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PROBLEM OF MATHEMATICAL INSTRUMENT SELECTION FOR THE PURPOSE OF INVESTIGATION OF EXHAUST GASES DYNAMICS IN URBAN AIR

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This work, by making specific references to investigation of ecological problems shows the legitimacy of two approaches: the apparatus of the theory of chance and mathematical statistics and the conceptually alternative approach connected with using the apparatus of the theory of differential equations in partial derivatives and equations of mathematical physics. The investigation comes to the conclusion that the second approach manages to penetrate deeper into the physical essence of the examined phenomena and yielding analytical results directly connected with the physical aspect of a particular process. Investigators face the prospects of receiving precise or approximate results with respect to the process in its dynamic development; they also get the possibility of forecasting the course of a particular physical phenomenon without the necessity of collecting a huge volume of statistical data. The second approach is deeper and more comprehensive – for example, if problems of motor traffic ecology are to be solved. On the other hand, the expediency of using approaches of the theory of chance and mathematical statistics is not argued. The two approaches, not conflicting with each other, are useful and mutually complementary. The task of the author was to show the legitimacy and the advantages of using a powerful apparatus of mathematical physics in cases where only the apparatus of probabilistic approaches is currently used as a rule.

Keywords: anthropogenic pollutions, exhaust gases, mathematical statistics, dynamics, partial differential equations, mathematical physics equations

1. Introduction

As a result of people's economic and engineering activities, large volumes of chemicals being in gas and aerosol state are injected into the atmosphere. On getting to the atmosphere, these substances are subject to a number of physical and chemical changes. All of those mechanisms are interconnected, each of them being a part of the overall complex problem of atmospheric disperse systems related to environmental protection.

The bulk of the modern tasks of the dynamics and kinetics of atmospheric disperse systems are described by multivariate non-linear equations in partial derivatives, and the solution of tasks of that class may be found only approximately – through approximation of the initial differential problem by finite-dimensional model. At that, the proof of the convergence of the approximate solution to the one of the initial differential model is quite difficult and even impossible in many cases. Therefore, it is extremely important to investigate the issue of a simplification of the initial complicated mathematical model – so that the received task would imply, first of all, an analytical and numerical solution and, secondly, would preserve the basic properties of the initial full mathematical model (the energy and mass conservation law and other fundamental laws).

City atmosphere is a complicated dynamic system implying various physical and chemical processes running; their intensity depends on specific features of the city in question. To describe these complicated atmospheric eddy processes, a complex mathematical model can be developed, allowing one to solve a broad spectrum of problems in the environmental protection field. Such a model contains the following:

1) the model of hydro-thermodynamics of urban and/or regional atmospheric eddy processes;
2) the models taking into account kinetic processes of nucleation, condensation, and coagulation, as well as mass exchange processes at the gas-particle phase;
3) the gas-borne particles (and aerosols) transfer and diffusion equations in city and/or region atmosphere, taking into consideration the photochemical and other transformations.

For example, the model of photochemical transformation should take into account man-made emissions with respect to substances typical to large industrial cities and regions; furthermore, the space-time dynamics of environmental attributes, brought about by the secondary photochemical pollution, should be investigated. As regards kinetic models of condensation, fluctuating nucleation processes and progressive growth of particles in saturated environment should be investigated, resulting in the birth and the subsequent development of disperse phase, and contamination of the environment.

In general, the methodology of investigating the urban ecologic problems looks as follows:

- formalization of concepts "environment", "environmental quality" and similar concepts related to the specific application domain;
- formulating environmental health criteria;
- revelation, refinement, and investigation of the main factors affecting health of urban population from ecological standpoint;
- development of methods of monitoring and full assessment of environmental quality;
- development of a substantiated scientific concept of urban environment quality control and a possibility of short-term or long-term forecast.

A peculiar feature of ecology-related problems is, first of all, an extreme uncertainty of the problem statement; another feature is the complex nature of tasks implying the necessity of taking into account the above-mentioned miscellaneous factors, ranging from geology to the alert level. According to engineering practice, the computation error may come up to hundreds of percent in this case. The purpose of applying formalized methods in ecologic design, mathematical modelling and forecasting is a more precise calculation of environmental damage volume, the forthcoming costs, and the evaluation of effectiveness of the respective measures applied.

