TRANSPORT AND TELECOMMUNICATION INSTITUTE

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# MODELLING OF THE URBAN TRANSPORT IMPACT ON THE CITY ATMOSPHERIC ENVIRONMENT USING APPARATUS OF MATHEMATICAL PHYSICS

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The thesis defense at a viva voce will take place in _________ at the promotional council of the Institute of transport and communication, room 130, 1 Lomonosov Street, Riga, telephone: (+371) 7100617, факс: (+371) 7100535.

CONFIRMATION

I hereby confirm that I have accomplished this thesis which has been submitted for approval as a Dr. thesis in Engineering Science (Dr.sc.ing.) to the Institute of transport and communication, This thesis has not previously been submitted for consideration to any other promotional councils.

___ ____________ 2012 S.A.Fradkin

In April 2011, the author of the thesis has officially changed his full name to Stanislav Aryeh Fradkin (latvian: Staņislavs Arie Fradkins). Before the quoted time, he went under the name Stanislav Grishin (latvian: Staņislavs Grišins).

The Thesis consists of the Introduction, 6 chapters, the Conclusions and 9 Appendices. It contains 36 drawings and 16 tables. Contains 180 pages of the basic text as well as Introduction; contains 12 pages, Appendices – 103 pages. Comprising 283 pages in total. The reference list includes 188 titles.

# ANNOTATION

The thesis accomplished by Stanislav Aryeh Fradkin is entitled “Modelling of the urban transport impact on the city atmospheric environment using apparatus of mathematical physics”. Academic advisers: Dr.sc.math., associated professor Sharif Guseynov, Dr.sc.ing., professor Eugen Kopytov and Dr.sc.ing., associated professor Oleg Schiptsov.

The thesis has been dedicated to investigation and research of mathematical models describing the dynamics of distribution of harmful substances produced by motor transport in urban canyons. Mathematical models for unambiguous definition of harmful substances concentration in a city block are developed in the thesis. Such determination of harmful substances concentration in city blocks under investigation enables one to make decisions concerning the improvement of environmental situation that exercises an essential influence upon health condition of local population; furthermore, it enables one to change controllable parameters of transport regulation tools accordingly – to achieve optimal parameter values characterizing environmental conditions. As a result of control actions, purposeful rational changing of ecological pollution levels at the investigated road section becomes possible, thus promoting a fair solution of city traffic ecology problems to a large extent and enhancing controllability of urban ecology.

The results obtained are of universal nature; they can be applied for planning and/or investigation of any city blocks and assessing the condition of environment formed under impact from motor transport.

**CONTENTS**

# 1. ACTUALITY OF THE PROBLEM

The state-of-art problematic relevant to the field of vehicular traffic flow ecology is observed in the thesis that is common and quite typical to the bulk of cities worldwide. Motor transport is ranked as a first among the series of anthropogenic air pollution sources of the large cities. The constantly growing number of vehicles brings about an increasingly large level of contamination by such a noise and electromagnetic pollution produced by motor vehicle emissions (substances emitted in the course of fuel combustion), pollution of urban territory by all kinds of solid waste, distortion of water and tectonic balance with the environment, some landscape distortions, etc. This, in its turn, results in some irresolvable consequences, which manifest themselves, for example, by an increasing disease incidence among the population, people’s inability to work, and early deaths. All that is confirmed by a permanent growth of the number of cases of oncologic, cardiovascular, and lung diseases, observed among inhabitants of large cities. For instance, cancer morbidity rate among the people residing close to a busy highway is up to ten times higher than the one typical to residents of quarters located at a distance from highways. Currently in major cities sets of measures are taken, aimed at cutting down air contamination level produced by urban vehicles. As a rule, the proposed solutions are based on the data of contaminating substance concentration in urban air. To evaluate concentrations, various types of approach are applied, first of all, the mathematical statistics means.

The usage of the apparatus of mathematical statistics (and other mathematical toolsets where development of a model is based on the results of single shots and integral environmental measurements in particular, those of urban atmospheric environment) aimed at determination of the dynamics of exhaust gases concentration in the urban atmosphere – is connected with the problem of low accuracy of measurements. According to engineering practice, a computation error may number in the hundreds of a per cent. Mathematical statistics normally uses interpolation methods of modeling and extrapolation models to estimate ecological parameters. It is shown in the thesis that neither of the abovementioned models is capable of presenting an acceptable quality level in turbulent environment of a city. However, those models may be used, for instance, to investigate laminar flows in nature where concentration changes are not so fast.

It has been shown in the thesis that methods of mathematical statistics are not capable of solving transport ecology problems in the optimal way. That is a main reason why the apparatus of mathematical physics is used in this thesis to investigate ecological problems as a major research tool applied to study the problem of urban atmospheric pollution produced by motor vehicles, – enabling one to achieve a resulting high accuracy based on initial data. Due to existing complexities in models’ development, number of such models is relatively law. Models of that kind enable one to investigate the ecology of urban quarters in a more profound and detailed way, revealing "sore" issues; after a decision has been made, those models also make it possible to validate mathematically the expected positive changes resulting from those decisions, and to test the expected results in practice.

In the investigations conducted, mathematical models aimed at finding concentrations of harmful substances in urban environment have been developed, taking into account the nature of the investigated phenomenon.

Areas have been developed that propose to conduct further research on the impact of transport on the environment of the city.

## 2. GOAL AND TASKS OF RESEARCH

The goal of the thesis is perfection of processes of monitoring and transport-rendered air pollution forecast by way of development and implementation of continuous mathematical models to calculate concentration of harmful substances in urban atmosphere at a limited number of measurements.

To meet the target, the following tasks have been resolve:

1. To assess problems related to transport system's impact on urban ecology. To define research targets of transport ecology for this work.
2. Analyzing conditions of functioning of continuous mathematical models generated on the grounds of discrete initial data application.
3. To substantiate the development of mathematical models based on the apparatus of mathematical physics as a tool for finding concentrations of harmful substances at a targeted point of the observed space at an arbitrary point of time.
4. To develop non-stationary mathematical models to determine the dynamics of exhaust gases concentrations in an urban quarter. To analyze their application areas, and reveal the main constraints and drawbacks of the developed mathematical models.
5. To find analytical solutions of the mathematical problems proposed allowing to determine exhaust gases concentrations in an urban quarter.
6. To develop the mathematical models on computer.
7. To conduct numerical experiments and test the validation of the mathematical models of vehicular traffic flows, developed on the basis of actual data with respect to urban environment as exemplified by the city of Riga.

## 3. DEGREE OF THE THEME STUDIES

Some well-known mathematical model approaches used when investigating gas dynamics have been observed in the thesis; some of the fundamental models describing air flow motion process were reviewed – like the approach based on probabilistic and statistical calculation methods (extrapolation and interpolation models) and models based on the apparatus of mathematical physics.

At present, the best known probabilistic and statistical methods of calculating the degree of propagation of impurities in urban aerial environment include some models based on normal distribution of random variables. According to results of numerous experiments, the above-mentioned model can be used only for estimating the maximum possible land-based concentration of emissions under the worst-case conditions of dissipation. Some more complicated models – such as hydro-thermodynamic (or/and hydro-gasdynamic) models (considering turbulent processes in the atmosphere of the city) are used to describe the terrain, the vertical temperature profile, and other weather conditions**.** However, the adequacy of the hydro-thermodynamic model is doubtful in respect to investigation of actual pollution fields under restrained urban conditions and complicated micro-meteorology conditions. Some other transfer models are known as well – such as the one from A.P. Kurkovsky that was developed by INDIC (a Swedish company), etc. To be able to apply any probabilistic and statistical model with a high degree of assurance, statistically-valid testing of those models is required. However, usually there is a lack of time for doing that since a huge volume of measurement data with uncontrolled errors needs to be processed a few times a day. Moreover, such statistical testing would make no economic sense. That's why the adequacy of such models in terms of modelling urban atmospheric pollution areas under actual urban conditions and pollutant transfers – has not been confirmed by practice so far.

Turbulence investigation history, the main achievements, their brief description, and semi-empirical mathematical models of turbulence have been set forth in fundamental works dedicated to statistical theory of turbulence by A.S. Monin and A.V. Yagloma, and in the review by Y.V.Lapin. The further presentation of turbulent flow process is largely based on the abovementioned works and, partially, on the works of G.I. Marchuk, L.D. Landau and E.M. Lifshitz, P. Bradshow, Zh. Smagorinsky, F.R. Menther, M. Kuntz, R. Langtry, L.G. Loitsansky, A.V. Garbaruk, Y.V. Lapin, M.H. Strelets, P.R. Spalart and S.R. Allmaras, I.J. Ackermaun, H. Hass, M. Memmesheimer, S.B. Pope (the corresponding references can be found in the text of the thesis).

The beginning of scientific research in the field of turbulence starts with the works of O.Reynolds published in 1883 – 1895. The Reynolds method is used for averaging equations describing low-velocity (i.e. incompressible) turbulent viscous flows. However, that approach does not provide any solution of a problem within the framework of a pure mathematics. This is connected to the fact that the equations obtained by Reynolds are non-closed ones.

Those equations contain tensors of convective strains – both Reynolds and turbulent shear strains occurring from averaged products of velocity fluctuations. Their nature and properties are defined by the characteristics of pulsating motion. To close the main equations, it is necessary to find the relations among convective strains and the mean characteristic of flow. At this point, it should be emphasized that many theoretical and experimental efforts made over the last 40 – 50 years focused on obtaining exactly that type of relations, which may be applied to the entire diversity of mean flows to find a universal correlation. However, there is no guarantee that such a correlation does exist. Instead relying on a universally-applied model of turbulent motion in atmosphere or a liquid, there is the ever-increasing assurance that formulation of an adequate theory of turbulence requires a far better understanding of the physical properties of turbulent flow.

The commonly accepted classification of turbulence models uses the aggregate of semi-empirical relations, including differential equations called "turbulence model", to close Reynolds Averaged Navier-Stokes equartions: RANS Models.

The Reynolds approach is used quite seldom (mainly in model problems) to model turbulent flows of compressible mediums (as for instance, urban aerial environment). Averaging of equations which describe compressed gas flows is normally made by applying the approach of A. Favr where weight-average parameters of flow are introduced so as to satisfy the requirements of equation of continuity. In the thesis, A. Favr's approach is supported when developing the mathematical model of defining velocity of motion of a turbulent atmosphere.

Algebraic models of turbulence (the so-called zero-order models) imply that the relation between Reynolds' stress tensors and properties of mean motion is set by algebraic correlations. For example, it is assumed in L. Prandgle's theory and in the works by T. Cartman, F. van Drist, F. Clauser, B. Baldwin, D. Johnson and L. King, H. Dryden, V. Nevzglyadov, R. Traky and D. Vilsoks, M. Kato, and B. Launder that local change of average velocity of turbulent flow is determined by the first-order derivative from the average velocity of flow on transverse coordinate. According to T. Karman's theory, turbulent shear stresses are determined by the first-order and the second-order derivatives of average velocity on transverse coordinate. Algebraic models also include such widely known models as Baldwin-Lomax, Johnson-King, etc. Algebraic models of turbulence feature computational efficiency and simplicity of modification. However, all of them are extremely sensitive to any changes of flow parameters, and they are neutral if convective and diffusion transfers are ascendant.

Therefore, they do not fit into the mathematical models offered in this thesis, since those models take into account essential influences of convective and diffusion transfer.

The semi-empirical theory of L. Prandtl and its modifications are based on the hypothesis of locality of turbulent transfer mechanism. According to that hypothesis, turbulent stresses are dependent only on local structure of mean flow. In this thesis, as in all differential models of turbulence, the above-stated considerations have become the initial pre-requisite for working out semi-empirical mathematical models of turbulence based on second moment transfer equations. Those equations include, in particular, the turbulence kinetic energy transfer equation and the Reynolds stresses tensor component transfer equations.

In differential models of turbulence, a fundamental role is rooted in engaging the second moment transfer equations. For the first time ever, such an idea was reflected in the works by A. Kolmogorov and L. Prandtl. Differential turbulence models also include the ones based not on Kolmogorov-Prandtl hypothesis but rather on the hypothesis developed in the work by H. Dryden and V.G.Nevzglyadov. The estimates of efficiency of differential models of turbulence, made at the second AFOSR-HTTM Stanford Conference on Complex Turbulent Flows, September 03-06, 1980, Stanford, CA, USA) in 1980 especially mark dissatisfaction with the efficiency of a numerous family of models in terms of calculation of wall-adjacent flows – in particular, the calculation of turbulent boundary layer separation under the impact of positive pressure relief.

Two-parameter differential models of turbulence are competed by one-parameter differential models – such as that of Spalart-Allmaras, Sekundov model , the model $v_t - 92$, which is an advanced version of Sekundov model, the two-band model of F. Menter, etc.

The turbulence models using the hypothesis of Zh. Bussinesk relate to the group of Eddy Viscosity Model: EVM. The major drawback of Zh. Bussinesk's concept is the assumption of turbulent viscosity isotropy, which is a strongly simplifying assumption, and which is hardly suitable for interpretation of complex flows in urban atmosphere.

The work by S.B. Pope "Ten questions concerning the large-eddy simulation of turbulent flows" raises and discusses a number of important questions for theoretical substantiation of the new method of Large Eddy Simulation (LES). In this work, not all the questions have answers, and some of the answers are purely debatable. However, the most original ideas and approaches set forth by S.B. Pope have been directly used in this thesis when developing mathematical models for harmful substances' concentration dynamics determination observing the aerial environment of urban turbulent atmosphere.

Air quality standards (as for instance, average annual figures or 98 percentiles) are normally based on statistical data. Consequently, a number of samplings of the respective meteorological parameters for a definite period of time (preferably, for ten years) is required. If no such a data series is available, a corresponding mathematical model of a meteorological flow may be used, which was described by O.M. Belotserkovsky (constructed jointly with A.V. Babakov, A.M. Oparin, V.M. Chechetkin, V.A. Ilyin, G.I. Marchuk in different years). The "tail-plume" model with Gaussian distribution of concentration (the so-called Gaussian plume model) was described, for instance, by R. Pielke. Situations where emissions and/or meteorological parameters vary significantly in time and/or space, may be described by Gaussian puff model (A.Venkatram, J.C. Wyngaard, A.F.Stein). With respect to accidental emission or individual case analysis, the Lagrange model or the particle model (S. Aubrun, B. Leitl) is recommended.

Under circumstances implying great fluctuations of wind properties within the simulated zone, fall-out simulation modeling should take into account the influence of topographic structure upon three-dimensional air flow. That can be done by using the Gaussian puff model of harmful substances (emission gases) or the Lagrange model. Another method is a more complicated Eulerian modelling. To determine the airflow direction on a terrain with a complex relief structure, mathematical modelling of air stability or diagnostic flow (R.A. Pielke) may be applied.

While investigating air pollution produced by sources located at a low level, one has to take into account an influence of the surrounding buildings. A significant part of emissions from motor vehicles may delay in deep "canyons" of streets. To describe that process, some empirical patterns have been developed in the fundamental work by J.E. Cermak and N. Isyumov, and in the work by Th. Karman.

Therefore, the analysis of works performed in the field under consideration shows that at present time no universal models exist that would equally fit for describing the process of turbulent dynamics of gases under urban conditions, and that would take into account a large variety of influencing factors.

It should be emphasized that the mathematical models suggested in this thesis for unique determination of harmful substances concentration in urban aerial environment, produced by urban traffic, with turbulent air flows running (both in the terrestrial and the upper layers of urban atmosphere) – do not neither lay claim on all-round pattern of the above-stated models nor pretend on overall generality of considered relationships; they should rather be treated as mathematical models for adequate description of the complicated process of turbulent flow in urban aerial environment.

## 4. METHODOLOGY AND METHODS OF INVESTIGATION

In the thesis the theory of partial differential equations was used to explore the tasks (an important element of the apparatus of mathematical physics), as well as methods of probability theory and mathematical statistics. The developed models describe continuous gas dynamics in a turbulent atmosphere of a street section.

The thesis investigations are based on the following:

- the evolutionary laws of physics;
- apparatus of mathematical physics;
- the theory of differential equations in partial derivatives;
- numerical methods;
- mathematical analysis.

Academic literature and study materials pertaining to the above-stated field have been used in the thesis. They included, in particular, thematic materials of periodic publications dedicated to the problems considered in the thesis, papers and proceedings from international conferences, results of research efforts performed in the Transport and telecommunications institute, technical documentation, scientific literature in the field of engineering sciences, physics and mathematics.

To validate and verify the developed models, numerical experiments have been conducted. For this purpose, statistical data describing urban areas and quarters holding vehicular traffic flows has been utilized and processed. The software coded using C/C++ programming languages has been worked out. Additional calculations using the software package MathCAD were performed.

## 5. SCIENTIFIC NOVELTY OF THE WORK

The scientific results obtained in the thesis comprise the following:

1. A non-stationary mathematical model, three-dimensional with regard to spatial variables, has been proposed for analytic determination of transport's exhaust gases concentration dynamics in a urban atmosphere, with a priori preset data describing vertical velocity of a turbulent air flow. In the developed model, the turbulent and molecular diffusion ratios are assumed to be known.
2. A non-stationary mathematical model, three-dimensional with regard to spatial variables, has been proposed for analytic determination of transport's exhaust gases concentration dynamics in a urban area, assuming that vertical velocity of turbulent flow is unknown and that molecular diffusion ratio is known.
3. A new approach (previously unknown) has been offered to solve Kolmogotov-Petrovsky-Piskunov equation for multi-layer environment. In this thesis, the equation (with the corresponding initial-boundary conditions) arises in the determination of an unknown vertical velocity of the turbulent flow.
4. The impact rendered by urban relief upon atmospheric air flow has been investigated (the urban topology essentially disturbs all the main characteristics of the environment). Moreover, it has been proved mathematically that, given some definite values of the main environmental parameters (velocity, density, pressure, temperature) a 'solid wall" tales shape in the atmosphere, wherein the developed mathematical model has singular points (velocity and/or pressure may grow "interminably").

## 6. WORK PRACTICAL VALUE AND ITS IMPLEMENTATION

The models developed in this dissertation are implemented as a set of programs, which allow:

1. Making calculations of harmful substances concentrations at a stated moment of time, at any point of calculated space, by using a small number of points of measurement of urban atmospheric condition that may be arranged conveniently (from the engineering and economic standpoint) within the urban area under investigation.
2. Forecasting a change in harmful substances concentration in exhaust gases components at any point of investigated area of a city block – both with actual and hypothetic initial data.
3. Solving operational issues of traffic control in city streets, taking into account the environmental pollution of urban air (first of all, in residential blocks of the city).
4. To analyze the environmental situation in the residential quarters of the city on the basis of the constructed models, and to make evidence-based proposals for the layout of streets and neighborhoods in terms of environmental safety.
5. Implement the constructed models in prospective planning of urban infrastructure development, in planning new residential blocks, for new motorways construction purpose, and for analyzing ecological situations in residential city blocks.

## 7. APPROBATION OF THE RESEARCH

The results obtained in the thesis of the research were reported at 16 research and practice conferences held in France, Israel, Italy, Latvia, Norway, Netherlands and Spain:

1. The Research and Practice and Educational and Methodological Conference "**Science and Technology is the Step into the Future**", Transport and Telecommunication Institute, Riga, Latvia, December 15-16, 2006;

2. The International Conference "Reliability and Statistics in Transportation and Communication" **RelStat'07**, Transport and Telecommunication Institute, Riga, Latvia, October 24-27, 2007;

3. The International Conference "**Modelling of business, industrial and transport systems**", Transport and Telecommunication Institute, Riga, Latvia, May 7-10, 2008;

4. The International Conference "Reliability and Statistics in Transportation and Communication" **RelStat'08**, Transport and Telecommunication Institute, Riga, Latvia, October 15-18, 2008;

5. The International Conference "Reliability and Statistics in Transportation and Communication" **RelStat'09**, Transport and Telecommunication Institute, Riga, Latvia, October 21-24, 2009;

6. The International Symposium "Stochastic models in reliability engineering, life sciences and operations management" **SMRLO'10**, Shamoon College of Engineering, Beer Sheva, Israel, February 8-11, 2010;

7. The International Conference "Reliability and Statistics in Transportation and Communication" **RelStat'10**, Transport and Telecommunication Institute, Riga, Latvia, October 20-23, 2010;

8. The International Conference "**Leonardo da Vinci**", European Communications of Enterprices (EUCOMEN), Pescara, Italja, June 15, 2010;

9. 2-nd International Specialized Symposium "Space & Global Security of Humanity" (**SGS 2010**), Transport and Telecommunication Institute, Riga, Latvia, July 5-9, 2010;

10. The International Conference "**EUCOMEN project conference for SME's**", European Communications of Enterprices, Riga, Latvia, July 28, 2010;

11. The International Conference "**Let's Adapt Project final conference**", TI-SAETO Project, Gandia, Spain, September 17, 2010;

12. The International conference "**Total Quality Managemant: A Need For Success and Development in 21st Century**", TI-SAETO Project, Riga, Latvia, November 25, 2010;

13. The International Conference "International Conference on Sustainable Urban Transport and Environment" (**ICSUTE 2011**), World Academy of Science, Engineering and Technology (WASET), Paris, France, June 28-30, 2011;

14. The International Conference "Reliability and Statistics in Transportation and Communication" **RelStat'11**, Transport and Telecommunication Institute, Riga, Latvia, October 19-22, 2011;

15. The International Conference "The Sustainable Urban Neighbourhoods Conference" (**SUN2012**), Norwegian University of Science and Technology (NTNU), Trondheim, Norway, June 20-21, 2012;

16. The International Conference “International Conference on Environmental Pollution and Remediation” (**ICEPR'12**), World Academy of Science, Engineering and Technology (WASET), Amsterdam, The Netherlands; July 25-26, 2012.

