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## USAGE OF PRODUCTION FUNCTIONS IN THE COMPARATIVE ANALYSIS OF TRANSPORT RELATED FUEL CONSUMPTION

***Adam Torok<sup>1</sup>, Arpad Torok<sup>2</sup>, Florian Heinitz<sup>3</sup>***

<sup>1</sup>*Budapest University of Technology and Economics*

*Department of Transport Economics,  
H-1111, Budapest, Muegyetemrkp 3  
E-mail: atorok@kgazd.bme.hu*

<sup>2</sup>*Budapest University of Technology and Economics*

*Department of Transport Economics,  
H-1111, Budapest, Muegyetemrkp. 3  
E-mail: artorok@kgazd.bme.hu*

<sup>3</sup>*Erfurt University of Applied Sciences,*

*InstitutVerkehr und Raum,  
AltonaerStraße 25, D - 99085 Erfurt; Germany  
E-mail: heinitz@fh-erfurt.de*

This contribution aims to examine the relationship between the transport sector and the macroeconomy, particularly in fossil energy use, capital and labour relations. The authors have investigated the transport related fossil fuel consumption 2003 - 2010 in a macroeconomic context in Hungary and Germany. The Cobb-Douglas type of production function could be justified empirically, while originating from the general CES (Constant Elasticity of Substitution) production function. Furthermore, as a policy implication, the results suggest that a solution for the reduction of anthropogenic CO<sub>2</sub> driven by the combustion of fossil fuels presupposes technological innovation to reach emission reduction targets. Other measures, such as increasing the fossil fuel price by levying taxes, would consequently lead to an undesirable GDP decline.

**Keywords:** production function, environmental pollution, transport emission, fossil fuels

### 1. Introduction

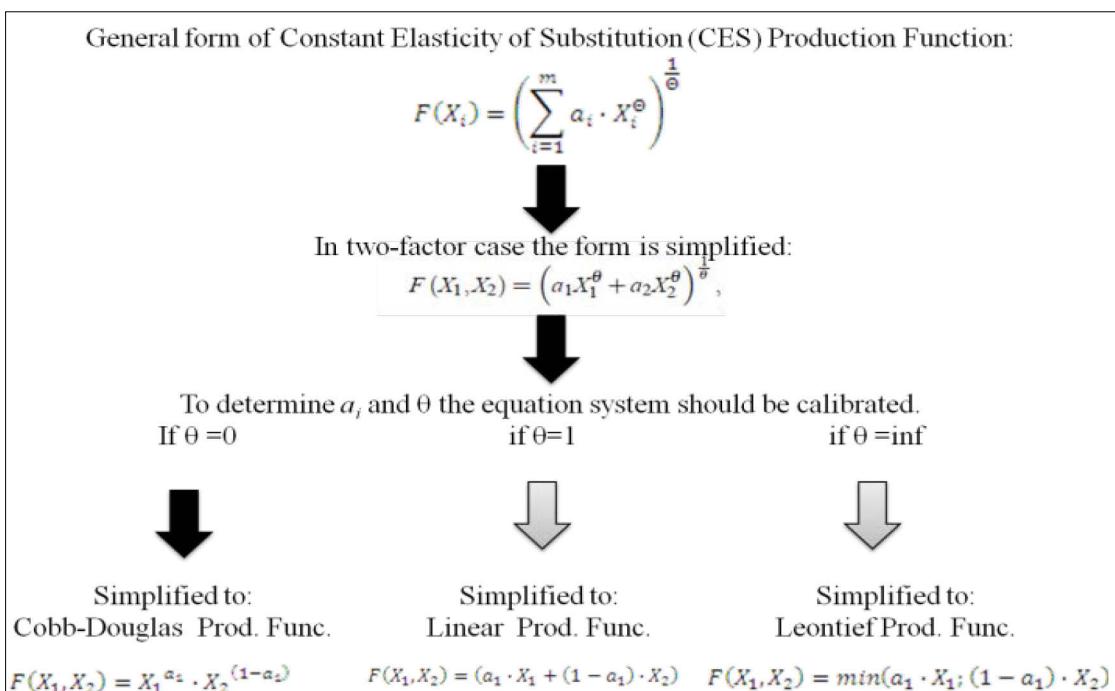
The neoclassical theory of economic growth and the development of constant elasticity substitution production functions in the 20<sup>th</sup> century are closely connected. Solow's pioneering contribution (Solow, 1956) did not only create a new dynamic macroeconomic theory, but a new type of aggregate production functions was created. Arrow et al. (Arrow et. al., 1961), de Brown and Canin (de Brown et. al., 1963) continued to develop the theory of constant elasticity of substitution production functions. "During the 1970s, many economists tried to generalize the CES function to allow variations of the elasticity of substitution. These attempts showed such generalization is possible under very limited circumstances. Aside from such generalizing attempts, the CES function (Constant Elasticity of Substitution) obtained large attention after the invention of the taste-for-variety model: Spence (Spence, 1976) for the continuum variety case and Avinash and Stiglitz (Avinash, Stiglitz, 1977) for the discrete variety case." (Saito, 2011) The concept of production function in economics is interpreted as the transformation of production factors (e.g. labour, capital goods, fossil fuels, land, and air) to other goods (emissions). In essence the "production decision" is the choice of production factors and the amount of outputs. The idealized assumption of production theory is that of the company as a unit of economy making rational decisions. This presupposed that every company possesses all available information and strives to the maximum achievable profit (Kerepesi, Romvari, 1993). "CGE models (Computable General Equilibrium) are now extensively used in studies of the economy-energy-environment nexus at the national (Bergman, 1988; Conrad, Schröder, 1993; Beauséjour et. al, 1995; Lee, Roland-Holst, 1997; Goulder, 1998; Conrad, 1999) and regional levels (Despotakis, Fischer, 1988; Li, Rose, 1995). The popularity of CGEs in this context reflects their multi-sectoral nature combined with their fully specified supply-side, facilitating the analysis of economic, energy and environmental policies." (Allan et.al., 2007) Based on the introduced theoretical background, this article aims to analyse the profit-oriented economic actors especially focusing on their transport related fossil fuel consumption in Hungary and

