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MULTILATERATION ERROR INVESTIGATION AND CLASSIFICATION. ERROR ESTIMATION

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The MLAT errors have their origin at TDOA measurement errors, as primary input information. These errors can be distinguished by many different criteria. The basic criterion is mechanism of error origin, for example. Under this criterion the errors can be split into the main categories – temperature dependence and aging of the HW component. In this article the analysis of errors of target localization using multilateration system have been made.

Keywords: *multilateration, accuracy, errors*

1. Abbreviations

ANS	Air Navigation Services
ATM	Air Traffic Management
GPS	Global Positioning System
HW	Hardware
MLAT	Multilateration
MODE S	Mode S Radar
MSSR	Monopulse Secondary Surveillance Radar
RMS	Root Mean Square
SNR	Signal to Noise Ratio
SSR	Secondary Surveillance Radar
TDOA	Time Difference of Arrival
TOA	Time Of Arrival
WAM	Wide Area Multilateration

2. Introduction

Errors of multilateration systems can be split into the main categories. For example:

- Errors, which are out of any user and/or system control ("natural" errors)
  - Signal to noise ratio, actual state of troposphere, terrain profile
  - Errors of SSR transponders of tracked targets
- Errors produced by the system (HW and SW implementation)
  - Quantization error
  - Antennas position error
  - Temperature dependence and aging of the HW components

3. Errors in MLAT Systems, Overall Description

From the TDOA measurements and MLAT data processing point of view it is more appropriate to categorize the corresponding errors by its statistical nature. Statistical model of MLAT error can distinguish 3 basic error categories:

1. Systematic errors σ_1.
2. Correlated errors σ_2.
3. Random errors σ_3.

Important mechanism is the speed of time variations of the corresponding error components.

3.1. Systematic Errors

Systematic errors are slowly varying in time and they are present in every TDOA measurement. Typically, these errors have the origin at additional delays' variations, regional and local troposphere changes and imperfect antennas position measurement, which are projected by various mechanisms to every TDOA measurement.

The additional delay is determined mainly by temperature dependence and/or aging of the HW components.

Projection of the errors corresponding to the antennas position measurement to the TDOA measurements strongly depends on actual positions of the tracked targets.

In the frame of the MLAT error statistical model this kind of errors (slowly time varying) can be significantly reduced by continuous calibration process.

3.2. Correlated Errors

This type of errors is slowly varying in time too, but it is connected only with TDOA measurements for individual targets located in the given domain of MLAT coverage region. These errors have typically origin at actual troposphere condition along the line of sight Target-Antenna. In the frame of the MLAT error statistical model it is only possible to monitor these errors and consequently exclude them from further MLAT data processing for targets located at "problematic" regions. For each MLAT receiving antenna and given MLAT geometry configuration it is possible to define map of "problematic" regions and subsequently exclude TDOA measurements corresponding to the targets in these regions.

Another type of correlated errors is caused by the multi-path signal propagation.

3.3. Random Errors

This type of errors has white noise character and is produced by additive external and/or internal noise at received signals (described quantitatively by SNR) and by quantization effects due to the digitalisation of the TDOA measurements.

The random error of the TDOA measurement has three main components:

Jitter of the leading edge of the pulse, caused by noise. Its standard deviation is in the optimal case:

σ_{SNR} = tr / √(2*SNR),

where

tr – the length of the pulse leading edge;

SNR – signal to noise ratio.

For the SSR pulse with typically tr = 70ns and SNR = 18dB:

σ_{SNR} = 70 / sqrt(2*10^1.8) = 6.2 ns.

Quantization error of the Measuring Unit (MU), where MU is unit which is used to quantize time period in the Central Processing Station where all pulses from all receiving station are come.

$\sigma_{MU} = \Delta t / \sqrt{12}$,

where

Δt is the discrete of the MU; Δt = 12.5 ns resp. 3.1 ns,

so the Quantization error is about σ_{MU} = 3.6 ns resp. 0.9 ns.