In addition, a number of reports have been carried out at the seminars dedicated to COST project:

1. The seminar within the framework of COST project: TSI (TTI) and RTU. ‘Research of Transport System Influence on Urban Environment’, Riga, Latvia, January 17, 2009.
2. Calculation of Distribution Dynamics of Hazardous Substances Emitted by Vehicle Engines in a Residential Quarter, Stanislav Aryeh Fradkin. *COST TUD Action TU0902, WG3*, Transport and Telecommunication Institute Riga, Latvia, 9th to 10th, November 2011. (plenary).

The materials of promotional research have been reported many times at the seminars of the Chair and at doctoral seminars of the ESF (European Social Fund) including:

1. Extended scientific seminar of the Chair of Software and Computation Systems, Transport and Telecommunication Institute, Riga, Latvia, April 26, 2012;
2. Doctoral seminar ESF, Transport and Telecommunication Institute, Riga, Latvia, March 27, 2012;
3. Doctoral seminar ESF, Transport and Telecommunication Institute, Riga, Latvia, February 14, 2011;
4. Doctoral seminar ESF, Transport and Telecommunication Institute, Riga, Latvia, April 22, 2011.

In the International competition "For the best doctor’s, master’s, student’s and school research work connected with problems of the Riga transport communication" organized by the Riga City Council on 29th December 2009, the research paper "Mathematical modeling of the dynamics of the exhaust gas in the turbulent atmosphere of the city" which was performed in the process of the study of scientific problems of the thesis, won the 2nd place in the doctoral students group (first place was left vacant).

# 8. PUBLICATIONS

Based on the results dissertation work research, 22 scientific papers, including 14 scientific articles and 8 conference abstracts, have been published at international conferences. All of them consider problems related to transport ecology and environmental monitoring, as well as development and research of non-stationary mathematic models of the dynamics of concentration of exhaust gases.

# 9. STRUCTURE OF THE THESIS

The Thesis consists of the Introduction, 6 chapters, the Conclusions and 9 Appendices. It contains 36 drawings and 16 tables. Contains 180 pages of the basic text as well as Introduction; contains 12 pages, Appendices – 103 pages. Comprising 283 pages in total. The reference list includes 188 titles.

*Introduction* is dedicated to considering the relevancy of the subject of Thesis, formulating the research goal and objectives; the scientific novelty and the practical value of the obtained results is shown and the summary of the work is presented.

*Chapter 1* shows the problems related to the environmental impact of transport system, and vehicular traffic flows on urban ecology. The specific features of global environmental standards are considered. The task assignment for the dissertation thesis is developed. The Author draws the conclusions that solution of problems related to the transport influence on urban air needs to select the means of research and mathematical models generation for which the initial input data (altered indicators of environmental situation) are discrete. The conditions of functioning provision of continuous mathematical models generated on the basis of discrete initial data.

*Chapter 2* analyzes the existing mathematical models. A mathematical apparatus is selected to resolve the posed objectives based on classification, analysis, and peculiar features of turbulent flow models in urban aerial environment, and their impact upon the quality of mathematical modeling.

*Chapter 3* constructing of hydro-thermodynamic equations of atmospheric processes in mesoscale; special numerical algorithms for its implementation are developed. The impact rendered by irregularities of the ground terrain on air mass flows is studied mathematically.

*Chapter 4* is dedicated to the development and research of a non-stationary mathematical model of exhaust gases concentration dynamics in urban aerial environment. The turbulent and molecular diffusion ratio is assumed to be known in the model, and changing subject to vertical distance from the earth surface. A non-stationary advanced mathematical model, three-dimensional in terms of space variables, assuming that a vertical velocity of turbulent flow is unknown and the molecular diffusion ratio is known – is developed and investigated for analytical determination of exhaust fumes concentration dynamics in a city.

*Chapter 5* presents a computer-based implementation of the developed mathematical model using software package MathCAD and program code using the programming language C/C++ in the framework of MS Visual Studio 2010 environment.

*Chapter 6* validation of generated models is carried out. Investigates some methods of practical implementation of the constructed mathematical model. The directions proposing to carry out further research of transport influence on the urban ecology are developed: the approach to evaluate the effect of contamination caused by urban transport upon human health; the models to evaluate risks of the effect of contamination caused by urban transport upon human health.

*Conclusion* presents the summary of the most significant results of the thesis.

*Appendix 1* contains the statement and the analysis of statistical data describing the ecological situation in Riga and Latvia, received from the Central statistical bureau of Latvia.

*Appendix 2* contains some proposals concerning the establishment of decision-making support system corresponding to the system of ecological monitoring of urban environment.

*Appendix 3* presents proposed versions of environmental risk evaluation for the system having various numbers of states.

*Appendix 4* presents developed probability analysis of environmental safety (PAES) and the set of tasks to evaluate the damage caused by contamination of urban environment by means of transport.

*Appendix 5* presents environmental standards Euro 0-6.

*Appendix 6* presents particularities of environmental situation development in Latvia in 2009-2015.

*Appendix 7* presents some algorithms in MathCAD software package.

*Appendix 8* presents the entire code for the program written in C/C++ programming language in the framework of MS Visual Studio 2010 environment – to calculate concentrations of any harmful substance at any specific time, at any point of the observed area of a city canyon; the entire program code has been created based on the decveloped non-stationary mathematical model, three-dimensional in terms of spatial variables.

*Appendix 9* presents a copy of the Diploma of International competition organized by the Riga City Council, entitled "For the best doctor's, master's, student's and school research work connected with problems of the Riga transport communication".

## 10. REVIEW OF PRINCIPAL RESULTS OF RESEARCH

***The first chapter*** of the thesis contains a rough analysis of the main existing problems of urban transport system. Problems connected with urban air pollution from motor transport, as well as those of traffic safety and environmental impact are highlighted as the ones inflicting the largest cumulative damage upon the environment. Table 1. demonstratively illustrates the percentage of that influence. From the table it can be seen that the main fault in environmental pollution is because of automobile transport and comprises more than 95%. A review of the main ecological components of damages inflicted by motor transport is proposed.

*Table 1.*

*Polluting substances influence depending on type of vehicles*

| Vehicle types | Polluting substances influence, % |
|---|---|
| Automobile transport | 95.6 |
| Air transport | 1.2 |
| Railway transport | 1.1 |
| Sea transport | 0.6 |
| River transport | 0.6 |
| Road servicing | 0.9 |
| *Total* | *100.0* |

A number of negative impacts from transport have been analyzed: the main kinds of transport influence upon natural and artificial ecosystems, environmental pollution from motor vehicles, environmental pollution from stationary sources in transport, noise and vibration impacts from transport, and consequences of transport accidents. To assess the ecological condition of Latvia (using monitoring), pollution parameters and the amount of financing the ecology programs in Latvia according to the Main statistical office of Latvia have been investigated. The investigation results are stated in Appendix 1 of Thesis.

To assess both the impact on human health rendered by urban traffic and the respective risks, financial losses due to anthropogenic pollution of atmospheric environment were shown. For example, the annual environmental costs spent by the UK with regard to air pollution make up 100,000,000 pound sterling, as against 700 billion Yen by Japan, 1,5 billion USD by USA. It is assumed in USA that clean air would allow one to cut down medical expenses by more than 2 billion USD per annum. It is quite obvious that huge money is spent word-wide as environmental costs to eliminate urban pollution.

The impact rendered by transport on the urban plant world results in polluting the habitat of animals and birds, reducing fertile soil acreage and depletion in numbers of animals and birds caused by noise, vibration, and urban air pollution; moreover, the transport-rendered impact leads to depletion in numbers of animals and birds due to pollution of their habitats, numerous road accidents killing animals, and affects human health etc. (see Fig.1).

Harmful substances behave in a specific way in a city (street) canyon, accumulating and changing their concentrations under the influence of a set of factors. The notion *"street canyon"* implies a motorway section congested by buildings from both sides. The less is the canyon width, the higher concentrations are observed. To assess pollution in street canyons, a large number of investigations have been conducted; they are next larger than the corresponding investigations of open-type motorways. In process of investigating pollutant ventilation within the boundaries of street canyons, a tendency of an abrupt reduction of admixture concentration from top to bottom of the canyon is observed. A small intensity of motor traffic (400 – 500 motor vehicles per hour) implies toxic substances affecting the 1$^{st}$ … the 4$^{th}$ floors of buildings (this fact is confirmed in Chapter 6 of the Thesis.)

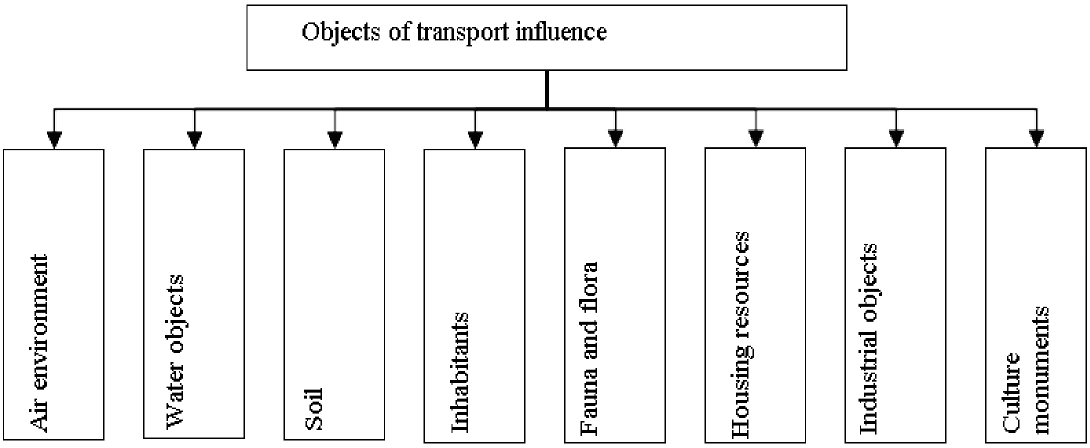


***Fig. 1.*** *Objects of destructing influence of vehicles*

All of the above-stated gives rise to the problem of safety of transport, which should be solved at the international level, by developing corresponding standards. The problem of motor vehicle structure security provision, in the context of a continuous increase in the number of manufactured vehicles, large-scale exhaust emissions, losses incurred by Society due to road traffic accidents (RTA), as well as the development of international trade, freight traffic and motor tourism – has become international. At present, the problem solution is a subject of activities run by a number of international agencies, with the leading ones being: Inland Transport Committee of UN Economic Commission for Europe (UNICE), International Standarts Organisation, European Economic Community (EEC), International Road Transport Union (MCAT). For instance, International Road Transport Union (MCAT), acknowledging the importance of cutting down emissions, has enforced realization of the practical decisions made by virtue of the introduced new harmful substance emission standards Euro – 0 (since 1990), Euro -1 (since 1993), Euro-2 (since 1996), Euro -3 (since 2001), Euro-4 (since 2006), and Euro - 5 (since 2009). According to the results of the measures taken as against 1990, CH emissions have been reduced by 81%, NOx – by 86%, CO – by 87% and particle pollutants – by 97%. The Appendix 5 of Thesis contains a short description of ecological standards Euro 0-6.

The Global document entitled „Planning of sustainable development of cities: global report on residential areas - 2009" contains an assessment of urban planning as a tool of solving unprecedented problems faced by cities in XXI century. It is pointed out that urban planning systems in many countries of the world are far from perfection, being a source of urban problems rather than a tool of environmental and people’s lives improvement. The existing approaches to planning should be changed; planning should be given a new niche in the provision of sustainable urban planning.

The Thesis, based on some examples of projects and documents pertaining to urban development (taking into account the priority orientations typical for the work of EU and other countries of the world), and based on determination of concentration of harmful substances related to urban transport operation) – has shown the necessity of developing mathematical models to be used in process of environmental monitoring. Those mathematical models may be used to solve a wide spectrum of transport ecology-related problems including, in particular, the refinement of planning of the prospective urban development.

In order to minimize pollution level, to create “ventilated” urban districts and minimize the impact rendered by city canyons upon harmful substances concentration etc. – the urban development taking into account transport ecology is currently becoming the leitmotif. To create a favorable urban environment taking into account the successes achieved in inter-sector cooperation, such a planning is actively realized according to the conclusions made by EU Topical strategy regarding urban environment. The circumstances pointed out in the Thesis account for selecting the problem of city canyon pollution as the main target of promotional research.

The Thesis shows that, in order to solve transport ecology problems, the corresponding mathematical models should be developed. As input parameters for such a class of models, actual data obtained from measurement stations of various types are used. All of that data is discrete (Tabular). In this connection, the burning issue of supporting the functioning of the above-mentioned transport ecology models should be discussed. An important aspect is the legitimacy of using discrete measurements of initial data for applying it in continuous mathematical models used to calculate harmful substances concentration in a city block.

Since in Chapters 3 and 4 of this Thesis some continuous and non-stationary mathematical models are developed and investigated as means of description of transport-produced exhaust gases concentration dynamics in urban atmosphere – the first chapter was dedicated to discussion of those models operation support. All the models developed in the Thesis have the form of initial boundary value problems of different level and complexity – for linear or non-linear partial differential equations (or systems of equations). Consequently, to ensure correctness of mathematical setting of those problems, some requirements have to be posed. First, some qualitative conditions should be observed (integrability by a degree, continuous differentiability, second-order continuous differentiability, sufficient smoothness etc.) both for the sought-for function and the initial functional data (for functional coefficients within equations, for initial functions, for boundary functions under initial boundary conditions). Secondly, it is necessary to highlight explicitly the natural function spaces wherein those initial boundary value problems operate.

To provide for the viability of the continuous mathematical models developed in the Thesis, one should pass over from initial discrete data to functions integrated with a degree, or continuous functions, or functions continuously differentiable a required number of times. In other words, to ensure functioning of both the continuous mathematical models developed in the Thesis and the algorithms developed for their analytical numerical solutions, it is necessary, first of all, to formulate and investigate the problem of interpolation of initial functions set by discrete method – i.e., tabularized ones. In the first Chapter, the interpolation problem is formulated, as well as some methods of its solution. At the same time, some examples are considered, illustrating the danger of a “blind” usage of interpolation formulas.

Various interpolation options are considered – such as generalized polynomial interpolation, Lagrange polynomial interpolation, and spline interpolation. A simple analysis of interpolation has shown that the improvement of desired precision of approximation of continuously differentiable function due to raising the degree of interpolation polynomial is connected with raising computational complexity essentially. Moreover, using high-degree interpolation polynomials requires some special precautionary measures to be taken already at the stage of selecting their notation form. Computations are accompanied by a dangerous round-off accumulation, leading to loss of stability. Therefore, piecewise-polynomial interpolation, using low-degree polynomials, is recommended in practice. However, this approximation method has an essential drawback: the variable, as a rule, has a discontinuity at the "junction point" of two adjacent polynomials (i.e., layers). This means that the important requirement of smoothness of approximating function turns out to be a large order, i.e., the simplest piecewise-polynomial interpolation becomes unacceptable.

A physical need of the availability of approximating functions that would combine the local simplicity of a low-degree polynomial and the global smoothness property throughout the entire section $[a,b]$ – lead to the occurrence of the so-called spline fuinctions. The term "spline" is to be interpreted as a "flexible line". Splines are smooth piecewise-polynomial functions specially developed; they became popular in the 60-ties of XX century as a means of interpretation of compound curves. By now, they have become an important constituent part of most variegated computation methods, and have become common use when solving various research-and-technology and engineering problems.

As a result of analysis, and with the aid of examples, it has been shown that the spline method enables one to solve the important problem of coordination between the boundary and the coordinated initial data (functions). This greatly simplifies initialization and setting of the initial development data to be used in ecological physical mathematical models applied to calculate concentrations of harmful substances, which are described in the next chapters of the Thesis.

***In the second chapter*** there is investigated mathematical means choice problem. For making this choice in the beginning there is considered urban atmosphere, the classification and analysis of turbulent flows in urban atmosphere are fulfilled. There are described properties of turbulent flows in atmosphere and of their influence on the simulation study quality. There was carried out the research of and was given the description of polluting substance movement process physics in urban atmosphere (the source of these substances is traffic). This has allowed to show statistics big volume processing necessity in use of the research model based on the probability theory and mathematical statistics means.

Taking into consideration process physics description, the alternative means of research are the offer of the use of differential equation means in partial derivatives (mathematical physics means). This approach could seem unacceptable from the point of view of specialists using in last decades only the probability theory means in solving similar tasks (using ecological monitoring). However, such an approach to the investigation of similar class tasks allows to reflect deeper the physics of the processes, to decrease the volume of necessary statistics and to increase result accuracy taking into account process dynamics.

There is shown in the work with the use of the specific examples of ecological problem research the validity of two approaches: one of the probability theory and mathematical statistics means and of fundamentally different one, connected with the use of the differential equations theory in partial derivatives (mathematical physics means). Advantages and drawbacks of both approaches are shown in Table 2.

To evaluate the implementation of solutions have traditionally involved apparatus of mathematical statistics. In connection with the above-stated, it should be noticed that application of mathematical statistics (and other mathematical apparatuses implying a model construction based on results of single shots and integral measurements of environmental quality – in particular, that of urban atmospheric air) to find exhaust gases concentration dynamics in urban air – is connected with the problem of accuracy of observation. As engineering practice shows, here the estimation and calculation error may constitute hundreds of a per cent.

Even if we improve accuracy by increasing measuring frequency as many times as we like – we will not manage to receive a plausible picture of urban air based on single shots anyway – especially, under dynamically changing conditions of a city. Consequently, neither should one be satisfied with the accuracy of integral criteria, indices, and indicators as atmospheric air quality assessment tools. Moreover, it is impossible to install measuring equipment in each point of residential quarter and at every demanding area. Therefore, one has to restore the general picture of urban aerial environment pollution, based on results of measurements performed at separate points, by means of using interpolation models.

*Table 2*

*Advantages and drawbacks of both ecological problem investigations' approaches*

| **Probabilistic statistical approach** | **Mathematical physics approach** |
|---|---|
| ***Advantages*** | ***Advantages*** |
| • Simplicity<br>• Availability of user experience<br>• Availability of experts | • Accuracy<br>• Stability<br>• Continuity<br>• Adequacy with regard to physics of processes<br>• Minimum statistical data required<br>• Taking into account all possible parameters and characteristics<br>• Forecast possibility<br>• Possibility of inputting any parameters (when designing) |
| ***Drawbacks*** | ***Drawbacks*** |
| • Inaccuracy<br>• Instability<br>• Too many factors<br>• Insufficient volume of data<br>• Extensive data volumes required<br>• Instrumentation errors | • Complexity of mathematical apparatus<br>• Complexity of implementation<br>• Resource intensity<br>• Lack of user experience<br>• Small number of experts |

That's why we have to reconstruct the overall picture of atmospheric air pollution, based on results of measurements made at individual points, by virtue of the so-called interpolation models. Impurities concentration values between given points are calculated according to interpolation formulas (that's why the models are called "interpolation models"); as a rule, linear interpolation is used in the following form:

$$F(n+x)=\frac{L-x}{L}\cdot F(n)+\frac{x}{L}\cdot F(n+L),\ x\in[0,L],\ n=\overline{1,N},$$

where $F(i)$ is the value of interpolating function into the function value at $i$ -th point, $L$ is the distance between datum points, while $N$ is the general number of points at which impurity concentration values on air were measured by stationary automatic stations.

So far, such interpolation models have not been developed to the extent of a wide practical application. Therefore, to apply the apparatus of mathematical physics for monitoring of atmospheric air, stationary automated points of atmospheric air quality control are normally installed and additionally equipped with movable means of inspection.

It's quite important to note that interpolation modeling methods are quite acceptable with respect to solid continuous media, like for instance, in the immovable water column. A "blind" mechanical application of interpolation methods in urban conditions, which may be characterized by a diversified irregularity of terrain between two atmospheric measurement units – may provide absurd results. For that reason, interpolation methods taking into account lack of terrain homogeneity and the availability of intermediate sources of emissions – are missing.

Extrapolative models imply that scattering calculation from point sources are developed on the basis of one of fundamental laws of physics – balance equations. The essence of a balance equation can be illustrated by a simple example as follows: let's assume that a source of the substance with specific concentration $C_{unit\ conc.}$ and the inflow rate $\vartheta_{\text{inflow}}$ is located inside a volume $V$ having the surface $S=\partial V$. Let us set the motion velocity of the substance directed at normal towards the surface $S$ as $\vartheta_{\text{movement}}$, the volume element – as $dV$, and the surface element – as $dS$. Then the scattering process is described by the following equation:

$$\int_V \frac{\partial\left\{\rho\cdot C_{unit\ conc.}\right\}}{\partial t}dV=\oint_S \rho\cdot\vartheta_{\text{movement}}dS+\int_V \vartheta_{\text{inflow}}\cdot C_{unit\ conc.}dV,$$

where $\rho$ denotes specific density of the substance investigated.

