Transport and Telecommunication, 2007, Volume 8, No 1, 12–19
Transport and Telecommunication Institute, Lomonosov 1, Riga, LV-1019, Latvia

SCHEDULING CHAIN MODELS WITH OPERATIONS OF RANDOM DURATIONS IN TRANSPORTATION SYSTEMS

Zohar Laslo[1], Dimitri Golenko-Ginzburg[2]

[1]Department of Industrial Engineering and Management, Negev Academic College of Engineering
Beer Sheva, 84100, Israel.
E-mail: zohar@nace.ac.il

[2]Department of Industrial Engineering and Management, Ben-Gurion University of the Negev
Beer Sheva, 84105, Israel.
E-mail: dimitri@bgumail.bgu.ac.il,

Department of Industrial Engineering and Management,
Academic College of Judea and Samaria
Ariel, 44837, Israel

Controlled transportation system to perform a set of operations with random time durations is considered in the given article. Three possible processing speeds can be introduced depending on the intensity of the transport resource consumption but not on the resource capacities. An optimal plan schedule to start all the operations is developed.

Keywords: *optimal plan schedule, optimistic pessimistic and planned speeds, decision-making at control points, problems of optimal control, transport resources*

1. The system's description

The authors consider a controlled transportation system, intended for realizing a finite set $\{O_i\}$ of transport operations to be done in a definite logical sequence. Assume that transportation program $\{O_i\}$ requires m types of transport resources repeatedly used in the course of their utilization and – without taking amortization or terminal breakdowns into account – practically unaltered. Each transport operation O_i, $1 \le i \le n$, consumes r_{ij} units of the j-th type of resource, $1 \le j \le m$, values r_{ij} being within preset limits of the intensity of resource consumption

$$r_{ij\,min} < r_{ij} < r_{ij\,max}\ ,\ 1 \le i \le n\ ,\ 1 \le j \le m\ . \tag{1}$$

Assume, further, that when satisfying restrictions (1) each transport operation O_i can be carried out (for the same capacity vector $\vec{r}_{ij}$) with the following three different speeds [1, 3]:

1. The maximal (tense) speed, which corresponds to the maximum degree of intensification of the transportation process (for instance, working with three shifts during twenty-four hours);

2. The non-tense (minimal) speed, characterized by the minimum degree of intensification of the transportation system (one-shift work load);

3. The planned speed which corresponds to some average transportation intensity (a two-shift work load, for instance).

Further on [1, 3] the authors henceforth call those speeds optimistic, pessimistic and planned ones. Note that a transfer to the tense speed can rarely be carried out when transportation falls behind the plan substantially. Permanent work of a controlled transportation system at the tense speed is impermissible because of the possibility of an irreversible wear down of the system.

Assume, further, that the system under observation functions with certain random influences, circumstances and interferences from the environment, so that the time t_i of performing O_i is random.

The distribution law parametrically comprises vector $\vec{r}_{ij}$, with the probability density function $P_i(t) = \varphi_i(\vec{r}_{ij}, t)$.

It is easy to see that the described model comprises the broadest spectrum of transportation systems. The latter include various transportation projects, experimental and small scale delivery orders, etc.

Taking into account the random nature of the time for carrying out O_i the corresponding formalization can be represented as follows.

When restrictions (1) hold, the density distribution of t_i takes the form

1. For the tense (optimistic) speed

$$p_{i\,opt.}(t) = \varphi_{i\,opt.}(\vec{r}_{ij}, t) , \qquad (2)$$

the finite interval $[a(\vec{r}_{ij})_{opt.}; b(\vec{r}_{ij})_{opt.}]$ is the domain of definition of $t_{i\,opt.}$

2. For the non-tense (pessimistic) speed, similar relations

$$p_{i\,pes.}(t) = \varphi_{i\,pes.}(\vec{r}_{ij}, t) , \qquad (3)$$

$$t_{i\,pes.} \in [a(\vec{r}_{ij})_{pes.}; b(\vec{r}_{ij})_{pes.}] \qquad (4)$$

hold.

3. The planned speed of carrying out O_i is characterized by the density distribution

$$t_{i\,pln.} = \varphi_{i\,pln.}(\vec{r}_{ij}, t) , \qquad (5)$$

$$t_{i\,pln.} \in [a(\vec{r}_{ij})_{pln.}; b(\vec{r}_{ij})_{pln.}] \qquad (6)$$

Note that when there is no control interference transport operations O_i are carried out with the planned speed. Assume that the controlled system possesses available total transport resource capacities R_j, $1 \le j \le m$, with evident restrictions

$$R_j \ge \underset{i}{Max}\, r_{ij\,min} , \qquad (7)$$

otherwise transportation program $\{O_i\}$ cannot be achieved at all.

