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DETERMINED MODEL AND SCALE DIAGRAMS TO INVESTIGATE PROBLEM OF TRANSPORT DELAYS

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In the paper the following question is investigated: what are the reasons of transport delays besides the diffusion of platoons? To answer the question a determined model is developed. It has a number of restrictions. A scale diagram of the highway state in discrete time points is created and described in details. The paper examines certain parameters of the determined model and its response to changes in these parameters with the help of scale diagrams. To be more precise all the places of delays can be detected according to the highway state diagram. Some of these delays can be substantially reduced by using phase offset of green traffic light from the side directions. So the paper contains proposals for the optimization of the developed model, aimed to reduce motor vehicle delays in front of the in-traffic light stop line along the main highway direction. Besides, the density of vehicles on the highway is considered.

Keywords: determined model, platoon, highway, intersection, traffic

1. Introduction

Constant and continuous growth of motor vehicles among the population, traffic growth leads to transport problems: appearance of traffic jams, traffic delay increase, increasing of accident number, environmental pollution, etc. These problems become apparent in the nodal points of the road network most tangibly.

Coordinated regulation makes it possible to smooth over these problems in part. The purpose of coordinated regulation is to ensure safe motor transport movement along a street or a highway. Coordination of traffic signals at adjacent intersections would reduce number of stops, braking and acceleration in the flow, and transport delays.

However coordinated regulation effectiveness often is low. Stochastic nature of speeds and accelerations in traffic flow leads to the diffusion of platoons causing serious difficulties for traffic management. Platoon edges significantly dilute during progression between two traffic lights. Only permanent platoon forms and platoon constant speeds would enable to predict the moment of intersection passage accurately, and make optimal program for traffic lights [1].

A question appears: what are the reasons of transport delays besides the diffusion of platoons? To answer the question, let ignore diffusion in the proposed model. All platoons of equal size, formed by a traffic light, travel with the same constant speed. This is the first position (postulate) of the model.

The object of study is a highway, along which *z+1* T-type intersections are located at different distances (Figure 1). The highway and all cross roads have one-way traffic. The direction of movement is indicated by arrows (Figure 1). Distances between intersections 0, 1, ..., Z are equal to $L_1, L_2, ..., L_z$ correspondingly. Intersections controlled by traffic lights working with two-phase cycle.

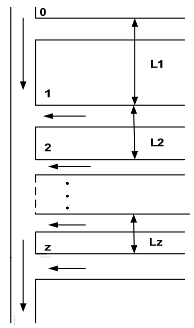

Figure 1. Highway plan with the adjacent T-type, controlled intersections

Let relate yellow times t_{y1} and t_{y2} of traffic light to the red signal (RS) in cycle length. Let take time duration of green signal (GS) of a crossing road, as a minimum time discrete $t_g=\Delta t$. Let make red time duration RS (with yellow) multiple of t_g, i. e. $t_g = k\, t_r$, where k – factor of multiplicity (integer). Thus the duration of the cycle C is:

$$C = t_g + t_r = (k+1)\, t_g \,. \quad (1)$$

Similarly, we introduce a distance discrete Δl. Let take $\Delta l = v\Delta t = v\ t_g$, where v is permissible speed on a highway. Let us assume that all the distances between intersections are multiples of the distance discrete Δl. Then the distance l_i between two intersections $I – 1$ and i $(i = 1, ..., Z)$ will be as follows:

$$l_i = p\Delta l = p\, v\, t_g \,, \quad (2)$$

where p is integer.

2. Structural Diagram

A structural diagram of discrete control of traffic lights is showed on Figure 2. The traffic light at the i-th intersection $(i = 0, ..., Z)$ can be represented by two keys K_{i1} and K_{i2}. The key K_{i1} opens or closes traffic on $(i, i+1)$ link. Key K_{i2} opens or closes transport flow into the highway from the crossing road (R_i).

Key management is carried out with register – a word of a highway state. Binary vector $P_j= (\delta_0, \delta_1, \dots \delta_i, \dots ,\delta_z)$, $j = 1, \dots ,2^{Z+1}$, δ_i € $0, 1, (i = 0, \dots ,Z)$ is putted into the register. Each vector's bit controls the corresponding crossroads. Management is carried out with paraphase signal, i.e. if one of the key pair, for example K_{i1} is opened, then key K_{i2} is closed, and vice versa. The combination of opened and closed keys defines the highway state. The state is uniquely determined by the vector P_j. Change of the vector P_j in the register with a command from the control computer changes the state and movement mode of highway and secondary roads R_0 - R_z.