This work investigates the choice of apparatuses for developing the model, as a tool of investigation of the urban atmosphere contamination from transportation vehicles. Some investigation tools applying methods of theory of chance and mathematical statistics are examined along with some models of the classic apparatus of differential equations in fraction derivatives and the powerful apparatus of mathematical physics.

2. Ecological Monitoring of Urban Environment Quality and Its Hypersensitivity to Measurement Error

Using the apparatus of mathematical statistics (and some other mathematical apparatuses implying model development based on results of one-time and integral measurements of environmental quality – in particular, that of urban outdoor air) to determine the dynamics of exhaust gases concentration in urban outdoor air – is related to the problem of accuracy of measurement. Namely, single-shot accuracy is quite low with respect to measurements of emission concentration in urban outside air. Even if we increase accuracy by increasing test frequency to any extent as high as possible – we will nevertheless be able to assess the condition of urban outside air after single shots – especially under dynamic urban conditions. Therefore, neither should we be satisfied by accuracy of integral criteria, indices, and indicators to assess the quality of urban outside air. Besides, it is quite impossible to install instrumentation at each point of the city. That's why we have to restore the general picture of outside air contamination according to measurement results from separate points – with the help of the so-called interpolation models. So far, such interpolation models have not been developed to the extent allowing their wide usage in practice. Therefore, to apply the mathematical statistics apparatus for outside air monitoring, the outside air quality automatic control points should be developed, supplementing them subsequently with mobile inspection tools. It is quite important to note that interpolation methods of modelling are quite acceptable in compact homogenous mediums – like, for instance, in fixed water column. As regards urban conditions that may imply various heterogeneities of the terrain between two points of atmospheric measurements – a "blind" mechanical application of interpolation methods may yield absurd results. For this reason, interpolation methods taking into account heterogeneity of terrain and the existence of intermediate emission sources – are absent. It is exactly the above-mentioned aspect, along with the impossibility of a quick measurement of hundreds of foreign matters in outside air, – plus the impossibility to interpret the results of measurement – urge ecology experts to develop some more general methods of environmental quality assessment. Therefore, to be able to use a statistic model steadily, one has to test them by statistically valid methods. However, there is no time to do it as a rule, since an incredibly large number of measurements would have to be processed a few times during the day – with uncontrolled errors (the so-called "white noise"). Moreover, such statistical tests would have been too costly from economic standpoint.

Since single-shots of foreign particle concentration measurement in urban outside air feature low accuracy – we will, first of all, make the very concept "air quality measurement" somewhat more precise in terms of the language of theory of sets – before we substantiate this fact. Measurement is single-valued transformation of a set of objects into a set of results, – retaining the relations existing on the set of objects. Measurement is a process of obtaining the value of measured quantity and comparing it to another quantity – conventional as a unit of measurement. Measurement as such is an integral part of control of any process or object. Since control is a directed impact of a controlling object upon a controlled one – we cannot assess the results of control without the knowledge of numerical characteristics of an object or process received by way of measurements. Normally, the quality of outside air, aquatic resources and soil, as well as environmental release from enterprises, the level of noise, irradiation and other physical factors – are chosen as measuring objects in urban environment. If we represent urban outside air as a homogenous innocuous environment containing foreign substances harmful to people – we will consider the unit of measurement as the volume of the foreign particle content with respect to a unit volume or unit weight, which is normally called emission concentration. In practice, this is expressed by mg/m^3 in the air, mg/l in water m and mg/kg in soil. If airborne emission or discharges to water are measured, the units of measurement will be g/c or t/year. According to [1, 2], one-time concentration is determined as the 20-minute averaging of instantaneous values of concentration with respect to the given kind of extraneous substance in outside air. At that, mean daily concentration is determined as the average concentration with respect to one-time values revealed during the day. Mean monthly concentration is determined as the average value of mean daily concentrations. Mean year concentration is the average of mean monthly concentrations. Background concentration in the given region is concentration of substances in air or water, determined by the global and the regional sum of natural and anthropogenic processes. All the units of measurement described above are called weight units. As a unit of measurement, just the number of contaminants is frequently used – i.e., the number of particles (moles) of the contaminant with respect to a million or a billion of air particles (ppm-part per million or, ppb-ppp-part per billion). For instance, under the air temperature $0^\circ C$ and $25^\circ C$ the correlations between units of measurements of concentration – mg/m3 and ppb with respect to some widely spread harmful contaminants polluting atmosphere are shown below in Table 1:

Table 1. Correlations between units of measurement of concentration of some harmful substances under temperature *0°C* and *25 C*.