Two problems can be formulated as follows:

1. To determine an optimal plan schedule for the moments to begin transport operations O_i – $t_{pln.\,beg.}[O_i]$, $1 \le i \le n$, to minimize either the mathematical expectation E or the $p_{pln.}$-th quantile $W_{p_{pln.}}$ of the total time duration for the transportation program $\{O_i\}$, i.e., of the value

$$T = \underset{i}{Max}\, t_{end.}[O_i] - \underset{i}{Min}\, t_{beg.}[O_i] , \qquad (8)$$

where symbols $t_{end.}[O_i]$ and $t_{beg.}[O_i]$ denote the actual moments of time when transport operation O_i ends and begins, respectively.

2. To construct an optimal policy for decision-making while controlling the transportation process. Note, that the very concept of "schedule" is directive in nature, and signifies that if in the course of realizing transportation process, transport operation O_i is *actually* ready to be done at moment $t_{beg.}[O_i] < t_{pln.\,beg.}[O_i]$, the operation is idle in the time interval $\{t_{beg.}[O_i], t_{pln.\,beg.}[O_i]\}$, even if there are free transport resources. But if the actual moment $t_{beg.}[O_i]$ exceeds the planned moment $t_{pln.\,beg.}[O_i]$ by more than a pregiven value Δ_i, i.e., relations $t_{pln.\,beg.}[O_i] + \Delta_i < t_{beg.}[O_i]$, $1 \le i \le n$, hold, the system must pay such a big penalty that time T increases practically unlimitedly. Restriction

$$t_{beg.}[O_i] - t_{pln.\,beg.}[O_i] \le \Delta_i \qquad (9)$$

is introduced in order to exclude the trivial solution $t_{pln.beg.}[O_i]=0$, $1\le i\le n$, which ensures the minimal time duration T but leads to unproductive idleness of transport resources in intervals $\{t_{pln.beg.}[O_i], t_{beg.}[O_i]\}$.

Note that determining the optimal schedule $t_{pln.beg.}[O_i]$ is carried out in view of the fact that all transport operations are carried out with the same planned speed. The first problem is solved at the stage of optimal planning, i.e., at the stage preceding optimal transportation control. The second one involves decision-making for transportation control only and boils down to a change of transportation speeds.

As far as we know there are no investigations combining different and variable transportation speeds with variable transport resource capacities for random time durations into an integrated transportation control model. From our point of view such a model may be more adequate than those used nowadays.

In paper [2] problems are solved to determine optimal schedules for a particular case of the general problem stated above – when there are no different production (in our case, transportation) speeds. This paper is devoted to a further development in the field of transportation control.

2. Notation

Let us introduce the following terms:

$a(\vec{r}_{ij})_{opt.}$	the lower bound of the optimistic speed for transport operation O_i;
$a(\vec{r}_{ij})_{pes.}$	the lower bound of the pessimistic speed for transport operation O_i;
$a(\vec{r}_{ij})_{pln.}$	the lower bound of the planned speed for transport operation O_i;
$b(\vec{r}_{ij})_{opt.}$	the upper bound of the optimistic speed for transport operation O_i;
$b(\vec{r}_{ij})_{pes.}$	the upper bound of the pessimistic speed for transport operation O_i;
$b(\vec{r}_{ij})_{pln.}$	the upper bound of the planned speed for transport operation O_i;
D	service discipline;
n	the number of transport operations O_i, entering the transportation system;
m	the number of transport resources, R_j, $1\le j\le m$;
R_j	the pregiven total capacity of the j-th type of transport resource at the company's disposal;
O_i	the i-th transport operation, $1\le i\le n$, entering the transportation system;
$p_{pln.}$	the planned chance constraint (pregiven);
P_j	priority index of the j-th type of transport resource, $1\le j\le m$, (pregiven);
r_{ij}	number of transport resource units of type j consumed by transport operation O_i, $1\le j\le m$, $1\le i\le n$;
$r_{ij\,min}$	lower bound of value r_{ij} (pregiven);
$r_{ij\,max}$	upper bound of value r_{ij} (pregiven);
M_s	simulation model of the transportation system;
$R_j(t)$	the total available j-type transport resources at moment $t\ge 0$;

$t_{beg.}[O_i]$ the actual moment transport operation O_i starts (a random value);

$t_{end.}[O_i]$ the actual moment transport operation O_i terminates (a random value);