Random error dependent on used time synchronisation technology.

Common Time (CT) – Jitter of the leading edge of the pulse, caused by the air (optic) data link: considered to be $\sigma_r = 7 / \sqrt{12}$ [ns] = 2 ns (σ_r = 1ns for optic).

Distributed Time (DT) – random error caused by the "common view" time synchronisation (GNSS / reference transponder).

Estimated values σ_s = 1 to 6 ns.

The overall standard deviation of the TDOA measurement consequently is as follows:

σ_{TDOA} = √(2 * σ2SNR + 2 * σ2MU + σ2r) using common time technology, or

σ_{TDOA} = √(2 * σ2SNR + 2 * σ2MU + σ2s) using distributed time technology.

The components σ_{2SNR} and σ_{2MU} are calculated twice, because two root squares are needed in creating a hyperbola.

When using common time, the typical overall random error for 1 pulse of SSR reply (e.g. F1 in case of A/C replies) consequently is the following:

σ_{TDOA} = 10.4 ns resp. 9.1 ns for the MU discrete Δt = 12.5ns resp. 3.1ns.

For the distributed technology, the figures are practically the same.

Using more SSR pulses enables better performance.

In the frame of MLAT error statistical model and subsequent data processing these errors can be significantly eliminate by two step processing:

averaging process, where corresponding standard deviation linearly decreases with the square root of measurements number:

- Kalman filtering process (tracking filter);
- Mode S replies/squitters generally provide better accuracy values.

4. Errors by Type of ITS ORIGIN

Up to now TOA errors of MLAT systems are not sufficiently classified and analysed, classifying them by place of origin, several types of errors might be identified. Each of them can influence on MLAT system performance namely:

1. Signal propagation errors –
    a. Propagation errors,
    b. Potential errors,
    c. Instrumental errors;
2. TDOA or timing errors;
3. Signal corruption;
4. Algorithm errors;
5. Survey errors.

Let's consider errors listed above more precisely.

4.1. Errors of Signal Propagation

This sort of errors can be sub classified as external errors and circuit errors. External errors are caused, for instance, by instability of radio wave propagation. Circuit errors can be divided on errors caused by noise and instrumental errors, which can appear due to imperfection of multilateration system nodes. Inappropriate adjustment/calibration of equipment can cause instrumental errors as well.

Noise acts a specific role. It is well known because of under the given shape of signal and signal to noise ratio (SNR), the optimal signal processing will provide a minimum error due to noise influence. Such minimum error is called potential error and characterizes the maximum accuracy under other ideal conditions.

Let us assume a normal distribution of random errors. Then the root-mean-square error (RMS) – σ(α) is as follows:

$$\sigma(\alpha) = \sqrt{\sum_{k=1}^{n} \frac{(\Delta\alpha_k)^2}{n-1}} \,. \qquad (1)$$

Here, $\Delta\alpha k = \alpha 0 - \alpha k$ is an error of k measurement and $\alpha 0 -$ is a true value.

Since random errors arise from different origins, their dispersions are coming into summation. The overall error of one receiving site is as follows:

$$\sigma_{\Sigma(td)} = \sqrt{\sigma_{prop(td)}{}^2 + \sigma_{pot(td)}{}^2 + \sum_i \sigma_{i(td)}{}^2} \,, \qquad (2)$$

where:

${\sigma_{prop(td)}}^2$ – propagation error due to conditions of electromagnetic waves propagation;

$\sigma_{pot}(td)^2$ – potential error due to noise;

$\sigma_i(td)^2$ – instrumental error, due to imperfection of receiver;

td – time of signal arrival to the receiving station (TOA).

It is easy to found the error depending on range based on TOA error using the following calculation:

$${\sigma_{r(td)}}^2 = c * \sigma^2_{(td)}. \tag{2.1}$$

4.1.1. Propagation Errors

Errors due to propagation effects are systematic: any random variation will manifest itself as variations in the TOA.

