully realizes a discrete rate approach. It should be pointed out that this library is used for logistics processes simulation and not for traffic simulation. The analysis of the block set of the library shows that only 6 of 11 blocks could be used for traffic flow

simulation. And these 6 blocks fully cover the functionality required to construct the model. The total number of blocks used in the model is approximately 132. Additional blocks are required to collect and to process data.

- To validate the constructed model, an output data from microsimulation model developed within the framework of the project with PTV VISION VISSIM 5.3 have been used. Among all possible data, volumes of traffic and average delay time in crossroad are used for validation. The difference in LOS levels in crossroads is not significant.
- It must be emphasized that development time of the mesoscopic model is approximately 5 times less than that of the same microscopic model. This can be treated as an advantage of mesoscopic simulation.

CONCLUSIONS

Statement of the main research results

The principal results of the study accomplished can be presented in terms of the conclusion:

1. The review of the current and the future role of simulation models in transportation sphere has been performed; this allows one to conclude that there will be a high demand for the simulation in future as it is a powerful and exact tool for traffic flow analysis.
2. The analysis of the existing traffic analysis tools and their role has been carried out with a detailed description; this allows one to show the place of simulation models among different kinds of traffic tools; it is shown that traditional simulation methods are based on microscopic and macroscopic approaches, which are characterized by the existence of certain non-removable disadvantages.
3. It has been found out that some known attempts to develop models combining positive characteristics of microscopic and macroscopic models, did not lead yet to the development of a new, well-defined class of models which are usually called mesoscopic; as a rule, these models are only an arbitrary combination of microscopic and macroscopic models; a review of the currently existing mesoscopic model has been performed.
4. The choice of a relatively new paradigm of flow systems, which is called the discrete rate approach, has been justified as a base for the development of mesoscopic models of traffic systems; an event planning opportunity was studied with respect to piecewise continuous processes, with the modelling being based on discrete rate approach; it is stated that the proposed event planning significantly accelerates the processing of flow systems mesoscopic models.
5. The discrete rate traffic reference model (DRTRM) is formulated and developed in mathematical form, showing the main ideas postulated in a proposed new mesoscopic traffic simulation model.
6. The traffic flow model development methodology was developed on DRTRM; some examples of the methodology application are described based on the demonstrative model of complex crossroads and the model of a real system.

7. Based on the data obtained from experiments, a validation of the developed models has been performed by using different methods, both qualitative and quantitative. Validation results prove that models developed by using DRTRM are valid; it is identified that practical benefits arising from the use of the new class of mesoscopic models lead to decrease of model development time and model processing time.
8. The example of using the proposed concept with respect to a real object located in Riga city has been presented, with a comparison of the real data obtained during the survey.

Further Research

Further research of DRTRM should be concentrated on different directions. Among these directions, the following must be mentioned as the most important:

- **Dynamic routing:** at the moment, DRTRM uses only static routing. It means that probabilities of turning movements are provided for each crossroad. But if the number of crossroads is high it could be a time-consuming problem. That is why the so-called dynamic routing is used in transport models, than the origin and destination zones for travellers in the form of OD matrix and route selection rules are provided. OD matrix is a standard form of presentation of movements and perfectly describes attraction and generation points in the network. The route selection rule is normally based on discrete choice theory. For discrete choice model [91] utility function should be defined. This utility function could integrate a lot of factors influencing route choice - like travel time, financial costs, and so on. So a good improvement of the presented model could be an integration of dynamic routing into it. It could provide more flexible simulation alternatives for user, from one point of view; from another point of view, it could extend the framework of model application. For instance, with the dynamic routing in the model, it could be possible to conduct a more extensive experiment with the model, like forecasting traffic volumes if some network elements will be closed. At the moment, such experiments are almost not possible with the provided model.
- **Collection of more reliable data:** as the proposed model describes a drivers' behaviour in general form, there is no need to operate high- precision data, like that for a microscopic, car-following principle-based models. But some data are required

anyway - e.g., the data describing how vehicles pass the crossroad. By obtaining and processing of that data, one could allow creating more valid models; this will eventually produce more precise output data.

- **Development of specialized simulation software or ExtendSim specialized traffic simulation library:** as it can be seen in Chapter 4. At the moment, two tools are available for creating the mesoscopic model universal programming language and ExtendSim discrete rate library. Unfortunately, in both cases the solution is not so good. With the growth of the number of intersections, the complexity of the code needed to write also increases; empirically, this approach could be good if the number of intersections is from 1 to 3. Using the discrete rate library allows one to model any number of crossroads, but the flexibility of model development will be lost within the framework of the functionality of discrete rate library blocks. That is why a good point of the future development is connected with development of specialized software, which will integrate a DRTRM and will provide a flexible interface for input and output data. This could be done either by creating a special traffic simulation library in ExtendSim or by development of standalone simulation software. The second option is more acceptable, as ExtendSim is not working properly with GIS. The standalone software could be directly based on GIS, like Google Maps or ArgGIS providing a very flexible network building and automatic collection of the some input data, like length of the road between crossroads, and so on. The input and output data interface could be provided for users in the form as it is accepted in traffic simulation software.

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APPENDIXES

Appendix 1

The list of scientific papers

1. M. Savrasovs, "Traffic Flow Simulation on Discrete Rate Approach Base", *Transport and Telecommunication*, Vol. 13, No. 2, 2012, pp. 167-173. [Citated in SCOPUS]
2. M. Savrasovs, "Urban transport corridor mesoscopic simulation", in *25th European Conference on Modelling and Simulation (ECMS'2011)*, Krakow, 2011, pp. 587-593. [Citated in SCOPUS]
3. M. Savrasovs, "The application of a discrete rate approach to traffic flow simulation," in *10th International Conference "Reliability and Statistics in Transportation and Communication*", Riga, 2010, pp. 433-439.
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17. M. Savrasovs, Y. Toluyew, "Application of Mesoscopic Modelling for Queuing Systems Research". *7th International Conference* "*Reliability and Statistics in Transportation and Communication*", Riga, 2007. pp. 94-99.
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Area of application: Analytical context

Analytical context	Analytical tools/Methodologies						
	Sketch Planning	Travel demand model	Analytical/ Deterministic tools	Traffic optimization	Macroscopic simulation	Mesoscopic simulation (DRTRM)	Microscopic simulation
Planning	●	●	∅	○	∅	○	○
Design	N/A	∅	●	●	●	●	●
Operations/ Construction	∅	○	●	●	●	●	●

The following notation is used for the table above:

● – specific context is generally addressed by the corresponding tools;

∅ – some tools address the specific context, some do not;

○ – the particular tools do not generally address the specific context;

N/A – the particular tool is not appropriate for use in addressing the specific context.

Area of application: Study area/Geographic scope

Analytical context/ Geographic scope	Analytical tools/Methodologies						
	Sketch Planning	Travel demand model	Analytical/ Deterministic tools	Traffic optimization	Macroscopic simulation	Mesoscopic simulation (DRTRM)	Microscopic simulation
Planning							
Isolated Locations	○	○	●	Ø	○	○	○
Segments	●	○	●	○	Ø	○	Ø
Corridor/ Small network	Ø	●	○	○	Ø	○	Ø
Region	Ø	●	N/A	N/A	N/A	N/A	N/A
Design							
Isolated Locations	N/A	N/A	●	●	●	●	●
Segments	N/A	○	●	Ø	●	●	●
Corridor/ Small network	N/A	Ø	○	○	●	●	●
Region	N/A	Ø	N/A	N/A	Ø	Ø	Ø
Operations/Constructions							
Isolated Locations	N/A	N/A	●	●	●	●	●
Segments	Ø	○	●	●	●	●	●
Corridor/ Small network	N/A	Ø	○	Ø	●	●	●
Region	N/A	Ø	N/A	N/A	Ø	Ø	Ø

The following notation is used for the table above:

● – specific context is generally addressed by the corresponding tools;

Ø – some tools address the specific context, and some do not;

○ – the particular tools do not generally address the specific context;

N/A – the particular tool is not appropriate for use in addressing the specific context.

Area of application: Facility type

Facility type	Analytical tools/Methodologies						
	Sketch Planning	Travel demand model	Analytical/ Deterministic tools	Traffic optimization	Macroscopic simulation	Mesoscopic simulation (DRTRM)	Microscopic simulation
Isolated intersections	○	Ø	●	●	●	●	●
Roundabouts	○	○	●	○	Ø	●	Ø
Arterial	●	●	●	●	●	●	●
Highway	●	●	●	Ø	●	●	●
Freeway	Ø	●	●	Ø	●	●	●
HOV Lanes	Ø	●	Ø	○	●	○	●
HOV Bypass Lane	○	●	○	Ø	Ø	○	●
Ramp	Ø	●	●	●	●	●	●
Auxibility lane	○	○	Ø	Ø	●	○	●
Reversible lane	○	Ø	○	○	○	○	Ø
Truck lane	○	●	Ø	Ø	Ø	○	●
Bus lanes	○	●	○	○	Ø	○	●
Toll Plaza	○	Ø	Ø	○	○	○	●
Light-Rail lane	○	●	○	○	○	○	●

The following notation is used for table above:

● – specific context is generally addressed by the corresponding tools;

Ø – some tools address the specific context, and some do not;

○ – the particular tools do not generally address the specific context;

N/A – the particular tool is not appropriate for use in addressing the specific context.