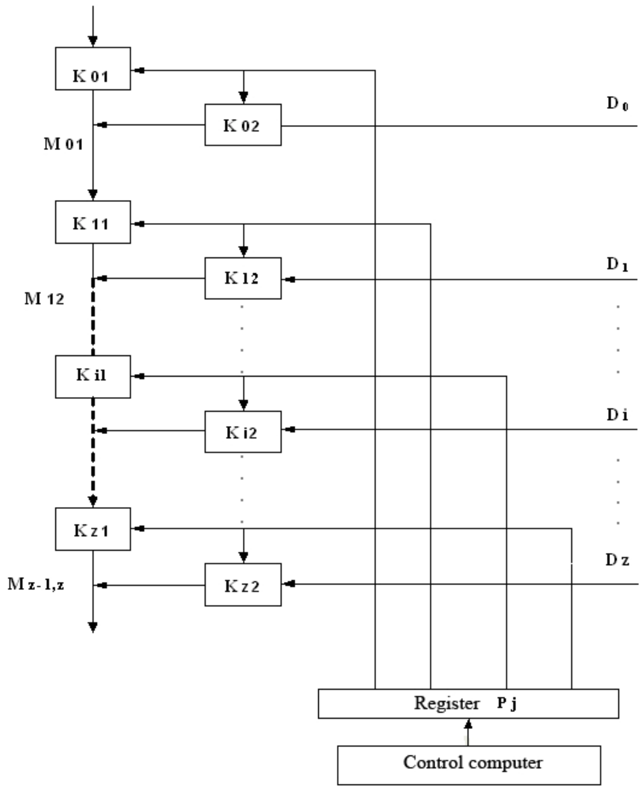

Figure 2. Structural diagram of discrete control of traffic lights

If rank i of vector P_j equals 1 ($\delta_i = 1$) then key K_{i1} state is opened, i.e. the passage through the *i*-th intersection is opened, otherwise ($\delta i = 0$) the intersection is closed for highway and opened for secondary road R_i.

Let consider the process of filling the highway with transport in the morning. The initial state of the highway is the state S_0 – the absence of motor transport. At the same time there are enough vehicles on crossing-roads to ensure that at the opening time, when $\Delta t = t_g$, n units of motor transport would enter the highway from any cross-road. Let us assume that platoon of n vehicles, that came in highway, will travel along it for a distance equal one distance discrete $\Delta l = v\Delta\ t= vt_g$ during discrete time $\Delta\ t = t_g$.

3. Scale Diagram

Let create a scale diagram of the highway state in discrete time points. To do this, time intervals of duration $\Delta t = t_g$ are plotted on the horizontal axis, distance intervals are plotted on the vertical axis. The scale for Δt and Δl is chosen so that they have equal length (e.g. a cell) on the diagram of states.

Figure 3 shows the state diagram for five intersections (with different distances between them) during 17 time discretes where k = 2. Only two control vectors are used P_j $(j = 1, 2)$. The first vector consists of all zeros and is in register P_j during t_r. Z +1 platoons enter the highway during this time. These platoons manage to pass distance of Δl along the highway. Then the highway goes from state S_0 to state S_1, characterized with the presence of five platoons of n cars (the first column in Figure 3). Here and in the future, highway state S_r means the number of platoons at the time t_r, and their composition, i.e. numbers of intersections from which they came. The darker vertical bars indicate red signals for all highway traffic lights, and vehicle emission into the highway. There are platoons numbered from 0 to 4 that came from the corresponding intersections in column 1. One platoon of n vehicles is from each intersection. Then the register P_j is filled with vector P_2, consisting of all ones. This vector set green signals for all highway traffic lights, and red signals for cross-roads. This is a phase of transit pass of platoons along the highway.