The systematic error is dependent on the lateral distance and the altitude of the target since both of these factors affect the shape of the actual propagation path. There are two characteristics of the signal propagation that produce conflicting effects: a larger mean speed (due to decreasing refractive index) and a longer propagation path (due to refraction).

At smaller lateral distances, the propagation path is approximately linear and hence the dominant effect is the underestimation of the propagation speed. This causes the calculated straight line separation length to be smaller than the actual straight line separation.

At larger lateral distances, the propagation path is not linear and the actual path taken by the signal is longer than the assumed path. This causes the calculated straight line separation to be larger than the actual straight line separation.

As it was mentioned above, propagation errors arise because of unstable conditions of electromagnetic wave propagation. The RMS error of range measurement due to speed deviation from the assumed value can be calculated by means of equation Δr/r=Δc/c, where r is a range and c is assumed speed of electromagnetic wave propagation in the medium.

Considering ΔR and Δc as systematic errors and exchanging them to the corresponding RMS value, we can get the following:

$$\sigma_{prop(r)} = r\sigma(c)/c. \tag{3}$$

According to [1]

$$\sigma(c)/c \approx 2,7 \cdot 10^{-4}. \tag{4}$$

From this

$$\sigma_{prop(r)} = 2,7 \cdot 10^{-4} \cdot r. \tag{5}$$

From (5) it is clear that propagation error is proportional to the target distance.

4.1.2. Potential errors

Potential errors are caused by noise influence in case of known signal shape, and optimal signal processing. It will define the maximum measurement accuracy. For real devices all the time there is a sum of signal and noise at the receiver input. Noise oscillations have random amplitude and phase allocation.

It is well known that the potential accuracy due to the optimal receiving depends on signal bandwidth and SNR only. The potential error can be calculated as:

$$\sigma_{pot(td)} = [\Delta f_e \sqrt{2E_s / N_o}]^{-1}, \tag{6}$$

where

E_s – energy of pulse signal with duration of τ_p till detector (in intermediate frequency);

N_o – spectral density of the noise (noise power in the spectral interval of 1 Hz);

Δf_e – signal effective bandwidth;

td – time of signal arrival at k – sensor/receiver.

So, for instance, if $2E_s / N_o = 10$ and $\Delta f_e = 25MHz$, $\sigma_{pot(td)} \leq 12,5ns$.

4.1.3. Instrumental Errors

Instrumental errors are not investigated deeply yet, but it is necessary to take them into account in the future.

4.2. TDOA or Timing Errors

The principle of multilateration system work is based on Time Difference of Arrival (TDOA) method (Figure 1). Where 1, 2, and 3 are location points of the receiving stations. Central Processing Station is located in the reference point O.

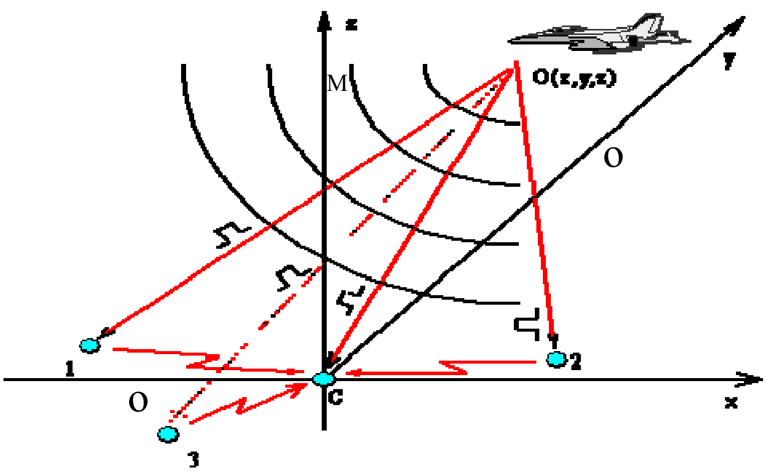

Figure 1. *TDOA principal of operation*

MLAT system calculates TDOA for each pairs of the receiver stations to locate an aircraft position. As an example for the receivers 1 and 2 range difference to the aircraft is:

$$\Delta r = c * (t_{d1} - t_{d2}), \tag{8}$$

where t_{d1} and t_{d2} are time of signal arrival for receiver 1 and 2 and c is a speed of light.