Substance	CO	NO	NO_2	SO_2	O_3
ppb	1	1	1	1	1
Mg/ m^3 $0^\circ C$	1.25	1.34	2.05	2.86	2.14
Mg/ m^3 $25^\circ C$	1.15	1.23	1.88	2.62	1.96

In general, the diversity of units of measurement of urban air quality is far less than that used with respect to aquatic medium and urban air. The main reason is a less developed state of exploration of the problem.

3. Accuracy of Measurement of Urban Outside Air Quality

Now we will consider some issues of urban outside air quality and increased sensitivity of statistic models used for determination of exhaust gases by the method of ecological monitoring. To do this, we will evaluate the accuracy of measurement by the example of a single substance – for example, nitrogen monoxide – at one point of urban outside air only. Let's consider a very simple method of measurement as follows: 1 m^3 of outside air is sampled into any enclosed space with a variable orifice – like a cylinder with a piston and variable orifice. Then, by squeezing out air from the cylinder – molecule after molecule – the pollution agent molecules, present in the air, are counted (in our example, the agent is nitrogen monoxide). The accuracy of this, purely theoretical method, is absolute – there is no error whatever. However, when this theoretical method is applied in actual practice – the observer, as a rule, is moving from the object and a relative reduction of object, as well as an increase of its motion speed and reduction of its life span take place – along with some other similar changes. Under these circumstances, the confidence in display of all calculated molecules of the pollution agent (i.e., the precise identification of the object type) - decreases. This means that a few molecules of nitrogen monoxide may be omitted by taking them for other molecules. For example, 5 out of 100 may be omitted, or, 105 may be calculated instead of 100. Therefore, when an in-situ measurement takes place, we can speak not about the absolute precision of the quantity measured, but only of the absolute error $\Delta R = R_{exact} - R_{real}$ of measurement of the value investigated, or its ratio error $\Delta R(\%) = \frac{R_{exact} - R_{real}}{R_{real}} \cdot 100(\%)$ where R_{exact} is the precise (ideal) meaning of the quantity measured, whereas R_{real} is the measurement result [3, 4]. Obviously, to increase the measurement accuracy, a few samples of 1 m^3 may be taken right away, measuring each of them subsequently. As a result, we will have a set of numbers with certain regularity embodying a single number, which is a precise (ideal) value of the quantity measured. As we know from the course of the theory of chance and mathematical statistics, this ideal and true value of the quantity measured (the concentration of nitrogen monoxide in 1 m^3 in this case) is for some objective physical reasons – a random variable that may be calculated based on results of separate measurements, with some likelihood ratio. In the specific case of ours, the error of one-time measurement of monoxide nitrogen concentration depends on a number of reasons of methodical, instrumental, and subjective nature: systematic errors occurs as a result of mistakes in measurement technique; instrumental errors are a consequence of inaccuracy of gauges and instrumentation, while individual errors are those made by experimentalist observer. According to engineering experience to date, errors of single shot-based measurements in gas dynamics and hydrodynamics come up to dozens of per cent. As it is known from the course of the theory of chance, in order to calculate the value of a random variable, one has to know its distribution function – i.e., the one showing the probability of the random variable assumes particular numeric values. This, in its turn, is defined by physical properties of the processes of foreign matters distribution in the environment – air, water, soil. As regards the processes of distribution of substances in the environment, they are described by fundamental laws of physics and chemistry, expressed in mathematical terms only – i.e. by correlations, equations, and models of mathematical physics. Therefore, in order to develop some more adequate, efficient, and universal models for determining concentration of various foreign substances in the environment (air, water, soil) by applying methods of theory of chance and mathematical statistics – one has to take into account the laws of environmental distribution of substances. At that, the turbulence of atmosphere by physical properties of the very environment etc. should also be taken into account; in other words, to develop adequate models of solution of the problem examined, the powerful apparatus of mathematical physics should also be used along with methods based on statistical distribution.