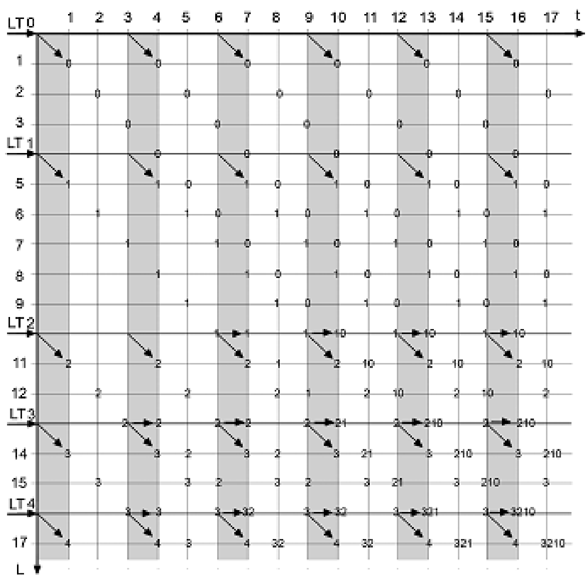

Figure 3. State diagram of highway with two control vectors P_1 and P_2

In the literature, e.g. [2], motor cars moving along the highway are of two types: transit and out of platoon. All motor cars are transit in the determined model. The process of platoon movement control is clearly determined. The effect of platoon "blurring" [2] on the links is not considered in the model.

Thus motor cars enter the highway from side directions in the time intervals 0 – 1, 3 – 4, 6 – 7, etc. as noted in Figure 3 with oblique arrows. All traffic lights on the highway have green phase in the time intervals 1 – 3, 4 – 6, 7 – 9 and so on. The flow crosses the highway without difficulties.

States S_1, S_2 and S_3 are equivalent, because they have equal and constant number of platoons (five). State S_4 is characterized by ten platoons of n vehicles in each platoon, because emission of motor cars occurred from side directions at time t_4. It should be noticed that emission is not done simultaneously, but in the interval between t_3 (start) and t_4 (finish). That is, green phase turns on for the side directions at the moment t_3 and turns off at the moment t_4.

Any platoon P_i *(t, l), (I = 1, ..., z)* is determined uniquely with two coordinates – time t and the distance *l* from the reference point – on the state diagram (Figure 3). The number of platoon in a column shows their number on the highway at the concrete moment of time , as well as the number of platoons on any link, and their structure, i.e. the number of source intersection and start time.

The actual end of the highway is the last traffic light. Platoon P_i moves bias within the next square. Increment of *Δt* corresponds to *Δl*. So, for example, platoon P_1 (1, 5) at the moment of time $t = 6$ (Figure 3) will face the traffic light number 2, which will be closed, as it is opened for secondary roads in the time interval 6–7. Platoon P_2 (7, 11) is being formed in this moment. Transit platoon P_1 (6, 10) will delay at the traffic light during *Δt* and goes into a state of P_1 (7, 10).

Later platoon P_1 (7, 10) moves without delays to the end of highway. It is joined with P_2 (9, 13) at the traffic light number 3. P_2 (9, 13) was forced to wait for the time *Δt* because of red light of traffic light number 3. Two fused platoons of P_1 (10, 13) and P_2 (10, 13) move to the traffic light number 4, where they are joined with platoon P_3 (13, 16). Then all three platoons reach the end of the highway.

Thus, all the places of delays can be detected according to the highway state diagram. For example, there are 14 such places in the Figure 3, and total delay is 14*Δt*.

This delay can be substantially reduced by using phase offset of green traffic light from the side directions. This process should start with the last traffic light, gradually moving to the beginning of the highway, will produce phase offset with control vectors P_j, i.e. the number of control vectors will be greater than two.

Let us show phase offset to the left of one time unit *Δt* for traffic light number 4 (see Figure 4). The offset is performed on the free diagonal. This ensures the passage of the platoons through the traffic light without delays. Since the empty diagonal was used (last row of the state diagram is completely filled), then the delays at traffic lights number 2 and number 3 persist. The number of delayed platoons has reduced from 14 to 9.