Let's assume that time of arrival errors are statistically independent for all sensors and therefore

$$\sigma_{1,2(\Delta r)}{}^2 = \sigma_{1(r)}{}^2 + \sigma_{2(r)}{}^2. \tag{9}$$

In practice location accuracy can be characterized with error ellipse (in plane) and with error ellipsoid (in space). For simplicity, let's take plane as it is shown on Fig. 2. Error ellipse is a locus of two components $l_x = \Delta x$ and $l_y = \Delta y$, which is equal to probability density of location errors in the plane.

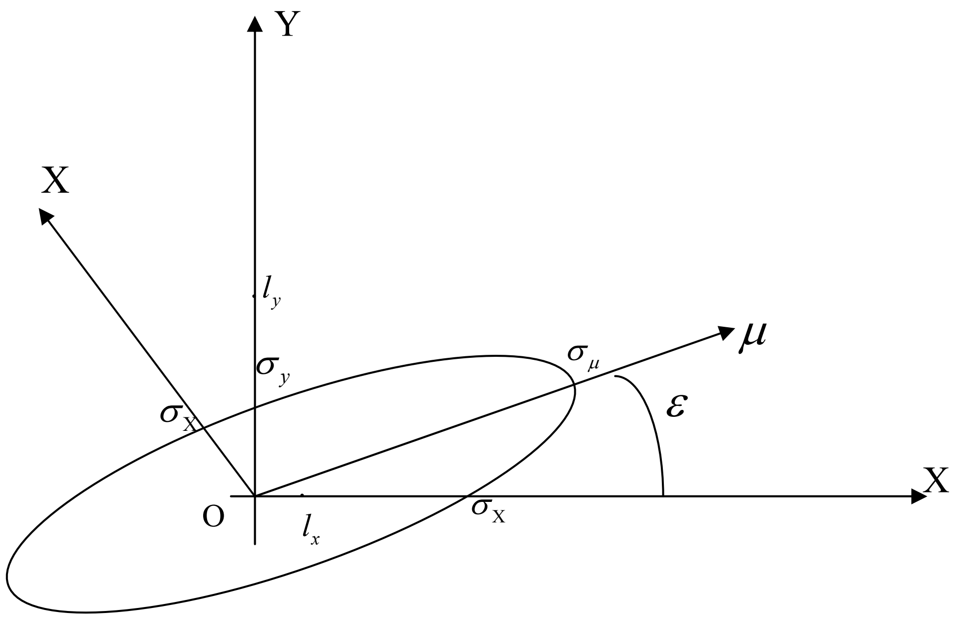

Figure 2. *Location accuracy*

Let's calculate position error for two receiving stations and one central processing station. For simplicity, let's assume the ideal situation when receiving stations (A and B) are situated on the reference axis and on the same distance from zero point (Figure 3) where the Central Processing Station is situated.