Germany. Further on authors show Cobb-Douglas Production Function (derived from CES production function) as a basic of CGE model.

## 2. Methodology

Cobb-Douglas production functions are widely used in consumption and production theory. Cobb and Douglas studied the contribution of each resource to the economic performance of the economic segments of the national economy (*Cobb, Douglas, 1928; Maeda, 2010*). In the below described model below described it is assumed that the economy is closed, and the input parameters for the produced output are capital, labour and fossil energy. The energy sources are imported. The economy is considered closed except for the energy sources (that is, export and import processes are excluded) and externalities were not taken into account. The usage of fossil fuel generates carbon dioxide emissions (Szendro, Torok, 2014; Shiwale et. al., 2014). Without the technological development, the efficiency of energy conversion for internal combustion, engines are constant so fossil energy consumption and carbon dioxide emission rates are linked (Vass, Német, 2013; Domanovszky, 2014). In this model fossil fuel consumptions were considered as a parameter of production.

*Figure 1* illustrates the conceptual approach of deriving, the simplified Cobb-Douglas production function from the CES production function. Further on the Cobb-Douglas production function is used. Following a formal deduction, such functions will be estimated using time series data from two European countries.



*Figure 1.* Visualisation of the model workflow  
(source: own representation)

The variables of the analysis and respective data sources are presented below. The time horizon of the analysis is 2003 through 2010. Despite of the on-going technological changes, in the investigated period, no significant technological changes were found, therefore production function is considered constant:

- Y: GDP [Billion HUF<sup>1</sup>/year] – source: *Eurostat*
- Q: number of consumed consumer basket [piece<sup>2</sup>/year] – *The World Factbook (ISSN 1553-8133)*

<sup>1</sup> HUF: currency of Hungary

<sup>2</sup> The consumer basket is a set of products that the population consume in one year. These are products and services purchased from the market. Partly methodological, partly for technical reasons, certain purchases for (property) are not included. In our model, the population is assumed proportional to the number of product units.