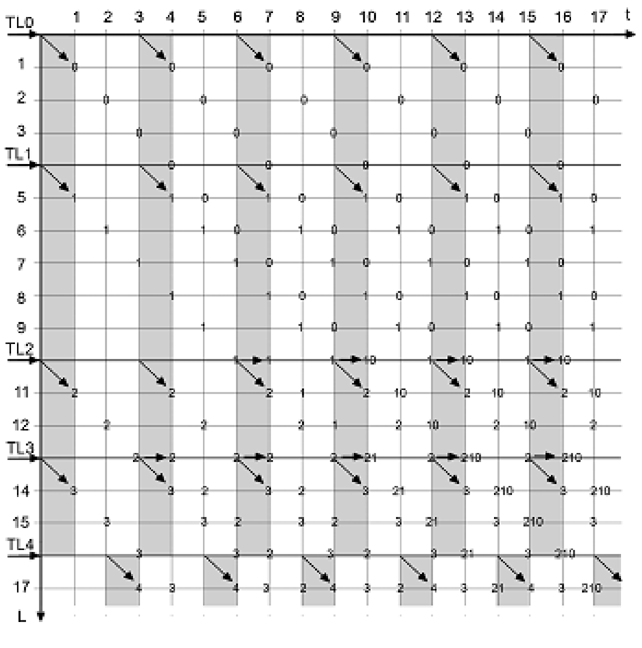

Figure 4. Highway state diagram with an increase in the number of control vectors

4. The Density of Vehicles on the Highway

Flow density of motor cars is a parameter characterizing traffic flow on the number of vehicles at any given time on a given section of road. In our case, the density will be called the number of platoons on a given section of road, not the number of individual vehicles.

Let us construct a density diagram of vehicles on the whole highway at every second for the developed model (see Figure 5).

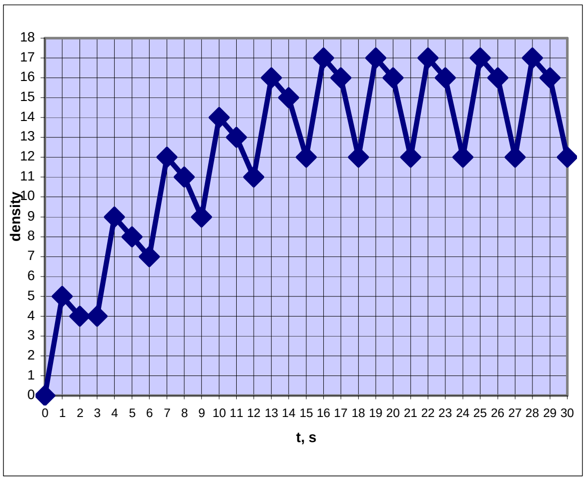

Figure 5. Density diagram

Figure 5 shows that the number of platoons increases jerkily during the highway is filling. After the 16th second condition is stabilized, the density on the highway is no longer changing. Thus, the highway state diagram line (Figure 3, 4), as well as all the highway state diagrams below, have the property that their state no longer change after the first platoon will have been travelled from the start to the end traffic lights on the highway.

5. The Optimal Choice of *k*

Let us create diagrams with different *k* using the same determined model, to find out how to choose the optimal *k* or how to change *k* to eliminate delays.

The highway state diagrams with *k* equals to 1, 2, 3 and 4 are showed on Figure 6. To calculate correctly the number of delays, the counting will be performed after the 20th second, when all the models will work in a stable mode. Also, the counting will be performed during a time interval equal to one cycle.

It is seen that:

- when $k = 1$, taking into account that one horizontal arrow can mean several platoon delays, there are 5 delay at the diagram, and the offset to reduce the number of delays is not possible;
- when $k = 2$, there are 3 delay, a free diagonal presents to make the phase offset in one discrete of time to the left for traffic light number 4. Then the number of delays will be equal to 2;
- when $k = 3$ we have 2 delay, there will be one delay after the phase offset in one discrete of time to the right for traffic light number 0;
- when $k = 4$ we have 1 delay, and there will not any delay after the phase offset in two discretes of time to the left for traffic light number 0.

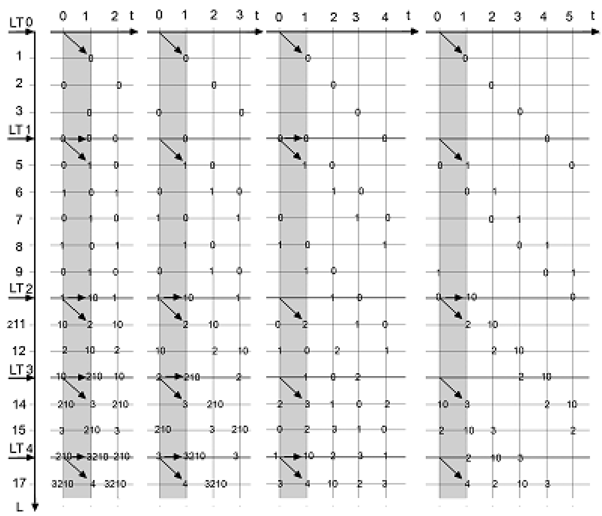

Figure 6. Determined models for 1) $k = 1$, 2) $k = 2$, 3) $k = 3$, 4) $k = 4$ in the stable mode

Diagram of the number of delays dependence on the coefficient k will illustrate the above (see Figure 7).