Some transformations can be made with the balance equation – to take the actual urban conditions into account. For example, we may set up various speeds of transfer along different directions, simulating the terrain. We can enter various kinds of roughness factors of geological substrate etc. At that, accuracy of all types of models is defined by the scale and the precision of description of reference and boundary conditions. A higher accuracy may be achieved through macro scale–based models. In this case, a smooth and even surface is examined, as it were, and the sole emission source is located at high altitude – so high that surface roughness can be neglected. The model will be simple and not completely inadequate, but its computational accuracy will be high. If we consider a town where the emission source is located at a low altitude – we have to take into account the turbulence occurring in street canyons, around buildings, and in natural depth shapes. Then the model will be getting more complicated and its adequacy will increase with respect to actual process of transfer of contaminants, - but its computational accuracy will drop.

This does not mean that extrapolative methods have no practical importance. They can be useful for working out a strategic planning of urban development, calculating different alternatives of highway construction, building new urban-industrial objects – but not for on-line computations of contaminated areas.

The above-stated circumstance and other (shown in Thesis), as well as the impossibility of a quick indication of many impurities in the atmosphere and the impossibility of processing of measurement results – urged ecologists to apply some more general methods of assessing environmental quality. Those works are currently under investigation. Therefore, using methods of mathematical statistics is not quite correct for solving ecological problems.

Due to lack of necessary statistical data, such an approach is not fit for making forecasts of ecological consequences of realization of prospective solutions; neither is it fit for analyzing any design alternatives of development and construction of urban residential districts and transportation network. For this purpose, as a rule, are used different mathematical models, modeling behavior of harmful substances in different conditions. The most of the modern tasks of dynamics and kinetics of atmospheric disperse systems are described by multilevel nonlinear equations in partial derivatives. The solution for the problems of this class can be found only roughly with the help of approximation of the initial differential problem by finite-dimensional model. At that, the evidence of convergence of approximate solution to the solution of the initial differential model in the most cases is difficult or impossible at all. For this reason it is extremely important to investigate the subject of such simplification of the initial complex mathematical model so that the obtained problem, first of all, allowed analytical-numeric solutions and, secondly, preserved basic properties of the initial complete mathematical model. The aforementioned refers also to the problem of approximation: it is extremely important to construct finite-difference analogues of the initial differential problem so that they preserved the basic properties of differential models.

Eventually the aim of use of formalized methods in ecological engineering, mathematical modeling and forecasting is the accurate calculation of ecological harm dimensions, of expected financial expenses and of evaluation of applied measures efficiency. But the methods of mathematical statistics do not supply necessary accuracy. Consequently it is necessary to apply some other, more accurate methods. For example, underexplored in ecology area mathematical physics means (including the means of differential equations in partial derivatives).

The choice of these means is connected with the following factors:

- The apparatus of mathematical physics allows one to use fundamental laws of physics – such as the laws of turbulent molecular scattering, turbulent atmosphere dynamics, the law of transfer in heterogeneous mediums, the laws of hydro- and gas dynamics, etc, and for developing the non-standard 3D-mathematical model to determine the dynamics of exhaust emission level in urban outside air – provided the air flow rate is not unknown apriori. Since all the above-mentioned fundamental laws of physics are described by linear and non-linear equations in partial derivatives, – the developed mathematical model is obviously described in terms of differential equations.
- The rich apparatus of differential equations in partial derivatives (an important component of mathematical physics apparatus) allows one to do the following, putting aside the specific object domain: analyze obtained differential equations (non-linear ones in general), - investigate their correctness, perform a qualitative analysis, develop solutions through numerical or analytical methods, investigate issues of unicity and solution stability with respect to small variations of input data.

These apparatuses deliver a desired accuracy; they are all-round and flexible, enabling one to make a forecast along the entire time axis. A direct connection of the theory of differential equations with the annexes, the nature, and the physics of processes should be emphasized. To construct the mathematical model in a form of differential equations it is necessary, as a rule, to know only local connections but information of the whole physical phenomenon is not needed. Mathematical model gives the possibility to study the phenomenon on the whole, to forecast its development and to make quantative evaluations of changes taking place in it with current of time. Applying the apparatus of mathematical physics enables one to determine theoretically (provided some initial data is available and some assumptions are made) the dynamics of concentration of any quantity for $N$ impurities at any point of the area investigated.

The author comes to the conclusion that the second approach enables one to make a deeper penetration into the physical essence of the examined phenomenon, and to receive analytical results directly connected with the physics of a particular process. There occur some opportunities of receiving precise or approximate findings for a dynamically developing process, with the possibility of forecasting the course of a particular physical phenomenon without having to collect a huge statistical material. On the other hand, the very appropriateness of using the approaches of theory of chance and mathematical statistics is not disputed. The two approaches are to be regarded as complementary rather than antagonistic; both of them are useful and complement each other. Nevertheless, taking all pros and cons of the two approaches into consideration, the second approach has been selected in the Thesis (using partial differential equations from the apparatus of mathematical physics equations).

It is marks in the second chapter, that gases flows observed in the atmosphere can be divided into two fundamentally different groups:

- Laminar flows (quiet, regular, smooth ones), which are rarely seen in nature.
- Turbulent flows (chaotic, vortex, unregulated temporally and in space), of which the city atmosphere consists of.

Urban atmosphere is a complicated dynamic system in which take place various physical-chemical processes the intensity of which depends on the specific characteristics of the considered city.

There are investigated in the dissertation the definitions of the „turbulent flow" and there are also provided unfold characteristics of turbulent movement properties. It is necessary because there are made in the dissertation, in the construction of the mathematical models, suggestions and admissions which are being based namely on the below stated characteristics of the turbulent movements: continuality, irregularity, whirling nature, Reynolds high rates, tridimensionality, nonlinear nature, dissipativity, diffusivity.

There is being developed ***in the third chapter*** of the dissertation formation of hypdrothermodynamical equations of atmospheric processes on the middle scale and worked out special numeric algorithms for its realization and also made investigation of the city terrain influence on airstream of atmosphere.

In the beginning there was conducted the investigation on influence of turbulence and meteorological parametres of urban atmosphere to modelling quality and further carried out the formation of hypdrothermodynamical equations on the middle scale.

From the beginning were developed in the work four equations which form a complete and closed hydrodynamical equation system describing atmospheric processes on the middle scale. In particular - urban atmosphere turbulent streams in microdiapason. The first equation (1) of four equations is shown below.

$$\rho \cdot \frac{d\vec{\vartheta}}{dt} = \rho \cdot g - \nabla p + \frac{\partial}{\partial x_j}\left\{ \mu \cdot \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{2}{3} \cdot \delta_{ij} \cdot \frac{\partial u_k}{\partial x_k} \right\}, \quad \left( i, j, k = \overline{1,3} \right). \tag{1}$$

Where $\rho$ is the fluid density, and $\vec{\vartheta}$ is the velocity vector, $\delta_{ij}$ is Kronecker delta, $u_i$ $\left( i = \overline{1,3} \right)$ is $i$-th component of velocity vector by the axes of reference $OX$, $OY$ and $OZ$ accordingly.

The equation (1) received is called Navier-Stokes equation. It describes atmospheric processes within the entire spectrum of atmospheric motions, claiming for some special transformations on the medium scale. Finally, we will note that the Navier-Stokes equation (1) obviously is not a closed equation and, therefore, some other closing equations are needed.

Putting field of meteoelements in a form of sum of average values describing large-scale flowing and fluctuations, representing turbulent flows of less scale in Navier-Stokes equation (1) and taking into consideration forces of Coriolis and also suggesting that atmosphere is incompressible, we will have:

$$\frac{\partial(\rho \cdot u_i)}{\partial t} + \frac{\partial\left(\rho \cdot \overline{u_i \cdot u_j}\right)}{\partial x_j} = -\frac{\partial p'}{\partial x_i} - \frac{\partial\left(\rho \cdot \overline{u_i' \cdot u_j'}\right)}{\partial x_j} + \left( \frac{\partial \tau_{ij}'}{\partial x_j} \right)_{лам.} - 2 \cdot \varepsilon_{i\alpha\beta} \cdot \Theta_\alpha \cdot \bar{u}_\beta - \delta_{i3} \cdot \frac{\rho}{\bar{\rho}} \cdot g, \tag{2}$$

where $\varepsilon$ – warmth inflow to singular air volume per time unit. The equation (2) is called Reynolds' equation. Further in the thesis is made a formation of warmth inflow equation and of specific humidity equation in turbulent atmosphere on the middle scale. Constructed of two equations (1) and (2) system does not create a closed system. For this reason we add the system of the two equations with equation for warmth inflow. The equation being constructed presents by itself a differentiating form of the first beginning of thermodynamics in the case of continuous body. After the transformations we will get the equation of warmth inflow in turbulent atmosphere:

$$\frac{\partial \tilde{V}}{\partial t} + \frac{\partial\left(\overline{u \cdot \tilde{V}}\right)}{\partial x} + \frac{\partial\left(\overline{\vartheta \cdot \tilde{V}}\right)}{\partial y} + \frac{\partial\left(\overline{w \cdot \tilde{V}}\right)}{\partial z} = -\sum_{i=1}^{3} \frac{\partial H_i}{\partial \xi_i}, \tag{3}$$

where

$$\xi_1 = x, \xi_2 = y, \xi_3 = z; \quad H_i = -\frac{\partial\left(\overline{u_i' V'}\right)}{\partial x_i} \quad \left( i = \overline{1,3} \right).$$

By acting exactly in the same way, was constructed the corresponding equation for specific humidity $q$:

$$\frac{\partial \tilde{q}}{\partial t} + \frac{\partial\left(\overline{u \cdot \tilde{q}}\right)}{\partial x} + \frac{\partial\left(\overline{\vartheta \cdot \tilde{q}}\right)}{\partial y} + \frac{\partial\left(\overline{w \cdot \tilde{q}}\right)}{\partial z} = -\sum_{i=1}^{3} \frac{\partial Q_i}{\partial \xi_i}, \tag{4}$$

where

$$\xi_1 = x, \xi_2 = y, \xi_3 = z; \quad Q_i = -\frac{\partial\left(\overline{u_i' \cdot q'}\right)}{\partial x_i} \quad \left( i = \overline{1,3} \right).$$

Thus there were concluded equations(1), (2), (3) и (4), which create a complete and closed hydrodynamic system of equations, describing atmospheric processes on the middle scale. In particular – urban atmosphere turbulent flows in microdiapason.

Also it may be noted that taking into consideration these four equations, it is possible to construct a complete hydrodynamic model in Cartesian system of coordinates for atmospheric processes on the middle scale.

If in Cartesian coordinate system to direct the axe $OX_1 = OX$ eastward, the axe $OX_2 = OY$ – northward, the axe $OX_3 = OZ$ – upright, and if to use the said boundaries for the order of magnitudes, then applying the conservation – of – momentum theorem, the first law of thermodynamics and the law of conservation of mass, we can write:

$$\frac{\partial \hat{u}}{\partial t} + \frac{\partial(u\hat{u}_i)}{\partial x} + \frac{\partial(v\hat{u}_i)}{\partial y} + \frac{\partial(w\hat{u}_i)}{\partial z} = -\frac{\partial p'}{\partial x_i} + l\hat{v}' + \frac{\partial \tau_{11}}{\partial x} + \frac{\partial \tau_{12}}{\partial y} + \frac{\partial \tau_{13}}{\partial z}, \tag{5}$$

$$\frac{\partial \hat{v}}{\partial t} + \frac{\partial(u\hat{v})}{\partial x} + \frac{\partial(v\hat{v})}{\partial y} + \frac{\partial(w\hat{v})}{\partial z} = -\frac{\partial p'}{\partial y} - l\hat{u}' + \frac{\partial \tau_{21}}{\partial x} + \frac{\partial \tau_{22}}{\partial y} + \frac{\partial \tau_{23}}{\partial z}, \tag{6}$$

$$\frac{\partial \hat{w}}{\partial t} + \frac{\partial(u\hat{w})}{\partial x} + \frac{\partial(v\hat{w})}{\partial y} + \frac{\partial(w\hat{w})}{\partial z} = -\frac{\partial P'}{\partial z} + \frac{g}{T}\vartheta'(1+0.61q) + \frac{\partial \tau_{31}}{\partial x} + \frac{\partial \tau_{32}}{\partial y} + \frac{\partial \tau_{33}}{\partial z}, \tag{7}$$

$$\frac{\partial \hat{\vartheta}}{\partial t} + \frac{\partial(u\hat{\vartheta})}{\partial x} + \frac{\partial(v\hat{\vartheta})}{\partial y} + \frac{\partial(w\hat{\vartheta})}{\partial z} + (\gamma_{wa} - \gamma)\hat{w}' = \frac{L_w \Phi \bar{\rho}}{c_p} + \frac{\partial H_1}{\partial x} + \frac{\partial H_2}{\partial y} + \frac{\partial H_3}{\partial z}, \tag{8}$$

$$\frac{\partial \hat{q}}{\partial t} + \frac{\partial(u\hat{q})}{\partial x} + \frac{\partial(v\hat{q})}{\partial y} + \frac{\partial(w\hat{q})}{\partial z} = -\hat{w}\frac{\partial Q}{\partial z} - \Phi\bar{\rho} + \frac{\partial Q_1}{\partial x} + \frac{\partial Q_2}{\partial y} + \frac{\partial Q_3}{\partial z}, \tag{9}$$

$$\frac{\partial \hat{u}'}{\partial x} + \frac{\partial \hat{v}'}{\partial y} + \frac{\partial \hat{w}'}{\partial z} = 0, \tag{10}$$

$$\hat{u}_i = \rho u_i \quad \left(i = \overline{1,3}\right),$$

$$\hat{v} = \rho v,$$

$$\hat{w} = \rho w,$$

$$\hat{\vartheta} = \rho\vartheta,$$

$$\hat{q} = \rho q,$$

$$p = \rho R T,$$

$$\theta = T\left(\frac{p_0}{p}\right)^{\frac{R}{c_p}},$$

$$H_i = -\frac{\partial\left(\overline{u_i' \cdot v'}\right)}{\partial x_i} \quad \left(i = \overline{1,3}\right),$$

$$Q_i = -\frac{\partial\left(\overline{u_i' \cdot q'}\right)}{\partial x_i} \quad \left(i = \overline{1,3}\right),$$

$$\Phi = \begin{cases} \dfrac{c_p}{L_w}(\gamma_a - \gamma_{ba} w) & if \quad q = q_n n, \\ 0 & if \quad q < q_n, \end{cases}$$

$$\gamma_{ba} = c_p \gamma_a \frac{R_n T^2 (p + 0.622 L_w E)}{c_p R_n T^2 p + 0.622 L_w^2 E},$$

where $u_i$ $\left(i=\overline{1,3}\right)$ is a component of velocity vector along a corresponding axe $OX_i$ $\left(i=\overline{1,3}\right)$; $p$ is pressure; $\rho$ is air density; $c_p$ is heat capacity at constant pressure; $g$ is a gravity factor; $T$ is absolute temperature; $R$ is universal dry air gas constant; $R_n$ is universal atmospheric steam gas constant; $L_w$ is specific heat of transformation; $\gamma_{wa}$ is adiabatic humidity gradient; $\gamma_{ba}$ is moisture and adiabatic gradient; $\gamma_a$ is adiabatic gradient; $\gamma$ is standard air vertical temperature gradient; $E$ is vapor tension; $\overline{\rho}(z)$ is background density; $\Phi$ is rate of formation of liquid phase in atmosphere; $P$ is so-called standard pressure; $q$ is specific air humidity.

Keep in mind that in (5)-(10) there was accepted an index record format, where the summation over repeated indices is performed. Equation (5) is an atmosphere dynamic equation, and it is due to the conservation – of – momentum theorem. Equation (6) is a $\theta$ potential temperature transfer equation. It is due to the first law of thermodynamics. Equation (7) is an atmospheric steam mass fraction transfer equation $q$ (specific air humidity). It is due to the continuity equation.

The spatial domain of the equation system (5)-(10) is a parallelepiped $D_{x,y,z} \overset{def}{\equiv} \left\{ \left(x_1 = x, x_2 = y, x_3 = z\right):\ a_i \le x_i \le b_i,\ i = 1,2;\ 0 \le x_3 \le c \right\}$. Consequently, the system (5)-(10) must be solved in the domain $D = D_{x,y,z} \times \left[0, t_{\text{observ.}}\right]$.

To the equations (5)-(10) the following initial and boundary conditions are added, as well as the conjugation conditions to the border of $x_3 = h$ layers: initial conditions, boundary conditions (Newmann Conditions), boundary conditions (Dirichlet Conditions), the conjugation conditions at the boundary of $x_3 = h$ layers (the complete conditions are in the text of the thesis).

The hydro-thermodynamic model (5)-(10) with the initial conditions, the mixed - type boundary conditions and the conjugation conditions, in an analytical form, unfortunately, can't be solved. For the time computational solution of the initial – boundary problem (5)-(10) there can be applied a splitting technique developed by Academician G.I.Marchouk. To this end the discretization of the set problem is required, and for the time discretization we will apply the method of fractional step weak approximation, developed by Academician N.N.Yanenko. Following the study of N.N.Yanenko, we introduce the fractional steps and the time approximation scheme so that in pursuance of splitting principles as regards physical processes on every time line $\left[t_j, t_{j+1}\right]$ $\left(j = \overline{0,N}\right)$: $t_0 = 0,\ t_N = t_{observ.}$ to get two main stages, namely, the stage of transference of substances along the paths and turbulent exchange and the stage of adjustment of atmospheric fields.

Further in the Thesis, a kind of system of equations (in differential setup) at the first and the second stages of equation splitting is presented. This system of equations (discretized equations formulated in differential setup) is solved under the boundary conditions stated in the Thesis.

Further, non-zero topography of the examined city street section was taken into account. As is known the Earth's relief has an impact on the character of air or liquid flow around a body. Ignoring the Earth's relief at construction of mathematical models of movement of admixtures in air and water environment prevents from differentiating the arising air or water flow disturbance. In many scientific articles, dedicated to studies of dynamics of admixtures in air /liquid, the corresponding mathematical models are constructed in canonical domains (1D is either finite or infinite bar; 2D is either a rectangle or a strip; 3D is either bounded or unbounded parallelepiped or a sphere etc.). Consequently, the presence of asperities of the area of the ground under the study, specifically, a studied section of a modern urban street with vehicular movement, has been ignored in these models.

Here the following task is being considered. Let there be two air masses $S1$ and $S2$, divided by a surface $\partial S$. It is supposed that $S1$ and $S2$ are moving along the earth plane, i.e., along a horizontal plane at different velocities $\vartheta_1$ and $\vartheta_2$ accordingly. Asperities of the ground cannot but entail disturbances of both air and water flow and, therefore, our objective in this Section is to differentiate an unperturbed state, which the air masses $S1$ and $S2$ could have in the absence of asperities, and a turbulent state, which takes place in reality. For this objective we will, first, speculate as regards an unperturbed state of the air masses $S1$ and $S2$:

– characteristic qualities, such as temperature, pressure, density etc., are changed only vertically, i.e., these characteristic qualities depend only upon the height, where the ground surface is taken as a reference mark;

– velocities $\vartheta_1$ and $\vartheta_2$ of air masses $S1$ and $S2$ are constants, at that: $\vartheta_1 \not\equiv \vartheta_2$;

– the temperature $T_1$ of the flow $S1$ changed in a linear fashion, namely:

$$T_1(x_3) \overset{def}{\equiv} T_{Land} + \alpha_1 \cdot x_3,$$

where $T_{Land}$ is a temperature on the ground surface, $\alpha_1 < 0$ is a temperature gradient, $x_3$ – is a height variable from the ground surface;

– the temperature $T_2$ of the flow $S2$ changed in a linear fashion, namely:

$$T_2(x_3) \overset{def}{\equiv} T_h + \alpha_2 \cdot x_3,$$

where $T_h$ is a temperature of the surface $\partial S$ of division of air masses $S1$ and $S2$, $\alpha_2 < 0$ is a temperature gradient, $h$ is height of the surface $\partial S$ of division from the ground, in other words, $h$ is the height of an unperturbed surface of division, $x_3$ is the height variable from the ground surface;

– the motion of the air masses $S1$ and $S2$ takes place equally in all vertical layers parallel to the flow $S1$ (Fig. 2).