- p: price of consumer basket [HUF/piece] - algebraically calculated
- $F_c$ : Consumed fossil fuel [million litre/year] – source: *Trading Economics* <http://www.tradingeconomics.com/>
- $P_f$ : Price of fossil fuel [HUF/litre/year] source: *The Automobile Association Limited* <http://www.theaa.com/>
- K: Number of companies [piece/year] – required to composite goods; source: *Eurostat*
- L: Number of employees [capita/year] – required to composite goods; source: *Eurostat*
- r: Estimated unit cost of capital investment, assuming a maximum investment readiness [HUF/company/year] – required to composite goods; source: *Economy Watch* (<http://www.economywatch.com/>)
- w: yearly gross wage [HUF/capita/year] – required to composite goods; source: *EuroMonitor* ([www.euromonitor.com](http://www.euromonitor.com))
- $\beta$ : price of composite goods [HUF/piece] – algebraically calculated
- R: composite goods [piece] – algebraically calculated
- $\Psi$ : Cobb Douglas parameter – algebraically calculated
- $\rho$ : technological parameter,  $\rho = \frac{1}{(1-\sigma)}$ , where  $\sigma$  is the parameter of constant elasticity

Let  $F$  be the production function (assuming a constant elasticity of substitution - CES), in general:

$$Q = F(K, L, F_c) \quad (1)$$

In other words, the output (Q) will change according to the used amount of capital (K), labour (L), and fossil fuels ( $F_c$ ). The decision problem of economic actors is described by a nested binary model in order to be able to use simplified tools of mathematics. Capital (K) and labour (L) are combined in a composite product (R). Then the CES production function can be rewritten as (2):

$$F(K, L, F_c) = G(R, F_c) \quad (2)$$

Let  $G$  and  $H$  be production functions with constant elasticity of substitution (3) (4):

$$G(R, F_c) = (\Psi \cdot R^\rho + (1 - \Psi)F_c^\rho)^{1/\rho} \quad (3)$$

where

$\rho$  is the elasticity rate of fossil fuels ( $F_c$ ) and composite product (R)

$$R = H(K, L) = (b_1 K^\theta + b_2 L^\theta)^{\frac{1}{\theta}} \quad (4)$$

where

$\theta$  is the elasticity rate of capital (K), labour (L)

$b_1, b_2$  are the parameters

Then CES production function as an embedded binary production function will be as below (5):

$$F(K, L, F_c) = (\Psi \left\{ (b_1 K^\theta + b_2 L^\theta)^{\frac{1}{\theta}} \right\}^\rho + (1 - \Psi)F_c^\rho)^{\frac{1}{\rho}} \quad (5)$$

### 3. Calibration of production function

Taking into account equations (1) - (5) and the related assumptions of constant elasticity of substitution production function, the model calibration can be done according to the below presented two equations. So the coefficient and exponent parameters of the production functions with constant elasticity of substitution can be determined based on (6), (7):

$$\Psi \cdot \left( \frac{F_c}{Q} \right)^\rho = \frac{P_f F_c}{Y} \quad (6)$$

$$(1 - \Psi) \cdot \left( \frac{R}{Q} \right)^\rho = \frac{R \cdot p}{Y} \quad (7)$$

Equation (6) and (7) describes the connection between the prices and the volumes of output, and the used resources (composite goods and fossil energy). The correctness of the applied equations can be accepted if general form of CES function can be derived from them:

$$\psi \cdot \left(\frac{F_c}{Q}\right)^\rho = \frac{P_f F_c}{Y} \quad (8)$$

$$\psi \cdot \left(\frac{F_c}{Q}\right)^\rho = \frac{F_c \cdot P_f}{Q \cdot p} \quad (9)$$

$$\psi \cdot \left(\frac{F_c}{Q}\right)^{\rho-1} = \frac{P_f}{p} \quad (10)$$

$$\psi = \left(\frac{F_c}{Q}\right)^{1-\rho} \frac{P_f}{p} \quad (11)$$

$$\left(\frac{F_c}{Q}\right)^{1-\rho} = p \frac{\psi}{P_f} \quad (12)$$

$$\frac{F_c}{Q} = p^{\frac{1}{1-\rho}} \cdot \left(\frac{\psi}{P_f}\right)^{\frac{1}{1-\rho}} \quad (13)$$

Finally, equation (13) is the general form of CES production function (Temple, 2012):

$$\frac{X_i}{Y} = p^{\frac{1}{1-\rho}} \cdot \left(\frac{\beta_i}{w_i}\right)^{\frac{1}{1-\rho}} \quad (14)$$

Hence the calibration equations can be accepted.