Area of application: Management strategy and application

Management strategy and applications	Analytical tools/Methodologies						
	Sketch Planning	Travel demand model	Analytical/ Deterministic tools	Traffic optimization	Macroscopic simulation	Mesoscopic simulation (DRTRM)	Microscopic simulation
Freeway management	●	∅	∅	●	●	●	●
Arterial intersections	○	○	●	●	●	●	●
Arterial management	∅	∅	∅	●	●	●	●
Incident management	∅	○	∅	○	∅	○	●
Emergency management	∅	○	∅	○	●	○	∅
Work zones	∅	○	●	○	∅	●	●
Special events	∅	○	●	○	∅	∅	∅
APTS	∅	○	○	○	○	○	∅
ATIS	∅	○	○	○	∅	○	∅
Electronic payment system	∅	○	○	○	○	○	●
Rail grade crossing monitors	∅	○	○	○	○	○	●
CVO	∅	○	○	○	○	○	∅
AVCSS	∅	○	○	○	○	○	∅
Weather management	○	○	○	○	∅	○	∅
TDM	●	●	∅	○	∅	○	∅

The following notation is used for table above:

● – specific context is generally addressed by the corresponding tools;

∅ – some tools address the specific context and some do not;

○ – the particular tools do not generally address the specific context;

N/A – the particular tool is not appropriate for use in addressing the specific context.