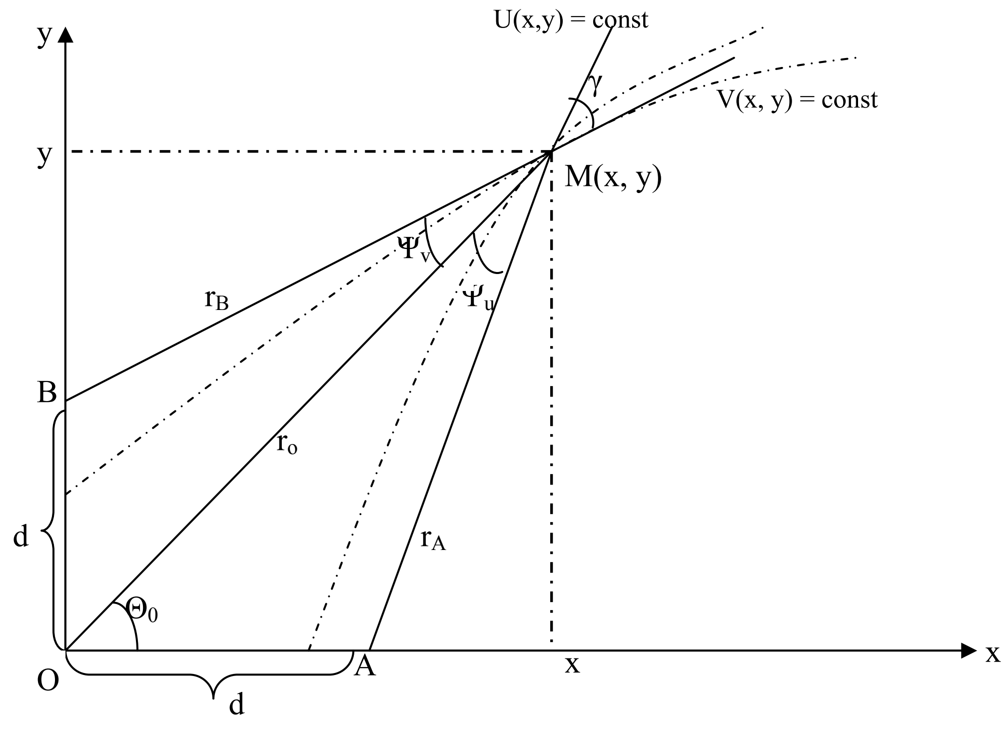

Figure 3. *Position of the aircraft*

The intersection of two positions lines u (x,y) and v (x,y) allows to calculate the position of an aircraft. Each position line (u (x,y) and v (x,y)) can be calculated by means of primary geometric parameter $n_u; n_v$ measurement.

RMS errors of position lines $\sigma_u; \sigma_v$ depend on primary measurements RMS errors.

To calculate position accuracy of the multilateration system it is necessary to calculate range differences nu = Δru = r0-rA; nv = Δrv = r0-rB from each receiving station to the aircraft M(x, y). The cross point of position lines (u,v = const) with the angle of γ gives position of the aircraft (see Figure 3).

The following calculations [2] help us to get range differences:

$$n_u(x,y) = u(x,y) = r_0 - r_A = \sqrt{x^2+y^2} - \sqrt{(x-d)^2+y^2}\,, \tag{10}$$

$$n_v(x,y) = v(x,y) = r_0 - r_B = \sqrt{x^2+y^2} - \sqrt{(y-d)^2+x^2}\,. \tag{11}$$

Assuming that $\sigma_n = \sigma_{\Delta r}$, the positional accuracy will be the following:

$$\frac{\sigma_r}{\sigma_{\Delta r}} = \left[1 - \frac{1-\frac{d}{r_0}(\cos\Theta_0+\sin\Theta_0)}{q_s}\right]^{-1/2} \times \left\{\left[1-\frac{(1-\frac{d}{r_0}\cos\Theta_0)}{q_c}\right]^{-1} + \left[1-\frac{(1-\frac{d}{r_0}\sin\Theta_0)}{q_s}\right]^{-1}\right\}^{1/2}, \tag{12}$$

where

σ_r – RMS of the location finding;

ΔR – RMS of the range difference, which in this case is the same for both receivers;

and

$$q_c = \left[1 - 2\frac{d}{r_0}\cos\Theta_0 + (\frac{d}{r_0})^2\right]^{1/2},$$
$$q_s = \left[1 - 2\frac{d}{r_0}\sin\Theta_0 + (\frac{d}{r_0})^2\right]^{1/2} \tag{13}$$

where r_0 and Θ_0 are polar aircraft coordinates from the same referent point.