#### 4. Results

The calibration process was carried out for every year and got the following results (Table 1) for Hungary and Germany as well:

**Table 1.** Results of calibration

Country	Hungary		Germany		
	Year	Cobb Douglas parameter ( $\psi$ )	Techno-logical parameter ( $\rho$ )	Cobb Douglas parameter ( $\psi$ )	Technological parameter ( $\rho$ )
		{-}	{-}	{-}	{-}
2003		.0350	-.0014	.0130	.0008
2004		.0322	.0006	.0140	-.0007
2005		.0332	.0026	.0142	.0005
2006		.0373	.0000	.0169	-.0003
2007		.0342	-.0018	.0153	-.0004
2008		.0338	-.0020	.0117	-.0011
2009		.0321	-.0015	.0150	-.0003
2010		.0329	-.0003	.0173	-.0002

(source: own results)

As it can be seen - according to the estimation that has been done - the constant elasticity substitution production function  $F(\rho)$  parameter value is varying close to zero, in case of both Germany and Hungary. The same results were found in Japan for fossil fuel consumption modelling (Maeda, 2010) and for  $SO_2$  emission in China (Xu, Masui, 2008). The Cobb-Douglas production function in this case is by definition:

$$F(K, L, F) = \left\{ (b_1 K^\theta + b_2 L^\theta)^{\frac{1}{\theta}} \right\}^\psi \cdot F_c^{(1-\psi)} \quad (15)$$

where  $\psi \in \{0,1\}$  is known to be the share of consumption of R (as composite goods, so that  $1-\psi$  is of  $F_c$ ). From (15) - if F is a production function - then the marginal production of R and  $F_c$  is (16) and (17):

$$\frac{\partial F}{\partial R} = \psi \cdot \left(\frac{F_c}{R}\right)^{1-\psi} \quad (16)$$

$$\frac{\partial F}{\partial F_c} = 1 - \psi \cdot \left(\frac{F_c}{R}\right)^{-\psi} \quad (17)$$

after that ratio of marginal production (RMP) is algebraically calculated (18):

$$RMP = \frac{\partial F / \partial R}{\partial F / \partial F_c} = \frac{\Psi}{1-\Psi} \cdot \frac{F_c}{R} \quad (18)$$

Since the relative change of a function is given by the derivative of its logarithm value (18) one obtains:

$$\sigma = \frac{d \ln(RMP)}{d(F_c/R)} = \frac{1}{F_c/R} \quad (19)$$

Therefore the consumption share is fixed and so  $\sigma = 1$ . This yields to the previously known conclusion that the CES function must converge to the Cobb-Douglas as  $\rho \rightarrow 0$  (Saunders, 2008). As expected, the parameter values are varying close to zero. This suggested to use the Cobb-Douglas type in the following (Wei, 2006), (van der Werf, 2007) in combination with the two estimated parameters.

## 5. Analysis and discussion

The resulting function is:

$$Q = R^\Psi \cdot F_c^{(1-\Psi)} \quad (20)$$

$$Y = F_c \cdot (R^\Psi \cdot F_c^{-\Psi} - P_f) \quad (21)$$

Figure 2 (a and b) visualises the data collected for the two cases investigated based on (21):