Source code for the 1st case study “One crossroad”

```
Option Explicit
Option Base 1
Dim tt As Long
Dim time As Long

Sub all_reset()
Dim zz As Double
zz = Rnd(-1)
Randomize (7)
Sheets("kernel").Select
Cells(2, 12) = ""
Cells(2, 14) = ""
End Sub

Sub full_run()
Dim maxsteps As Integer
Application.ScreenUpdating = False
maxsteps = Worksheets("input_data").Cells(4, 10)
Call all_reset
  Do
    Call meso_step
  Loop Until Worksheets("kernel").Cells(2, 12) >= maxsteps
End Sub

Sub meso_step()
Dim maxsteps As Integer
Dim timestep As Long
Dim i As Integer
Dim yy As Single

Application.ScreenUpdating = False
Sheets("kernel").Select
maxsteps = Worksheets("input_data").Cells(4, 10)
If Cells(2, 12) = "" Then
' Init
  Range("B3:I22").Select
  Selection.ClearContents
  For i = 3 To 22
    Cells(i, 9) = Cells(i, 10)
  Next
  Sheets("output_data").Select
  Range(Cells(5, 3), Cells(5, 50)).Select
  Selection.ClearContents
  i = 6
  Do
    i = i + 1
  Loop Until Cells(i + 1, 1) = ""
  Range(Cells(7, 1), Cells(i, 50)).Select
  Selection.ClearContents
  Cells(7, 2) = 0
  Cells(7, 11) = Worksheets("kernel").Cells(4, 9)
  Cells(7, 12) = Worksheets("kernel").Cells(5, 9)
  Cells(7, 13) = Worksheets("kernel").Cells(6, 9)
  Worksheets("kernel").Cells(7, 9) = Cells(7, 11) + Cells(7, 12) + Cells(7, 13)
  Cells(7, 14) = Worksheets("kernel").Cells(7, 9)

  Cells(7, 23) = Worksheets("kernel").Cells(9, 9)
  Cells(7, 24) = Worksheets("kernel").Cells(10, 9)
  Cells(7, 25) = Worksheets("kernel").Cells(11, 9)
  Worksheets("kernel").Cells(12, 9) = Cells(7, 23) + Cells(7, 24) + Cells(7, 25)
  Cells(7, 26) = Worksheets("kernel").Cells(12, 9)

  Cells(7, 35) = Worksheets("kernel").Cells(14, 9)
  Cells(7, 36) = Worksheets("kernel").Cells(15, 9)
  Cells(7, 37) = Worksheets("kernel").Cells(16, 9)
  Worksheets("kernel").Cells(17, 9) = Cells(7, 35) + Cells(7, 36) + Cells(7, 37)
  Cells(7, 38) = Worksheets("kernel").Cells(17, 9)

  Cells(7, 47) = Worksheets("kernel").Cells(19, 9)
  Cells(7, 48) = Worksheets("kernel").Cells(20, 9)
```

```
    Cells(7, 49) = Worksheets("kernel").Cells(21, 9)
    Worksheets("kernel").Cells(22, 9) = Cells(7, 47) + Cells(7, 48) + Cells(7, 49)
    Cells(7, 50) = Worksheets("kernel").Cells(22, 9)

    Sheets("kernel").Select
    tt = 0
    Cells(2, 12) = tt
    time = 0
    Cells(2, 14) = time

  ElseIf Cells(2, 12) < maxsteps Then
    tt = Cells(2, 12) + 1
    time = Cells(2, 14)
    timestep = Worksheets("input_data").Cells(7, 5)
    time = time + timestep
    Cells(2, 12) = tt
    Cells(2, 14) = time

  ' *********************************************
    Dim garant As Single
    Dim kolich As Single
    Dim faktich As Single
    Dim dopvozm As Single
    Call rigt_straight(1)
    Call rigt_straight(2)

    Cells(6, 8) = Cells(6, 5) - Cells(10, 8)    ' real time for left 1
    garant = Worksheets("input_data").Cells(27, 3)
    kolich = Cells(6, 4)
    If kolich <= garant Then
      faktich = kolich
    End If
    If kolich > garant Then
      faktich = garant
      kolich = kolich - garant
      If Abs(Cells(6, 8)) > 0.001 Then
        dopvozm = y_diagr(Cells(6, 8), 3)
        If kolich <= dopvozm Then
          faktich = faktich + kolich
        Else
          faktich = faktich + dopvozm
        End If
      End If
    End If
    Cells(6, 7) = faktich
    Cells(6, 9) = Cells(6, 4) - Cells(6, 7)
    Cells(7, 7) = Cells(4, 7) + Cells(5, 7) + Cells(6, 7)
    Cells(7, 9) = Cells(4, 9) + Cells(5, 9) + Cells(6, 9)

    Cells(11, 8) = Cells(11, 5) - Cells(5, 8)    ' real time for left 2
    garant = Worksheets("input_data").Cells(27, 4)
    kolich = Cells(11, 4)
    If kolich <= garant Then
      faktich = kolich
    End If
    If kolich > garant Then
      faktich = garant
      kolich = kolich - garant
      If Abs(Cells(11, 8)) > 0.001 Then
        dopvozm = y_diagr(Cells(11, 8), 6)
        If kolich <= dopvozm Then
          faktich = faktich + kolich
        Else
          faktich = faktich + dopvozm
        End If
      End If
    End If
    Cells(11, 7) = faktich
    Cells(11, 9) = Cells(11, 4) - Cells(11, 7)
    Cells(12, 7) = Cells(9, 7) + Cells(10, 7) + Cells(11, 7)
    Cells(12, 9) = Cells(9, 9) + Cells(10, 9) + Cells(11, 9)

    Call rigt_straight(3)
    Call rigt_straight(4)

    Cells(16, 8) = Cells(16, 5) - Cells(20, 8)    ' real time for left 3
```

```
  garant = Worksheets("input_data").Cells(27, 5)
  kolich = Cells(16, 4)
  If kolich <= garant Then
    faktich = kolich
  End If
  If kolich > garant Then
    faktich = garant
    kolich = kolich - garant
    If Abs(Cells(16, 8)) > 0.001 Then
      dopvozm = y_diagr(Cells(16, 8), 9)
      If kolich <= dopvozm Then
        faktich = faktich + kolich
      Else
        faktich = faktich + dopvozm
      End If
    End If
  End If
  Cells(16, 7) = faktich
  Cells(16, 9) = Cells(16, 4) - Cells(16, 7)
  Cells(17, 7) = Cells(14, 7) + Cells(15, 7) + Cells(16, 7)
  Cells(17, 9) = Cells(14, 9) + Cells(15, 9) + Cells(16, 9)

  Cells(21, 8) = Cells(21, 5) - Cells(15, 8)    ' real time for left 4
  garant = Worksheets("input_data").Cells(27, 6)
  kolich = Cells(21, 4)
  If kolich <= garant Then
    faktich = kolich
  End If
  If kolich > garant Then
    faktich = garant
    kolich = kolich - garant
    If Abs(Cells(21, 8)) > 0.001 Then
      dopvozm = y_diagr(Cells(21, 8), 12)
      If kolich <= dopvozm Then
        faktich = faktich + kolich
      Else
        faktich = faktich + dopvozm
      End If
    End If
  End If
  Cells(21, 7) = faktich
  Cells(21, 9) = Cells(21, 4) - Cells(21, 7)
  Cells(22, 7) = Cells(19, 7) + Cells(20, 7) + Cells(21, 7)
  Cells(22, 9) = Cells(19, 9) + Cells(20, 9) + Cells(21, 9)

  Call protocol
  'Worksheets("output_data").Cells(7 + tt, 1) = tt
  'Worksheets("output_data").Cells(7 + tt, 2) = time
End If
End Sub

Sub rigt_straight(source As Integer)  ' source=1,2,3,4
'Sub rigt_straight()  ' source=1,2,3,4
Dim p1 As Integer
Dim p2 As Integer
Dim p3 As Integer
Dim p4 As Integer
Dim p5 As Integer
'Dim source As Integer
'source = 1
Sheets("kernel").Select
Select Case source
  Case 1
  p1 = 0
  p2 = 1  ' flow1
  p3 = 2  ' flow2
  p4 = 3  ' flow3
  p5 = 1  ' column 1 or 3
  Case 2
  p1 = 5
  p2 = 4  ' flow1
  p3 = 5  ' flow2
  p4 = 6  ' flow3
  p5 = 1  ' column 1 or 3
  Case 3
  p1 = 10
```

```
  p2 = 7  ' flow1
  p3 = 8  ' flow2
  p4 = 9  ' flow3
  p5 = 3  ' column 1 or 3
  Case 4
  p1 = 15
  p2 = 10  ' flow1
  p3 = 11  ' flow2
  p4 = 12  ' flow3
  p5 = 3  ' column 1 or 3
End Select
  Cells(4 + p1, 2) = Cells(4 + p1, 9)
  Cells(5 + p1, 2) = Cells(5 + p1, 9)
  Cells(6 + p1, 2) = Cells(6 + p1, 9)
  Cells(7 + p1, 2) = Cells(7 + p1, 9)
  Cells(4 + p1, 3) = inp_objem(p2)
  Cells(5 + p1, 3) = inp_objem(p3)
  Cells(6 + p1, 3) = inp_objem(p4)
  Cells(7 + p1, 3) = Cells(4 + p1, 3) + Cells(5 + p1, 3) + Cells(6 + p1, 3)
  Cells(4 + p1, 4) = Cells(4 + p1, 2) + Cells(4 + p1, 3)
  Cells(5 + p1, 4) = Cells(5 + p1, 2) + Cells(5 + p1, 3)
  Cells(6 + p1, 4) = Cells(6 + p1, 2) + Cells(6 + p1, 3)
  Cells(7 + p1, 4) = Cells(7 + p1, 2) + Cells(7 + p1, 3)
  Cells(4 + p1, 5) = Worksheets("input_data").Cells(7, p5)
  Cells(5 + p1, 5) = Worksheets("input_data").Cells(7, p5)
  Cells(6 + p1, 5) = Worksheets("input_data").Cells(7, p5)
  Cells(4 + p1, 6) = y_diagr(Cells(4 + p1, 5), p2)
  Cells(5 + p1, 6) = y_diagr(Cells(5 + p1, 5), p3)
  Cells(6 + p1, 6) = y_diagr(Cells(6 + p1, 5), p4)
  Cells(7 + p1, 6).Value = Cells(4 + p1, 6) + Cells(5 + p1, 6) + Cells(6 + p1, 6)
  If Cells(4 + p1, 4) <= Cells(4 + p1, 6) Then
    Cells(4 + p1, 7) = Cells(4 + p1, 4)
  Else
    Cells(4 + p1, 7) = Cells(4 + p1, 6)
  End If
  If Cells(5 + p1, 4) <= Cells(5 + p1, 6) Then
    Cells(5 + p1, 7) = Cells(5 + p1, 4)
  Else
    Cells(5 + p1, 7) = Cells(5 + p1, 6)
  End If
  If Cells(4 + p1, 7) = Cells(4 + p1, 6) Then
    Cells(4 + p1, 8) = Cells(4 + p1, 5)
  Else
    Cells(4 + p1, 8) = x_diagr(Cells(4 + p1, 7), p2)
  End If
  If Cells(5 + p1, 7) = Cells(5 + p1, 6) Then
    Cells(5 + p1, 8) = Cells(5 + p1, 5)
  Else
    Cells(5 + p1, 8) = x_diagr(Cells(5 + p1, 7), p3)
  End If
  Cells(4 + p1, 9) = Cells(4 + p1, 4) - Cells(4 + p1, 7)