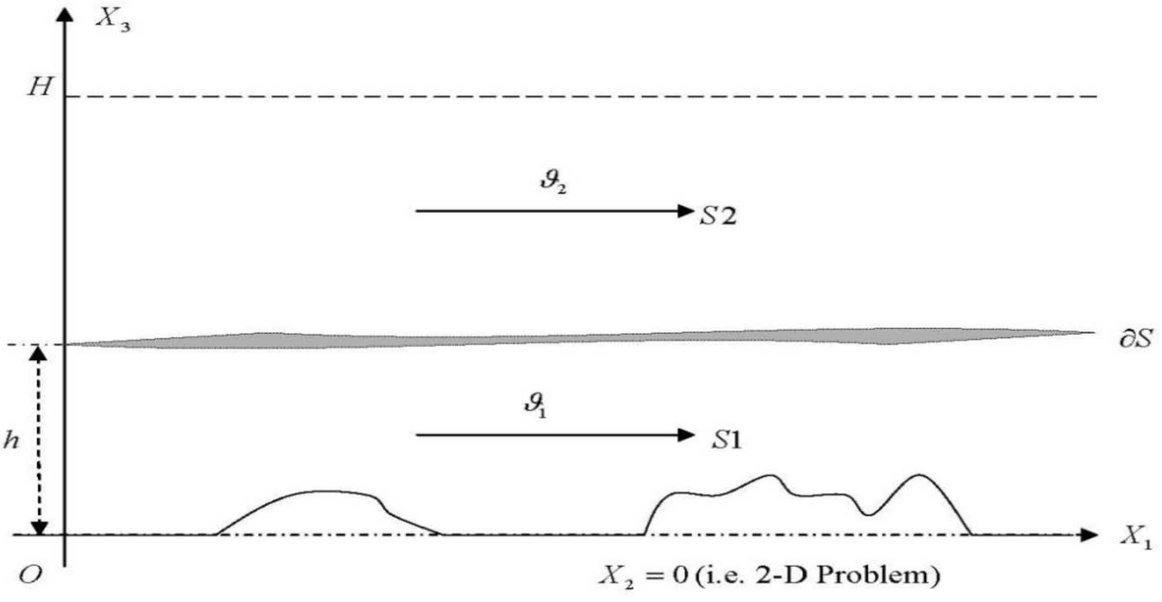


***Fig.2.*** *Motion of air masses $S1$ and $S2$ inclusive of urban relief*

For constructing an adequate mathematical model of the above-described problem at the made in Thesis speculations, let's denote by $p_1(x_3)$ and $p_2(x_3)$ the pressure of the air masses $S1$ and $S2$, accordingly; by $\rho_1(x_3)$ and $\rho_2(x_3)$ the density of the air masses $S1$ and $S2$, accordingly; by $\vartheta_1(x_3)$ and $\vartheta_2(x_3)$ the velocity of the air masses $S1$ and $S2$, accordingly. As shown above, the temperature of the flows $S1$ and $S2$ is denoted by $T_1(x_3)$ and $T_2(x_3)$, accordingly.

The values $p_i$, $\rho_i$, $\vartheta_i$ and $T_i$ $(i=1,2)$ are the characteristic qualities of unperturbed flows of air masses $S1$ and $S2$. Our objective goal is to determine (i.e., differentiate) analogous characteristic qualities of turbulent flows, where disturbance is entailed by the presence of asperities on the ground surface (to be more exact, on the surface of the studied section of urban street with vehicular movement). Therefore alongside with the symbols $p_i$, $\rho_i$ and $\vartheta_i$ $(i=1,2)$ we will introduce also such symbols as $\tilde{p}_i$, $\tilde{\rho}_i$, $\tilde{\vartheta}_i$ and $w_i$ $(i=1,2)$, where $\tilde{p}_i$ is the disturbance in pressure of the flow $i$ $Si$ $(i=1,2)$; $\tilde{\rho}_i$ – disturbance of thickness of *i*-flow $Si$ $(i=1,2)$; $\tilde{\vartheta}_i$ – disturbance of "horizontal" speed of *i*-flow $Si$ $(i=1,2)$; $w_i$ is the disturbance in the "vertical" velocity of the $i$ $Si$ $(i=1,2)$. Then the correct (real, true) values of these characteristic qualities obviously will be equal to $p_i+\tilde{p}_i$, $\rho_i+\tilde{\rho}_i$, $\vartheta_i+\tilde{\vartheta}_i$ and $w_i$ $(i=1,2)$, and with respect to these true characteristic qualities we can write corresponding motion equations:

$$\frac{1}{\rho_i+\tilde{\rho}_i}\cdot\frac{\partial(p_i+\tilde{p}_i)}{\partial x_1}+\left(\vartheta_i+\tilde{\vartheta}_i\right)\cdot\frac{\partial\left(\vartheta_i+\tilde{\vartheta}_i\right)}{\partial x_1}+w_i\cdot\frac{\partial\left(\vartheta_i+\tilde{\vartheta}_i\right)}{\partial x_3}=0;\quad i=1,2, \tag{11}$$

$$\left(\vartheta_i+\tilde{\vartheta}_i\right)\cdot\frac{\partial w_i}{\partial x_1}+w_i\cdot\frac{\partial w_i}{\partial x_3}+\frac{1}{\rho_i+\tilde{\rho}_i}\cdot\frac{\partial(p_i+\tilde{p}_i)}{\partial x_3}+g=0;\quad i=\overline{1,2}, \tag{12}$$

$$\frac{\partial\left\{(\rho_i+\tilde{\rho}_i)\cdot\left(\vartheta_i+\tilde{\vartheta}_i\right)\right\}}{\partial x_1}+\frac{\partial\left\{w_i\cdot(\rho_i+\tilde{\rho}_i)\right\}}{\partial x_3}=0,\quad i=\overline{1,2}, \tag{13}$$

where

$$0<x_1<L;\quad 0<x_3<h\ ,\ i=1;$$
$$0<x_1<L;\quad h<x_3<H\ ,\ i=2;$$
$$p_i=p_i(x_3),\ \rho_i=\rho_i(x_3),\ \vartheta_i=\vartheta_i(x_3),\ \tilde{p}_i=\tilde{p}_i(x_1,x_3),$$
$$\tilde{\rho}_i=\tilde{\rho}_i(x_1,x_3),\ \tilde{\vartheta}_i=\tilde{\vartheta}_i(x_1,x_3)\ \text{and}\ w_i=w_i(x_1,x_3)\quad(i=1,2).$$

To the (11)-(13) systems it was necessary to add the heat balance equation. Since we assumed that the arising disturbances are entailed solely by a relief of the studied section of a city (of course, there are also other impacts that generate disturbances of the cited characteristic qualities of air flow), then heat inflow should be equal to zero. Consequently, to the equations (11)-(13) it is necessary to add a condition of lack of heat inflow, namely,

$$\left(\vartheta_i+\tilde{\vartheta}_i\right)\cdot\frac{\partial}{\partial x_1}\left(\frac{p_i+\tilde{p}_i}{(\rho_i+\tilde{\rho}_i)^\beta}\right)+w_i\cdot\frac{\partial}{\partial x_3}\left(\frac{p_i+\tilde{p}_i}{(\rho_i+\tilde{\rho}_i)^\beta}\right)=0, \tag{14}$$

where $\beta=1.41$.

So, the true characteristic qualities of turbulent airflows are determined (of course, in the presence of corresponding boundary conditions and conjugation conditions on the surface $\partial S$ of division of the air masses $S1$ and $S2$) by a system of equations (11)-(14). Ditto as in the study, in this Section we will be restricted to the study of a question of finding of asymptotic expression of solutions of the system (11)-(14) on below assumptions:

- the flow disturbances are taken as small that their degrees above the first order might be ignored;
- smallness of irregularity of the surface of the urban street section under consideration is taken (against the maximum spatial dimensions of the area under consideration).

These two assumptions allow dividing the system (11)-(14) into two parts, namely, the first system contains equation of unperturbed motion, but the second part of the system contains equation of turbulent motion. The idea of division of the system (11)-(14) into the above – said parts is based on the idea of equating in the system of equations (11)-(14) of finite summands and first – order infinitesimals that allows reducing the initial system (11)-(14) to below two equation systems:

- a linear equation system for unperturbed air mass;
- a linear equation system for disturbed air mass.

Further in the Thesis, the two systems of equations with matching conditions are presented. At that, the configuration of the surface of the city street section investigated is given by the equation as follows: $x_3 = X_3(x_1) = \cos(\gamma \cdot x_1)$.

In the process of solving the problem the following ordinary linear differential equation of the second order concerning the desirable function $\xi_i^{(1)}(x_3)$ was derived:

$$\begin{gathered}
\beta \cdot R \cdot T_i(x_3) \cdot \vartheta_i^2(x_3) \cdot \left(\beta \cdot R \cdot T_i(x_3) - \vartheta_i^2(x_3)\right) \cdot \left\{\xi_i^{(1)}(x_3)\right\}'' + \\
+\left\{g \cdot \beta^2 \cdot R \cdot T_i(x_3) \cdot \vartheta_i^2(x_3) + 2 \cdot \beta \cdot R \cdot \alpha_i \cdot \vartheta_i^4(x_3) - g \cdot \beta \cdot \vartheta_i^4(x_3) - \alpha_i \cdot \beta^2 \cdot R^2 \cdot T_i(x_3) \cdot \vartheta_i^2(x_3)\right\} \cdot \left\{\xi_i^{(1)}(x_3)\right\}' + \\
+\left\{g \cdot \left\{(\beta-1) \cdot g - \beta \cdot R \cdot \alpha_i\right\} \cdot \left(\beta \cdot R \cdot T_i(x_3) - \vartheta_i^2(x_3)\right) - \beta^2 \cdot \vartheta_i^2(x_3) \cdot \left(\beta \cdot R \cdot T_i(x_3) - \vartheta_i^2(x_3)\right) + \right. \\
\left. + \beta \cdot R \cdot \alpha_i \cdot \left\{(\beta-1) \cdot g - \beta \cdot R \cdot \alpha_i\right\} \cdot \vartheta_i^2(x_3)\right\} \cdot \xi_i^{(1)}(x_3) = 0.
\end{gathered} \tag{15}$$

We will study the derived equation (15). In the first instance let's single out that the coefficient at the top-order derivative $\left\{\xi_i^{(1)}(x_3)\right\}''$ will equal zero at two points, namely, where $\beta \cdot R \cdot T_i(x_3) \cdot \vartheta_i^2(x_3) = 0$, and at the points, where $\beta \cdot R \cdot T_i(x_3) - \vartheta_i^2(x_3) = 0.$ In other words, the equation (15) is the equation with two regular singular points.

Considering that $\vartheta_i(x_3) \neq 0 \ \forall x_3 \quad (i = 1, 2)$, then a point at which $T_i(x_3) = 0$ corresponds to the first singular point. Since $T_{Land} \neq 0$ (i.e. the value is not equal to zero in all points of earth's surface) in the expression $T_1(x_3) \overset{def}{\equiv} T_{Land} + \alpha_1 \cdot x_3$, then we will get that for the second medium $S2$ this singular point locates at its upper boundary and, consequently, it cannot reside inside of the first medium $S1$.

The second singular point, for which

$$\vartheta_i^2\left(x_3\right)=\beta\cdot R\cdot T_i\left(x_3\right)\quad\left(i=1,2\right),\tag{16}$$

might reside either inside the first or the second layer. Therefore at this singular point we bring in the roundedness of the functions $\xi_i^{(1)}\left(x_3\right)$, $\eta_i^{(1)}\left(x_3\right)$, $\tilde{p}_i^{(1)}\left(x_3\right)$ и $\tilde{\rho}_i^{(1)}\left(x_3\right)$. In order to study the second singular point, which according to the formula (16) might be interpreted as a point at the flow velocity $\vartheta_i\left(x_3\right)$ equal to sound velocity $\beta\cdot R\cdot T_i\left(x_3\right)$, we will introduce a new independent variable $\varphi$ according to the formula

$$\varphi\overset{def}{\equiv}\frac{\beta\cdot R\cdot T_i\left(x_3\right)-\vartheta_i^2\left(x_3\right)}{\vartheta_i^2\left(x_3\right)}.\tag{17}$$

The introduced by convention (17) variable $\varphi$, as we will see later, essentially simplifies the procedure of statement of conditions contributing to the finiteness of the functions $\xi_i^{(1)}\left(x_3\right)$, $\eta_i^{(1)}\left(x_3\right)$, $\tilde{p}_i^{(1)}\left(x_3\right)$ и $\tilde{\rho}_i^{(1)}\left(x_3\right)$.

Considering the new independent variable $\varphi$, determined by the equality (17), the equation (15) takes the following more simplified form:

$$\begin{gathered}\varphi\cdot\left(\varphi+1\right)\cdot\left\{\xi_i^{(1)}\left(\varphi\right)\right\}_\varphi''+\frac{\alpha_i\cdot R\cdot\left(\varphi-1\right)-g\cdot\varphi}{\alpha_i\cdot R}\cdot\left\{\xi_i^{(1)}\left(\varphi\right)\right\}_\varphi'+\\+\frac{\beta\cdot\left(g-\alpha_i\cdot R\right)\cdot\left(\beta\cdot R\cdot\alpha_i+g\cdot\varphi\right)-g\cdot\left(\beta\cdot R\cdot\alpha_i+g\cdot\varphi\right)-\gamma^2\cdot\vartheta_i^4\left(\varphi\right)\cdot\varphi^2}{\beta^2\cdot R^2\cdot\alpha_i^2}\cdot\xi_i^{(1)}\left(\varphi\right)=0.\end{gathered}\tag{18}$$

Now then, the characteristic qualities of the equation (18) are $\lambda_1=0$ and $\lambda_1=2$. Directly one can easily make sure that the solution of the equation (18), which corresponds to the characteristic quality $\lambda_1=0$, has a logarithmic singularity. If we pass on to the earlier notations of variable $x_3$ (17), then according to the formula (30) we will derive that at the point $\varphi=0$ the pressure $\tilde{p}_i^{(1)}\left(x_3\right)$ becomes infinite. Consequently, it is necessary to take only such solution of the equation (18), which corresponds to the characteristic quality $\lambda_1=2$. A solution corresponding to this characteristic quantity is effective in the second layer $S2$ and it reduces the function $\xi_i^{(1)}\left(x_3\right)$ to zero, which means reduction of the "vertical" velocity $w_2\left(x_1,x_3\right)$ to zero. This may be interpreted as the presence of a certain surface in a form of a solid boundary on which there is a singular point $\varphi=0$, i.e. whereon there is

$$\varphi=\frac{\beta\cdot R\cdot T_2\left(x_3\right)-\vartheta_2^2\left(x_3\right)}{\vartheta_2^2\left(x_3\right)}=0,$$

that means

$$\vartheta_2^2\left(x_3\right)=\beta\cdot R\cdot T_2\left(x_3\right).\tag{19}$$

In other words, the singular point $\varphi\in S2$, at which there is the equality (19), defines a certain solid boundary, i.e. the surface described by the equality (19).

Later in the dissertation is shown the unconventional solution of this system of equations as a result of which can be found the solution of the whole problem which may be written in the form of Fourier integral in the following form:

$$\xi_i^{(General)}\left(x_1,x_3\right)=\frac{1}{\pi}\cdot\int\limits_0^{+\infty}d\sigma_1\int\limits_{-\infty}^{+\infty}\xi_i^{(1)}\left(\sigma_1,x_3\right)\cdot X_3\left(\sigma_2\right)\cdot\sin\left(\gamma\cdot\left(x_1-\sigma_2\right)\right)d\sigma_2, \tag{20}$$

$$\eta_i^{(General)}\left(x_1,x_3\right)=\frac{1}{\pi}\cdot\int\limits_0^{+\infty}d\sigma_1\int\limits_{-\infty}^{+\infty}\eta_i^{(1)}\left(\sigma_1,x_3\right)\cdot X_3\left(\sigma_2\right)\cdot\cos\left(\gamma\cdot\left(x_1-\sigma_2\right)\right)d\sigma_2, \tag{21}$$

$$\tilde{\rho}_i^{(General)}\left(x_1,x_3\right)=\frac{1}{\pi}\cdot\int\limits_0^{+\infty}d\sigma_1\int\limits_{-\infty}^{+\infty}\tilde{\rho}_i^{(1)}\left(\sigma_1,x_3\right)\cdot X_3\left(\sigma_2\right)\cdot\cos\left(\gamma\cdot\left(x_1-\sigma_2\right)\right)d\sigma_2, \tag{22}$$

$$p_i^{(General)}\left(x_1,x_3\right)=\frac{1}{\pi}\cdot\int\limits_0^{+\infty}d\sigma_1\int\limits_{-\infty}^{+\infty}p_i^{(1)}\left(\sigma_1,x_3\right)\cdot X_3\left(\sigma_2\right)\cdot\cos\left(\gamma\cdot\left(x_1-\sigma_2\right)\right)d\sigma_2, \tag{23}$$

$$X_{3,\partial S}\left(x_1\right)=\frac{1}{\pi}\cdot\int\limits_0^{+\infty}d\sigma_1\int\limits_{-\infty}^{+\infty}X_{3,\partial S}\left(\sigma_1\right)\cdot Z\left(\sigma_2\right)\cdot\cos\left(\gamma\cdot\left(x_1-\sigma_2\right)\right)d\sigma_2, \tag{24}$$

where $\tilde{p}_i$, $\tilde{\rho}_i$, $\xi_i^{(1)}$, $\eta_i^{(1)}$ and $X_{3,\partial S}$ are the solutions of the same problem, on condition (16).

There is worked out in ***the fourth chapter*** tridimensional on extensive variables instable mathematical model for analytical determination of urban exhaust air concentration dynamics at a priori defined information of air flow speed.

In the worked-out model it is supposed that the coefficient of turbulent and molecular diffusion changes in dependence on vertical farness from earth's surface. In other words in the basis of the proposed model is put the important condition of urban air vertical lamination because of accumulation in it of harmful substances ventholed by traffic. This supposition of the urban air vertical lamination corresponds to the reality: there are accumulated harmful substances in urban atmosphere air ventholed by traffic and other stable and/or unstable sources ( hard, aerosolic and gaseous poisonous factory waste; polluted aquatic medium; city garbage dumps, etc. sources).

Therefore, we suggest the following mathematical formulation of the problem described above: it is required to define concentrations $C^{\{n\}}\left(x_1,x_2,x_3,t\right)$ $n^{\text{th}}$ $\left(n=\overline{1,N}\right)$ of harmful substance at any spatial point $\left(x_1,x_2,x_3\right)$ of the area $\left[0,l_1\right]\times\left[0,l_2\right]\times\left[0,l_3\right]$, at any moment of time $t\in\left[0,T\right]$ - stemming from the equation:

$$\begin{gathered}\frac{\partial C^{\{n\}}\left(x,t\right)}{\partial t}=div\left(D\left(\vec{\vartheta}\left(x,t\right)\right)\cdot\overrightarrow{grad}\,C^{\{n\}}\left(x,t\right)\right)-\vec{\vartheta}\left(x,t\right)\cdot\overrightarrow{grad}\,C^{\{n\}}\left(x,t\right),\ t\geq 0,\\ x=\left(x_1,x_2,x_3\right):\ 0<x_i<l_i\ \left(i=\overline{1,3}\right);\end{gathered} \tag{25}$$

from initial condition

$$C^{\{n\}}\left(x,t\right)\Big|_{t=0}=C_0^{\{n\}}\left(x\right),\ x=\left(x_1,x_2,x_3\right):\ 0\leq x_i\leq l_i\ \left(i=\overline{1,3}\right); \tag{26}$$

from boundary conditions (at each fixed $j=\overline{0,M-1}$)

$$\begin{gathered}\gamma_{i,1,j}^{\{n\}}\cdot\frac{\partial C^{\{n\}}\left(x,t\right)}{\partial x_i}\bigg|_{x_i=a_{i,j}}-\gamma_{i,2,j}^{\{n\}}\cdot C^{\{n\}}\left(x,t\right)\Big|_{x_i=a_{i,j}}=C_{i,j}^{\{n\}}\left(x/\left\{x_i\right\},t\right),\\ a_{i,j}\leq x_i\leq b_{i,j}\ \left(i=\overline{1,3}\right),\ t\geq 0,\end{gathered} \tag{27}$$

$$\gamma_{i,3,j}^{\{n\}}\cdot\left.\frac{\partial C^{\{n\}}(x,t)}{\partial x_i}\right|_{x_i=b_{i,j}}+\gamma_{i,4,j}^{\{n\}}\cdot\left.C^{\{n\}}(x,t)\right|_{x_i=b_{i,j}}=C_{i+3,j}^{\{n\}}\left(x/\{x_i\},t\right),$$
$$a_{i,j}\le x_i\le b_{i,j}\ \left(i=\overline{1,3}\right),\ t\ge 0, \tag{28}$$

from the coupling conditions as follows:

$$\left.C^{\{n\}}(x,t)\right|_{x_3=l_3^{\{j\}}-0}=\left.C^{\{n\}}(x,t)\right|_{x_3=l_3^{\{j\}}+0},\ \ j=\overline{1,M-1},\ 0\le x_i\le l_i\ \ (i=1,2), \tag{29}$$

$$D\left(\vec{\vartheta}(x,t)\right)\cdot\left.\frac{\partial C^{\{n\}}(x,t)}{\partial x_3}\right|_{x_3=l_3^{\{j\}}-0}=D\left(\vec{\vartheta}(x,t)\right)\cdot\left.\frac{\partial C^{\{n\}}(x,t)}{\partial x_3}\right|_{x_3=l_3^{\{j\}}+0},$$
$$j=\overline{1,M-1},\ \ 0\le x_i\le l_i\ \ (i=1,2). \tag{30}$$

