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SOME FEATURES OF THE HOP MODEL OF EARTH-IONOSPHERE WAVEGUIDE

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The problem of electromagnetic pulse radiator location from single station observation is considered, based on the hop model of pulse propagation in spherical waveguide 'Earth-ionosphere'. Pulse characteristics both ground wave and hop (sky) waves of that waveguide are modelled as functions of the distance from radiation source. The waveguide parameters, namely, the distance from the radiator and the effective reflecting heights of the ionosphere, can be evaluated through the delays of the waves reflected by the ionosphere with respect to the ground wave. The errors in values of delays give rise to the errors in the waveguide parameters. These connections are investigated. It is demonstrated how the waveguide parameter errors depend on the distances and heights of reflections. A method to reinterpret the hop model features by using some equivalent antenna array conception is proposed.

Keywords: electromagnetic radiation, ionosphere, waveguide, hop model, distance, effective heights, delays, lightning discharge, atmospherics

1. Introduction

Ionospheric layers and the Earth's surface which create a spherical waveguide with both semiconducting walls should have a significant influence on features of electromagnetic (EM) fields propagating in it. The rigorous method to find the EM field structure in this waveguide is based on a wave equation. The solution of it is an infinite series which divergence is too slow for distances lesser than 700-1000 km from radiation source. However those are the distances representing the region of interest for many technical applications, for instance, finding of lightning discharges positions by analysis of their own EM pulses named atmospherics only.

At relatively low frequencies (ELF, VLF and LF bands) the higher layers of ground and the lower ionospheric layer (D in daylight or E in night time) are good conductors. It is known that EM fields cannot penetrate deeply into those media. Consequently, geometric dimensions, namely the distance between the radiating and receiving points and the effective heights of the waveguide ‘Earth-ionosphere’, become the main factors, which determine the characteristics of propagation.

If the distance is not larger than 1500–1700 km, a hop model of propagation could be considered. It is supposed that the received signal consists of a ground wave and some waves reflected from the D or E ionospheric layer (Fig. 1). This model permits some methods for evaluating waveguide dimensions from a receiving signal only.

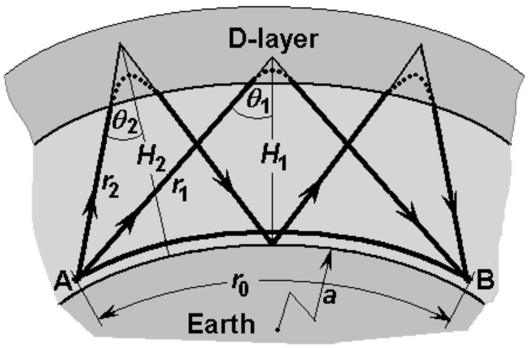

Figure 1. The hop model of ionospheric waveguide (in daytime)

EM radiation initiated from point *A* propagates to point *B*. The receiver in *B* registers the process of interaction of the ground wave $E_g(t)$ which has to pass the distance r_0 along the surface of the Earth, and a few ionospheric waves. Only two waves from them are shown on Figure 1. The single-hop wave $E_{i1}(t)$ and the two-hop wave $E_{i2}(t)$ pass the ways signed as r_1 and r_2 being reflected from the effective heights H_1 and H_2 with angles θ_1 and θ_2 respectively.

The receiving signal in point B can be written as

$$u(t) = E_g(t) + \sum_{n=1}^{N} E_{in}(t) = I(0,t) * [h_c(0,t) * h_g(t) + \sum_{n=1}^{N} h_c(\theta_n, t) * h_{in}(t)], \tag{1}$$

where sign $*$ is symbol of convolution, $I(0, t)$ is the temporal form of the current pulse at antenna input points; $h_c(\theta, t)$ is the pulse function of antenna radiation in the direction set by angle θ; $h_g(t)$ and $h_{in}(t)$ are pulse functions of traces for ground wave and n-th ionospheric wave respectively.

Evidently, the reflected waves $E_1(t)$ and $E_2(t)$ are delayed in relation to the ground wave by the times indicated as τ_1 and τ_2 respectively.