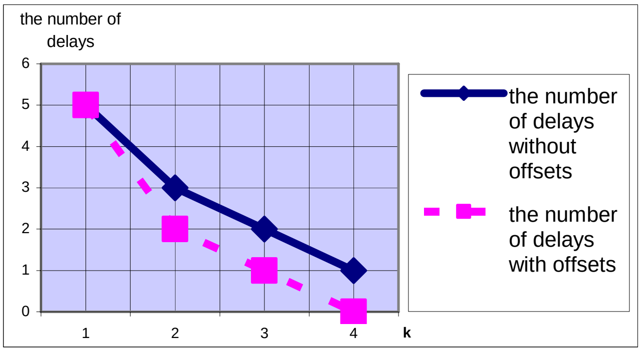

Figure 7. Diagram of the number of delays dependence on the coefficient k

Thus, Figure 7 shows that the smaller k, the greater delays, and vice versa. When $k = 4$, all delays can be eliminated. Phase offsets allow reducing the number of delays.

However, it should be noted that for large k there is little time for the passage from the side streets. And also a great cycle time may provoke massive violations of rules of the road. Therefore, when choosing k, reasonable ratio should be observed between the duration of green signal on the highway and on the cross-streets. [2] gives a formula for calculating the length of green signal for the transport of cross roads:

$$t_z = \frac{q \cdot C min}{q_s \cdot x_{lim}} \geq 14s, \tag{3}$$

where q – movement intensity for a secondary direction for given band, veh / s;
q_s – the flow of saturation for a given traffic flow, veh / s;
C_{min} – the minimum duration of the cycle;
x_{lim} – limit (recommended) value of the load factor of a band for a secondary direction (≈ 0,6).

Equation (3) has approximate and recommended character, but you can use it to find the upper limit of k. For more detailed information about the formula and its parameters see the source [2].

In the determined model it should be taken into account that the appearance of delays may depend not only on k, but also on the relations presented below.

There is a delay at intersection b if

$$\left(\sum_{i=a+1}^{b} p_i + \mu\right) mod(k+1) = 0, \tag{4}$$

There is not a delay at intersection b if

$$\left(\sum_{i=a+1}^{b} p_i + \mu\right) mod(k+1) \neq 0, \tag{5}$$

where mod is a remainder of the division;
a – number of the intersection, from which a platoon came;
b – number of the intersection, which is tested for presence / absence of delay;
μ – the number of delays has already happened for this platoon;
p_i – the distance between $(i-1)$-th and the i-th intersections (in distance discretes).

With the help of formulas 4 and 5 it is possible not to build a determined model to define the presence or absence of delays at the intersection for a platoon. For example in Figure 6 when $k = 1$, let us check presence of delay at the first traffic light (TL1). The second summand in the numerator is not included, as just one cycle is considered. Thus $a = 0$, $b = 1$. Then we calculate the sum of discretes between the traffic light number 0 and the traffic light number 1. The sum is equal to 4. The denominator is equal to 2. There is no remainder after division of the numerator by the denominator. Therefore, the delay is present.

The same pack when k = 2 will pass TL1 without delay, because the sum of discretes still equal to 4, while the denominator is equal to 3. There is remainder after the division. When k = 1 lets check the delay for the 1st platoon at LT2. a = 1, b = 2, the sum is 6, denominator is 2. There will be delay for the first platoon. While there will not be delay for the platoon number 0, because sum is 6, denominator is 2, but μ = 1 (for the platoon number 0 there has been delay at TL1), so the numerator is equal to 7, and the denominator is 2. I.e. division is with a remainder, no delay. And so on.

6. A Determined Model for Bidirectional Movement

Let's extend the model.