In the non-stationary, initial boundary problem (25)-(30), $D\left(\vec{\vartheta}(x,t)\right)$ denotes the coefficient of turbulent and molecular diffusion, which, according to the assumption made above, is a piecewise constant function of the type as follows:

$$0<d_{\min}\le D\left(\vec{\vartheta}(x,t)\right)\overset{def}{\equiv}\begin{cases}D_1=const & if\quad 0=l_3^{\{0\}}\le x_3\le l_3^{\{1\}},\\ D_2=const & if\quad l_3^{\{1\}}\le x_3\le l_3^{\{2\}},\\ \dots\dots\dots\dots\dots\dots\dots\dots & \\ D_M=const & if\quad l_3^{\{M-1\}}\le x_3=l_3^{\{M\}},\end{cases}$$

for $\forall x_i\in[0,l_i]$, $(i=1,2)$, where $M$ is the number of stratified mediums parallel to the plane $X_1OX_2$ along the vertical axis $OX_3$; by $\vec{\vartheta}(x,t)$ is denoted air flow speed vectorial function, which in this work is supposed to be experimentally known and also piecewiese-constant function of a form

$$\vec{\vartheta}(x,t)\overset{def}{\equiv}\begin{cases}\vartheta_1^{\{average\}}=const & if\quad 0=l_3^{\{0\}}\le x_3\le l_3^{\{1\}},\\ \vartheta_2^{\{average\}}=const & if\quad l_3^{\{1\}}\le x_3\le l_3^{\{2\}},\\ \dots\dots\dots\dots\dots\dots\dots\dots & \\ \vartheta_M^{\{average\}}=const & if\quad l_3^{\{M-1\}}\le x_3=l_3^{\{M\}},\end{cases}$$

for $\forall x_i\in[0,l_i]$, $(i=1,2)$; the frontier points of strata are denoted by $a_{i,j}$ and $b_{i,j}$ – namely,

$$a_{i,j}\overset{def}{\equiv}\begin{cases}0 & if\quad i=1,2,\\ l_3^{\{j\}} & if\quad i=3;\ j\ne 0,\\ l_3^{\{0\}} & if\quad i=3;\ j=0,\end{cases}\qquad b_{i,j}\overset{def}{\equiv}\begin{cases}l_i & if\quad i=1,2;\ \ \forall j,\\ l_3^{\{j+1\}} & if\quad i=3;\ \ \ \forall j.\end{cases}$$

In the problem(25)-(30), the known initial data is the constants $g\approx 9.8\ m/c^2$ (free fall acceleration), $N\in\mathbb{N}$, $M\in\mathbb{N}$, $T\in\mathbb{R}_+^1$, $D_j\in\mathbb{R}_+^1$ $\left(j=\overline{1,M}\right)$, $l_i^{\{j\}}\in\mathbb{R}_+^1\left(i=\overline{1,3};\ j=\overline{0,M}\right)$, $\vartheta_j^{\{average\}}$ $\left(j=\overline{1,M}\right)$, $\gamma_{i,k,j}^{\{n\}}\in\mathbb{R}_+^1$ $\left(i=1,2;\ \ k=\overline{1,4};\ \ j=\overline{0,M-1}\right)$, for $\forall n=\overline{1,\tilde{N}}$

$$\gamma_{i,k,j}^{\{n\}} = \begin{cases} 1 & if\ j=0;\ \ i=1;\ \ k=1,3, \\ 0 & if\ j=0;\ \ i=1;\ \ k=2,4, \\ \gamma_{i,k,j}^{\{n\}} > 0 & if\ \ \ j \neq 0;\ \ \forall i,k; \end{cases}$$

as well as the functions $C_0^{\{n\}}(x)$ $\left(\forall n = \overline{1,\tilde{N}}\right)$, $C_{i,j}^{\{n\}}(\bullet,t)$ $\left(i=\overline{1,6};\ \ j=\overline{0,M-1}\right)$, at that, for $\forall n = \overline{1,\tilde{N}}$ the following takes place:

$$C_{i,j}^{\{n\}}(\bullet,t) = \begin{cases} C_{i,j}^{\{n\}} & if\ \{j=0\} \wedge \{i=3,6\}; \\ 0 & otherwise\ . \end{cases}$$

Finally, it is assumed in tasks (25)-(30) with respect to concentration that the initial function $C_0^{\{n\}}(x)$ and the boundary functions $C_{i,j}^{\{n\}}(\bullet,t)$ meet the corresponding consistency conditions - with respect to each $n=\overline{1,\tilde{N}}$ of the exhaust gas.

Using standard technique one can show that the problem formulated in (25)-(30) has the unique solution. Further on we shall suppose that the number of vertical layers from the earth surface to the height $l_3$, which we are interested in, equals to four, i.e. $M=4$ and limits of each layer are determined empirically. The upper limit of the layer most distant from the ground surface (i.e. the upper limit of the last fourth layer) equals 30 meters that is considered to be an average height of buildings in Riga city.

So, the formulated model (25)-(30) implies that the axis $OX_1$ is directed along the examined section of the city street with motor traffic, - i.e., it is assumed that the length of the examined section of the city street is located along this axis; $OX_2$ axis is directed across the width of the street, while $OX_3$ axis is directed along the altitude of the investigated city area. In other words, the space domain of the mathematical model (25)-(30) is stratified along $OX_3$ axis - a parallelepiped where length/width/height are characterized by axes of reference $OX_1/OX_2/OX_3$ accordingly, and, secondly, all the strata are parallel to the plane $X_1OX_2$. The stratified “parallelepiped” domain described above, taken as a simulated area of the city street section with intense motor traffic, is schematically shown below on Fig.3 and Fig.4.

The first layer (from down up) is of height 0.2 meters from the ground. In this layer the vehicle exhaust fumes emission is going on. The level of the second layer is 2 meters. There is observed a pedestrian flow at this layer. Next two layers divide the height left into 2 levels of equal height. The upper limit of the layer most distant from the ground surface layer (i.e., the upper limit of the last fourth layer) equals to 30 meters.

This formulated task (25) – (30) is solved in this chapter of the thesis. The solution comes in a form:

$$C^{\{n\}}(x_1,x_2,x_3,t) = e^{\frac{\vartheta_j^{\{average\}}}{2\cdot D_j}\cdot\sum_{i=1}^{3} x_i - \frac{3\cdot\left(\vartheta_j^{\{average\}}\right)^2}{4\cdot D_j}\cdot t} \cdot w^{\{n\}}(x_1,x_2,x_3,t),\ t\geq 0,\ 0\leq x_i\leq l_i\ \left(i=\overline{1,3}\right),$$

wherein $w^{\{n\}}(x_1,x_2,x_3,t)$ is a very complicated function containing in itself some other functions. Some of them contain roots of transcendental equations (stated in the Thesis).

From the formula we can find concentrations $C^{\{n\}}(x_1,x_2,x_3,t)$ of each $N$ of harmful substances in any time $t\in[0,T]$ in any point $(x_1,x_2,x_3)\in[0,l_1]\times[0,l_2]\times[0,l_3]$ of urban atmospheric laminated air.

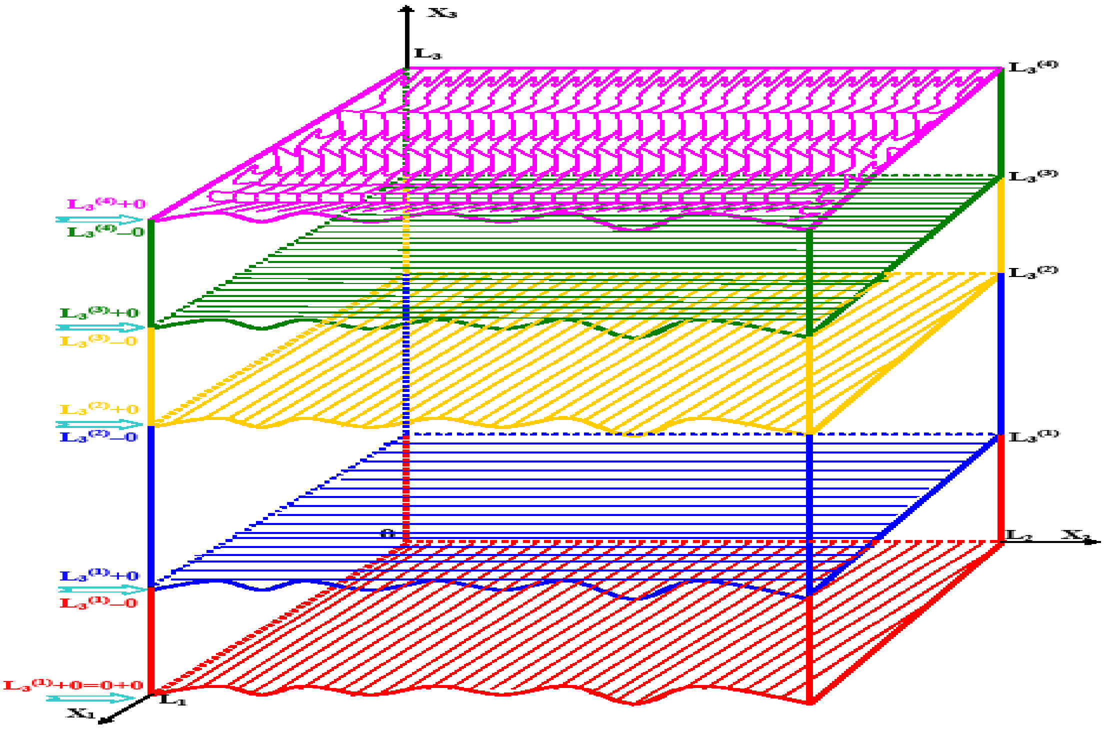


***Fig. 3.*** *The stratified "parallelepiped" domain of the simulated area of the city street section*

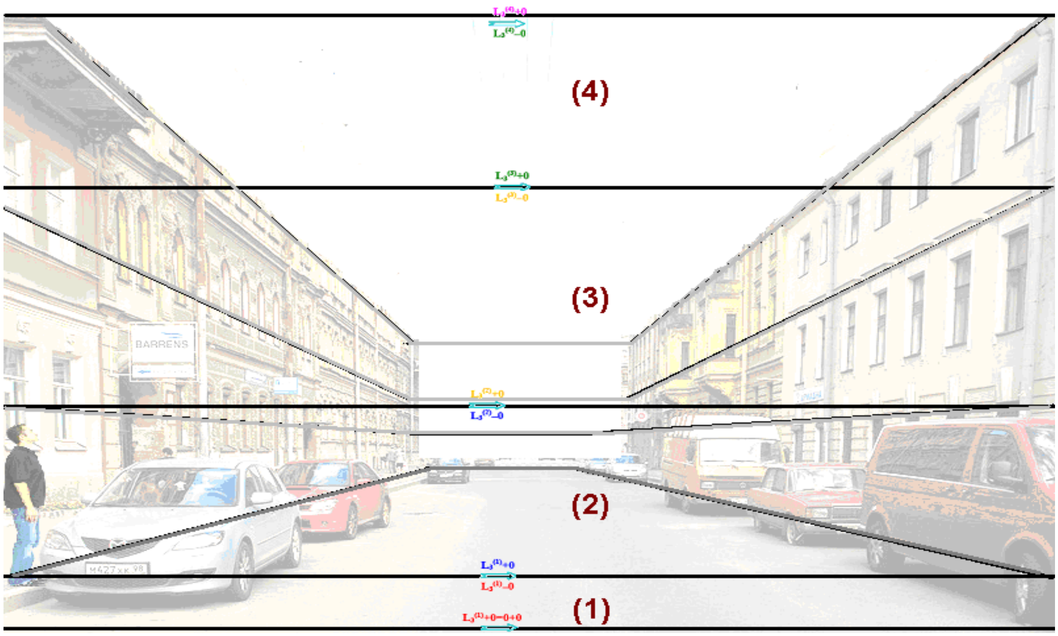


***Fig. 4.*** *The stratified domain of the simulated area of the city street section with intense motor traffic*

In the Thesis was investigated a complete mathematical model at unknown vertical speed of turbulent flow and at the known coefficient of molecular diffusion. Let us suppose that, like in the first model, the coefficient of turbulent and molecular diffusion changes in dependence of the vertical farness from earth's surface.. There is also a supposition of the urban air vertical lamination there Let us bring in the model, being constructed, one more simplifying condition that unstable speed of turbulent air flow depends only on vertical extensive coordinate. This simplification allows to construct one-dimensional on extensive variable (on vertical component) non-homogeneous nonlinear equation in relation to turbulent air flow speed. Taking into consideration with some simplifications the basic fundamental physics laws, such as: the laws of turbulent and molecular diffusion, the laws of turbulent atmosphere dynamics, the laws of transfer in non-homogeneous environments, the laws of hydro- and gaseoues dynamics, with aforementioned suppositions, we obtain the mathematical model of dynamics of nonlinear damping environments of various physical nature, situated in external field (environment with „pumping of energy").

Thus, like in the first model, there is proposed the following mathematical setting of the described problem: it is required to define concentrations $C^{\{n\}}(x_1,x_2,x_3,t)$ $n^{\text{th}}$ $\left(n=\overline{1,N}\right)$ of harmful substance at any spatial point $(x_1,x_2,x_3)$ of the area $[0,l_1]\times[0,l_2]\times[0,l_3]$, at any moment of time $t\in[0,T]$ - stemming from the equation

$$\begin{gathered}\frac{\partial C^{\{n\}}(x,t)}{\partial t}=div\left(D\left(\vec{\vartheta}(x,t)\right)\cdot\overrightarrow{grad}\,C^{\{n\}}(x,t)\right)-\vec{\vartheta}(x,t)\cdot\overrightarrow{grad}\,C^{\{n\}}(x,t),\ t\geq 0,\\ x=(x_1,x_2,x_3):\ \ 0<x_i<l_i\ \ \left(i=\overline{1,3}\right);\end{gathered}\tag{31}$$

from initial condition

$$C^{\{n\}}(x,t)\Big|_{t=0}=C_0^{\{n\}}(x),\ \ x=(x_1,x_2,x_3):\ \ 0\leq x_i\leq l_i\ \ \left(i=\overline{1,3}\right);\tag{32}$$

from boundary conditions (at each fixed $j=\overline{0,M-1}$)

$$\begin{gathered}\gamma_{i,1,j}^{\{n\}}\cdot\frac{\partial C^{\{n\}}(x,t)}{\partial x_i}\Bigg|_{x_i=a_{i,j}}-\gamma_{i,2,j}^{\{n\}}\cdot C^{\{n\}}(x,t)\Big|_{x_i=a_{i,j}}=C_{i,j}^{\{n\}}\left(x/\{x_i\},t\right),\\ a_{i,j}\leq x_i\leq b_{i,j}\ \ \left(i=\overline{1,3}\right),\ \ t\geq 0,\end{gathered}\tag{33}$$

$$\begin{gathered}\gamma_{i,3,j}^{\{n\}}\cdot\frac{\partial C^{\{n\}}(x,t)}{\partial x_i}\Bigg|_{x_i=b_{i,j}}+\gamma_{i,4,j}^{\{n\}}\cdot C^{\{n\}}(x,t)\Big|_{x_i=b_{i,j}}=C_{i+3,j}^{\{n\}}\left(x/\{x_i\},t\right),\\ a_{i,j}\leq x_i\leq b_{i,j}\ \ \left(i=\overline{1,3}\right),\ \ t\geq 0,\end{gathered}\tag{34}$$

from the coupling conditions as follows:

$$C^{\{n\}}(x,t)\Big|_{x_3=l_3^{\{j\}}-0}=C^{\{n\}}(x,t)\Big|_{x_3=l_3^{\{j\}}+0},\ \ j=\overline{1,M-1},\ \ 0\leq x_i\leq l_i\ \ (i=1,2),\tag{35}$$

$$\begin{gathered}D\left(\vec{\vartheta}(x,t)\right)\cdot\frac{\partial C^{\{n\}}(x,t)}{\partial x_3}\Bigg|_{x_3=l_3^{\{j\}}-0}=D\left(\vec{\vartheta}(x,t)\right)\cdot\frac{\partial C^{\{n\}}(x,t)}{\partial x_3}\Bigg|_{x_3=l_3^{\{j\}}+0},\\ j=\overline{1,M-1},\ \ 0\leq x_i\leq l_i\ \ (i=1,2).\end{gathered}\tag{36}$$

In the non-stationary, initial boundary problem (31)-(36), $D\left(\vec{\vartheta}(x,t)\right)$ denotes the coefficient of turbulent and molecular diffusion, which, according to the assumption made above, is a piecewise constant function of the type as follows:

$$0<d_{\min}\leq D\left(\vec{\vartheta}(x,t)\right)\overset{def}{\equiv}\begin{cases}D_1=const & if\quad 0=l_3^{\{0\}}\leq x_3\leq l_3^{\{1\}},\\ D_2=const & if\quad l_3^{\{1\}}\leq x_3\leq l_3^{\{2\}},\\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots & \\ D_M=const & if\quad l_3^{\{M-1\}}\leq x_3=l_3^{\{M\}},\end{cases}$$

for $\forall x_i \in [0, l_i]$, $(i=1,2)$, where $M$ is the number of stratified mediums parallel to the plane $X_1OX_2$ along the vertical axis $OX_3$; to simplify the formulation, the frontier points of strata are denoted by $a_{i,j}$ and $b_{i,j}$ - namely,

$$a_{i,j} \overset{def}{\equiv} \begin{cases} 0 & if \quad i=1,2, \\ l_3^{\{j\}} & if \quad i=3;\ j \neq 0, \\ l_3^{\{0\}} & if \quad i=3;\ j=0, \end{cases} \qquad b_{i,j} \overset{def}{\equiv} \begin{cases} l_i & if \quad i=1,2;\ \forall j, \\ l_3^{\{j+1\}} & if \quad i=3; \quad \forall j; \end{cases} \tag{37}$$

while $\vec{\vartheta}(x,t) \equiv \vartheta(x_3,t)$ denotes an unknown vector function of the averaged velocity of turbulent atmospheric air flow, which is the solution of the non-linear problem as follows:

$$\frac{\partial \vartheta(x_3,t)}{\partial t} = \frac{\partial}{\partial x_3}\left( D(\vartheta(x_3,t)) \cdot \frac{\partial \vartheta(x_3,t)}{\partial x_3} \right) - \vartheta(x_3,t) \cdot \frac{\partial \vartheta(x_3,t)}{\partial x_3} + P(x_3,t;g),\ 0 < x_3 < l_3,\ t > 0; \tag{38}$$

$$\vartheta(x_3,t)\big|_{t=0} = \vartheta_{initial}(x_3),\ 0 \le x_3 \le l_3; \tag{39}$$

$$\vartheta(x_3,t)\big|_{x_3 = l_3^{\{i\}}} = \vartheta_i(t),\ t \ge 0,\ i = \overline{0,M}; \tag{40}$$

$$\vartheta_{initial}\left(l_3^{\{i\}}\right) = \vartheta_i(0),\ i = \overline{0,M}; \tag{41}$$

$$\vartheta(x_3,t)\big|_{x_3 = l_3^{\{i\}}-0} = \vartheta(x_3,t)\big|_{x_3 = l_3^{\{i\}}+0},\ i = \overline{1,M-1},\ t \ge 0; \tag{42}$$

$$D(\vartheta(x_3,t)) \cdot \frac{\partial \vartheta(x_3,t)}{\partial x_3}\bigg|_{x_3 = l_3^{\{i\}}-0} = D(\vartheta(x_3,t)) \cdot \frac{\partial \vartheta(x_3,t)}{\partial x_3}\bigg|_{x_3 = l_3^{\{i\}}+0},\ i = \overline{1,M-1},\ t \ge 0. \tag{43}$$

In the problem (31)-(43), the known initial data is the constants $g \approx 9.8\ m/c^2$ (free fall acceleration), $N \in \mathbb{N}$, $M \in \mathbb{N}$, $T \in \mathbb{R}_+^1$, $D_j \in \mathbb{R}_+^1$ $\left(j = \overline{1,M}\right)$, $l_i^{\{j\}} \in \mathbb{R}_+^1$ $\left(i = \overline{1,3};\ j = \overline{0,M}\right)$, $\gamma_{i,k,j}^{\{n\}} \in \mathbb{R}_+^1 \left(i=1,2;\quad k = \overline{1,4};\quad j = \overline{0,M-1}\right)$, at that, for $\forall n = \overline{1,N}$, the following takes place:

$$\gamma_{i,k,j}^{\{n\}} = \begin{cases} 1 & if\ j=0;\ i=1;\ k=1,3, \\ 0 & if\ j=0;\ i=1;\ k=2,4, \\ \gamma_{i,k,j}^{\{n\}} > 0 & if \quad j \neq 0;\ \forall i,k; \end{cases} \tag{44}$$

as well as the functions $\vartheta_{initial}(x_3)$, $\vartheta_i(t)$ $\left(i = \overline{1,M}\right)$, $P(x_3,t;g)$ (the power of external sources), $C_0^{\{n\}}(x)$ $\left(\forall n = \overline{1,N}\right)$, $C_{i,j}^{\{n\}}(\bullet,t)$ $\left(i = \overline{1,6};\quad j = \overline{0,M-1}\right)$, at that, for $\forall n = \overline{1,N}$ the following takes place

$$C_{i,j}^{\{n\}}(\bullet,t) = \begin{cases} C_{i,j}^{\{n\}}(\bullet,t)\ if\ \{j=0\} \wedge \{i=3,6\}; \\ 0\ otherwise\ . \end{cases} \tag{45}$$

Finally, it is assumed in tasks (31)-(45) with respect to concentration that the initial function $C_0^{\{n\}}(x)$ and the boundary functions $C_{i,j}^{\{n\}}(\bullet,t)$ meet the corresponding consistency conditions - with respect to each $n = \overline{1,N}$ of the exhaust gas. Thus, in the complete model (31)-(45) the sought functions are the concentration $C^{\{n\}}(x_1,x_2,x_3,t)$ of $n$-th $\left(n = \overline{1,N}\right)$ harmful substance and the averaged velocity $\vartheta(x_3,t)$ of turbulent flow of the atmosphere.

These problems was solved by complicated nontrivial methods. In the Thesis is shown a uniqueness of the solutions.. To resolve these problems in the dissertation there are listed many challenging features found by numeric means, which has been skipped in this work. For example, extraction of roots of transcendence equations for mathematical model ( of the first and of the second type). Also there has been made a determination of quasioptimal model parametres (31)-(45) at limitation in computer resources.. There has been carried out investigation of hydrothermodynamic model of atmospheric processes on the middle scale and worked out special numeric algorithmes for its realization (the complete code of the obtained programme is given in Appendix 3 of the Thesis).