2. Modelling the Waveguide Pulse Functions

The procedures for calculating of pulse functions $h_g(t)$ and $h_{in}(t)$ in Eq. (1) are briefly exposed below. Detailed description is in preparation to be published. Spectra all of these functions are evaluated in frequency domain in the band [500 - 0.25×10^6] Hz for different distances with subsequent return and restoration to the temporal forms by inverse fast Fourier transformation (IFFT).

a) Modelling of the ground wave

Representation proposed by V.A. Fok [1, 2] includes the infinite series with members which contain coefficients described by Airy functions. These series diverge slowly for short distances, and too many members (up to 40) may be necessary to keep in even if simplification for good conducting ground commonly employed in low frequencies bands is used in order to force calculations.

The graphs of the ground wave pulse functions $h_g(t)$ for different distances up to 600 km are represented on Figure 2 a. As one can look forward, the ground wave pulse function suffers distortions (becomes blurred) as the distance from EM radiation source is increased.

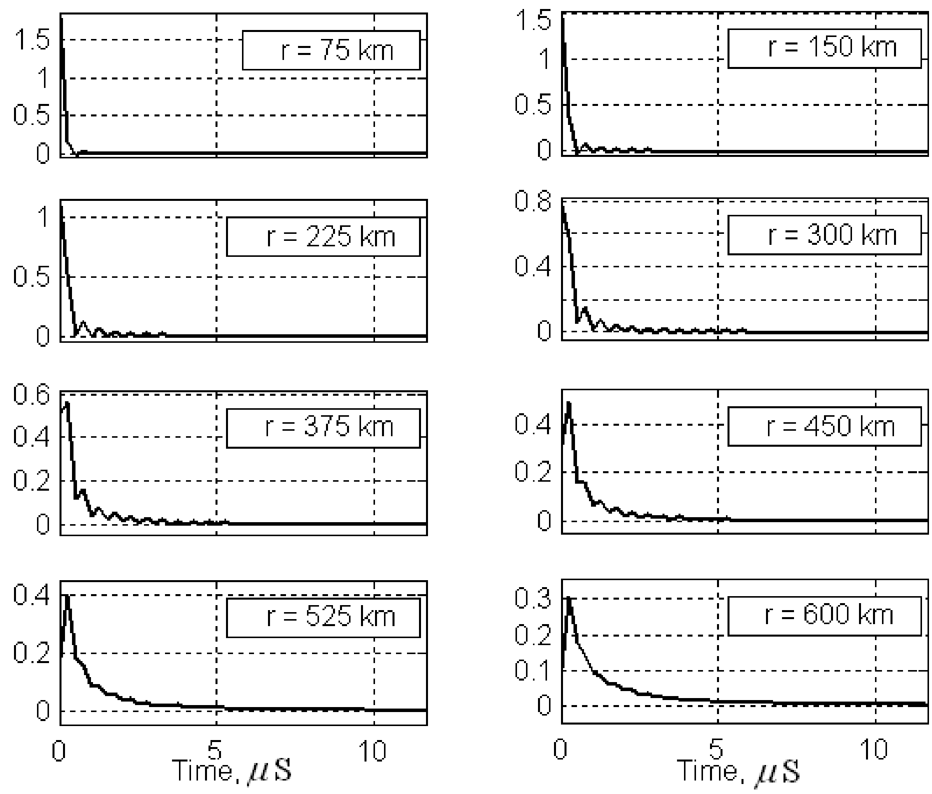

Figure 2 a. Ground wave pulse function

b) Modelling of the hop waves

Below the simplified procedure for calculation of first hop pulse function $h_{i1}(t)$ and in the event of vertical polarisation is considered only. If an inner spherical surface of ionospheric layer is approximated locally as a plane then reflected waves may be evaluated by Fresnel coefficients (see, e.g., [3]) with due regard for complex dielectric permeability of reflected ionospheric layer. The first hop function $h_{i1}(t)$ is IFFT of the frequency spectrum of named Fresnel coefficients.