  Cells(5 + p1, 9) = Cells(5 + p1, 4) - Cells(5 + p1, 7)

End Sub

Sub diagr_podgot()
Dim i1 As Integer
Dim i2 As Integer
Dim j As Integer
Dim k As Single
Sheets("input_data").Select
' Init
  Range("B53:N65").Select
  Selection.ClearContents
i1 = 36
i2 = 52
Do
i1 = i1 + 1
i2 = i2 + 1
Cells(i2, 2) = Cells(i1, 3)
For j = 3 To 14
  k = Cells(22, j)
  Cells(i2, j) = Cells(i1, 4) * k
Next j
Loop Until Cells(i1 + 1, 3) = ""
End Sub
```

```
Function inp_objem(flow As Integer) As Single
Dim vnr As Integer
Dim a As Single
Dim b As Single
Dim c As Single
Dim d As Single
Dim sred As Single
Dim rr As Double
Dim rr0 As Double
Dim x As Double
Dim timestep As Long

vnr = Worksheets("input_data").Cells(13, flow + 2)
Select Case vnr
  Case 1
    c = Worksheets("input_data").Cells(14, flow + 2)
    a = c
    b = c
    x = c
  Case 2
    rr = Rnd
    a = Worksheets("input_data").Cells(15, flow + 2)
    b = Worksheets("input_data").Cells(16, flow + 2)
    c = (a + b) / 2
    Worksheets("input_data").Cells(14, flow + 2) = c
    x = a + rr * (b - a)
  Case 3
    rr = Rnd
    a = Worksheets("input_data").Cells(15, flow + 2)
    b = Worksheets("input_data").Cells(16, flow + 2)
    sred = Worksheets("input_data").Cells(14, flow + 2)
    c = 3 * sred - a - b
    rr0 = (c - a) / (b - a)
    If rr <= rr0 Then
      x = a + Sqr(rr * (c - a) * (b - a))
    Else
      d = c * (2 * b - c) + (rr - rr0) * (b - c) * (b - a)
      x = b - Sqr(b * b - d)
    End If
End Select
timestep = Worksheets("input_data").Cells(7, 5)
inp_objem = x * timestep / 60
End Function

Function y_diagr(x As Single, flow As Integer) As Single
Dim i As Integer
Dim y As Single
Dim x1 As Single
Dim x2 As Single
Dim y1 As Single
Dim y2 As Single
Sheets("input_data").Select
i = 52
Do
  i = i + 1
Loop Until Cells(i, 2) >= x Or Cells(i + 1, 2) = ""
x1 = Cells(i - 1, 2)
x2 = Cells(i, 2)
y1 = Cells(i - 1, flow + 2)
y2 = Cells(i, flow + 2)
y = y1 + (x - x1) * (y2 - y1) / (x2 - x1)
y_diagr = y
Sheets("kernel").Select
End Function

Function x_diagr(y As Single, flow As Integer) As Single
Dim i As Integer
Dim x As Single
Dim x1 As Single
Dim x2 As Single
Dim y1 As Single
Dim y2 As Single

Sheets("input_data").Select
i = 52
```

```
Do
  i = i + 1
Loop Until Cells(i, flow + 2) >= y Or Cells(i + 1, 2) = ""
x1 = Cells(i - 1, 2)
x2 = Cells(i, 2)
y1 = Cells(i - 1, flow + 2)
y2 = Cells(i, flow + 2)
x = x1 + (y - y1) * (x2 - x1) / (y2 - y1)
x_diagr = x
Sheets("kernel").Select
End Function

Sub protocol()
Dim summa As Double
Dim i As Integer
Dim j As Integer
Dim timestep As Long
Dim tau As Single
Dim lambda As Single

  timestep = Worksheets("input_data").Cells(7, 5)
  Sheets("output_data").Select
  Cells(7 + tt, 1) = tt
  Cells(7 + tt, 2) = time
' source 1
  Cells(7 + tt, 3) = Worksheets("kernel").Cells(4, 3)
  Cells(7 + tt, 4) = Worksheets("kernel").Cells(5, 3)
  Cells(7 + tt, 5) = Worksheets("kernel").Cells(6, 3)
  Cells(7 + tt, 6) = Worksheets("kernel").Cells(7, 3)
  Cells(7 + tt, 7) = Worksheets("kernel").Cells(4, 7)
  Cells(7 + tt, 8) = Worksheets("kernel").Cells(5, 7)
  Cells(7 + tt, 9) = Worksheets("kernel").Cells(6, 7)
  Cells(7 + tt, 10) = Worksheets("kernel").Cells(7, 7)
  For j = 3 To 10
    summa = 0
    For i = 7 To 7 + tt
      summa = summa + Cells(i, j)
    Next
    Cells(5, j) = summa
  Next
  tau = timestep - Worksheets("kernel").Cells(4, 5)
  lambda = Worksheets("kernel").Cells(4, 3) / timestep
  Cells(7 + tt, 11) = Worksheets("kernel").Cells(4, 2) + lambda * tau
  tau = timestep - Worksheets("kernel").Cells(5, 5)
  lambda = Worksheets("kernel").Cells(5, 3) / timestep
  Cells(7 + tt, 12) = Worksheets("kernel").Cells(5, 2) + lambda * tau
  tau = timestep - Worksheets("kernel").Cells(6, 8)
  lambda = Worksheets("kernel").Cells(6, 3) / timestep
  Cells(7 + tt, 13) = Worksheets("kernel").Cells(6, 2) + lambda * tau
  Cells(7 + tt, 14) = Cells(7 + tt, 11) + Cells(7 + tt, 12) + Cells(7 + tt, 13)
  For j = 11 To 14
    If Cells(7 + tt, j) > Cells(5, j) Then
      Cells(5, j) = Cells(7 + tt, j)
    End If
  Next
' source 2
  Cells(7 + tt, 15) = Worksheets("kernel").Cells(9, 3)
  Cells(7 + tt, 16) = Worksheets("kernel").Cells(10, 3)
  Cells(7 + tt, 17) = Worksheets("kernel").Cells(11, 3)
  Cells(7 + tt, 18) = Worksheets("kernel").Cells(12, 3)
  Cells(7 + tt, 19) = Worksheets("kernel").Cells(9, 7)
  Cells(7 + tt, 20) = Worksheets("kernel").Cells(10, 7)
  Cells(7 + tt, 21) = Worksheets("kernel").Cells(11, 7)
  Cells(7 + tt, 22) = Worksheets("kernel").Cells(12, 7)
  For j = 15 To 22
    summa = 0
    For i = 7 To 7 + tt
      summa = summa + Cells(i, j)
    Next
    Cells(5, j) = summa
  Next
  tau = timestep - Worksheets("kernel").Cells(9, 5)
  lambda = Worksheets("kernel").Cells(9, 3) / timestep
  Cells(7 + tt, 23) = Worksheets("kernel").Cells(9, 2) + lambda * tau
  tau = timestep - Worksheets("kernel").Cells(10, 5)
  lambda = Worksheets("kernel").Cells(10, 3) / timestep
```

```
  Cells(7 + tt, 24) = Worksheets("kernel").Cells(10, 2) + lambda * tau
  tau = timestep - Worksheets("kernel").Cells(11, 8)
  lambda = Worksheets("kernel").Cells(11, 3) / timestep
  Cells(7 + tt, 25) = Worksheets("kernel").Cells(11, 2) + lambda * tau
  Cells(7 + tt, 26) = Cells(7 + tt, 23) + Cells(7 + tt, 24) + Cells(7 + tt, 25)
  For j = 23 To 26
    If Cells(7 + tt, j) > Cells(5, j) Then
      Cells(5, j) = Cells(7 + tt, j)
    End If
  Next
' source 3
  Cells(7 + tt, 27) = Worksheets("kernel").Cells(14, 3)
  Cells(7 + tt, 28) = Worksheets("kernel").Cells(15, 3)
  Cells(7 + tt, 29) = Worksheets("kernel").Cells(16, 3)
  Cells(7 + tt, 30) = Worksheets("kernel").Cells(17, 3)
  Cells(7 + tt, 31) = Worksheets("kernel").Cells(14, 7)
  Cells(7 + tt, 32) = Worksheets("kernel").Cells(15, 7)
  Cells(7 + tt, 33) = Worksheets("kernel").Cells(16, 7)
  Cells(7 + tt, 34) = Worksheets("kernel").Cells(17, 7)
  For j = 27 To 34
    summa = 0
    For i = 7 To 7 + tt
      summa = summa + Cells(i, j)
    Next
    Cells(5, j) = summa
  Next
  tau = timestep - Worksheets("kernel").Cells(14, 5)
  lambda = Worksheets("kernel").Cells(14, 3) / timestep
  Cells(7 + tt, 35) = Worksheets("kernel").Cells(14, 2) + lambda * tau
  tau = timestep - Worksheets("kernel").Cells(15, 5)
  lambda = Worksheets("kernel").Cells(15, 3) / timestep
  Cells(7 + tt, 36) = Worksheets("kernel").Cells(15, 2) + lambda * tau
  tau = timestep - Worksheets("kernel").Cells(16, 8)
  lambda = Worksheets("kernel").Cells(16, 3) / timestep
  Cells(7 + tt, 37) = Worksheets("kernel").Cells(16, 2) + lambda * tau
  Cells(7 + tt, 38) = Cells(7 + tt, 35) + Cells(7 + tt, 36) + Cells(7 + tt, 37)
  For j = 35 To 38
    If Cells(7 + tt, j) > Cells(5, j) Then
      Cells(5, j) = Cells(7 + tt, j)
    End If
  Next
' source 4
  Cells(7 + tt, 39) = Worksheets("kernel").Cells(19, 3)
  Cells(7 + tt, 40) = Worksheets("kernel").Cells(20, 3)
  Cells(7 + tt, 41) = Worksheets("kernel").Cells(21, 3)
  Cells(7 + tt, 42) = Worksheets("kernel").Cells(22, 3)
  Cells(7 + tt, 43) = Worksheets("kernel").Cells(19, 7)
  Cells(7 + tt, 44) = Worksheets("kernel").Cells(20, 7)
  Cells(7 + tt, 45) = Worksheets("kernel").Cells(21, 7)
  Cells(7 + tt, 46) = Worksheets("kernel").Cells(22, 7)
  For j = 39 To 46
    summa = 0
    For i = 7 To 7 + tt
      summa = summa + Cells(i, j)
    Next
    Cells(5, j) = summa
  Next
  tau = timestep - Worksheets("kernel").Cells(19, 5)
  lambda = Worksheets("kernel").Cells(19, 3) / timestep
  Cells(7 + tt, 47) = Worksheets("kernel").Cells(19, 2) + lambda * tau
  tau = timestep - Worksheets("kernel").Cells(20, 5)
  lambda = Worksheets("kernel").Cells(20, 3) / timestep
  Cells(7 + tt, 48) = Worksheets("kernel").Cells(20, 2) + lambda * tau
  tau = timestep - Worksheets("kernel").Cells(21, 8)
  lambda = Worksheets("kernel").Cells(21, 3) / timestep
  Cells(7 + tt, 49) = Worksheets("kernel").Cells(21, 2) + lambda * tau
  Cells(7 + tt, 50) = Cells(7 + tt, 47) + Cells(7 + tt, 48) + Cells(7 + tt, 49)
  For j = 47 To 50
    If Cells(7 + tt, j) > Cells(5, j) Then
      Cells(5, j) = Cells(7 + tt, j)
    End If
  Next
  Sheets("kernel").Select
End Sub
```