There is shown program realization, ***in the fifth chapter*** of the dissertation, on the created mathematical model. There are demonstrated solutions of transcendence equations of the first and of the second type developed in the mathematical model for determination of harmful substances concentrations in urban quarter, general solution is being found and quasioptimal parameters of this model are being determined at limitation in computer resources.

In the Thesis in the final formula, to find the $n$-th concentration $C^{\{n\}}(x_1,x_2,x_3,t)$ there is the function $w^{\{n\}}(x_1,x_2,x_3,t)$ defined by the expression and the parameters $\overline{\lambda}_i^{\{n\}}$, $\overline{\overline{\lambda}}_j^{\{n\}}$, $\overline{\overline{\overline{\lambda}}}_k^{\{n\}}$ $(i,j,k\in\mathbb{N})$ included into this expression, are defined through the solution of the corresponding transcendental equations which are one-type equations (let's call them the equations of the first type). Moreover, the expression includes the functions $W_{i,j,k}^{\{n\}}(x_1,x_2,x_3)$, defined by the formula (8.64), containing the parameters $\overline{\beta}_i^{\{n\}}$ и $\overline{\overline{\beta}}_i^{\{n\}}$ $(i\in\mathbb{N})$, which are positive roots of the two one-type transcendent equations (let's call them transcendent equations of the second type). Since we have to solve transcendent equations of the first and the second type approximately, – the analytical formula yields not the accurate, but the approximate solution of the initial task. Here, we have to note that the sough-for concentration $C^{\{n\}}(x_1,x_2,x_3,t)$, as it is perceptive even with respect to relatively small errors of the roots of these transcendent equations – especially, with respect to the roots of the transcendent equations of the second type. In dissertation work has developed some stable numerical algorithms allowing one to find all the roots of transcendental equations of the first type and all the positive roots of the transcendental equations of the second type.

The following subsection sets forth the algorithm of localization and finding all the roots of transcendent equations of the first type. Since all the three transcendent equations differ only by their constants – we can confine ourselves only to detailed investigation of only one of those equations.

Let's denote

$$A\overset{def}{\equiv}\frac{\gamma_{2,2,0}^{\{n\}}}{\gamma_{2,1,0}^{\{n\}}}+\frac{\gamma_{2,4,0}^{\{n\}}}{\gamma_{2,3,0}^{\{n\}}},\quad B\overset{def}{\equiv}\frac{\gamma_{2,2,1}^{\{n\}}}{\gamma_{2,1,1}^{\{n\}}}+\frac{\gamma_{2,4,1}^{\{n\}}}{\gamma_{2,3,1}^{\{n\}}},\quad C\overset{def}{\equiv}\frac{\gamma_{2,2,2}^{\{n\}}}{\gamma_{2,1,2}^{\{n\}}}+\frac{\gamma_{2,4,2}^{\{n\}}}{\gamma_{2,3,2}^{\{n\}}}\ \text{и}\ D\overset{def}{\equiv}\frac{\gamma_{2,2,3}^{\{n\}}}{\gamma_{2,1,3}^{\{n\}}}+\frac{\gamma_{2,4,3}^{\{n\}}}{\gamma_{2,3,3}^{\{n\}}}.$$

Then, we will have the equation as follows

$$tg\left(\overline{\overline{\lambda}}^{\{n\}}\cdot l_2\right)=\overline{\overline{\lambda}}^{\{n\}}\cdot\left\{\frac{A}{\overline{\overline{\lambda}}^{\{n\}}-A}+\frac{B}{\overline{\overline{\lambda}}^{\{n\}}-B}+\frac{C}{\overline{\overline{\lambda}}^{\{n\}}-C}+\frac{D}{\overline{\overline{\lambda}}^{\{n\}}-D}\right\}. \tag{46}$$

The equation (46) has the property of transitivity and has a countable number of roots due to the existence of trigonometric function $tg(\bullet)$ in its left hand-side. It is obvious that the function $tg\left(\overset{=\{n\}}{\lambda}\cdot l_2\right)$ is a periodic one, while the function being in the right hand-side of the equation (46), and is not periodic. Consequently, the roots of the equation are not located with constant period on the number axis. However, the special discontinuity points of the equation (8.69) have quasi-periodicity, and their locations on the numerical axis coincide with the locations of special points of the function $tg\left(\overset{=\{n\}}{\lambda}\cdot l_2\right)$ - except for the four points where the common denominators in the right hand-side of the equation go to zero. Moreover, the special points of the function $tg\left(\overset{=\{n\}}{\lambda}\cdot l_2\right)$ pre-dominate over the special points of the function of the right side of the equation – and that's why the equation (46) differs from the equation $tg\left(\overset{=\{n\}}{\lambda}\cdot l_2\right)=0$ only by shifting along y-direction (i.e., the vertical axis) when distancing from the beginning of the coordinates. Consequently, there is solely and only one root between each couple of the neighboring discontinuity points. Then, if we know precise values of the four special discontinuity points of the right-hand side of the equation (46), - we can analytically calculate all the discontinuity points of the equation and hence will be able to define the root localization segments. Indeed, special points of the function $tg\left(\overset{=\{n\}}{\lambda}\cdot l_2\right)$ are $\frac{\pi}{2\cdot l}+n\cdot\frac{\pi}{l},\ n\in\mathbb{N}$.

Now, by stating the beginning of the root search range, we can find as many discontinuities as possible – including the required number $M_{\text{discontinuous}}$ is the number of summands in the sum. Obviously, what we have to add to the points found is the special points of the right hand side of the equation lying in the root search range preset by us. Furthermore, we have to implement the process of finding the sought-for roots, consisting of the following: all the neighboring couples of discontinuities are defined; then a numerical algorithm – for instance, the dichotomy method - is applied to find the roots of the equation (46) in each section of the localization; the ends of t his section will be exactly these couples of the neighboring discontinuities.

The described idea of localization algorithm and finding all the roots of transcendent equations of the first type was realized through the application software package MathCAD and MS Visual Studio C/C++.

Then composed the algorithm of localization and finding positive roots of second-type transcendental equations. Since both transcendental equations differ from each other by invariables only, - we may confine ourselves by a detailed investigation of only one of these equations – for instance, the transcendental equation.

Through substitutions similar to the substitutions, we may examine the following equation instead:

$$D_1\cdot\sqrt{\left(\frac{\overline{\beta}^{\{n\}}}{D_1}-c\right)}\cdot ctg\left(\sqrt{\left(\frac{\overline{\beta}^{\{n\}}}{2\cdot D_1}-c\right)}\right)=D_2\cdot\sqrt{\left(\frac{\overline{\beta}^{\{n\}}}{D_2}-c\right)}\cdot ctg\left(\sqrt{\left(\frac{\overline{\beta}^{\{n\}}}{2\cdot D_2}-c\right)}\right). \qquad (47)$$

At the first glance, the functions with the graphs shown on Fig.5 seem somewhat "chaotic"; however, we can show that, depending on the value of $\Delta D\overset{def}{=}D_1-D_2$, the number and the location of the roots of the equation (47) vary according to a definite law. Namely, the roots of the equation (47) are close to the special points (discontinuities) of the equation, having various signs. Consequently, by finding the function discontinuity points in the left-hand and the right-hand sides of the equation and by arranging these discontinuities, we can find all the couples of the neighboring discontinuities where the sign on the left side of the interval is opposite to that from the right side of the interval. By arranging and numbering the received discontinuity couples, we may claim that, due to the sign difference between any two neighboring couples, the root of the equation (47) is between these two neighboring discontinuities. In other words, such intervals between the found neighboring discontinuities are exactly the sought-for localization of roots. The discontinuities of the equation (47) are obviously to be found from the equation:

$$\sqrt{\left(\frac{\overline{\beta}^{\{n\}}}{2\cdot D_1}-c\right)}=n\cdot\pi,$$

where $n$ is the index number of discontinuity. Hence, we receive

$$\overline{\beta}_{D_1}^{\{n\}}=2\cdot\left\{D_1\cdot(n\cdot\pi)^2+c\right\}. \tag{48}$$

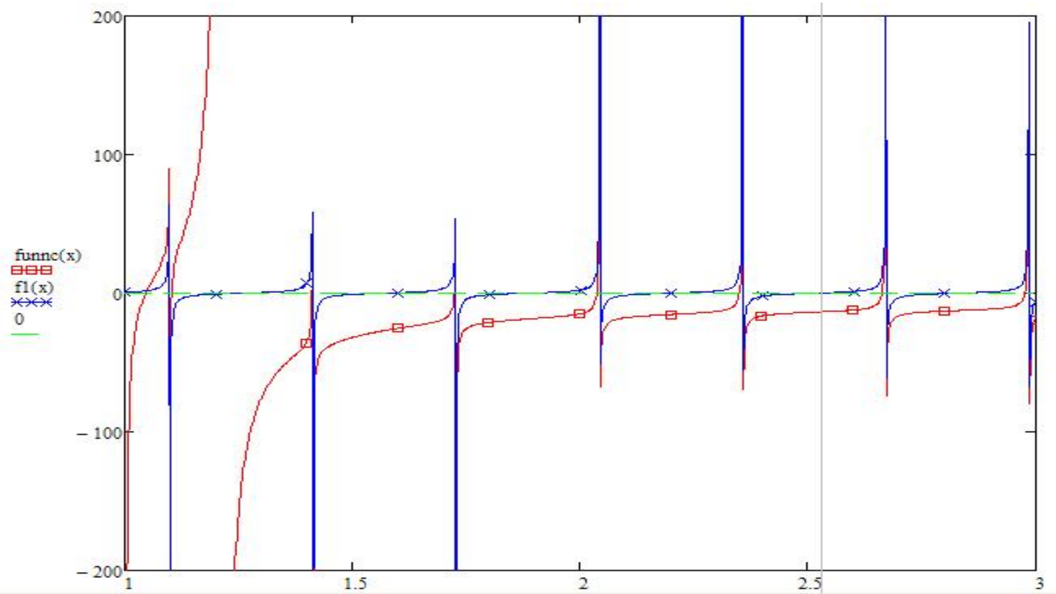


***Fig. 5.*** *Function graphs included into the left-hand and the right-hand sides of the transcendental equation (46) of the first type*

But acting exactly the similar way with respect to the function from the right-hand side of the equation (47), we shall receive:

$$\overline{\beta}_{D_2}^{\{n\}}=2\cdot\left\{D_2\cdot(n\cdot\pi)^2+c\right\}. \tag{49}$$

The $y$-dimension displacement of trigonometric functions located in the right-hand side and the left-hand side of the equation (47) is show below on the diagram (Fig. 6).

Now, using the formulas (48) and (49), we shall receive the ordered list of discontinuities. So, we can neglect the discontinuity couples where the section edges imply the function's having the same sign. This means that the two functions does not intersect the abscissa. i.e., there are no roots at such sections. The non-disregarded discontinuity couples form some sections of localization of the roots of the equation (47) the ends of which consist of discontinuity points. We can search for the roots of the equation (47) on the sections received through the dichotomy method.

Based on the above-stated idea of localization of the roots of the equation (47), an algorithm has been developed in the dissertation work. The algorithm recursively finds the discontinuity points of the equation; it normalizes the received sequence of discontinuity points; it also checks if there exist some roots between any two neighboring elements of this normalized sequence; furthermore, the algorithm finds the sought-for roots through dichotomy method.

Further is carried out determination of quasioptimal parameters of this model at limitation in computer resources.

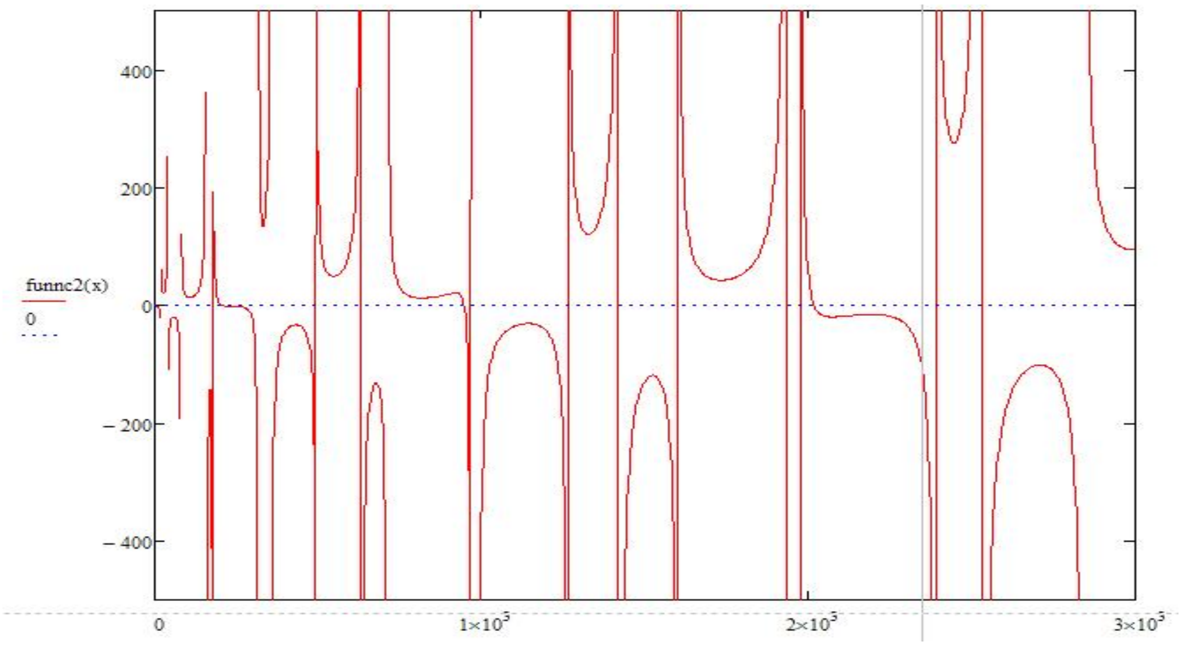


***Fig. 6.*** *The function graphs included into the left and the right parts of the transcendent equation (47) of the second type*

In the computer implementation of the model, the main initial characteristic parameters of the modelled area (layered, "parallelepiped" area) are: the dimensions of the modelled area in space and in time and values of constants and the parameters of the model. In this section of the Thesis, the particular example of a model problem clarifies the abovementioned major theoretical parameters: e.g. a specific street of Riga city with the following initial values of the parameters of the modelled area is observed. In particular, a specific street of the city of Riga, with initial parameter points of the area simulated, is examined.

So, for a given capacity of 4 GB of the main storage by the formula (5.7), we shall get the following quasioptimal values of the model parameters:

- maximum possible length of the considered section of the street holding the heavy traffic flow is 20 meters long;
- maximum possible width of the observed road section is 10 meters wide;
- maximum possible height of the studied "parallelepiped" area of the city is 30 meters high;
- maximum possible time interval during which dynamics of concentration of harmful substance is investigated is 5 minutes;
- maximum possible step in spatial variables is 1/5 metre;
- maximum possible step in time is 1/2 second.

The obtained data show that the desired parameters values of discrete analogue to the mathematical model are not reached. However, we would like to highlight that in our opinion, there is no other way how to overcome the problem, concerning the restriction of resources of modern computing systems. Therefore, the abovementioned parameter values of the discrete analogue of the mathematical model may be accepted as a result of rational and scientific approach applying which the maximum possible coefficients of the search parameters under the condition of reasonable restrictions in computer resources.

***In the sixth chapter*** of the dissertation there are being constructed 3D – nomograms and is checked the adequacy of the mathematical model to the real data.

For practical application of the suggested model, a more complicated program with a higher-level language C/C++ has been worked out; the process was demonstrated in this Chapter.

To check the adequacy of a mathematical model, some statistical data for results verification is necessary. To that end, the author has taken statistical data describing the motor traffic flow intensity and structure at the examined street section and the urban air pollution levels per 1 day. The data has been received from the measuring station located next to the building at 43 Valdemar Street (Fig.7).

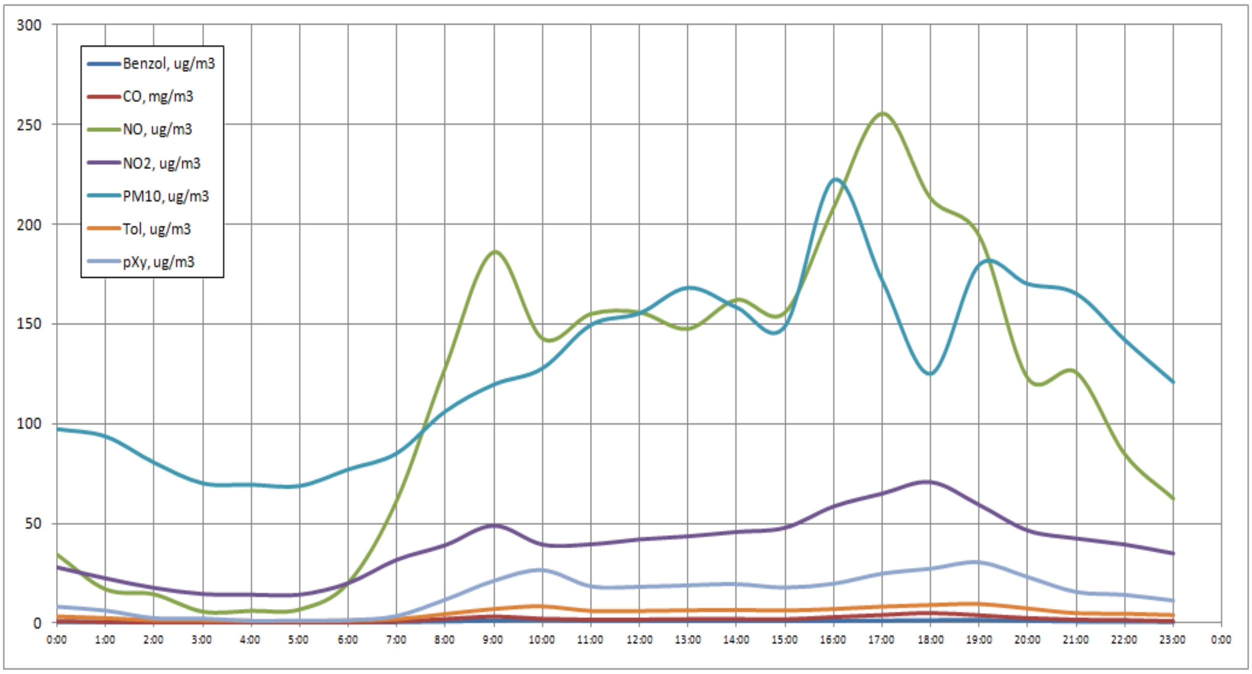


***Fig.7.*** *The concentrations of harmful substances 00:00-23:00*

Fig. 8 shows the street traffic intensity at the analyzed section of the street – i.e., the overall number of motor vehicles (in this case, all kinds of transport taking part in the traffic along that street are taken into account, with the share percentage split-up as follows: 86,9% - motor cars, 0,8% - commercial vehicles (trucks), 8,3% - minibuses, 0,7% - buses, 3,3% - the rest), - and the total concentration of harmful substances within 24 hours, derived based on the data received from the measuring station.

It can be seen from the graph that, although the correlation between the number of transport vehicles and the levels of harmful substances exists, it is still far from a direct dependence. In particular, one can see that the number of motor vehicles in the street is decreased between 3 p.m. and 7 p.m., but concentration levels start fluctuating (very intensively). The traffic flow intensity essentially increases between 6.00 a.m. and 3.00 p.m., while the pollution level increase is not so significant. At the same time, some asymmetric dips and peaks, being in phase opposition, are present. Consequently, the levels of contaminants is influenced by other factors, not only the number of vehicles. For example, the following should be mentioned primarily: turbulent flows in the city block under investigation, depending on wind velocity, and temperature, humidity and etc.

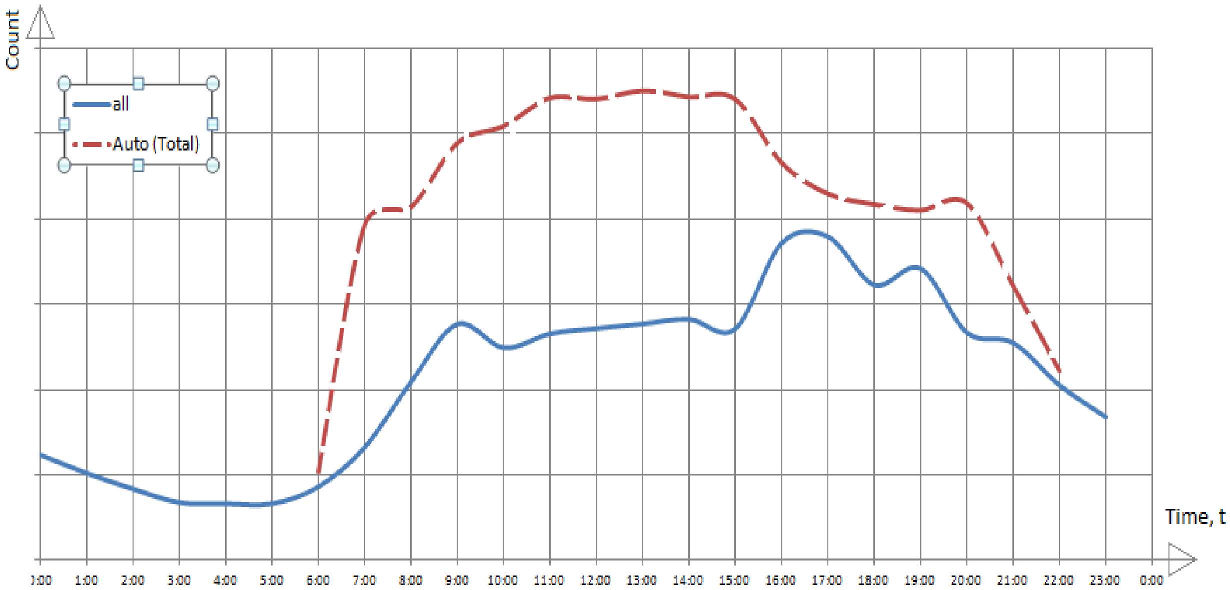


***Fig.8.*** *Number of transport (upper line) and the sum of the weights of hazardous substances (lower line)*

It the thesis considered the example of calculation of concentration change of harmful substance on a different height above the road area at the initiation fixed time moments 1, 2, 6 and 12 hours, respectively, for a hypothetical road section, by virtue of a simplified model.