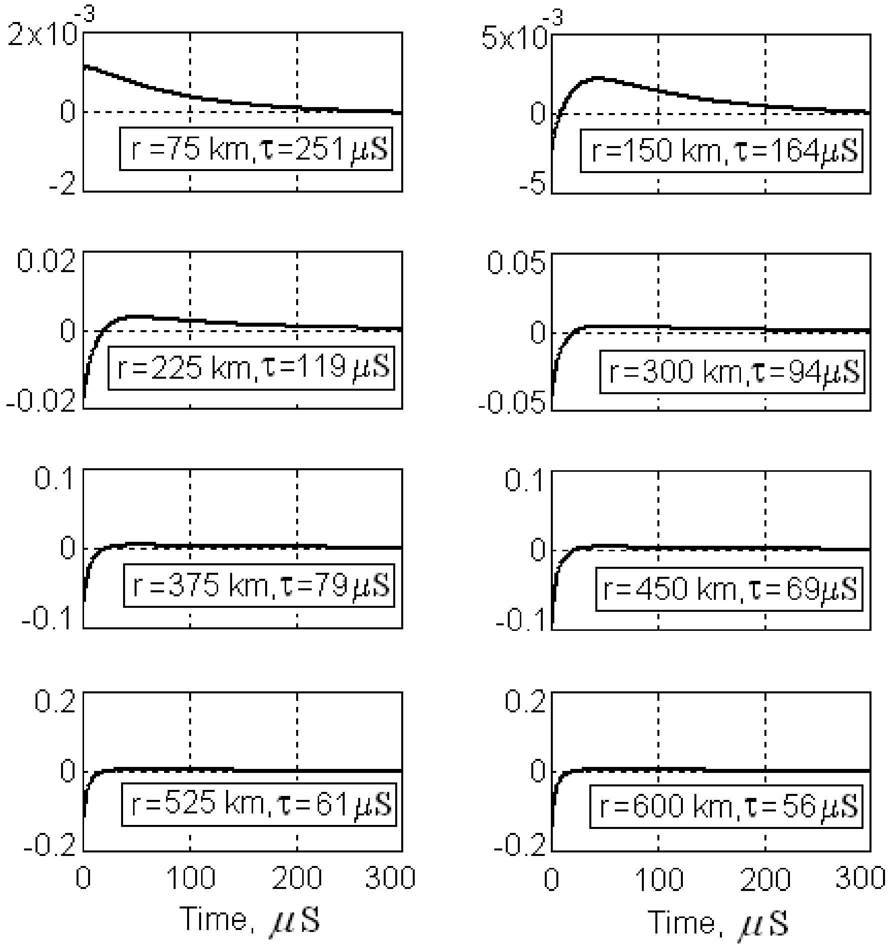

Figure 2 b. First hop sky wave pulse function

The graphs of $h_{i1}(t)$ represented on Figure 2 b for different distances are non-trivial. One can see that the pulse has to change the own sign as the distance is among 150-300 km from EM radiation source. Most likely, it has to be explained by pseudo-Brewster effect having a place in the event of electromagnetic waves with vertical polarisation.

3. Evaluation of Delays of Ionospheric Waves

The waveguide inherent features should appear as delays, or arrival times of those reflected waves. From the geometrical properties of the spherical waveguide with good conducting walls, one can obtain the system of equations

$$T_n = (1 - 2Z_n \cos R_n + Z_n^2)^{1/2} - R_n, \quad n = 1, 2, \ldots, \tag{2}$$

where $T_n = c\tau_n / 2na$, $R_n = r_0 / 2na$, $Z_n = 1 + H_n / a$ are normalized delays, distances and effective reflection heights respectively; $c = 2{,}998 \cdot 10^8$ *m/sec* is light velocity in a vacuum, $a = 6378$ *km* is the radius of the

Earth. Previously we have proposed to estimate these delays from the received signal with a pseudocepstral method using Huang – Hilbert decomposition [4].

As the two-hop model is considered, the system (2) consists of two equations, but three unknown quantities are in it, namely, the distance r_0 and the ionospheric waves' effective heights H_1 and H_2. Therefore, this system is undetermined, since the number of equations is less in 1 than the number of unknown quantities. The algorithms have been examined in [5] to expand the system based on some common physical considerations; Fermat‘s theorem, in particular.

Errors in the solving of the undetermined system have decisive influences on precision in the evaluation of waveguide parameters. The errors in estimation of ionospheric waveguide length and effective heights are considered below.

If digital processing of receiving signals is used these errors depend strongly on the sampling rate.