According to observational experience generalized in literature, random values of emission concentration in atmosphere and water comply with the Gaussian law. As we know, the values of random variables X, characterized by the Gaussian law $\rho(x) = \frac{1}{\sigma \cdot \sqrt{2 \cdot \pi}} \cdot e^{-\frac{(x-m)^2}{2 \cdot \sigma^2}}$ are determined by mathematical expectation $m = M\{X\}$ and the variance (dispersion) $\sigma^2 = D\{X\}$ [3,4]. Moreover, arithmetic mean set derived from n of measured values may also be used as the measured quantity value – provided the number of measurements is sufficiently large. At that, the accuracy of the mean derived from n measurements of the value complying with the Gaussian law is by $\sqrt{n}$ times higher than the accuracy of single measurement. As a rule [2], the relative net inaccuracy of measurement of mass concentration in outside air should not exceed $\pm 25\%$ Therefore a single measurement of the extraneous substance concentration in outside air features quite a low accuracy. Even if we increase accuracy by taking a few samples, we will nevertheless be unable to judge on the condition of outside air after single measurements – especially in urban conditions. Experiments show that cities populated by 1-1, 5 million inhabitants feature daily fluctuations of extraneous substances concentration in central parts of the city (2-3 times). Taking this fact into consideration, let's get back to the considered example of determining the nitrogen monoxide concentration at one point of town: we will try to determine the daily mean concentration of nitrogen monoxide at one point of town. Let's assume that the measurement takes place 3 times a day. If extraneous substance concentration values change by 2 times daily, the maximum error of a single measurement will be equal to 100% at the absolute instrumental accuracy. Then, the pooled error of the three measurements will be $\frac{100}{\sqrt{3}}\% \approx 57.8\%$. Consequently, in order to measure the daily mean concentration of extraneous substances in outside air downtown with the required accuracy (for instance, it makes $\pm 25\%$ according to National Standard – see [5], – at least 16 measurements should be performed, each time with absolute precision. If the one-time measurement error is $\pm 20\%$ which is considered quite acceptable with respect to modern instrumentation – the necessary number of measurements to determine the daily mean concentration of extraneous substances in the downtown outside air with the required accuracy $\pm 25\%$ – will be at least 24. In other words, measurements should be performed hourly. It should be noted however that, if we manage to know the long-standing average daily curve of extraneous substance concentration values at each point of the city – the number of measurements in these specific city points can be slightly decreased. The process of measuring daily mean concentration of the main extraneous substances (CO, NO, NO_2, SO_2 и O_3) by automatic instrumentation, currently applied in most of European cities, is based on even more complicated methods to achieve the accuracy needed. The impossibility of a quick and urgent measurement of extraneous substances in natural environment – in particular, in urban outside air, – and the impossibility of the adequate interpretation of measurement results made experts look for some more general ways of environment quality assessment[6]. By this time, these works are at the stage of investigation, and so far, experts have used to manage just by concentration meanings of up to ten extraneous substances in air, water, and soil.

If just one concentration meaning could have been chosen as an environmental criterion (for instance, only nitrogen monoxide concentration as air quality criterion, bromine concentration – as water quality criterion, and lead volume as soil quality criterion – it would have been simple enough to solve the problem of measurement and regulatory actions for environmental quality. However, the environment contains a lot of substances extremely harmful to human's health if their concentration is high. Even if we assume that we have managed to measure concentration of each of them in the environment – it would be practically impossible anyway to make the general assessment of the environment condition. That's why scientists and ecology experts are facing the tasks of developing some general integral estimations of environmental quality. The above-stated information on the accuracy of measurement of environmental quality allows us to pass over to a brief discussion of integral criteria, integral indices, and integral indicators to assess the environmental quality. When the influence of various airborne harmful substances on human health is investigated, the notion of Maximum Permissible Concentration (MPC) is normally introduced and included into the respective sanitary standards. MPC is the maximum concentration of an extraneous substance in natural environment, attributed to a definite average time. This concentration, influencing a human either periodically or throughout the human's life, does not exercise any harmful influence upon the human and the environment in general (neither does it cause any long-term effects). The MPC notion includes the maximum one-time MPC and the maximum average daily MPC. The maximum one-time MPC is set to preclude any reflect responses from humans, while the average daily MPC is determined as the pollutant concentration in the air not rendering any harmful effect upon humans inhaling the air round-the-clock. Finally, we remind that the maximum permissible concentrations are determined as a result of medical investigations and are approved by the respective public health authorities [7]. Currently the total number of rated concentrations in water, soil, and air has been set with respect to 8 000 substances worldwide.