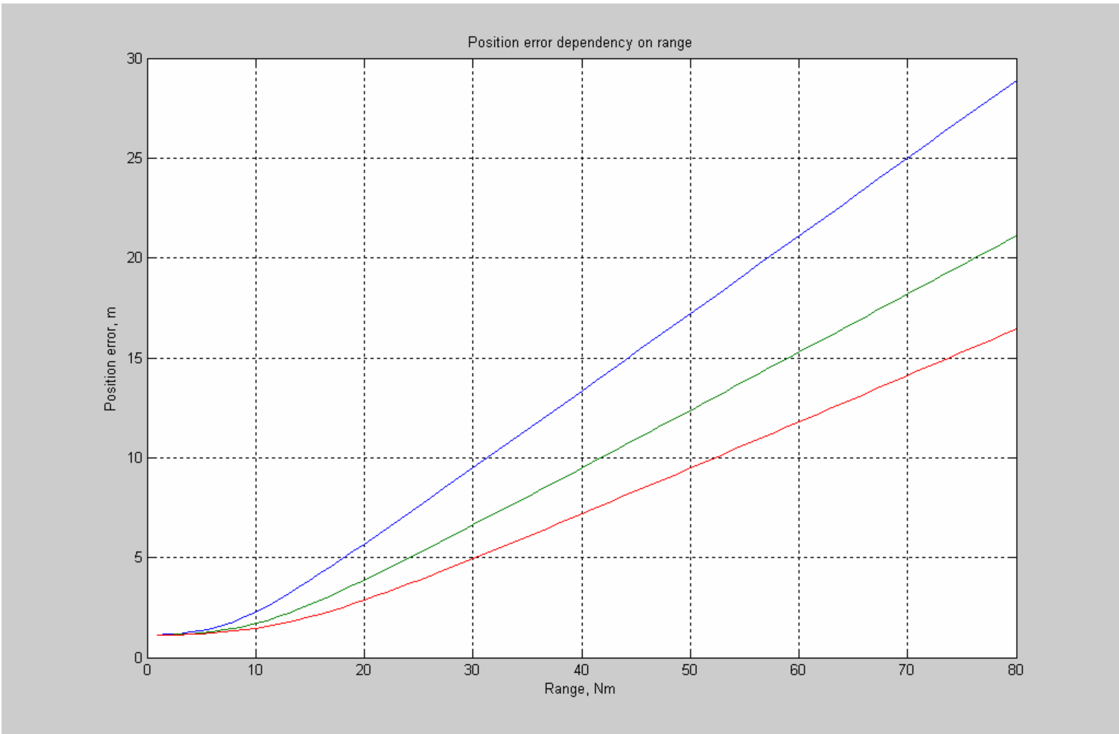

Figure 4. *Position – dependency on range*

Figure 4 shows the position error dependency on range for receiver station allocation like it is shown on Figure 3. Base between each receiving station and Central Processing Station – d is equal to 15, 20 and 25 km (red, green and blue curves correspondingly). As it was expected the position error is increasing with distance to aircraft and base between two receiving stations (from 15 to 25 km). As it is stated in this case the target is situated on the azimuth of 45 degrees. In case of calculation total errors of MLAT systems it will be necessary to take into account all other errors, which were considered in the first chapter of this article.

4.3. Signal Corruption

The transponder signal received by the system may be corrupted. This can be caused by a combination of multipath, garbling and potentially malicious or unintentional interference (jamming) conditions.

Multipath is where multiple copies of the same signal are received due to reflections from objects such as the ground, water, buildings or other aircraft. Antenna choice can help to reduce multipath.