Source code for the 2nd case study “Two connected crossroads”

```
Option Explicit
Option Base 1
Dim tt As Long
Dim tt1 As Long
Dim tt2 As Long
Dim time As Long
Dim node_nr As Integer
Dim faza1 As String
Dim faza2 As String
Dim kapa4 As Integer
Dim kapa7 As Integer

Sub all_reset()
Dim zz As Double
zz = Rnd(-1)
Randomize (10)
Sheets("kernel").Select
Cells(2, 12) = ""
Cells(2, 14) = ""
Cells(24, 12) = ""
Cells(24, 14) = ""
faza1 = "12"
'faza1 = "34"
faza2 = "56"
kapa4 = Worksheets("input_data").Cells(27, 11)
kapa7 = Worksheets("input_data2").Cells(27, 10)
End Sub

Sub full_run()
Dim maxtime As Integer
Application.ScreenUpdating = False
maxtime = Worksheets("input_data").Cells(4, 10)
Call all_reset
  Do
    Call meso_step
  Loop Until Worksheets("kernel").Cells(2, 14) >= maxtime Or _
  Worksheets("kernel").Cells(24, 14) >= maxtime
End Sub

Sub meso_step()
Dim maxtime As Integer
Dim timestep1 As Long
Dim timestep2 As Long
Dim i As Integer
Dim yy As Single

Application.ScreenUpdating = False
Sheets("kernel").Select
maxtime = Worksheets("input_data").Cells(4, 10)
If Cells(2, 12) = "" Then
' Init Cross-road1
  Range("B3:I22").Select
  Selection.ClearContents
  For i = 3 To 22
    Cells(i, 9) = Cells(i, 10)
  Next
  Sheets("output_data").Select
  Range(Cells(5, 3), Cells(5, 50)).Select
  Selection.ClearContents
  i = 6
  Do
    i = i + 1
  Loop Until Cells(i + 1, 1) = ""
  Range(Cells(7, 1), Cells(i, 50)).Select
  Selection.ClearContents
  Cells(7, 2) = 0
  Cells(7, 11) = Worksheets("kernel").Cells(4, 9)
  Cells(7, 12) = Worksheets("kernel").Cells(5, 9)
  Cells(7, 13) = Worksheets("kernel").Cells(6, 9)
  Worksheets("kernel").Cells(7, 9) = Cells(7, 11) + Cells(7, 12) + Cells(7, 13)
  Cells(7, 14) = Worksheets("kernel").Cells(7, 9)
```

```
  Cells(7, 23) = Worksheets("kernel").Cells(9, 9)
  Cells(7, 24) = Worksheets("kernel").Cells(10, 9)
  Cells(7, 25) = Worksheets("kernel").Cells(11, 9)
  Worksheets("kernel").Cells(12, 9) = Cells(7, 23) + Cells(7, 24) + Cells(7, 25)
  Cells(7, 26) = Worksheets("kernel").Cells(12, 9)

  Cells(7, 35) = Worksheets("kernel").Cells(14, 9)
  Cells(7, 36) = Worksheets("kernel").Cells(15, 9)
  Cells(7, 37) = Worksheets("kernel").Cells(16, 9)
  Worksheets("kernel").Cells(17, 9) = Cells(7, 35) + Cells(7, 36) + Cells(7, 37)
  Cells(7, 38) = Worksheets("kernel").Cells(17, 9)

  Cells(7, 47) = Worksheets("kernel").Cells(19, 9)
  Cells(7, 48) = Worksheets("kernel").Cells(20, 9)
  Cells(7, 49) = Worksheets("kernel").Cells(21, 9)
  Worksheets("kernel").Cells(22, 9) = Cells(7, 47) + Cells(7, 48) + Cells(7, 49)
  Cells(7, 50) = Worksheets("kernel").Cells(22, 9)

' Init Cross-road2
  Sheets("kernel").Select
  Range("B25:I44").Select
  Selection.ClearContents
  For i = 25 To 44
    Cells(i, 9) = Cells(i, 10)
  Next
  Sheets("output_data2").Select
  Range(Cells(5, 3), Cells(5, 50)).Select
  Selection.ClearContents
  i = 6
  Do
    i = i + 1
  Loop Until Cells(i + 1, 1) = ""
  Range(Cells(7, 1), Cells(i, 50)).Select
  Selection.ClearContents
  Cells(7, 2) = 0
  Cells(7, 11) = Worksheets("kernel").Cells(26, 9)
  Cells(7, 12) = Worksheets("kernel").Cells(27, 9)
  Cells(7, 13) = Worksheets("kernel").Cells(28, 9)
  Worksheets("kernel").Cells(29, 9) = Cells(7, 11) + Cells(7, 12) + Cells(7, 13)
  Cells(7, 14) = Worksheets("kernel").Cells(29, 9)

  Cells(7, 23) = Worksheets("kernel").Cells(31, 9)
  Cells(7, 24) = Worksheets("kernel").Cells(32, 9)
  Cells(7, 25) = Worksheets("kernel").Cells(33, 9)
  Worksheets("kernel").Cells(34, 9) = Cells(7, 23) + Cells(7, 24) + Cells(7, 25)
  Cells(7, 26) = Worksheets("kernel").Cells(34, 9)

  Cells(7, 35) = Worksheets("kernel").Cells(36, 9)
  Cells(7, 36) = Worksheets("kernel").Cells(37, 9)
  Cells(7, 37) = Worksheets("kernel").Cells(38, 9)
  Worksheets("kernel").Cells(39, 9) = Cells(7, 35) + Cells(7, 36) + Cells(7, 37)
  Cells(7, 38) = Worksheets("kernel").Cells(39, 9)

  Cells(7, 47) = Worksheets("kernel").Cells(41, 9)
  Cells(7, 48) = Worksheets("kernel").Cells(42, 9)
  Cells(7, 49) = Worksheets("kernel").Cells(43, 9)
  Worksheets("kernel").Cells(44, 9) = Cells(7, 47) + Cells(7, 48) + Cells(7, 49)
  Cells(7, 50) = Worksheets("kernel").Cells(44, 9)

  Sheets("kernel").Select
  time = 0
  tt1 = 0
  Cells(2, 12) = tt1
  Cells(2, 14) = time
  tt2 = 0
  Cells(24, 12) = tt2
  Cells(24, 14) = time

  'Bunker 4
  Cells(20, 11) = kapa4 - Cells(22, 9)
  Cells(22, 11) = 0
  'Bunker 7
  Cells(37, 11) = kapa7 - Cells(39, 9)
  Cells(39, 11) = 0

ElseIf Cells(2, 14) < maxtime And Cells(24, 14) < maxtime Then ' Beginn step
```

```
  Dim time1 As Long
  Dim time2 As Long
  If faza1 = "12" Then
    timestep1 = Worksheets("input_data").Cells(7, 1) + _
    Worksheets("input_data").Cells(7, 2)
  Else
    timestep1 = Worksheets("input_data").Cells(7, 3) + _
    Worksheets("input_data").Cells(7, 4)
  End If
  If faza2 = "56" Then
    timestep2 = Worksheets("input_data2").Cells(7, 1) + _
    Worksheets("input_data").Cells(7, 2)
  Else
    timestep2 = Worksheets("input_data2").Cells(7, 3) + _
    Worksheets("input_data").Cells(7, 4)
  End If
  time1 = Cells(2, 14) + timestep1
  time2 = Cells(24, 14) + timestep2
  If time1 <= time2 Then
    node_nr = 1
    time = time1
    Cells(2, 14) = time
    If faza1 = "12" Then
      Call rigt_straight1(2)
      Call rigt_straight1(1)
      Call real_left(1)
      Call real_left(2)
      'tt = tt1
      'Call protocol1
      faza1 = "34"
    Else
      Call rigt_straight1(3)
      Call rigt_straight1(4)
      Call real_left(3)
      Call real_left(4)
      tt1 = Cells(2, 12) + 1
      Cells(2, 12) = tt1
      tt = tt1
      Call protocol1
      faza1 = "12"
    End If
  Else
    node_nr = 2
    time = time2
    Cells(24, 14) = time
    If faza2 = "56" Then
      Call rigt_straight2(5)
      Call rigt_straight2(6)
      Call real_left(5)
      Call real_left(6)
      'tt = tt2
      'Call protocol2
      faza2 = "78"
    Else
      Call rigt_straight2(7)
      Call rigt_straight2(8)
      Call real_left(7)
      Call real_left(8)
      tt2 = Cells(24, 12) + 1
      Cells(24, 12) = tt2
      tt = tt2
      Call protocol2
      faza2 = "56"
    End If
  End If
' *********************************************

End If
End Sub

Sub real_left(source As Integer)        ' source=1,2,3,4,...,8
Dim p1 As Integer
Dim p2 As Integer
Dim p3 As Integer
Dim p4 As Integer
  Dim garant As Single
```

```
  Dim kolich As Single
  Dim faktich As Single
  Dim dopvozm As Single
Sheets("kernel").Select
Select Case source
  Case 1
  p1 = 6
  p2 = 10
  p3 = 3
  p4 = 7
  Case 2
  p1 = 11
  p2 = 5
  p3 = 4
  p4 = 12
  Case 3
  p1 = 16
  p2 = 20
  p3 = 5
  p4 = 17
  Case 4
  p1 = 21
  p2 = 15
  p3 = 6
  p4 = 22
  Case 5
  p1 = 28
  p2 = 32
  p3 = 3
  p4 = 29
  Case 6
  p1 = 33
  p2 = 27
  p3 = 4
  p4 = 34
  Case 7
  p1 = 38
  p2 = 42
  p3 = 5
  p4 = 39
  Case 8
  p1 = 43
  p2 = 37
  p3 = 6
  p4 = 44
End Select
  Cells(p1, 8) = Cells(p1, 5) - Cells(p2, 8)    ' real time for left 1
  If node_nr = 1 Then
    garant = Worksheets("input_data").Cells(27, p3)
  Else
    garant = Worksheets("input_data2").Cells(27, p3)
  End If
  kolich = Cells(p1, 4)
  If kolich <= garant Then