4. Inaccuracy in the Estimation of Ionospheric Waveguide Length

It is important to find specific relations for inaccuracy in the estimation of waveguide length r_0 with effective heights H_n and delays τ_n in equation (2). Differentiating the Eq. (2) by τ_n as an implicit function which relates the distances in spherical waveguide with the delay, one can obtain the relative inaccuracy of the waveguide length estimation

$$\delta \boldsymbol{r}_0 = \frac{\Delta \boldsymbol{r}_0}{\boldsymbol{r}_0} = -\frac{\boldsymbol{c}\Delta\tau_\mathrm{n}}{2\boldsymbol{naR}_\mathrm{n}}\left(1 - \frac{\boldsymbol{Z}_\mathrm{n}\sin\boldsymbol{R}_\mathrm{n}}{\mathrm{T}_\mathrm{n} + \boldsymbol{R}_\mathrm{n}}\right)^{-1}, \tag{3}$$

where value $\Delta\tau_\mathrm{n}$ is the absolute error of the estimation of the delay τ_n. It is obvious that the potential accuracy in the τ_n estimation by means of digital processing procedures has to correspond to the considered signal sampling rate.

The graphs of Eq. (3) for different reflection heights are represented on Figure 3. One can establish that the relative inaccuracy δr_0 depends on the evaluated distance r_0 non-monotonously. There are minima in middle distances (100–700 km) with positions depending on reflecting heights.

If ionospheric conditions are unperturbed (i.e. lower ionosphere edge H_0 is higher than perhaps 50 km) then the value $\delta r_0 \leq 0.05$, as the sampling interval is no more than 1 microsecond. The errors increase noticeably as the height of reflecting layer is reduced. Thus, they would be larger in the daylight when the D-layer plays a main part in reflection process than in the night time when the D-layer disappears and the E-layer becomes the essentials. It would be expected in the last case that the value δr_0 does not exceed 0.02.

The area painted over on Figure 3 corresponds to the ordinary locations of reflecting D and E layers in the ionosphere.

Extremely great errors of distance estimation should be observed when ionospheric perturbations are influenced by strong ionisation factors, and reflecting layers can rise to smaller heights than in unperturbed conditions. The hypothetic case for $H_0 = 20$ km is also illustrated on Figure 3. It is found in this situation that the errors for middle distances have to increase very sharply up to more than four times in contrast to errors for ordinary conditions.

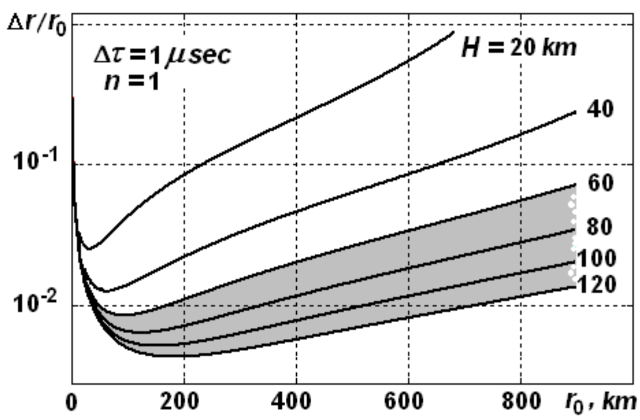

Figure 3. Relative inaccuracy of waveguide length estimation

As the value δr_0 depends on the evaluated distance r_0 non-monotonously, it can find the minimum position of r_0 for every reflection height. Examining Eq. (3) for extreme points, one can obtain the transcendental equation for the distance r_{0min}

$$[(Z_n\cos(r_{0min}/2na)-1](r_{0min}+c\tau_n)r_{0min}+[2naZ_n\sin(r_{0min}/2na)-r_{0min}-\tau_n]c\tau_n=0. \tag{4}$$

The required value r_{0min} is determined by the positive root in it. As the argument of trigonometric functions is small, then expression (4) can be approximated after corresponding simplifications by the quadratic equation in the form

$$r_{0min}^2+2c\tau_n\, r_{0min}-(c\tau_n)^2 a/H_n\approx 0. \tag{5}$$

The positive root of it is

$$r_{0min}=c\tau_n[(1+a/H_n)^{1/2}-1]\approx c\tau_n\,(a/H_n)^{1/2}\approx 0., \tag{6}$$

where it is taken into account that a/H_n>>1.