Short path differences cause the same reply to arrive at multiple times with the pulses overlapping. Typically the direct and earliest path will be at a higher level than the reflected paths. These overlapping but attenuated pulses cause deformation of the pulse shape of the direct received signal. It usually has negative impact on TOA accuracy.

Long path differences result in multiple copies of the same reply to be received. If this is undetected it can cause ghost tracks, although due to the nature of the system, they do not last for long.

Garbling occurs, when two or more different signals are received that overlap in time. The probability of garble occurring on any given signal increases with the density of the SSR signal environment.

Both multipath and garble have an impact on the accuracy of multilateration system as well as affecting probability of detection. In many cases, especially with multipath, the signal itself can be recovered sufficiently for identification purposes. However the deformation of the signal affects the accuracy of TOA measurement or quality of the correlation. Accuracy can be maintained by rejecting these signals but at the expense of probability of detection.

If higher than expected levels of interference occur at a receiver this will also degrade accuracy. This is because the SNR of the received signal has a direct influence upon accuracy. If the SNR is particularly poor, the probability of detection and decoding ability may also be affected. In general multilateration receivers are relatively narrowband, being restricted to the 1090MHz signals, and thus interference is either directly in-band (typically malicious) or unintentional sidebands of other systems (e.g. DME).

Especially in large and complex airports, shadowing and multipath effects become important to the propagation of the SSR transponder signals. Multilateration systems use the SSR downlink frequency (1090MHz) and this is sufficiently high that the performance is limited to line-of-sight signal paths. Thus, there the blind spots or areas of unreliable coverage behind buildings may occur, etc. Reflections and multipath can cause degraded accuracy of position measurements or even ghost targets.

To limit the impact of these propagation effects, more than the minimum of 4 receiver sensors (the minimum required to derive 3-D position) will be required. It enables elimination of the false TOA caused by multipath and reflection.

Area Management mechanism for eliminating reflected signals from unreal locations (buildings etc.) is also applied.

Variable sensitivity of receiver dependent on the amplitude of received signals is also applied. The sensitivity is decreased to the level corresponding to the best signal.

Garbled replies are not being processed by the MLAT – the first or stronger reply is processed. If this creates a false reply, it is effectively eliminated in the Measuring Unit (based on TOA of F1 and F2 pulse or on the parity information). The probability of having the garbled replies received at all receivers at the same time is very low, which also helps eliminating the impact of garbling. Whisper-shout technique can be applied in case if there is a high garbling rate in the area.

Interleaved replies are far more probable than the garbled ones. Interleaved replies are detected by the Measuring Unit and are further processed.

4.4. Algorithm Errors

The main issue is to determine which type of algorithms for TDOA calculation will be used in the system. Typically this is being considered at survey stage and the influence of errors will be defined according to chosen algorithm: common clock system or distributed clock system.

- Common clock architecture benefits from a simple receiver with low power consumption and most of the complexity in the central multilateration processor. However the signal delay between the antenna and the multilateration processor puts stringent requirements on the type and range of the link. Typically a single hop custom microwave link is used, or dedicated fibre is laid between the sites as illustrated below. The location of the multilateration processor must typically be at the centre of the system to minimise communication link distances. So the errors which can arise in data link between receiving station and central processing station shall be taken into account.
- Distributed clock systems use a more complex receiver to reduce the demands on the data link. The RF signal is down-converted to a baseband or video signal and then the digitisation, code extraction and TOA measurement are all done at the receiver. This gives great flexibility in the data link as any digital data link can be used and the link latency is not critical. However a mechanism must be used to synchronise the clocks at the local sites.

4.5. Survey Errors

Survey errors will introduce uncertainties in the measured position of sensors and hence the construction of the hyperbolae. There may be some common systematic component of positional uncertainty which partially cancels out when establishing the aircraft position with respect to the sensors. A common systematic component will manifest itself when trying to refer the calculated position to a wider frame of reference.

Measuring the unknown position of a sensor will introduce both systematic and random errors. The random errors occur because a measurement is being made of an unknown quantity. Systematic errors occur due to unknown effects in the measurement system.