Any integral estimation of environmental quality is essentially a convolution of a set of measurement results, performed at the intuitive and the formal level. In terms of mathematical statistics, numeric or graphic convolution is construed as any convolution of information extracted from a data file to reveal at least one of its inherent features. Academician N.N.Moiseev in his works dedicated to simulation of natural economic systems (the 70-ies of the XX century) had developed mathematical models of determination of environmental status indicator. These models have won international acclaim. Currently they are used actively both in Russia and the majority of European countries. These models form a basis for many technical measuring systems currently used to determine the environmental quality – in particular, that of urban outside air. Before these works by N.N. Moiseev were published, the following integral estimation of environmental quality had been used with respect to the environment containing a few contaminants with synergistic effect:

$$I_n = \sum_{i=1}^{n} \frac{C_i}{\mathrm{MAC}_i}, \tag{1}$$

where C_i is the concentration of i-th contaminant, while MAC_i is maximum allowable concentration of i-th substance.

Practical application of this estimate is quite difficult since, if the estimate is to meet the accuracy requirements similar to those posed to its components, - the frequency and the accuracy of one-time measurements will go beyond technically feasible parameters at $n > 3$ already. To get convinced in the fact, we will consider the example as follows: let's assume we deal with three kinds of gaseous impurities in outside air, that is to say, $n = 3$. Then,

$$I_n = \frac{C_1}{\mathrm{MAC}_1} + \frac{C_2}{\mathrm{MAC}_2} + \frac{C_3}{\mathrm{MAC}_3}. \tag{2}$$

Since all the three random variables in (2) are independent, the common error is formed by three errors of individual values. If the common error of computation should not exceed 20%, the computation error when calculating the meaning of one value should not exceed 65%. This means that calculations are to be performed every 4,2 min. which is impossible under the current technical conditions of many developing countries. Here we will note that measurements are fixed each 3 minutes in Berlin, allowing one to achieve the measuring accuracy of 54% and use the simplest methods of integration of separate indices. At the current level of instrumentation and measuring techniques even simple and easily interpreted indices yield the necessary accuracy only for average annual values. Accuracy issues are a stumbling block actually for all indicators of environmental quality; however, this does not hamper scientists and ecology experts to continue their development. Now, we will formulate the conditions at which convolution may serve as indication:

- indication should guarantee sensitivity of meanings of received values to be able to trace the space-time variability of the variability related to the process of determining factors. Otherwise it should allow to prove that no changes have taken place;
- indication should be developed so that any observation might be admitted to take part in it;
- indication should be expressed so that at least some of the people interested in the subject might be able to understand its interpretation;
- indication should provide for the possibility of receiving the estimate for the entire city, according to estimates of its part or objects;
- indication should represent values and numbers, mathematical operations, with which they are stated at least in one of natural spaces.

The bulk of well-known indicators have been developed for natural ecological systems. Indicators are subdivided into marker indicators and analytical indices, which, in their turn, are subdivided into conventional functionals and desirability functions [6]. It is important to note that a city essentially differs from a natural ecosystem, the main differences being as follows:

- contamination of natural territory, outside air, surface waters is a lot more intensive and inhomogeneous with respect to extent area than the one typical to natural ecosystems;
- the time history of contamination level of outside air within the daytime (the so-called diurnal course) is by hundred times more intensive than the one in nature;
- the number of harmful impurities makes dozens – unlike units in nature;
- the dynamics and the location of emission points are changing faster than in natural ecosystems.

The environmental quality control in cities, unlike natural complexes, imply taking control decisions within days, and control actions – within the month's time. In Berlin, for instance, at elevated concentration of ozone in outside air person drivers are recommended to limit the traffic in town. Scientific approach to quality control implies choosing the time, directions, and the term of control actions based on regular measurements and forecasts of ecological situation development. The above-stated quality indices are not applicable for day-to-day management of urban environment for the following reasons:

- low accuracy or achieving a high accuracy too long;
- Inadaptability for ecologic conditions.

However, the above-stated does not mean that indicators can not be applied in a city at all. Just like in natural eco-systems, they can be used to measure processes lasting for years, and to control these processes – for instance, construction and reconstruction of highway systems, construction of purification works. For this purpose, well-known indicators should be adapted to urban conditions and, possibly, a number of special models describing load exercised on urban territories, public health, and similar factors.