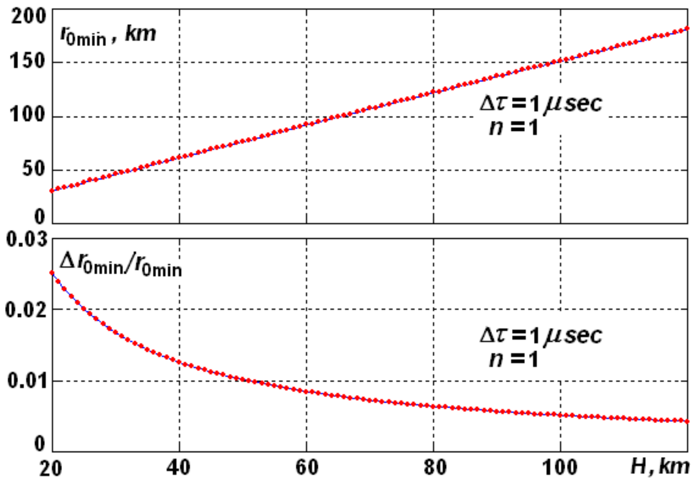

Figure 4. Solutions of Eq. (4)

Positions of the minima of Eq. (4) are displayed on Figure 4 in relation to the reflection height H_1. It follows from it that the approximation of the function $r_{0min}(H)$ with a straight line is rather good. In that time, errors of distance estimation at minima points are very small.

It is seen from Figure 2 that the errors of distance estimation become especially great if r_0 is very small (r_0<10 km) or if r_0 is near maximum distances (r_0~1800 km) still satisfied the hop model. In order to reduce these errors the usual method can be used, namely, to scale down the sampling interval in received signal digital processing. However, a necessary condition for this is the extension of the receiver frequency band. Complex upgrading of the receiver set may be necessary because of this. An alternative is simulated frequency band dilation using digital processing methods which leads to an evaluation of delays with a higher resolution than the sampling interval.

It is necessary to note that one can use Eq. (3) as an additional condition to redefine the undetermined system (2). Setting the maximum tolerable limit δr_{0tol} for distance estimation error, Eq. (3) may be apply as the inequality in a form $\delta r_0 < \delta r_{0tol}$ which restricts additionally the decision space of (2) by limiting r_0 and H_0 coupling.

5. Inaccuracy in Estimation of Ionospheric Waveguide Reflection Height

By analogy with (3) the error in the estimation of waveguide reflection height may be expressed via waveguide length r_0 and reflected wave delay τ_n. Supposing $H_n = H_n(\tau_n)$ and differentiating Eq. (2) by τ_n as an implicit function which relates the reflection height with the delay, one can obtain the relative inaccuracy of the estimation for H_n

$$\delta H_{\mathrm{n}} = \frac{\Delta H_{\mathrm{n}}}{H_{\mathrm{n}}} = \frac{c\Delta\tau_{\mathrm{n}}}{2nH_{\mathrm{n}}} \frac{T_{\mathrm{n}} + R_{\mathrm{n}}}{Z_{\mathrm{n}} - \cos R_{\mathrm{n}}}. \tag{7}$$

If r_0 is increased, the error (7) enlarges monotonically as it is seen on Figure 5. The painted over area corresponds with the ordinary heights of reflecting D and E layers in ionosphere.

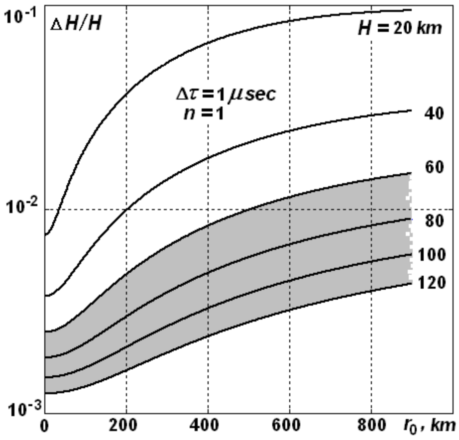

Figure 5. Relative inaccuracy of waveguide reflection height estimation

For the small $r_0 \to 0$ it may be discovered that τ_{n} tends to $2nH_{\mathrm{n}}/c$. Thus, the error (7) achieves the minimum value

$$\delta H_{\mathrm{n}\,min} = \delta H_{\mathrm{n}}\Big|_{r_0 \to 0} = \frac{c\Delta\tau_{\mathrm{n}}}{2nH_{\mathrm{n}}}. \tag{8}$$

If sampling interval $\Delta\tau$ is no more than 2 microseconds and conditions in the ionosphere are unperturbed, error values (8) have not to exceed $0.5 \cdot 10^{-2}$. This is better by up to approximately one order than the errors (3) of waveguide length estimation.