$t_{pln.\,beg.}[O_i]$ the planned moment for transport operation O_i to start (to be determined);

T the total duration of carrying out program $\{O_i\}$ (a random value);

$W_{p_{pln.}}$ the planned p-quantile;

t_i the random time of performing transport operation O_i, $1 \le i \le n$;

$t_{i\,opt.}$ the optimistic time to perform transport operation O_i;

$t_{i\,pes.}$ the pessimistic time to perform transport operation O_i;

$t_{i\,pln.}$ the planned time to perform transport operation O_i;

$\varphi_{i\,opt.}(\vec{r}_{ij}, t)$ probability density function of $t_{i\,opt.}$ with transport resources $\vec{r}_{ij}$;

$\varphi_{i\,pes.}(\vec{r}_{ij}, t)$ probability density function of $t_{i\,pes.}$ with transport resources $\vec{r}_{ij}$;

$\varphi_{i\,pln.}(\vec{r}_{ij}, t)$ probability density function of $t_{i\,pln.}$ with transport resources $\vec{r}_{ij}$;

Δ_i the accuracy estimate for transport operation O_i, $1 \le i \le n$ (pregiven).

3. Optimal planned schedule without decision-making at control points

Note that in the case of unrestricted transport resources

$$R_j \ge r_{ij\,max}, \quad 1 \le j \le m, \tag{10}$$

the problem to determine an optimal schedule is simplified and in some cases has a trivial solution. In particular, for a set of independent transport operations carried out in an arbitrary order, optimal values $t_{pln.\,beg.}[O_i]$ sought are equal to zero. For the case of a chain of consecutive transport operations or for the case of a network graph the problem is more complicated but may be solved using simulation modelling.

In the case of restricted transport resources

$$\underset{i}{Max}\, r_{ij\,min} \le R_j < \sum_i r_{ij\,max}, \quad 1 \le j \le m, \tag{11}$$

we need some additional assumptions:

1. Introducing a service discipline D, which signifies a choice of determined rules, recommendations, and relations making it possible at any moment of time t to distribute free available transport resources $R_j(t)$, $1 \le j \le m$, (not participating in transport operations at moment t) between K transport operations $\{O_{\ell_\eta}\}$, $1 \le \eta \le K$, $1 \le \ell_\eta \le n$, claiming realization. At moment t they have not yet started and are in fact ready to be carried out and be supplied with required transport resources according to evident relations

$$r_{ij\,min} < r_{ij} < Min\{r_{ij\,max}, R_j(t)\}, \quad t \ge t_{pln.\,beg.}[O_{\ell_\eta}]. \tag{12}$$

2. Developing a simulation model M_s making it possible to determine the value of a random variable – the duration T of carrying out the whole set of transport operations $\{O_\ell\}$ – in the process of a single realization. Model M_s includes service discipline D given with a set of planned terms $t_{pln.\,beg.}[O_i]$ for starting transport operations O_i and all the probabilities and resource characteristics

determined above for operations O_i, while a special graph and certain logical links determine the flow chart and the sequence of transport operations. Note that simulation model M_s can be used effectively for redistributing free resources between operations in conflicting situations, which are in fact control points.

By a conflicting situation we mean the following relation for a subset of operations $\{O_{\ell_\eta}\}$ to be performed: for at least one type j of resources relation

$$R_j(t) < \sum_{\eta=1}^{K} r_{\ell_\eta j \max} \tag{13}$$

holds, where $R_j(t)$ is the number of units of the j-th resource ready to be fed in for subset $\{O_{\ell_\eta}\}$ at time t. It should be noted that value $R_j(t)$, $1 \le j \le m$, can change with time t due to idleness from a breakdown, or because of resetting after repair. An estimate of values $R_j(t)$, $t = 1, 2....$ is one of the tasks of the simulation model M_s too.

In paper [4] the following optimisation problem is solved: determine values $t_{p\ln.\,beg.}[O_i]$, $1 \le i \le n$, minimizing objective function

$$T = E\{M_s(D, R_j, t_{p\ln.\,beg.}[O_i]\}, \tag{14}$$

or

$$T = W_{p_{p\ln.}}\{M_s(D, R_j, t_{p\ln.\,beg.}[O_i]\}, \tag{15}$$

subject to restrictions (1) and (9). In (14–15) model M_s simulates the value calculated by equation (8).

4. Algorithms for problems of optimal control

We have pointed out that at the stage of planning, deadlines for performing transport operations O_i, must be pregiven. Thus, values $t_{p\ln.\,beg.}[O_i]$ have to be taken into account.