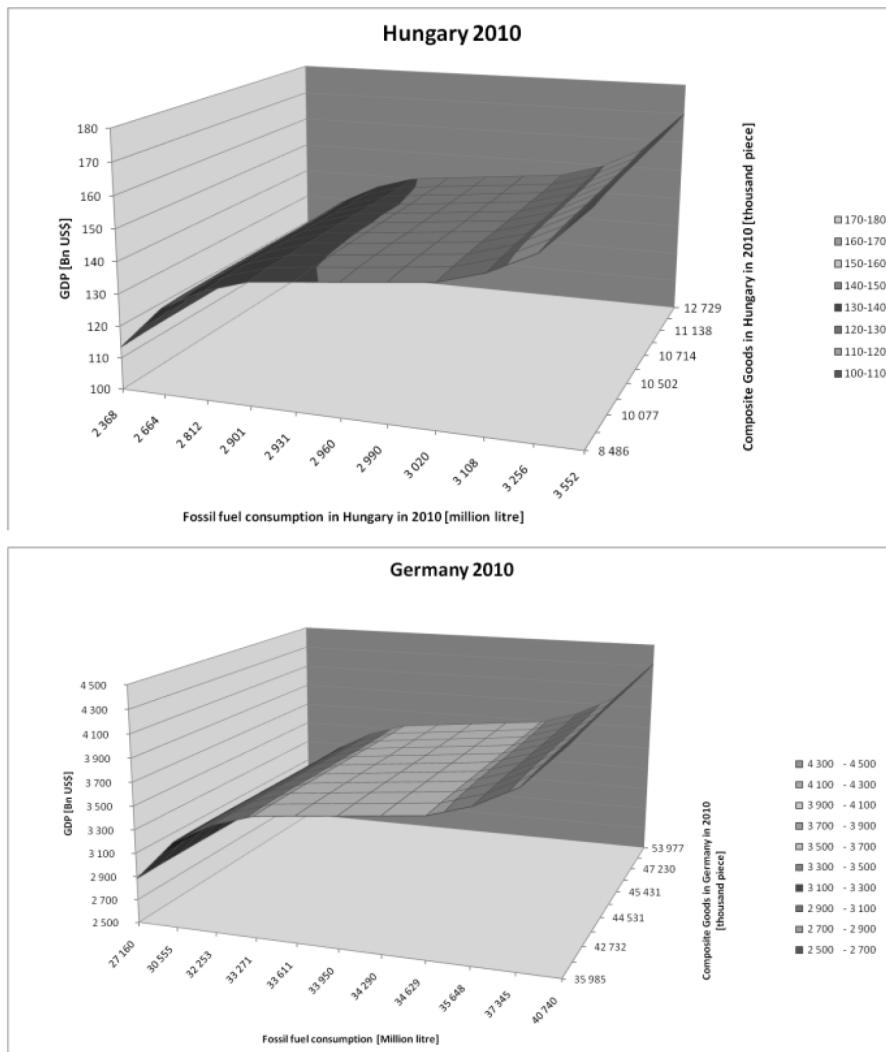
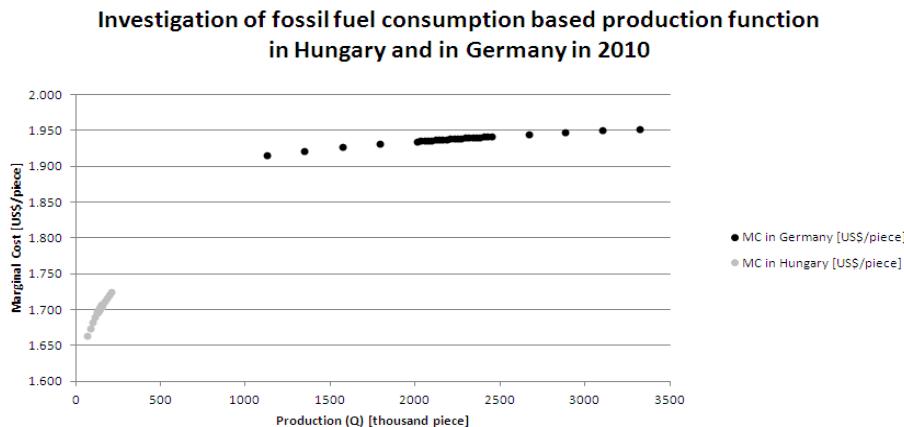


Figure 2 (a and b). Visualisation of Cobb-Douglas production function in case of Hungary and in case of Germany  
(source: own representation)

*Figure 3* shows that the fossil fuel used for transport purposes has a strictly increasing marginal production cost curve. That means, if the production increasing then the unit cost is increasing, it is more costly to produce goods. **With other words, increasing the fuel price by imposing taxes in order to increase fuel price to reach emission targets will hurt the economy.**



*Figure 3.* Investigation of fossil fuel consumption based on production function in 2010  
(source: own representation)

In consequence it can be stated that the obvious solution to reach emission reduction target could be technological innovation to avoid the undesirable trends of GDP decline. It has to be taken into account that technological innovations and R&D activities have their own costs. **However, only the technological development can complete the emissions reduction without harming the economy.** Accordingly, the common objectives to reduce the environmental pollution (Torok, 2014, Mitsakis, 2014), in particular greenhouse gas emissions, will depend on the substitution of composite goods (that contains labour work and investment) and fossil fuels. Further statistical analysis indicates that the Cobb-Douglas coefficients have not changed significantly in this case.

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