For a MLAT System, it is reasonable to assume that sensor positions will be determined using GPS or an equivalent satellite based system. The random errors for GPS measurements originate in the variation of the clocks in the satellites and the fluctuations in the propagation path due to the atmosphere.

Systematic errors originate in the variation of satellite orbits from the modelled orbit: real time position measurements must assume some knowledge of satellite position and there is often a deviation from this. Systematic errors also include any deliberate reduction in accuracy imposed by the GPS operators. Although Selective Availability (SA) has been switched off, the true accuracy of the GPS system is still reserved for the military and is not generally available.

Generally, sensor positions are surveyed infrequently and hence their impact on position measurement does not vary with time. Sensor surveying is mainly a one-off set-up measurement. The result of this, is that systematic errors may remain in the system for a long time, if not indefinitely.

Incorrect sensor positioning leads to an error in the calculated separation of a sensor pair and subsequently to the creation of the wrong hyperbola. The size of the positional error is more significant the further from the sensor axis. In addition to the creation of the wrong hyperbola, the surveying error may cause the hyperbola to be rotated with respect to the sensor pair.

Averaging the GPS position over a period of 24 hours is sufficient to reduce the random element of the uncertainty to below 1cm. Further improvements can be obtained by recalculating position after adjusting for the actual satellite positions. This data is generally available a day or two after the event but the calculations are non-trivial.

An absolute calibration is not strictly possible on the other hand it is possible to minimise or characterise systematic errors by surveying known locations

5. Error Estimation

Multilateration systems consist of several sensor elements distributed in space at which transmissions from aircraft can be detected. Detecting these transmissions and measuring the arrival times enables the difference in arrival time to be calculated for each pair of sensors. Combining the Time Difference of Arrival (TDOA) from several pairs of sensors allows the position of the aircraft, relative to the sensors, to be determined. Finally, knowledge of the sensor positions within a wider co-ordinate system allows the position of the aircraft to be determined absolutely.

At each point in this process there are opportunities for uncertainties and errors to enter the system and introduce an error in the calculated position. The purpose of this work is to identify types of errors at each point of the multilateration process and finally to derive a formula of error distribution of MLAT system in total.

In frames of work two types of TDOA measurement shall be taken into account. As it was said in the first part of the article, synchronization methods in MLAT systems can be classified as follows:

- Common clock systems. In these types of MLAT system the signal delay between the antenna and the multilateration processor puts stringent requirements on the type and range of the link.
- Distributed clock systems. The distributed clock systems use a more complex receiver to reduce the demands on the data link. The RF signal is down-converted to a base band or video signal and then the digitisation, code extraction and TOA measurement are all done at the receiver. It is possible to analyse and estimate error distribution by means of two ways:

1) Analytical way;
2) Statistical modelling.

Analytical estimation means to develop the mathematical derivation of error distribution formula. Analytical way of error estimation almost is not possible to do, because of difficulties in formulas which will be derived and differences of error sources and laws of its distribution (random or non-random).

Error distribution by means of statistical modelling will be analysed in the given work.

The method of statistical modelling will represent to be the model of signal plus noise passing through MLAT system's receiving tract. At the first stage it will be TOA errors or time of arrival errors. Forming the model it is necessary to take into account errors and its distribution laws in propagation medium (propagation errors) as well as errors, which can arise from the receiving point till Central Processing station.

Let's assume that at the input of receiver we have random process, which consists of mixture of useful signal and noise with zero average of distribution and correlation function. The main task of the model is to determine the average distribution and dispersion of the process at the output as well as law of error distribution for signal time of arrival at the output of receiver.

The same models shall be examined at each point of MLAT system. Forming the model it is necessary to use one of the mathematical packages, f.i. Mathcad and Matlab. To form the model of noise-like signal the measuring transducer with Weibull's law of distribution will be used.

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