Now the highway and all cross-roads have two-way traffic, and intersections are of X-shaped type (Figure 8).

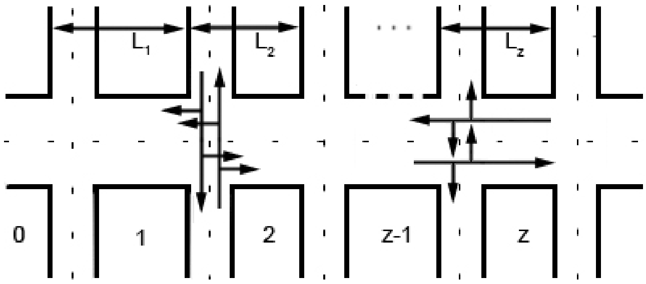

Figure 8. Highway plan with the adjacent X-shaped, controlled intersections

On Figure 8 distances between the intersections 0, 1, ..., Z are equal to $L_1, L_2, ..., L_z$. Possible for all the intersections, movement directions from the cross-roads are shown with the arrows at the intersection number 1. Possible for all the intersections, movement directions from the highway are shown with the arrows at the intersection number Z – 1. All the intersections are equipped with traffic lights. We assume that all traffic lights are working with two-phase traffic light cycle. Then the structural diagram of a discrete control of traffic lights (see Figure 2) is also suitable for the case.

Let's create a scale diagram of the highway state in discrete time points for bidirectional movement (Figure 9). As before k = 2, two control vectors are used: P_1 consists of all zeros and is in register P_j during t_r, P_2 consists of all ones. This vector set green signals for all highway traffic lights, and red signals for cross-roads.

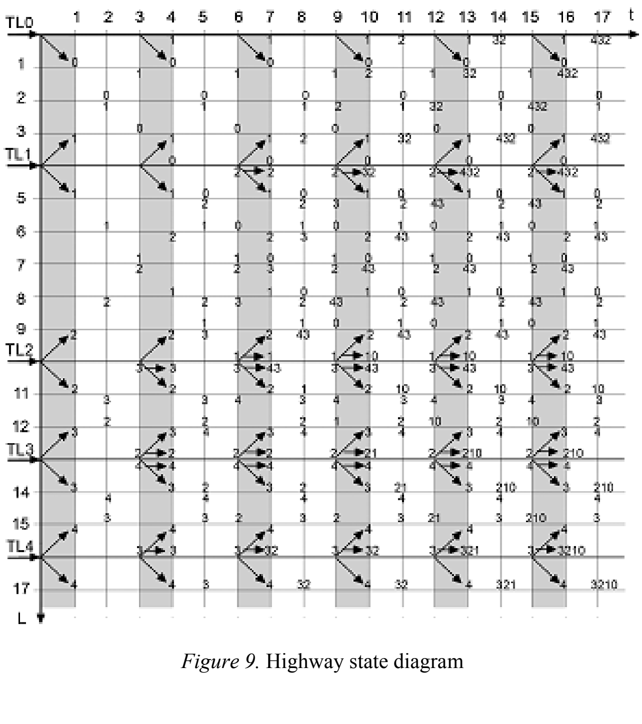

Figure 9. Highway state diagram

Figure 9 shows the 5 traffic lights with 17 distance discretes for 17 time discretes. Arrows pointing down mean the next emission of platoons into one side of the highway (direction I) from the side streets. These platoons are placed above the horizontal lines. Arrows pointing upward mean the next emission of platoons into the other side of the highway (direction II). These platoons are placed under horizontal lines in the diagram. Horizontal arrows indicate platoon delay, because their state is changed only on the time axis and the axis of the distance remains the same. All platoons are numbered in accordance with the number of traffic lights, from which they entered the highway. Several consecutive numbers mean that the packets from different traffic lights merged into one.

Analysing the chart shown on Figure 9, we can conclude that, it is not possible to avoid delays at any intersection for a given location of traffic lights and $k = 2$. And traffic lights 2 and 3 create delays in both directions.

Solutions to this situation are:

- to increase the duration of green phase (k),
- to make phase offset.