4. Extrapolative Models

Extrapolative models development technique is as follows: characteristics of the main emission points are described for urban outside air; the urban landscape, the site development parameters and the meteorological situation are described; then concentrations of contaminating impurities are calculated at each point, and areas subject to contamination are calculated.

Extrapolative models imply that scattering calculation from point sources are developed on the basis of one of fundamental laws of physics – balance equations. The essence of a balance equation can be illustrated by a simple example as follows: let's assume that a source of the substance with specific concentration $C_{unit\ conc.}$ and the inflow rate $\vartheta_{\text{inflow}}$ is located inside a volume V having the surface $S = \partial V$. Let us set the normal motion speed of the substance towards the surface S by $\vartheta_{\text{movement}}$, the volume element – by dV, and the surface element – by dS. Then the scattering process is described by the equation as follows [8].

$$\int_V \frac{\partial\left\{\rho \cdot C_{unit\ conc.}\right\}}{\partial t} dV = \oint_S \rho \cdot \vartheta_{\text{movement}} dS + \int_V \vartheta_{\text{inflow}} \cdot C_{unit\ conc.} dV, \tag{3}$$

where ρ denotes specific gravity of the substance investigated.

In (3), the volume integral in the left hand side of the equation denotes the changing speed of the quantity of investigated substance in the given volume; the surface integral in the right hand side of the equation is the complete flow of the substance through the surface S, while the volume integral in the right hand side (3) describes the influx of the substance in the volume V.

Some transformations can be made with the balance equation (3) – to take the actual urban conditions into account. For example, we may set up various speeds of transfer along different directions, simulating the terrain; we can enter various kinds of roughness factors of geological substrate etc. It should be noted that direct computation of integrals is very cumbersome and, therefore, various numerical models are applied in practice. At that, accuracy of all types of models is defined by the scale and the precision of description of reference and boundary conditions. A higher accuracy may be achieved through macro scale–based models. In this case, a smooth and even surface is examined, as it were, and the sole emission source is located at high altitude – so high that surface roughness can be neglected. The model will be simple and not completely inadequate, but its computational accuracy will be high. If we consider a town where the emission source is located at a low altitude – we have to take into account the turbulence occurring in street canyons, around buildings, and in natural depth shapes. Then the model will be getting more complicated and its adequacy will increase with respect to actual process of transfer of contaminants, – but its computational accuracy will drop.

At present, the most well-known probabilistic statistical methods of calculating the expansion of extraneous substances in urban outside air are models based on normal distribution of random variables [5]. Many experiments show that these models can be used only to assess the maximum possible overland concentration of impurities at the worst dispersion conditions. For terrain records, vertical temperature profile, and other weather conditions some more complicated models are worked out – like hydro-thermodynamic model. However, as far as the latter is concerned, its adequacy with respect to actual contamination areas remains doubtful under the conditions of urban high-density zone and complicated micro-meteorology.

5. Interpolation Models

Interpolation models development technique is as follows: contaminating impurities concentration levels in the air are measured at some city points by stationary automatic recording stations. Afterwards, the impurity concentration levels are calculated at some points located between each two adjacent stations, taking into account the urban landscape and weather conditions. As it was stated above, interpolation models use actual values of impurities concentration at given points as baseline data. Impurities concentration values between given points are calculated according to interpolation formulas (that's why the models are called "interpolation models" [9]); as a rule, linear interpolation is used of the kind as follows:

$$F\left(n+x\right) = \frac{L-x}{L} \cdot F\left(n\right) + \frac{x}{L} \cdot F\left(n+L\right),\ x \in \left[0, L\right],\ n = \overline{1, N}, \tag{4}$$

where $F\left(i\right)$ is the value of interpolating function into the function value at i -th point, L is the distance between datum points, while N is the general number of points at which impurity concentration values on air were measured by stationary automatic stations.

It is obvious that, at linear interpolation of the kind (4), the mean square deviation will be:

$$ERROR = \frac{x}{L} \cdot \Delta\left(n, L-x\right) + \frac{L-x}{L} \cdot \Delta\left(n, x\right) - \frac{x \cdot \left(L-x\right)}{L^2} \cdot \Delta\left(n, L\right),$$

where

$$\Delta(n,\bullet) \overset{def}{\equiv} \left\{F(n) - F(n+\bullet)\right\}^2 .$$

As it was mentioned in the introduction, interpolation methods of modelling are more or less acceptable with respect to homogenous environment – like for example, still water column. In a city where some variegated irregularity of terrain may be located between two measuring points, a "blind" mechanical use of interpolation methods may yield absurd results. For that reason, interpolation methods taking into account terrain irregularity and the existence of intermediate emission sources – are missing.