If at the stage of optimal control the actual course of production lags behind the plan, decision-making must correct the course of transportation with corresponding modifications for the speed of carrying out individual transport operations. The results described below boil down to a modification of model M_s by way of simulating separate operations O_i, including simulation of their speed changes.

It is important to note that such a modified model can also be used in the process of solving optimal problems (14–15) making it possible to combine optimal planning and optimal control models into a general integrated model.

To begin with, note that as a result of solving problem (14) or (15), we are in fact able to plan both the moments for beginning transport operations O_i and the moments for completing them, though the duration of those transport operations is random in nature.

Indeed, if according to the logical links, transport operation O_{i_2} follows *directly* after transport operation O_{i_1}, and $t^*_{i_1} = t_{p\ln.\,beg.}[O_{i_1}]$ and $t^*_{i_2} = t_{p\ln.\,beg.}[O_{i_2}]$ are the planned moments for beginning operations O_{i_1} and O_{i_2} (obtained, for instance, as a result of optimising problems (14-15), then, in principle, we have the right to equalize the planned duration of operation O_{i_1} to the length of interval $[t^*_{i_1}; t^*_{i_2}]$:

$$t_{i\,p\ln.} = t^*_{i_2} - t^*_{i_1}. \tag{16}$$

Assume relations

$$t_{i\,p\ln.} = E[\varphi_{i\,p\ln.}(\vec{r}^{\,*}_{ij}, t)], \tag{17}$$

or

$$t_{i\,p\ln.} = W_{p_{p\ln.}}[\varphi_{i\,p\ln.}(\vec{r}^{\,*}_{ij}, t)], \tag{18}$$

depending on the characteristics being optimised. Here symbol $\vec{r}_{ij}^{*}$ denotes the set of transport resource capacities satisfying restriction (1) and determined as the solution of equation (17) or (18), while value $t_{i\,pln.}$ is obtained beforehand.

Let us consider the procedure for obtaining $\vec{r}_{ij}^{*}$ in greater detail, since the problem stated does not have a definite solution and needs supplementary restrictions and definitions. There are several ways to formalize the problem, and in our view, the following statement is the most expedient one.

In the process of solving optimisation problems (14–15) we use a serial block, which carries out repeated simulation of planned terms $t_{pln.\,beg.}[O_i]$ by use of Monte Carlo methods with subsequent application to the simulation model M_s. With regard to that stated above, we modify the simulation model M_s as follows: after simulating the planned terms $t_{pln.\,beg.}[O_i]$, $1 \le i \le n$, we calculate planned durations $t_{i\,pln.}$ by equation (16). Afterwards, to determine vectors $\vec{r}_{ij}^{*}$, we solve the following optimisation problem independently for all O_i – determine values r_{ij}^{*} minimizing objective function:

$$\sum_{j=1}^{m} P_j r_{ij} \Rightarrow \text{Min} \tag{19}$$

subject to restrictions (1), and

$$Q[\varphi_{i\,pln.}(\vec{r}_{ij}^{*}, t)] \le t_{i\,pln.}\,. \tag{20}$$

Here Q denotes either the mathematical expectation E or the $p_{pln.}$-th quantile $W_{p_{pln.}}$, and P_j is a priority index of the j-th transport resource (for instance, the cost of consuming a resource unit per time unit).

Though the definition of vectors $\vec{r}_{ij}^{*}$ is in itself a result of the optimisation problem (19–20), solving the problem for a case of small m's does not present any substantial difficulty, and can be carried out with either statistical or heuristic methods.

Thus, we can plan not only duration $t_{i\,pln.}$ but also, subsequently, transport resource capacities $\vec{r}_{ij}^{*}$.

Note that if:

A. transport operation O_i begins at the planned moment $t_{pln.\,beg.}[O_i]$, but is supplied with resources $r_{ij} \ne r_{ij}^{*}$; or

B. transport operation O_i actually begins after the planned moment $t_{beg.}[O_i] > t_{pln.\,beg.}[O_i]$, the process of carrying out operation O_i can proceed with a speed different from the planned one.

Now consider each of these situations arising in the process of simulating.

A. Moments $t_{beg.}[O_i]$ and $t_{pln.\,beg.}[O_i]$ coincide

In this case, at the beginning of any transport operation O_i, an information input is formed and transferred to the supremal hierarchical level of distributing resources, the input being a request for the capacity $\vec{r}_{ij}^{**}$ of transport resources. If, as a result of applying service discipline D, the operation actually receives $r_{ij} \ne r_{ij}^{*}$ resources, the simulation model M_s must additionally include a special unit with the following step-by-step structure:

STEP 1. If vector $\vec{r}_{ij}^{*}$ does not satisfy restriction (1), operation O_i does not begin and the request for transport resources is not recalled. Otherwise, go to the next step.