    faktich = kolich
  End If
  If kolich > garant Then
    faktich = garant
    kolich = kolich - garant
    If Abs(Cells(p1, 8)) > 0.001 Then
      If node_nr = 1 Then
        dopvozm = y_diagr1(Cells(p1, 8), 3)
      Else
        dopvozm = y_diagr2(Cells(p1, 8), 3)
      End If
      If kolich <= dopvozm Then
        faktich = faktich + kolich
      Else
        faktich = faktich + dopvozm
      End If
    End If
  End If
  Cells(p1, 7) = faktich

  ' input for Funnel 4
  If source = 6 Then
```

```
    If Cells(p1, 7) > Cells(20, 11) Then
      Cells(p1, 7) = Cells(20, 11)
    End If
    Cells(20, 11) = Cells(20, 11) - Cells(p1, 7)
    Cells(22, 11) = Cells(22, 11) + Cells(p1, 7)
  End If
  ' input for Funnel 7
  If source = 1 Then
    If Cells(p1, 7) > Cells(37, 11) Then
      Cells(p1, 7) = Cells(37, 11)
    End If
    Cells(37, 11) = Cells(37, 11) - Cells(p1, 7)
    Cells(39, 11) = Cells(39, 11) + Cells(p1, 7)
  End If

  Cells(p1, 9) = Cells(p1, 4) - Cells(p1, 7)
  Cells(p4, 7) = Cells(p4 - 3, 7) + Cells(p4 - 2, 7) + Cells(p4 - 1, 7)
  Cells(p4, 9) = Cells(p4 - 3, 9) + Cells(p4 - 2, 9) + Cells(p4 - 1, 9)

  If source = 4 Then
    Cells(20, 11) = kapa4 - Cells(p4, 9)
  End If
  If source = 7 Then
    Cells(37, 11) = kapa7 - Cells(p4, 9)
  End If
End Sub

Sub rigt_straight1(source As Integer)  ' source=1,2,3,4
'Sub rigt_straight()  ' source=1,2,3,4
Dim p1 As Integer
Dim p2 As Integer
Dim p3 As Integer
Dim p4 As Integer
Dim p5 As Integer
Dim pribitie As Single

'Dim source As Integer
'source = 1
Sheets("kernel").Select
Select Case source
  Case 1
  p1 = 0
  p2 = 1  ' flow1
  p3 = 2  ' flow2
  p4 = 3  ' flow3
  p5 = 1  ' column 1 or 3
  Case 2
  p1 = 5
  p2 = 4  ' flow1
  p3 = 5  ' flow2
  p4 = 6  ' flow3
  p5 = 1  ' column 1 or 3
  Case 3
  p1 = 10
  p2 = 7  ' flow1
  p3 = 8  ' flow2
  p4 = 9  ' flow3
  p5 = 3  ' column 1 or 3
  Case 4
  p1 = 15
  p2 = 10  ' flow1
  p3 = 11  ' flow2
  p4 = 12  ' flow3
  p5 = 3  ' column 1 or 3
End Select
  Cells(4 + p1, 2) = Cells(4 + p1, 9)
  Cells(5 + p1, 2) = Cells(5 + p1, 9)
  Cells(6 + p1, 2) = Cells(6 + p1, 9)
  Cells(7 + p1, 2) = Cells(7 + p1, 9)

  If source <> 4 Then
    Cells(4 + p1, 3) = inp_objem1(p2)
    Cells(5 + p1, 3) = inp_objem1(p3)
    Cells(6 + p1, 3) = inp_objem1(p4)
  Else
    pribitie = Cells(22, 11)
```

```
    Cells(22, 11) = 0
    Cells(20, 11) = kapa4
    Cells(4 + p1, 3) = 0.25 * pribitie
    Cells(5 + p1, 3) = 0.6 * pribitie
    Cells(6 + p1, 3) = 0.15 * pribitie
  End If

  Cells(7 + p1, 3) = Cells(4 + p1, 3) + Cells(5 + p1, 3) + Cells(6 + p1, 3)
  Cells(4 + p1, 4) = Cells(4 + p1, 2) + Cells(4 + p1, 3)
  Cells(5 + p1, 4) = Cells(5 + p1, 2) + Cells(5 + p1, 3)
  Cells(6 + p1, 4) = Cells(6 + p1, 2) + Cells(6 + p1, 3)
  Cells(7 + p1, 4) = Cells(7 + p1, 2) + Cells(7 + p1, 3)
  Cells(4 + p1, 5) = Worksheets("input_data").Cells(7, p5)
  Cells(5 + p1, 5) = Worksheets("input_data").Cells(7, p5)
  Cells(6 + p1, 5) = Worksheets("input_data").Cells(7, p5)
  Cells(4 + p1, 6) = y_diagr1(Cells(4 + p1, 5), p2)
  Cells(5 + p1, 6) = y_diagr1(Cells(5 + p1, 5), p3)
  Cells(6 + p1, 6) = y_diagr1(Cells(6 + p1, 5), p4)
  Cells(7 + p1, 6).Value = Cells(4 + p1, 6) + Cells(5 + p1, 6) + Cells(6 + p1, 6)

  ' right
  If Cells(4 + p1, 4) <= Cells(4 + p1, 6) Then
    Cells(4 + p1, 7) = Cells(4 + p1, 4)
  Else
    Cells(4 + p1, 7) = Cells(4 + p1, 6)
  End If
  If source = 2 Then
    If Cells(4 + p1, 7) > Cells(37, 11) Then
      Cells(4 + p1, 7) = Cells(37, 11)
    End If
    Cells(37, 11) = Cells(37, 11) - Cells(4 + p1, 7)
    Cells(39, 11) = Cells(39, 11) + Cells(4 + p1, 7)
  End If

  ' straight
  If Cells(5 + p1, 4) <= Cells(5 + p1, 6) Then
    Cells(5 + p1, 7) = Cells(5 + p1, 4)
  Else
    Cells(5 + p1, 7) = Cells(5 + p1, 6)
  End If
   If source = 3 Then
    If Cells(5 + p1, 7) > Cells(37, 11) Then
      Cells(5 + p1, 7) = Cells(37, 11)
    End If
    Cells(37, 11) = Cells(37, 11) - Cells(5 + p1, 7)
    Cells(39, 11) = Cells(39, 11) + Cells(5 + p1, 7)
  End If


  If Cells(4 + p1, 7) = Cells(4 + p1, 6) Then
    Cells(4 + p1, 8) = Cells(4 + p1, 5)
  Else
    Cells(4 + p1, 8) = x_diagr1(Cells(4 + p1, 7), p2)
  End If
  If Cells(5 + p1, 7) = Cells(5 + p1, 6) Then
    Cells(5 + p1, 8) = Cells(5 + p1, 5)
  Else
    Cells(5 + p1, 8) = x_diagr1(Cells(5 + p1, 7), p3)
  End If
  Cells(4 + p1, 9) = Cells(4 + p1, 4) - Cells(4 + p1, 7)
  Cells(5 + p1, 9) = Cells(5 + p1, 4) - Cells(5 + p1, 7)

End Sub

Sub rigt_straight2(source As Integer)  ' source=5,...,8
Dim p1 As Integer
Dim p2 As Integer
Dim p3 As Integer
Dim p4 As Integer
Dim p5 As Integer
Dim pribitie As Single

Sheets("kernel").Select
Select Case source
  Case 5
  p1 = 22
```

```
    p2 = 1  ' flow1
    p3 = 2  ' flow2
    p4 = 3  ' flow3
    p5 = 1  ' column 1 or 3
    Case 6
    p1 = 27
    p2 = 4  ' flow1
    p3 = 5  ' flow2
    p4 = 6  ' flow3
    p5 = 1  ' column 1 or 3
    Case 7
    p1 = 32
    p2 = 7  ' flow1
    p3 = 8  ' flow2
    p4 = 9  ' flow3
    p5 = 3  ' column 1 or 3
    Case 8
    p1 = 37
    p2 = 10  ' flow1
    p3 = 11  ' flow2
    p4 = 12  ' flow3
    p5 = 3  ' column 1 or 3
  End Select
    Cells(4 + p1, 2) = Cells(4 + p1, 9)
    Cells(5 + p1, 2) = Cells(5 + p1, 9)
    Cells(6 + p1, 2) = Cells(6 + p1, 9)
    Cells(7 + p1, 2) = Cells(7 + p1, 9)

    If source <> 7 Then
      Cells(4 + p1, 3) = inp_objem2(p2)
      Cells(5 + p1, 3) = inp_objem2(p3)
      Cells(6 + p1, 3) = inp_objem2(p4)
    Else
      pribitie = Cells(39, 11)
      Cells(39, 11) = 0
      Cells(37, 11) = kapa7
      Cells(4 + p1, 3) = 0.25 * pribitie
      Cells(5 + p1, 3) = 0.6 * pribitie
      Cells(6 + p1, 3) = 0.15 * pribitie
    End If

    Cells(7 + p1, 3) = Cells(4 + p1, 3) + Cells(5 + p1, 3) + Cells(6 + p1, 3)
    Cells(4 + p1, 4) = Cells(4 + p1, 2) + Cells(4 + p1, 3)
    Cells(5 + p1, 4) = Cells(5 + p1, 2) + Cells(5 + p1, 3)
    Cells(6 + p1, 4) = Cells(6 + p1, 2) + Cells(6 + p1, 3)
    Cells(7 + p1, 4) = Cells(7 + p1, 2) + Cells(7 + p1, 3)
    Cells(4 + p1, 5) = Worksheets("input_data2").Cells(7, p5)
    Cells(5 + p1, 5) = Worksheets("input_data2").Cells(7, p5)
    Cells(6 + p1, 5) = Worksheets("input_data2").Cells(7, p5)
    Cells(4 + p1, 6) = y_diagr2(Cells(4 + p1, 5), p2)
    Cells(5 + p1, 6) = y_diagr2(Cells(5 + p1, 5), p3)
    Cells(6 + p1, 6) = y_diagr2(Cells(6 + p1, 5), p4)
    Cells(7 + p1, 6).Value = Cells(4 + p1, 6) + Cells(5 + p1, 6) + Cells(6 + p1, 6)

    ' right
    If Cells(4 + p1, 4) <= Cells(4 + p1, 6) Then
      Cells(4 + p1, 7) = Cells(4 + p1, 4)
    Else
      Cells(4 + p1, 7) = Cells(4 + p1, 6)
    End If
    If source = 5 Then
      If Cells(4 + p1, 7) > Cells(20, 11) Then
        Cells(4 + p1, 7) = Cells(20, 11)
      End If
      Cells(20, 11) = Cells(20, 11) - Cells(4 + p1, 7)
      Cells(22, 11) = Cells(22, 11) + Cells(4 + p1, 7)
    End If

    ' straight
    If Cells(5 + p1, 4) <= Cells(5 + p1, 6) Then
      Cells(5 + p1, 7) = Cells(5 + p1, 4)
    Else
      Cells(5 + p1, 7) = Cells(5 + p1, 6)
    End If
     If source = 8 Then
      If Cells(5 + p1, 7) > Cells(20, 11) Then
```

```
      Cells(5 + p1, 7) = Cells(20, 11)
    End If