Let us set our choice on the latter. Let us find out what changes are possible to minimize the number of delays, if we consider both directions independently of each other. In the direction I it is possible to make phase offset in one time discrete Δt to the left for traffic light number 4. There is no alternative, because there are not free diagonals. In the direction II it is admissible to make phase offset in one time discrete Δt to the left for traffic light number 1 and to the right for traffic light number 4. Now you have to track how these changes are coordinate with each other. Offset in the direction I at the fourth traffic light will lead to the fact that traffic lights will operate asynchronously in the direction I and II. In our case, this situation is not possible, because there are only 3 time discretes in the cycle. If the traffic light is asynchronous, then at least one discrete would be dedicated to the green signal in the direction I, another discrete of green would be dedicated in the direction II. If the third discrete dedicate to the green signal, it would mean that the green light does not ever light up for the cross road. If the third discrete dedicate to the red light, it would mean that the greater part of time the red signal will be turned on at this intersection on the highway. That is also unacceptable. Therefore, phase moving in one direction require phase moving into another one. At the same time we should check if such an offset is possible in the direction II with the diagram. There is a free diagonal. But, having taken it, we no longer have the possibility to shift the phase at the traffic light number 1. As a result, we obtain a new diagram shown in Figure 10. 3 control vectors are needed to manage it. The number of delays has decreased after the phase offset.

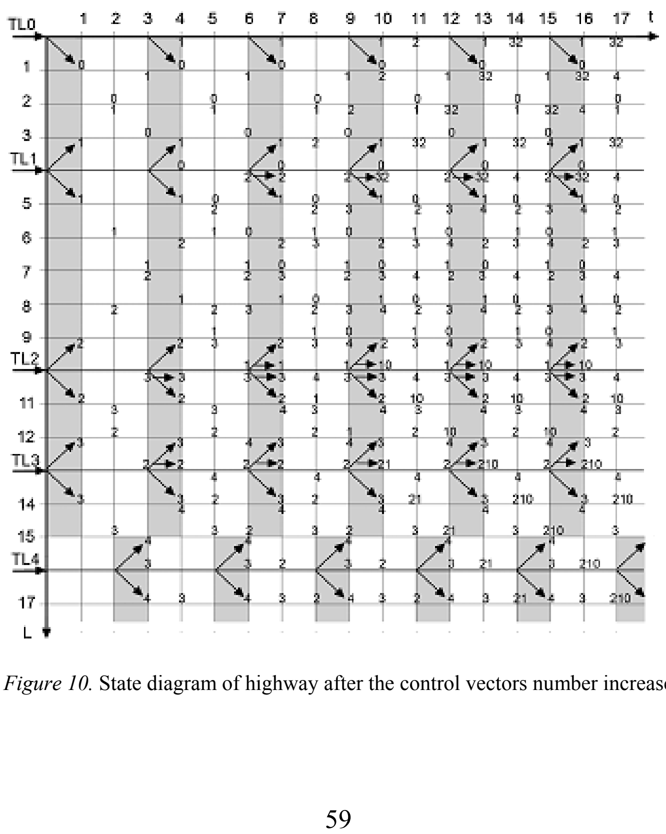

Figure 10. State diagram of highway after the control vectors number increase

7. Conclusions

Diffusion of platoons is not the only a reason of transport delays and probably is not one of the main reasons in the coordinated regulation. The revealed reason for delays is platoon competition from different lateral directions when crossing highway traffic lights. Further research should evaluate the impact of each factor over delays of vehicles. For this it is necessary to introduce an element of chance in the deterministic model, which would dither platoons.

The highway state diagrams in the idealized case move in a stable mode some time after the start. In the real life, such stable modes may exist only for some limited period of time, because traffic flow has stochastic nature. When choosing the coefficient k a balance should be observed. The absence of delays can be achieved in the highway deterministic model with increasing of k, but it should be considered what length of the green signal for the side streets will turn out. It is much more difficult to avoid delays of vehicles with bidirectional movement. Ways to struggle against delays are the following: phase offset, increase of k, phase offset with the asynchronous operation of traffic lights.

References

1. Kremenets, Y. A. and Pechorski, M. P. *Technical means of traffic regulation.* Moscow: Transport, 1981. 137 p.
2. Lukonin, V. N. et al. *Vehicular flows and surrounding environment.* Moscow: Infra-M, 2001. 317 p.
3. Vrubel, Y. A. *Traffic organization. Part 2.* Minsk: Byelorussian Fund of Traffic Safety, 1996. 204 p.