STEP 2. Check the fulfilment of the inequalities

$$Q[\varphi_{i\,pes.}(\vec{r}_{ij}, t)] \le t_{i\,pln.}, \quad (21)$$

$$Q[\varphi_{i\,pln.}(\vec{r}_{ij}, t)] \le t_{i\,pln.}. \quad (22)$$

If inequality (21) holds, perform the operation with the non-tense speed. If inequality (21) does not hold, but relation (22) is satisfied, process the operation with the planned speed. If inequalities (21) and (22) are both false, apply the next step.

STEP 3. Check inequality

$$Q[\varphi_{i\,opt.}(\vec{r}_{ij}, t)] \le t_{i\,pln.} \quad (23)$$

If the inequality holds, operation O_i is performed with the tense speed. Otherwise, go to the next step.

STEP 4. Solve an optimisation problem as follows: determine transport resource values r_{ij}^{**}, $1 \le j \le m$, to minimize the objective function:

$$F_i = \sum_{j=1}^{m} P_j r_{ij}^{**} \quad (24)$$

subject to restrictions (1) and

$$Q[\varphi_{i\,opt.}(\vec{r}_{ij}^{**}, t)] \le t_{i\,pln.}. \quad (25)$$

STEP 5. Transport operation O_i is performed with the tense speed and starts to be processed with available transport resources. At the same time, a new request is formed to the supremal level, to supply operation O_i with additional resources

$$\Delta r_{ij}^{***} = r_{ij}^{**} - \vec{r}_{ij}. \quad (26)$$

If for some j relation $\Delta_{ij}^{***} < 0$ holds, surplus capacities can be removed while adding the missing ones. Meanwhile, it should be noted that if the request is satisfied in $\Delta t_i > 0$, the work of the unit described above for introducing decision-making will be repeated by applying procedure B.

B. Relation $t_{beg.}[O_i] > t_{pln.\,beg.}[O_i]$ holds

In this case, the planned duration $t_{i\,pln.}$ is to be reduced by value Δt_i determined by the equation:

$$\Delta t_i = t_{beg.}[O_i] - t_{pln.\,beg.}[O_i], \quad (27)$$

after which Step 1 is applied for the unit described above, but for the corrected value $t_{i\,pln}^{*}$:

$$t_{i\,pln.}^{*} = t_{i\,pln.} - \Delta t_i. \quad (28)$$

We believe that such a modification of the simulation model makes it possible to employ service discipline D, combined with different types of control, (see [2]). In our view, the corresponding integrated model of optimal planning and control can be used in practical calculations.

5. Conclusions

To conclude, note that for a broad class of actual transportation systems, the construction of mathematical models stated above does not cause any fundamental difficulties. In particular, for transportation systems of individual items, as well as for various service systems of small serial type, when the controlled installations are mapped by network models, the introduction of tense, planned, and non-tense speeds can signify the use of three-shift, two-shift or one-shift forms of work, respectively. As for the choice of permissible boundaries $r_{ij\,min}$ and $r_{ij\,max}$ of transport resource capacities for transport operations O_i, they can be determined by proceeding from an analysis of transportation technology. Finally, the domains of definition $[a(\vec{r}_{ij}); b(\vec{r}_{ij})]$ can be estimated on the basis of expert estimates, as it is done everywhere in network planning systems when the duration of individual transport operations is random in nature [1, 2]. The corresponding optimistic and pessimistic estimates are provided by the personnel responsible for carrying out each of the operations.

6. Acknowledgment

This research has been partially supported by the Paul Ivanier Center for Robotics Research and Production Management, Ben-Gurion University of the Negev.

References

1. Golenko-Ginzburg, D. I. *Statistical Methods in Network Planning*. Moscow: Nauka, 1968. (In Russian)
2. Golenko-Ginzburg, D. I. *Statistical Methods in Economic Systems*. Moscow: Nauka, 1970. (In Russian)
3. Golenko-Ginzburg, D. I. and Sinuany-Stern, Z. Production control models with variable speeds and inspection points, *International Journal of Production Research*, Vol. 27(4), 1989, pp. 629–636.
4. Golenko-Ginzburg, D. I. Planning and controlling production systems with operations of random duration, *Journal of Terotechnology OMO*, Vol. 5, 1997, pp. 217–222.