    Cells(20, 11) = Cells(20, 11) - Cells(5 + p1, 7)
    Cells(22, 11) = Cells(22, 11) + Cells(5 + p1, 7)
  End If

  If Cells(4 + p1, 7) = Cells(4 + p1, 6) Then
    Cells(4 + p1, 8) = Cells(4 + p1, 5)
  Else
    Cells(4 + p1, 8) = x_diagr2(Cells(4 + p1, 7), p2)
  End If
  If Cells(5 + p1, 7) = Cells(5 + p1, 6) Then
    Cells(5 + p1, 8) = Cells(5 + p1, 5)
  Else
    Cells(5 + p1, 8) = x_diagr2(Cells(5 + p1, 7), p3)
  End If
  Cells(4 + p1, 9) = Cells(4 + p1, 4) - Cells(4 + p1, 7)
  Cells(5 + p1, 9) = Cells(5 + p1, 4) - Cells(5 + p1, 7)

End Sub

Sub diagr_podgot()
Dim i1 As Integer
Dim i2 As Integer
Dim j As Integer
Dim k As Single
Sheets("input_data").Select
' Init
  Range("B53:N65").Select
  Selection.ClearContents
i1 = 36
i2 = 52
Do
i1 = i1 + 1
i2 = i2 + 1
Cells(i2, 2) = Cells(i1, 3)
For j = 3 To 14
  k = Cells(22, j)
  Cells(i2, j) = Cells(i1, 4) * k
Next j
Loop Until Cells(i1 + 1, 3) = ""

Sheets("input_data2").Select
' Init
  Range("B53:N65").Select
  Selection.ClearContents
i1 = 36
i2 = 52
Do
i1 = i1 + 1
i2 = i2 + 1
Cells(i2, 2) = Cells(i1, 3)
For j = 3 To 14
  k = Cells(22, j)
  Cells(i2, j) = Cells(i1, 4) * k
Next j
Loop Until Cells(i1 + 1, 3) = ""
End Sub

Function inp_objem1(flow As Integer) As Single
Dim vnr As Integer
Dim a As Single
Dim b As Single
Dim c As Single
Dim d As Single
Dim sred As Single
Dim rr As Double
Dim rr0 As Double
Dim x As Double
Dim timestep As Long

vnr = Worksheets("input_data").Cells(13, flow + 2)
Select Case vnr
  Case 1
    c = Worksheets("input_data").Cells(14, flow + 2)
    a = c
```

```
    b = c
    x = c
  Case 2
    rr = Rnd
    a = Worksheets("input_data").Cells(15, flow + 2)
    b = Worksheets("input_data").Cells(16, flow + 2)
    c = (a + b) / 2
    Worksheets("input_data").Cells(14, flow + 2) = c
    x = a + rr * (b - a)
  Case 3
    rr = Rnd
    a = Worksheets("input_data").Cells(15, flow + 2)
    b = Worksheets("input_data").Cells(16, flow + 2)
    sred = Worksheets("input_data").Cells(14, flow + 2)
    c = 3 * sred - a - b
    rr0 = (c - a) / (b - a)
    If rr <= rr0 Then
      x = a + Sqr(rr * (c - a) * (b - a))
    Else
      d = c * (2 * b - c) + (rr - rr0) * (b - c) * (b - a)
      x = b - Sqr(b * b - d)
    End If
End Select
timestep = Worksheets("input_data").Cells(7, 5)
inp_objem1 = x * timestep / 60
End Function

Function inp_objem2(flow As Integer) As Single
Dim vnr As Integer
Dim a As Single
Dim b As Single
Dim c As Single
Dim d As Single
Dim sred As Single
Dim rr As Double
Dim rr0 As Double
Dim x As Double
Dim timestep As Long

vnr = Worksheets("input_data2").Cells(13, flow + 2)
Select Case vnr
  Case 1
    c = Worksheets("input_data2").Cells(14, flow + 2)
    a = c
    b = c
    x = c
  Case 2
    rr = Rnd
    a = Worksheets("input_data2").Cells(15, flow + 2)
    b = Worksheets("input_data2").Cells(16, flow + 2)
    c = (a + b) / 2
    Worksheets("input_data2").Cells(14, flow + 2) = c
    x = a + rr * (b - a)
  Case 3
    rr = Rnd
    a = Worksheets("input_data2").Cells(15, flow + 2)
    b = Worksheets("input_data2").Cells(16, flow + 2)
    sred = Worksheets("input_data2").Cells(14, flow + 2)
    c = 3 * sred - a - b
    rr0 = (c - a) / (b - a)
    If rr <= rr0 Then
      x = a + Sqr(rr * (c - a) * (b - a))
    Else
      d = c * (2 * b - c) + (rr - rr0) * (b - c) * (b - a)
      x = b - Sqr(b * b - d)
    End If
End Select
timestep = Worksheets("input_data2").Cells(7, 5)
inp_objem2 = x * timestep / 60
End Function

Function y_diagr1(x As Single, flow As Integer) As Single
Dim i As Integer
Dim y As Single
Dim x1 As Single
Dim x2 As Single
```

```
Dim y1 As Single
Dim y2 As Single
Sheets("input_data").Select
i = 52
Do
  i = i + 1
Loop Until Cells(i, 2) >= x Or Cells(i + 1, 2) = ""
x1 = Cells(i - 1, 2)
x2 = Cells(i, 2)
y1 = Cells(i - 1, flow + 2)
y2 = Cells(i, flow + 2)
y = y1 + (x - x1) * (y2 - y1) / (x2 - x1)
y_diagr1 = y
Sheets("kernel").Select
End Function

Function x_diagr1(y As Single, flow As Integer) As Single
Dim i As Integer
Dim x As Single
Dim x1 As Single
Dim x2 As Single
Dim y1 As Single
Dim y2 As Single

Sheets("input_data").Select
i = 52
Do
  i = i + 1
Loop Until Cells(i, flow + 2) >= y Or Cells(i + 1, 2) = ""
If i = 53 Then
  x_diagr1 = 0
Else
  x1 = Cells(i - 1, 2)
  x2 = Cells(i, 2)
  y1 = Cells(i - 1, flow + 2)
  y2 = Cells(i, flow + 2)
  x = x1 + (y - y1) * (x2 - x1) / (y2 - y1)
  x_diagr1 = x
End If
Sheets("kernel").Select
End Function

Function y_diagr2(x As Single, flow As Integer) As Single
Dim i As Integer
Dim y As Single
Dim x1 As Single
Dim x2 As Single
Dim y1 As Single
Dim y2 As Single
Sheets("input_data2").Select
i = 52
Do
  i = i + 1
Loop Until Cells(i, 2) >= x Or Cells(i + 1, 2) = ""
x1 = Cells(i - 1, 2)
x2 = Cells(i, 2)
y1 = Cells(i - 1, flow + 2)
y2 = Cells(i, flow + 2)
y = y1 + (x - x1) * (y2 - y1) / (x2 - x1)
y_diagr2 = y
Sheets("kernel").Select
End Function

Function x_diagr2(y As Single, flow As Integer) As Single
Dim i As Integer
Dim x As Single
Dim x1 As Single
Dim x2 As Single
Dim y1 As Single
Dim y2 As Single

Sheets("input_data2").Select
i = 52
Do
  i = i + 1
Loop Until Cells(i, flow + 2) >= y Or Cells(i + 1, 2) = ""
```

```
If i = 53 Then
  x_diagr2 = 0
Else
  x1 = Cells(i - 1, 2)
  x2 = Cells(i, 2)
  y1 = Cells(i - 1, flow + 2)
  y2 = Cells(i, flow + 2)
  x = x1 + (y - y1) * (x2 - x1) / (y2 - y1)
  x_diagr2 = x
End If
Sheets("kernel").Select
End Function

Sub protocol1()
Dim summa As Double
Dim i As Integer
Dim j As Integer
Dim timestep As Long
Dim tau As Single
Dim lambda As Single

  timestep = Worksheets("input_data").Cells(7, 5)
  Sheets("output_data").Select
  Cells(7 + tt, 1) = tt
  Cells(7 + tt, 2) = time
' source 1
  Cells(7 + tt, 3) = Worksheets("kernel").Cells(4, 3)
  Cells(7 + tt, 4) = Worksheets("kernel").Cells(5, 3)
  Cells(7 + tt, 5) = Worksheets("kernel").Cells(6, 3)
  Cells(7 + tt, 6) = Worksheets("kernel").Cells(7, 3)
  Cells(7 + tt, 7) = Worksheets("kernel").Cells(4, 7)
  Cells(7 + tt, 8) = Worksheets("kernel").Cells(5, 7)
  Cells(7 + tt, 9) = Worksheets("kernel").Cells(6, 7)
  Cells(7 + tt, 10) = Worksheets("kernel").Cells(7, 7)
  For j = 3 To 10
    Cells(5, j) = Cells(5, j) + Cells(tt + 7, j)
  Next
  Cells(7 + tt, 11) = Worksheets("kernel").Cells(4, 2) + _
  Worksheets("kernel").Cells(4, 3)
  Cells(7 + tt, 12) = Worksheets("kernel").Cells(5, 2) + _
  Worksheets("kernel").Cells(5, 3)
  Cells(7 + tt, 13) = Worksheets("kernel").Cells(6, 2) + _
  Worksheets("kernel").Cells(6, 3)

  Cells(7 + tt, 14) = Cells(7 + tt, 11) + Cells(7 + tt, 12) + Cells(7 + tt, 13)
  For j = 11 To 14
    If Cells(7 + tt, j) > Cells(5, j) Then
      Cells(5, j) = Cells(7 + tt, j)
    End If
  Next
' source 2
  Cells(7 + tt, 15) = Worksheets("kernel").Cells(9, 3)
  Cells(7 + tt, 16) = Worksheets("kernel").Cells(10, 3)
  Cells(7 + tt, 17) = Worksheets("kernel").Cells(11, 3)
  Cells(7 + tt, 18) = Worksheets("kernel").Cells(12, 3)
  Cells(7 + tt, 19) = Worksheets("kernel").Cells(9, 7)
  Cells(7 + tt, 20) = Worksheets("kernel").Cells(10, 7)
  Cells(7 + tt, 21) = Worksheets("kernel").Cells(11, 7)
  Cells(7 + tt, 22) = Worksheets("kernel").Cells(12, 7)
  For j = 15 To 22
    Cells(5, j) = Cells(5, j) + Cells(tt + 7, j)