Considered the road section with the width of 21 m and the length of 165 m. Computations will be performed for an "imaginary vertical column", the foundation of which is exactly in the middle of the considered road and it is determined for the point $(x_1 = 10.5\,\text{m};\ x_2 = 82.5\,\text{m})$. Numerical implementation of considered mathematical model has been used the packaged MathCAD. The results of calculations for the different moments of time, passing after the beginning of turbulent diffusion process, are presented in Fig. 9.

Fig. 9*a* shows a change of concentration $C(x_1 = 10.5, x_2 = 82.5, x_2, t = 1)$ depending on the variable $x_3$, i.e. the constructed curve reflects a change of harmful substance concentration depending on a height $x_3$ in 1 hour after the beginning of process of supervision of harmful substance turbulent diffusion in the fixed point of the road area $(x_1 = 10.5\,\text{m};\ x_2 = 82.5\,\text{m})$.

Changes of concentration depending on a height in the same point at the moments 2, 6 and 12 hours after the beginning of turbulent diffusion process are presented in Fig. 9*b*, 9*c* and 9*d*, respectively. Note that in Fig. 9*a* scale of ordinates is compressed $10^2$ times less, and in the other figures scales of ordinates are taken 10 times less, i.e. there is the graph of function $C(x_3) = 10^{-2} \cdot C(x_1, x_2, x_3, t)\big|_{x_1=10.5;\, x_2=82.5;\, t=1}$ in Fig. 9*a*, and there are the graphs of functions $10^{-1} \cdot C(x_1, x_2, x_3, t)\big|_{x_1=10.5;\, x_2=82.5;\, t=2}$, $10^{-1} \cdot C(x_1, x_2, x_3, t)\big|_{x_1=10.5;\, x_2=82.5;\, t=6}$ and $10^{-1} \cdot C(x_1, x_2, x_3, t)\big|_{x_1=10.5;\, x_2=82.5;\, t=12}$ in Fig. 9*b*, 9*c* and 9*d*, respectively.

It should be noted that this example is merely illustrative (its solution was made under some simplifying assumptions).

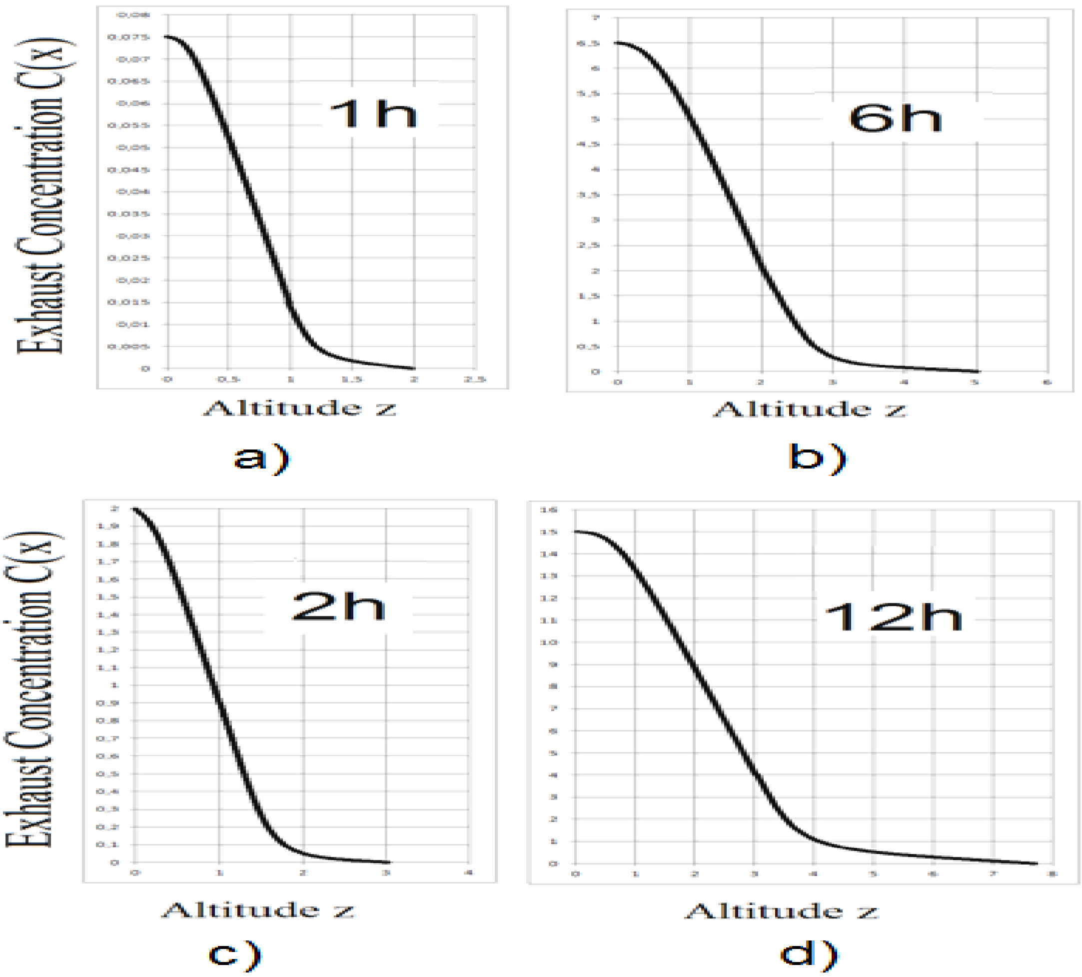


***Fig. 9.*** *Change of the concentration of harmful matter on a different height above a road area*

Then an analysis of the part of road will be carried out with the use of the developed model. According to the obtained initial data we find that the concentration of harmful substances. The initial data: the width of the site – 21 m, the length of the site – 165 m, height of buildings – 30 meters. Concentrations are measured at the height of 2.5 meters (the height of the measuring station on Kr.Valdemara street, Riga), the coordinates of the measuring element at the measuring station in three-dimensional quarter: $x_1$=25m, $x_2$=5m, $x_3$=2.5m.

For the evaluation of the quality of the resulting model the analysis for the mean values of three selected substance: benzene, NO and $NO_2$, is done. These gases were chosen due to the fact that they are the most dangerous among other automobile exhaust gases. Benzene is toxic and dangerous to the environment, a strong carcinogen. NO – non salifiable nitric oxide, it is a colorless gas. Nitric oxide (II) is toxic. $NO_2$ (Nitric oxide (IV) (nitrogen dioxide) - gas, red-brown in color, with a pungent smell. Nitric oxide (IV) is highly toxic.

The dynamics of changing the actual concentration of harmful substances and the concentration calculated according to the model is presented at Fig.10. The figure shows that the model badly takes into account 3 parameters: the initial concentration of harmful substances (which was taken as 0), as well as two "rush hours" of the road traffic (which in the model were not considered). Model also does not take into account the street terrain (vehicles motion velocities, its height, etc.).

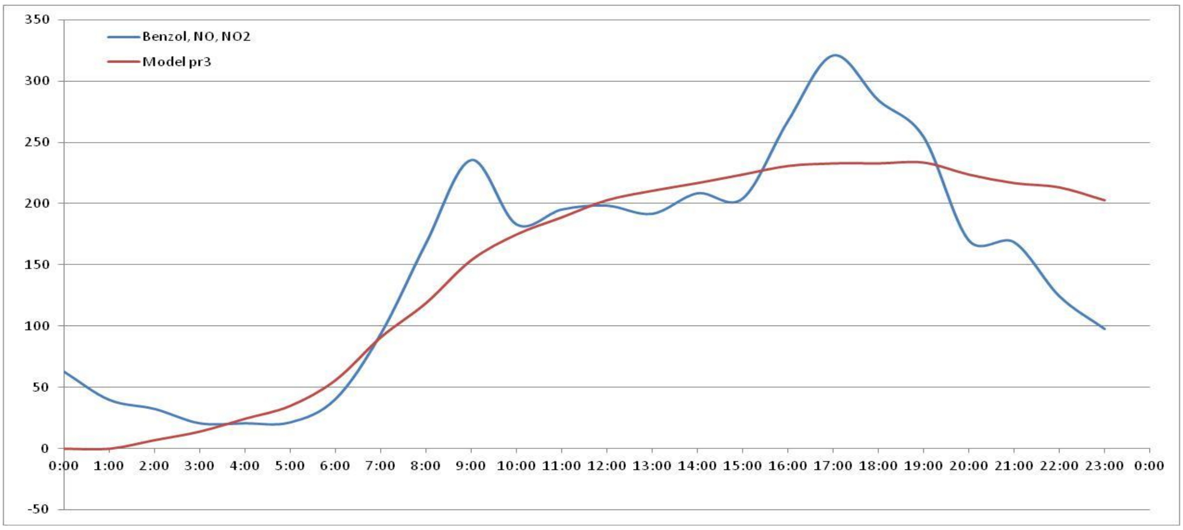


***Fig.10.*** *The concentrations sum of three pollutants (benzene, NO, NO2) and reduced concentrations computed by the model*

As an example was fixed the coordinates of the point measured by the width and length of the studied road section (see Fig.11). It shows that in case of increasing the height, the concentrations of harmful substances is reducing. During the time the concentration are also growing vertically.

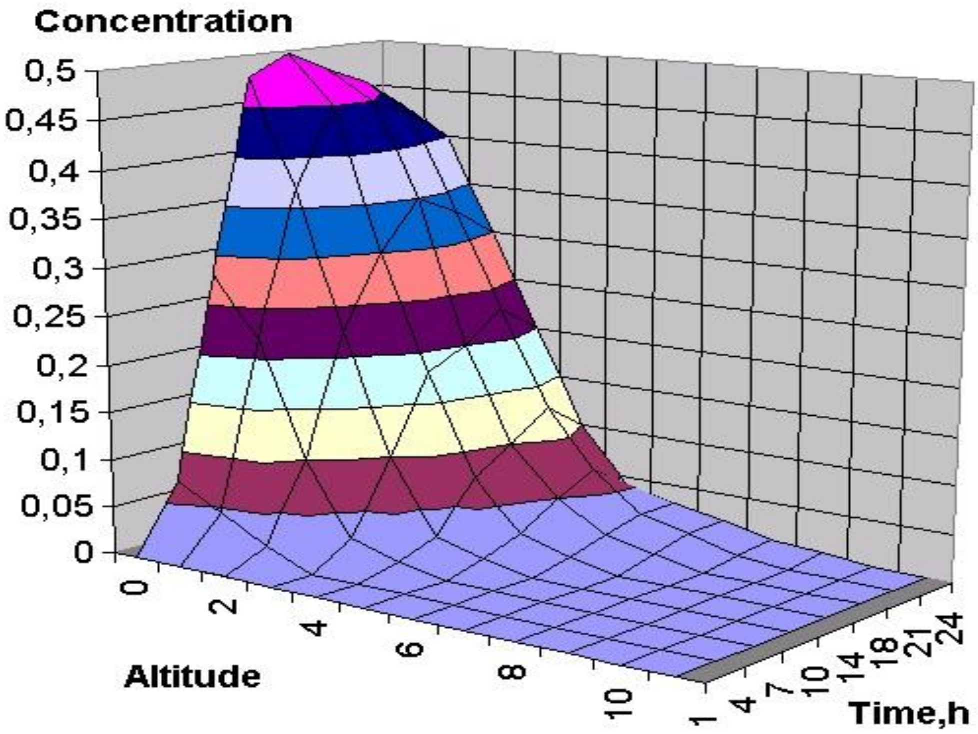


***Fig. 11.*** *Distribution of concentration at a fixed length and width of the measured points*

In the Thesis, distribution of concentrations with respect to each of the harmful substances investigated (CO, NO, $NO_2$, benzene) was calculated separately. As one can see from the graphs (Fig. 12–15), the mathematical model reflects the actual (real) concentration of each harmful substance in a fairly good way. More accurate data can be performed in further studies, taking into account the values of molecular masses of each substance of the investigated gases. In addition, it is necessary to take into account two automobile "rush hours" (morning and evening). With further studies, it is assumed to obtain results considering the influence of the number of vehicles and meteorological parameters.

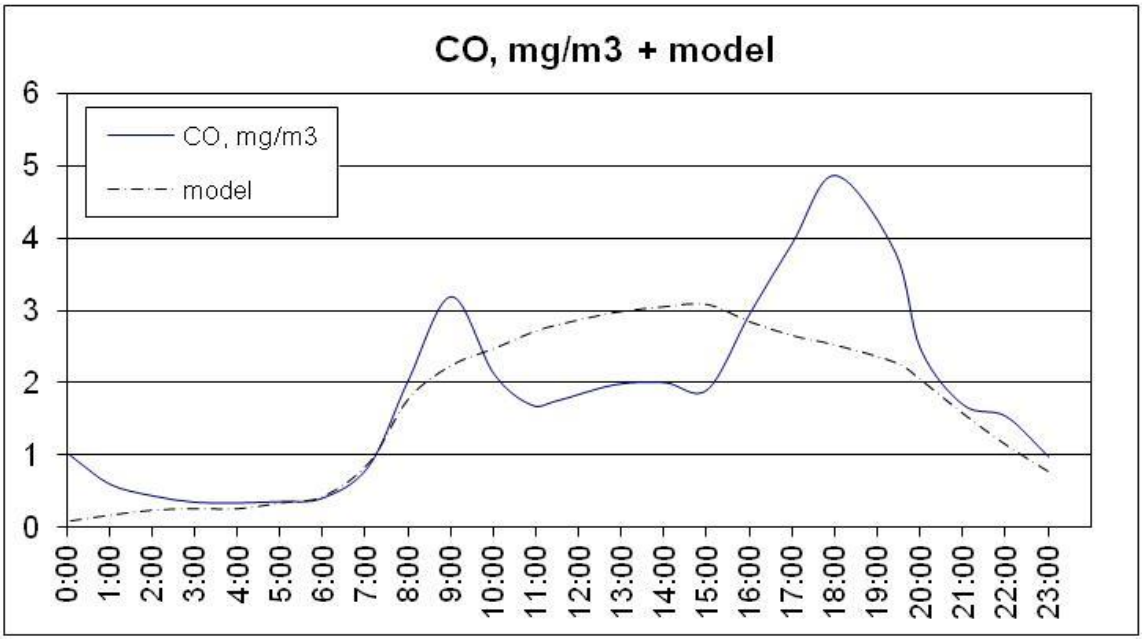


***Fig. 12.*** *Concentration levels of CO substance*
*Actual concentration and concentration calculated according to the model*

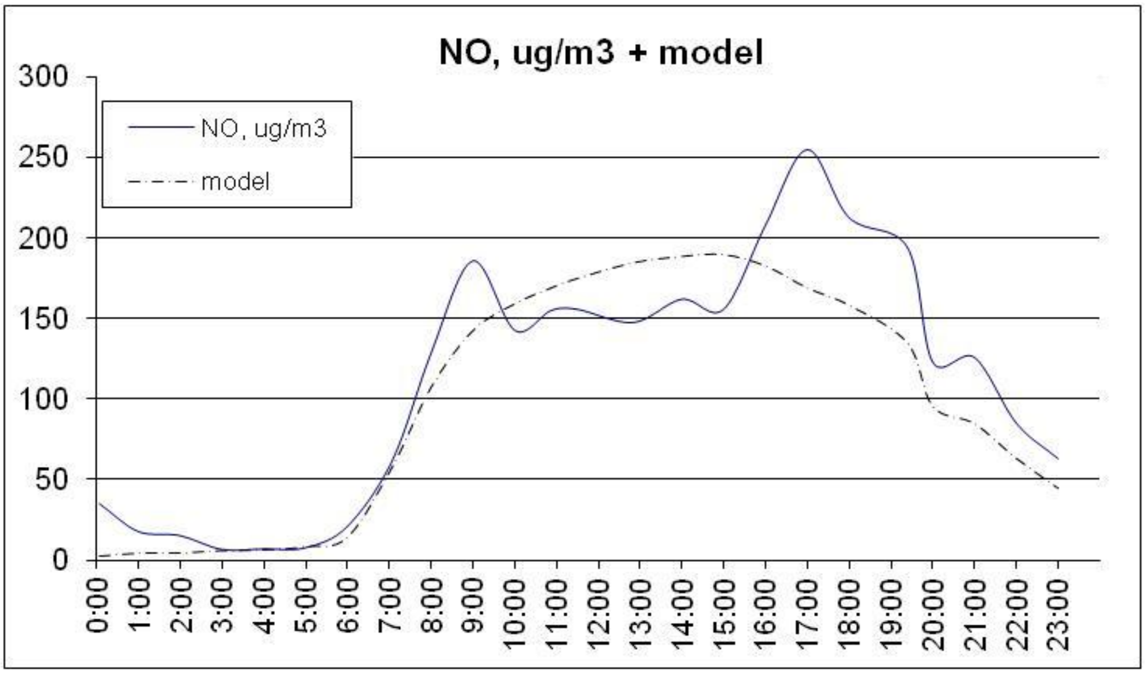


***Fig. 13.*** *Concentration levels of NO substance*
*Actual concentration and concentration calculated according to the model*

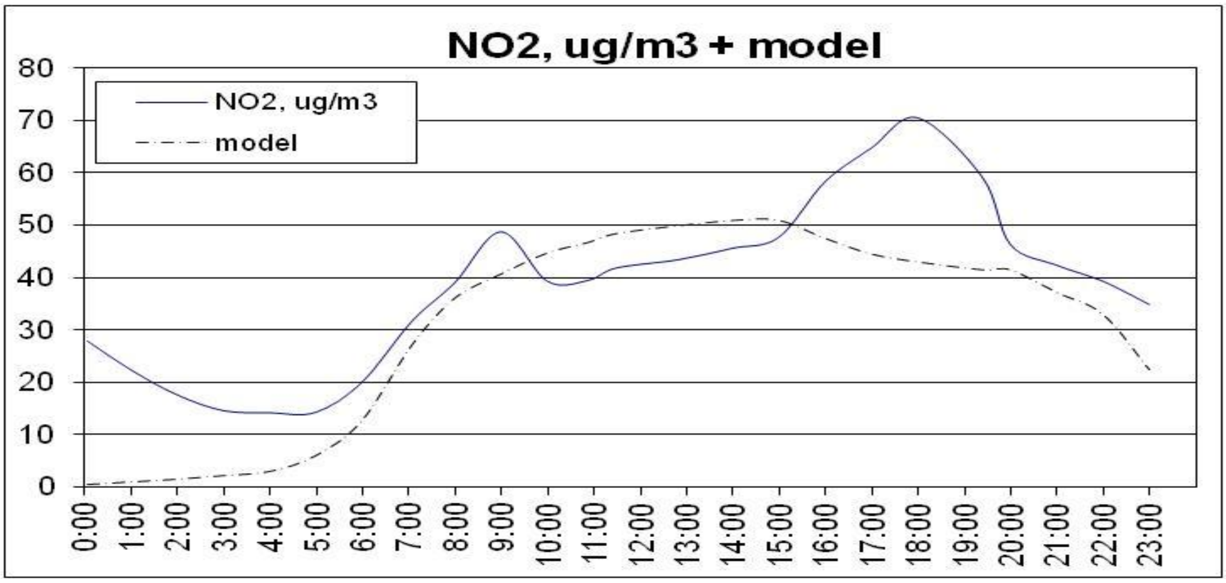


***Fig. 14.*** *Concentration levels of $NO_2$ substance*
*Actual concentration and concentration calculated according to the model*

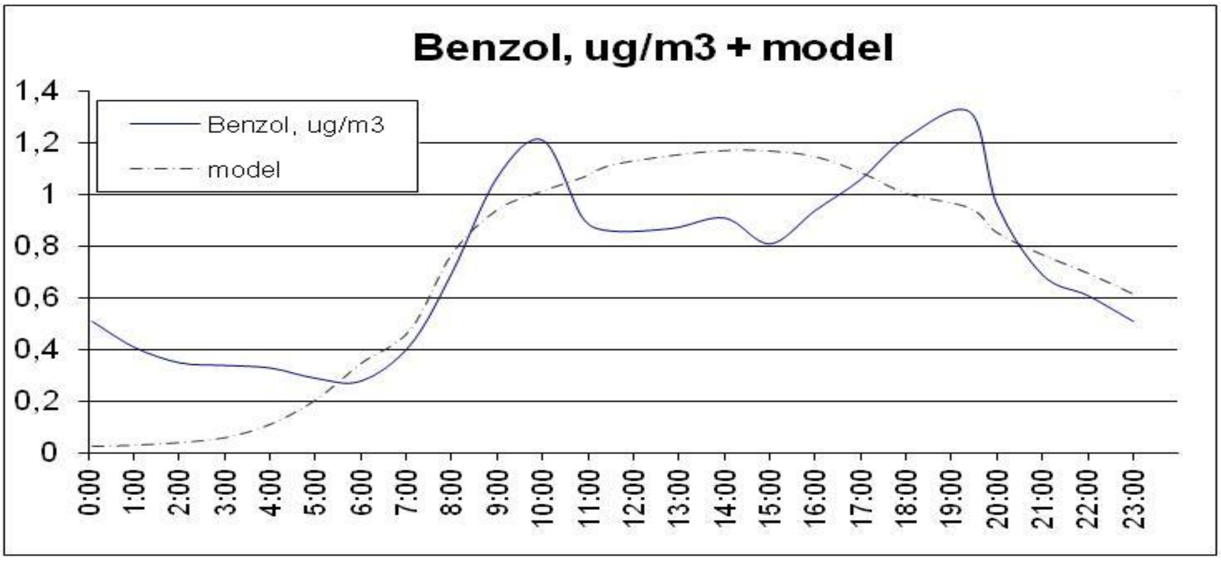


***Fig. 15.*** *Concentration levels of benzol,*
*Actual concentration and concentration calculated according to the model*

The sixth Chapter shows that all the investigations performed within the framework of the Thesis are related to transportation ecology problems. In particular, they are connected with the transport-rendered impact on urban air. The research efforts concerning investigation of ecological state of environment, including, in particular, selection of mathematical apparatus, construction of new mathematical models with the analysis of conditions of their functioning and practical application – are shown at the diagram (Fig. 16). The text blocks used in the Thesis are highlighted by subdued color. For obtaining *The ecological state of environment*, two kinds of sources of pollution (emitters) have been highlighted – the *Internal* and the *External* ones. In order to support the objective posed, the internal sources have been selected, with respect to which, some kinds of internal sources of pollution have been considered - in particular, *Transport* was investigated. *Motor transport* was highlighted among all kinds of transport. Among the kinds of environmental impact from motor transport, the kind *Pollutant emissions into airborne environment of the City* was highlighted.