6. Array Antenna Model of EM Waves Excitation in the Ionospheric Waveguide

The explanation for the fairly high accuracy of waveguide parameters estimation in hop model frames may be proposed as follows. Let us suppose that every wave which is a part of the sum in (1) is excited by a separate source S_0, S_1, S_2, as it is on Figure 5 for two-hop model. These waves pass from the mentioned sources to receiving point *B*. The ways r_0, r_1, r_2 are different but they are the same as in the initial hop model (see Fig. 1). The sources form a discrete antenna array S_0, S_1 and S_2 represented on Figure 6.

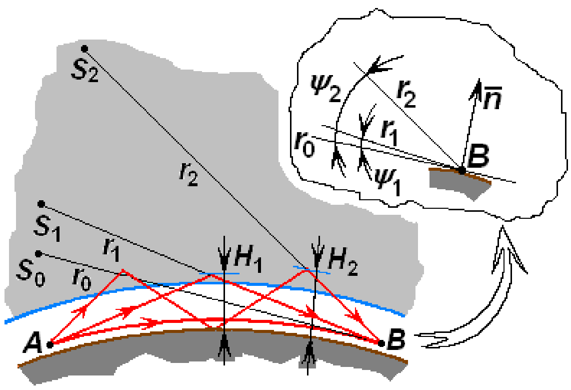

Figure 6. The equivalent antenna array included in the hop model

If the origin of the spherical coordinates system is selected at the point B then the positions of the sources have to be described by the equations:

$$R_n = (Z_n^2 - 2Z_n \cos(r_0/2na) + 1)^{1/2}, \tag{9}$$

$$\psi_n = \arc\cos \frac{\boldsymbol{Z}_n \sin(\boldsymbol{r}_0/2\boldsymbol{n}\boldsymbol{a})}{\boldsymbol{R}_n}, \qquad \boldsymbol{n} = 1, 2 \tag{10}$$

The distance r_0 is read off along the tangent line direction to the Earth's surface at the point B. The spacing d_{10} and d_{21} among radiators S_1, S_0, and S_2, S_1 are correspondingly

$$\boldsymbol{d}_{10} = (\boldsymbol{r}_0^2 + \boldsymbol{r}_1^2 - 2\boldsymbol{r}_0\boldsymbol{r}_1\cos\psi_1)^{1/2}, \tag{11}$$

$$\boldsymbol{d}_{21} = [\boldsymbol{r}_1^2 + \boldsymbol{r}_2^2 - 2\boldsymbol{r}_1\boldsymbol{r}_2\cos(\psi_2 - \psi_1)]^{1/2}. \tag{12}$$

In general, these spacings are not small in terms of wavelengths even if radiation frequencies are relatively low (ELF, VLF or LF bands). Hence, the antenna array has to possess obviously pronounced directivity. Perhaps, that is the feature which allows us to explain the high potential accuracy in waveguide parameters estimation using the hop model.

This situation is typical for lightning discharge location when an EM pulses of lightning (atmospherics) are received at a single point only.

7. Conclusions

In this research the hop model of the propagation of an electromagnetic pulse from a source allocated in the Earth-ionosphere spherical waveguide is considered. The procedures for calculating of pulse functions of ground wave and hopped sky waves are briefly exposed. The results are represented for distances up to 600 km. The pseudocepstral method is recommended for evaluating the delays of reflected ionospheric waves. Specific dependencies are established for waveguide parameters estimation accuracy in the hop model propagation frames. If ionospheric conditions are not perturbed, this accuracy is acceptable for many operational applications. A modification of the hop model is offered by introducing of a discrete antenna array based on conservation of the lengths for all the rays as in the initial hop model.

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