  Next
  Cells(7 + tt, 23) = Worksheets("kernel").Cells(9, 2) + _
  Worksheets("kernel").Cells(9, 3)
  Cells(7 + tt, 24) = Worksheets("kernel").Cells(10, 2) + _
  Worksheets("kernel").Cells(10, 3)
  Cells(7 + tt, 25) = Worksheets("kernel").Cells(11, 2) + _
  Worksheets("kernel").Cells(1, 3)

  Cells(7 + tt, 26) = Cells(7 + tt, 23) + Cells(7 + tt, 24) + Cells(7 + tt, 25)
  For j = 23 To 26
    If Cells(7 + tt, j) > Cells(5, j) Then
      Cells(5, j) = Cells(7 + tt, j)
    End If
  Next
' source 3
```

```
  Cells(7 + tt, 27) = Worksheets("kernel").Cells(14, 3)
  Cells(7 + tt, 28) = Worksheets("kernel").Cells(15, 3)
  Cells(7 + tt, 29) = Worksheets("kernel").Cells(16, 3)
  Cells(7 + tt, 30) = Worksheets("kernel").Cells(17, 3)
  Cells(7 + tt, 31) = Worksheets("kernel").Cells(14, 7)
  Cells(7 + tt, 32) = Worksheets("kernel").Cells(15, 7)
  Cells(7 + tt, 33) = Worksheets("kernel").Cells(16, 7)
  Cells(7 + tt, 34) = Worksheets("kernel").Cells(17, 7)
  For j = 27 To 34
    Cells(5, j) = Cells(5, j) + Cells(tt + 7, j)
  Next
  Cells(7 + tt, 35) = Worksheets("kernel").Cells(14, 2) + _
  Worksheets("kernel").Cells(14, 3)
  Cells(7 + tt, 36) = Worksheets("kernel").Cells(15, 2) + _
  Worksheets("kernel").Cells(15, 3)
  Cells(7 + tt, 37) = Worksheets("kernel").Cells(16, 2) + _
  Worksheets("kernel").Cells(16, 3)

  Cells(7 + tt, 38) = Cells(7 + tt, 35) + Cells(7 + tt, 36) + Cells(7 + tt, 37)
  For j = 35 To 38
    If Cells(7 + tt, j) > Cells(5, j) Then
      Cells(5, j) = Cells(7 + tt, j)
    End If
  Next
' source 4
  Cells(7 + tt, 39) = Worksheets("kernel").Cells(19, 3)
  Cells(7 + tt, 40) = Worksheets("kernel").Cells(20, 3)
  Cells(7 + tt, 41) = Worksheets("kernel").Cells(21, 3)
  Cells(7 + tt, 42) = Worksheets("kernel").Cells(22, 3)
  Cells(7 + tt, 43) = Worksheets("kernel").Cells(19, 7)
  Cells(7 + tt, 44) = Worksheets("kernel").Cells(20, 7)
  Cells(7 + tt, 45) = Worksheets("kernel").Cells(21, 7)
  Cells(7 + tt, 46) = Worksheets("kernel").Cells(22, 7)
  For j = 39 To 46
    Cells(5, j) = Cells(5, j) + Cells(tt + 7, j)
  Next
  Cells(7 + tt, 47) = Worksheets("kernel").Cells(19, 2) + _
  Worksheets("kernel").Cells(19, 3)
  Cells(7 + tt, 48) = Worksheets("kernel").Cells(20, 2) + _
  Worksheets("kernel").Cells(20, 3)
  Cells(7 + tt, 49) = Worksheets("kernel").Cells(21, 2) + _
  Worksheets("kernel").Cells(21, 3)

  Cells(7 + tt, 50) = Cells(7 + tt, 47) + Cells(7 + tt, 48) + Cells(7 + tt, 49)
  For j = 47 To 50
    If Cells(7 + tt, j) > Cells(5, j) Then
      Cells(5, j) = Cells(7 + tt, j)
    End If
  Next
  Sheets("kernel").Select
End Sub

Sub protocol2()
Dim summa As Double
Dim i As Integer
Dim j As Integer
Dim timestep As Long
Dim tau As Single
Dim lambda As Single

  timestep = Worksheets("input_data2").Cells(7, 5)
  Sheets("output_data2").Select
  Cells(7 + tt, 1) = tt
  Cells(7 + tt, 2) = time
' source 5
  Cells(7 + tt, 3) = Worksheets("kernel").Cells(26, 3)
  Cells(7 + tt, 4) = Worksheets("kernel").Cells(27, 3)
  Cells(7 + tt, 5) = Worksheets("kernel").Cells(28, 3)
  Cells(7 + tt, 6) = Worksheets("kernel").Cells(29, 3)
  Cells(7 + tt, 7) = Worksheets("kernel").Cells(26, 7)
  Cells(7 + tt, 8) = Worksheets("kernel").Cells(27, 7)
  Cells(7 + tt, 9) = Worksheets("kernel").Cells(28, 7)
  Cells(7 + tt, 10) = Worksheets("kernel").Cells(29, 7)
  For j = 3 To 10
    Cells(5, j) = Cells(5, j) + Cells(tt + 7, j)
  Next
```

```
    Cells(7 + tt, 11) = Worksheets("kernel").Cells(26, 2) + _
    Worksheets("kernel").Cells(26, 3)
    Cells(7 + tt, 12) = Worksheets("kernel").Cells(27, 2) + _
    Worksheets("kernel").Cells(27, 3)
    Cells(7 + tt, 13) = Worksheets("kernel").Cells(28, 2) + _
    Worksheets("kernel").Cells(28, 3)

    Cells(7 + tt, 14) = Cells(7 + tt, 11) + Cells(7 + tt, 12) + Cells(7 + tt, 13)
    For j = 11 To 14
      If Cells(7 + tt, j) > Cells(5, j) Then
        Cells(5, j) = Cells(7 + tt, j)
      End If
    Next
  ' source 6
    Cells(7 + tt, 15) = Worksheets("kernel").Cells(31, 3)
    Cells(7 + tt, 16) = Worksheets("kernel").Cells(32, 3)
    Cells(7 + tt, 17) = Worksheets("kernel").Cells(33, 3)
    Cells(7 + tt, 18) = Worksheets("kernel").Cells(34, 3)
    Cells(7 + tt, 19) = Worksheets("kernel").Cells(31, 7)
    Cells(7 + tt, 20) = Worksheets("kernel").Cells(32, 7)
    Cells(7 + tt, 21) = Worksheets("kernel").Cells(33, 7)
    Cells(7 + tt, 22) = Worksheets("kernel").Cells(34, 7)
    For j = 15 To 22
      Cells(5, j) = Cells(5, j) + Cells(tt + 7, j)
    Next
    Cells(7 + tt, 23) = Worksheets("kernel").Cells(31, 2) + _
    Worksheets("kernel").Cells(31, 3)
    Cells(7 + tt, 24) = Worksheets("kernel").Cells(32, 2) + _
    Worksheets("kernel").Cells(32, 3)
    Cells(7 + tt, 25) = Worksheets("kernel").Cells(33, 2) + _
    Worksheets("kernel").Cells(33, 3)

    Cells(7 + tt, 26) = Cells(7 + tt, 23) + Cells(7 + tt, 24) + Cells(7 + tt, 25)
    For j = 23 To 26
      If Cells(7 + tt, j) > Cells(5, j) Then
        Cells(5, j) = Cells(7 + tt, j)
      End If
    Next
  ' source 7
    Cells(7 + tt, 27) = Worksheets("kernel").Cells(36, 3)
    Cells(7 + tt, 28) = Worksheets("kernel").Cells(37, 3)
    Cells(7 + tt, 29) = Worksheets("kernel").Cells(38, 3)
    Cells(7 + tt, 30) = Worksheets("kernel").Cells(39, 3)
    Cells(7 + tt, 31) = Worksheets("kernel").Cells(36, 7)
    Cells(7 + tt, 32) = Worksheets("kernel").Cells(37, 7)
    Cells(7 + tt, 33) = Worksheets("kernel").Cells(38, 7)
    Cells(7 + tt, 34) = Worksheets("kernel").Cells(39, 7)
    For j = 27 To 34
      Cells(5, j) = Cells(5, j) + Cells(tt + 7, j)
    Next
    Cells(7 + tt, 35) = Worksheets("kernel").Cells(36, 2) + _
    Worksheets("kernel").Cells(36, 3)
    Cells(7 + tt, 36) = Worksheets("kernel").Cells(37, 2) + _
    Worksheets("kernel").Cells(37, 3)
    Cells(7 + tt, 37) = Worksheets("kernel").Cells(38, 2) + _
    Worksheets("kernel").Cells(38, 3)

    Cells(7 + tt, 38) = Cells(7 + tt, 35) + Cells(7 + tt, 36) + Cells(7 + tt, 37)
    For j = 35 To 38
      If Cells(7 + tt, j) > Cells(5, j) Then
        Cells(5, j) = Cells(7 + tt, j)
      End If
    Next
  ' source 8
    Cells(7 + tt, 39) = Worksheets("kernel").Cells(41, 3)
    Cells(7 + tt, 40) = Worksheets("kernel").Cells(42, 3)
    Cells(7 + tt, 41) = Worksheets("kernel").Cells(43, 3)
    Cells(7 + tt, 42) = Worksheets("kernel").Cells(44, 3)
    Cells(7 + tt, 43) = Worksheets("kernel").Cells(41, 7)
    Cells(7 + tt, 44) = Worksheets("kernel").Cells(42, 7)
    Cells(7 + tt, 45) = Worksheets("kernel").Cells(43, 7)
    Cells(7 + tt, 46) = Worksheets("kernel").Cells(44, 7)
    For j = 39 To 46
      Cells(5, j) = Cells(5, j) + Cells(tt + 7, j)
    Next
    Cells(7 + tt, 47) = Worksheets("kernel").Cells(41, 2) + _
```

```
  Worksheets("kernel").Cells(41, 3)
  Cells(7 + tt, 48) = Worksheets("kernel").Cells(42, 2) + _
  Worksheets("kernel").Cells(42, 3)
  Cells(7 + tt, 49) = Worksheets("kernel").Cells(43, 2) + _
  Worksheets("kernel").Cells(43, 3)

  Cells(7 + tt, 50) = Cells(7 + tt, 47) + Cells(7 + tt, 48) + Cells(7 + tt, 49)
  For j = 47 To 50
    If Cells(7 + tt, j) > Cells(5, j) Then
      Cells(5, j) = Cells(7 + tt, j)
    End If
  Next
  Sheets("kernel").Select
End Sub
```