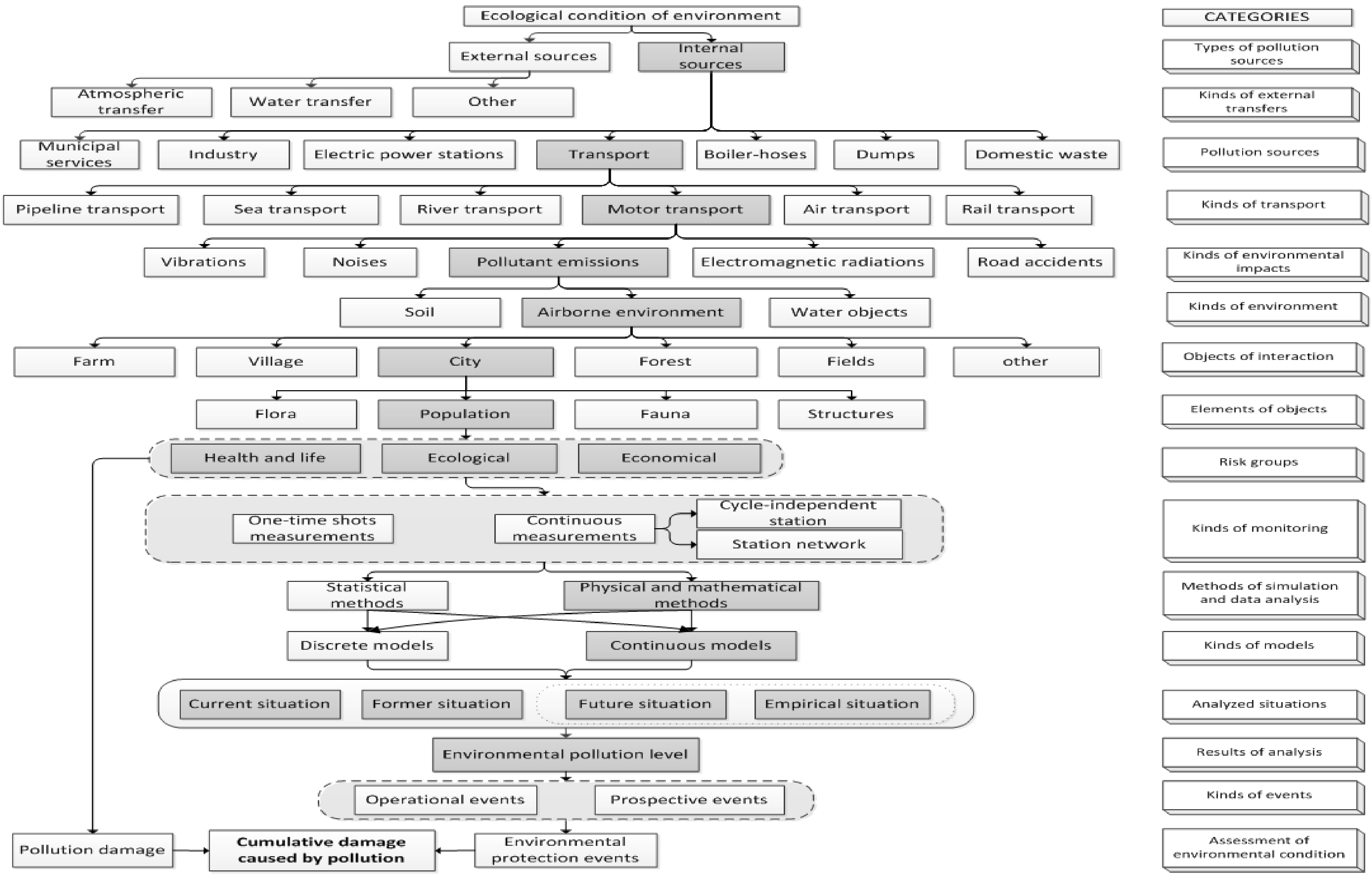


***Fig. 16***. *Schematic structure of research*

The impact rendered by emissions is characterized by the environmental state models with regard to the environment affecting the health condition of the *Population* of the city. Those models allow one to carry out risk analysis (the category: *Risk groups).* Calculation analysis is possible, based on *Monitoring* of the ecological state of air environment. To analyze situations, the main approaches to simulation of pollutant contamination of the urban air were examined.

Selection of *Physical mathematical apparatus* enabling one to simulate any necessary situations to obtain a more accurate (as against *Statistical methods*) result of measuring concentration of harmful substances in turbulent urban atmosphere, has been substantiated. This technique implies the existence of two kinds of models – *Continuous* and *Discrete.* The author has chosen continuous models; using them allows one to obtain *Air environment pollution levels.*

By virtue of the developed mathematical models based on partial differential equations, *Air environment pollution levels* are determined. This allows one to suggest organizing of environment protection events and to assess the actual change in environmental conditions.

The cost of the environment protection actions and the environmental pollution damage (including the cost of medical services aimed at health resumption etc.) enable one to assess the total loss experience connected with lack of ecological safety – in the city in this case.

Based on the data illustrating the cost of the environment protection actions and the damage calculated by applying risk models, we obtain a possibility of assessing the integral air *Pollution damage* in the city. In this connection, it seems expedient to apply to the algorithm scheme of an action aimed at minimizing of (and adaptation to) urban air pollutions, developed by us.

Fig. 17 shows the organization chart of the events aimed at minimizing urban air pollution from motor vehicles; the chart conststs of three units:

1. Initial data (basic characteristics of the ecological transport system).
2. Decision-making support (measuring the motor vehicles-produced pollutants concentration dynamics).
3. Mitigation and adaptation (technological and socio-economic preventive care and adaptive control).

First of all, we shall briefly explain the content of the first unit, i.e., the base characteristics of ecological transport system. Local information on movable sources of urban atmospheric pollution includes the overall number of motor vehicles registered in the city, the number of „active" motor vehicles split up according to seasons, working days, days-off, and holidays, - as well as the averaged number of motor vehicles coming to and going from the city, split up accordiing to seasons; kinds of liquid and gaseous fuel for motor vehicles, and their structural chemical, carburettor-related, and fractional characteristics; the city fleet of motor vehicles modernization dynamics split up according to MV types. The main characteristics of traffic flow are density, intensity, and velocity. The traffic flow structure is characterized by structural characteristics: homogeneity an inhomogeneity of traffic flow (i.e. presence of motor vehicles of various types in the traffic flow), types of inhomogeneity and their share percentage: motor vehicles (according to vehicle size, gross vehcile weight, pollution properties), and commercial trucks ((according to vehicle size, gross vehcile weight, pollution properties). The same unit includes composition and characteristics of exhaust gases (the main substances and their chemical, kinematic, dynamic, and provisional properties) supplemented by the main characteristics of the atmospheric environment of the city (velocity and direction of turbulent air flow; temperature stratification, pressure, density, and humidity

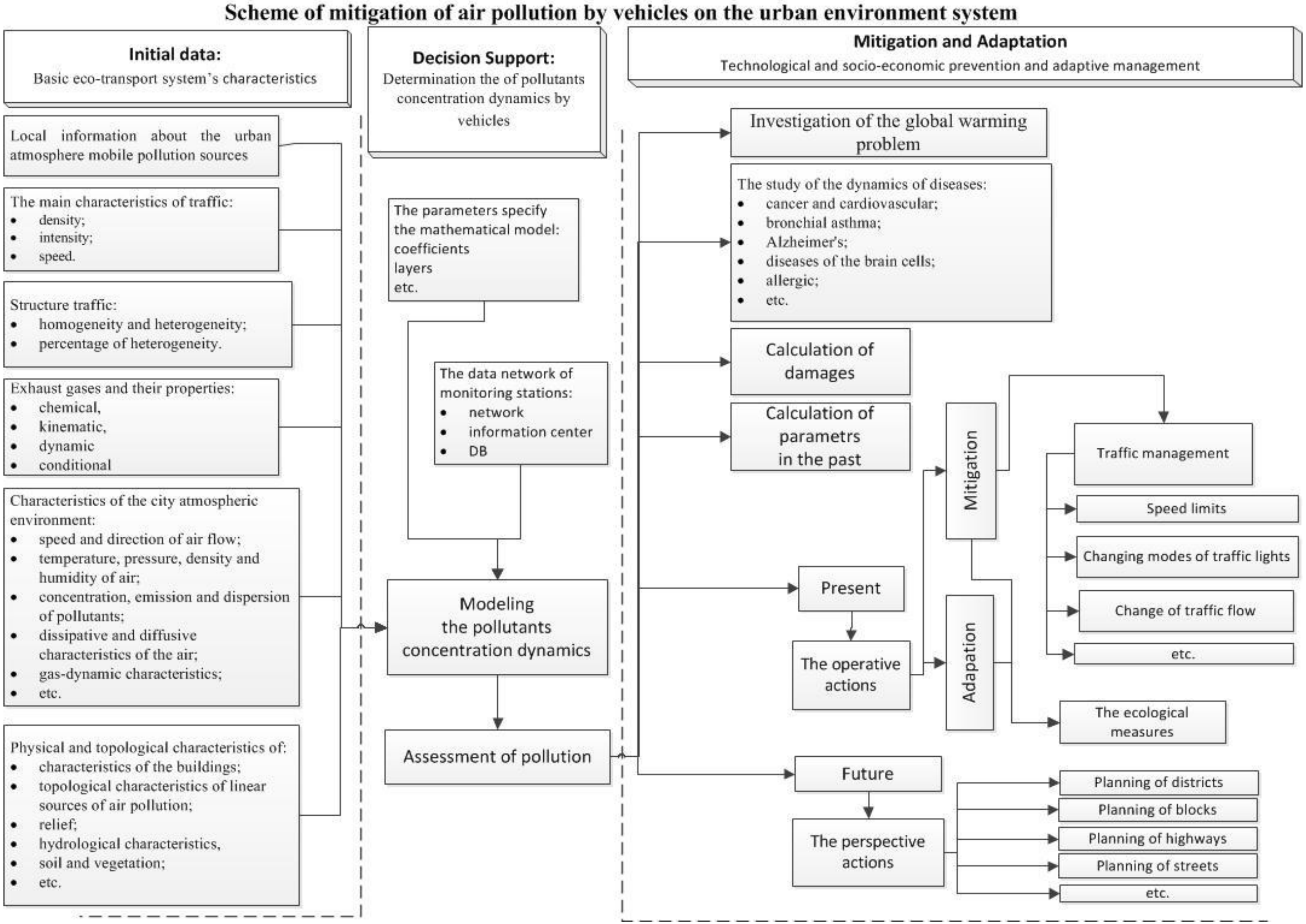


***Fig. 17.*** *The scheme of minimization of urban air pollution from motor vehicles*

of airborne environment; concentration, emission, and dissipation of pollutants; dissipative and diffusion characteristics of air basin; gas dynamics characteristics, etc.), - and by physical and topological characeristics of the city (characteristics of its geological structure, topological characteristics of linear sources of air basin pollution, land surface pattern, hydrological characteristics, soil mantle and growth (plantations) etc.)).

Now let us male a few remarks with regard to the second unit – the decision-making support. Determination of motor vehicle-produced pollutants concentration dynamics (assessment of exhaust gases concentration in the city air basin is one of the significant attributes for making Pareto optimal decisions (i.e. effective or unimprovable trade-off decisions): 1) when measuring the urban air quality; 2) when solving operational issues concerning road traffic organization within the boundaries of urban residential districts; 3) in process of perspective planning of urban development; 4) when designing new residential blocks, 5) when building new motorways; 6) when investigating the dynamics of oncological and cardiovascular diseases, bronchial asthma, Alzheimer’s disease, various types of diseases of brain cells and allergic diseases, 7) when investigating the problem of global warming.

Technological and socioeconomic preventive care and adaptive control (Mitigation and Adaptation) represent the third unit. It includes different goals: investigation of the global warming problem; studying the dynamics of various diseases (connected with ecology), calculation of damage (both in the past and in the future), operational and prospective actions aimed at adaptation and/or mitigation of ecological consequences.

The sixth chapter also outlines the directions for future research of the assessment of risks connected with transport-rendered impact on human health. In Appendix 4 enclosed to the promotional paper, some risk assessment models have been developed, taking into account various numbers of system (man) statuses and magnification of changes within a system (a human body). Appendix 3 shows some materials illustrating the linkage of problems when assessing the urban environmental pollution damage rendered by transport, and using probabilistic analysis of ecological safety (PAES). PAES may be one of the efficient approaches when assessing the state of environment and human body subject to an essential influence from urban transport.

## CONCLUSIONS

1. The entire set of studies is carried out in the Thesis in order to cope with the significant problems of urban transport ecology. The Thesis is devoted to the studies, which unconditionally approve innovative idea of using the apparatus of mathematical physics, despite its complexity, for solving urban environmental problems. The applied scientific approach opens new prospective ways of solving urban transport ecological problems, covering the urban quarters and highway design, planning the traffic routes within the existing urban quarters, etc.
2. In the first stage of research within the Thesis, the problems related to the influence upon transport system on the city ecology were analysed. The main kinds of transportation impact rendered upon natural and artificial ecological systems have been investigated. As the main application object of the findings obtained by the promotional work research, the problem of city canyons pollution by transport has been highlighted and examined.
3. To provide for an analytical numerical solution of continuous mathematical models constructed in the Thesis and the algorithms developed, the problem of interpolation of initial funcitons set in discrete way (in tabular style) has been investigated. Various interpolation scenarios have been discussed. The examples cited in the Thesis illustrate the danger of a „blind” usage of interpolation formulas. It has been shown that spline technique enables one to solve the important problem of interfacing the boundary and coordinated initial data (functions), which greatly simplifies the setting of initial conditions and data.

4. The existing mathematical models, which are used for solving problems of transport ecology were analysed. Non-sufficiency of exclusively the mathematical statistics methods application for evaluating pollution problems of urban atmosphere caused by the anthropogenic sources was justified due to the existing problem of single measurements accuracy in urban dynamic conditions having a turbulent atmosphere; the fact of hypersensitivity towards the measurement error of urban atmosphere ecological quality monitoring was scientifically grounded. The fundamental use of the toolset of mathematical physics and it`s major component – the theory of differential equations in partial derivatives for solving transport ecology problems was substantiated. The apparatus of mathematical physics allows obtaining more accurate results without demanding a huge accumulation of statistical data.

5. The classification and analysis of the turbulent flows models in the urban aerial environment was performed. It was shown that the complex problem solving of harmful substance spread for both micro- and macroscale processes is advisable to carry out on the basis of the modified hydrodynamic and hydro-thermodynamic models. The influence of the Earth's terrain irregularity on the flow of air masses was investigated. Taking into account the specific set of turbulent flows in an urban atmosphere, the closed system of hydrodynamic equations in a mesoscale was derived from scratch, it consists from the Navier-Stokes equations, the Reynolds equation, heat inflow equation and the specific humidity equation.

6. A three-dimensional mathematical model in respect to spatial variables aimed for determination of the concentration dynamics of harmful substance in the stratified turbulent urban atmosphere was developed and analyzed based on the understanding that a turbulent atmospheric airflow velocity is known. Various boundary conditions are formulated for the constructed mathematical model as well as the correctness of the mathematical statements is proved, which were obtained from different initial-boundary problems.

7. A 3D mathematical model of determination of the concentration dynamics of harmful substance in the stratified turbulent urban atmosphere was developed and investigated, taking into account that velocity of a flow of a turbulent atmospheric air is not the desired value. The model is constructed, taking into account assumptions that the transient velocity of the turbulent air flow depends only on a vertical spatial coordinate.

8. While constructing a 3D mathematical model the following complementary results were also obtained:

- various boundary conditions were formulated and the correctness of the mathematical statements of the obtained various initial-boundary problems was proved; formulating of the initial-boundary conditions and clear evidence are required in order to substantiate the measurement complex, technical means for measurers-experimenters who are taking measurements of the concentration of harmful substance;
- with the assistance of a non-degenerate transformation, the initial 3D model of determination of the harmful substance concentration dynamics in the stratified turbulent urban atmosphere is reduced to the equal, but more simple model and the uniqueness of it`s solution is proved;
- the finite-difference approximation issue actual for the model of determination of turbulent flow velocity is investigated, the numerical solvability was studied, the conditions-assumptions required for the effective computer implementation of the discrete model towards a priori unknown velocity of the turbulent flow were formulated.

9. The Thesis studies are carried out in order to reduce the labour-consumption of the computational procedures’ algorithms:

- independent additional problems are studied, related to finding the roots of two different types of transcendental equations and solving systems of non-linear algebraic equations for the implementation of numerical algorithms;
- the conditions-requirements, reducing the computational complexity of the implemented algorithms for several values that depend on the steps value of the constructed spatial-time grid and on dimension of the investigated layered surroundings were formulated;
- multicriteria multidimensional conventional extremum problem of decision-making for the quasioptimal parameters of the finite-difference model under the condition of restricted modern computer resources was formulated and analytically solved;
- the software tool aimed at planning and conducting of numerical experiments was developed.

10. The following significant results are obtained in the process of the research:
    - for evaluating the airflow disturbances caused by near-earth terrain layer it is necessary to analyse the layered "parallelepiped" area of urban environment on the basis of appropriate motion equation and heat inflow balance equation; it is the required condition for constructing a complete system of equations of the airflow disturbances complemented by the issue study of finding the asymptotic solutions of the constructed complete system;
    - it is mathematically proved that the presence of irregularities on the ground surface has an effect on flowing around the object airflow or liquid and therefore ignoring the Earth terrain in the construction of mathematical models of the impurities motion in the aerial and in aqueous environments does not allow identifying and evaluating the resulted disturbances of water or air flow;
    - in a Cartesian coordinate system, the hydro-thermodynamic model of the atmospheric processes in the mesoscale is constructed;
    - the corresponding initial-boundary conditions of the mixed type are constructed as well as the consistency conditions due to the fact that the layered "parallelepiped" area is considered to be the studied area for solving the urban transport ecology problem;
    - for the numerical solution of the obtained initial-boundary problem, its discretization in time has been made in accordance with the rules of the splitting method;
    - a two-cyclic splitting scheme for the numerical solution of the discrete analogue of the hydro-thermodynamic model of the atmospheric processes in the mesoscale was developed.

11. The developed algorithms were tested on the basis of specific examples, involving the application software package MathCAD and C/C++ programming language using Visual Studio 2010 environment. The statistical data of the ecology monitoring station owned by the ecology and transport structures of Riga City Council was analyzed and processed for models validating purpose. The results of calculations performed on the basis of the developed models go well together with real life data.

12. The results of the thesis have practical application in solving ecological problems of a city and allow:

- to produce modeling of pollution by exhaust gases in urban quarter neighborhoods of the city;
- to provide an analysis of the environmental situation in the residential quartersof the city;
- to predict the change in concentrations of the exhaust gas at any point of the tested city quarter;

- to address operational issues related to the ecology of the city;
- to make evidence-based proposals for the layout of streets and neighborhoods in terms of environmental safety;
- to make long-term planning of urban infrastructure development (new districts, neighborhoods) as well as for the construction of new urban highways.

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