Summing up we can say that, to be able to use one or another probabilistic statistic model with assurance, they should be subject to a compulsory statistically valid testing. As a rule, there is lack of time to do it since it is necessary to process an incredibly high volume of current measurements – with uncontrolled errors ("white noise"). Moreover, such statistical investigations would have been too costly from the standpoint of economy. Therefore, the reliability of such models of calculating contaminated areas of urban outside air and contaminants transfer in cities is so far questionable.

6. The Classic Apparatus of Differential Equations in Partial Derivatives and Mathematical Physics Apparatus

The choice of a classical apparatus of differential equations in partial derivatives and a powerful apparatus of mathematical physics – as means of investigating the problem of urban atmosphere contamination by vehicles, is connected with the following factors [4]:

- the apparatus of mathematical physics allows one to use fundamental laws of physics – such as the laws of turbulent molecular scattering, turbulent atmosphere dynamics, the law of transfer in heterogeneous mediums, the laws of hydro- and gas dynamics, – and for developing the non-standard 3D-mathematical model to determine the dynamics of exhaust emission level in urban outside air – provided the air flow rate is not unknown *apriori.* Since all the above-mentioned fundamental laws of physics are described by linear and non-linear equations in partial derivatives, - the developed mathematical model is obviously described in terms of differential equations;
- the rich apparatus of differential equations in partial derivatives allows one to do the following, putting aside the specific object domain: analyse obtained differential equations (non-linear ones in general), investigate their correctness, perform a qualitative analysis, develop solutions through numerical or analytical methods, investigate issues of unicity and solution stability with respect to small variations of input data.

A direct connection of differential equations theory with the nature and physics of the processes should be emphasized. Describing mathematics as a method of penetration into the secrets of nature, investigator, first of all, creates its mathematical idealization. In other words, investigator neglecting the minor characteristics of a phenomenon expresses the main laws governing the phenomenon, in calculus notation. Quite frequently, these laws can be expressed as differential equations. This applies to models of various phenomena of continuum mechanics, chemical reactions, etc.

7. Example of Application of the Second Approach

As an example, the model used by the author in the works [10, 11] is presented. In [10, 11], the desired concentration of each harmful substance at any point of a confined 3D-area, and at any point of time interval within which the process of the exhaust gases concentration dynamics is investigated. In the example, the received theoretical mathematical results are used with respect to a specific district of Riga, where the road traffic is one of the most intensive. The task of determining the concentration dynamics of each of the seven harmful substances – CO, CO_2, NO, NO_2 and some others at various altitudes from the ground surface, i.e. in layers parallel to the ground surface, is investigated. The boundaries of each layer have been determined empirically, while the upper boundary of the layer the most distant from the ground surface was taken as equal to 75 m. taking into account the specific character of buildings in the city of Riga: $75\,m = M_{floor} \times 3\,m$ where M_{floor} is the number of floors of highest building in Riga. Kr. Valdemara Street has been taken as an example.

With respect to this task, let $C^{\{n\}}\left(x_1,x_2,x_3,t\right)$ denote the concentrations of n-th substance at the point $\left(x_1,x_2,x_3\right)\in\left[0,l_1\right]\times\left[0,l_2\right]\times\left[0,l_3\right]$ at any moment of time $t\in\left[0,T\right]$, where $n=\overline{1,\tilde{N}}$, $l_i\in R_+^1$ $\left(i=\overline{1,3}\right)$, T – time during which the concentration changing process is investigated, while $\tilde{N}$ is the number of substances the concentration of which should be determined. For each fixed $n=\overline{1,\tilde{N}}$ in the closed region $\left[0,l_1\right]\times\left[0,l_2\right]\times\left[0,l_3\right]\times\left[0,T\right]$, the function $C^{\{n\}}\left(x_1,x_2,x_3,t\right)$, consistent with the equation

$$\begin{gathered}\frac{\partial C^{\{n\}}\left(x,t\right)}{\partial t}=div\left(D^2\left(x\right)\cdot\overrightarrow{grad}C^{\{n\}}\left(x,t\right)\right)-\vec{\vartheta}\left(x,t\right)\cdot\overrightarrow{grad}C^{\{n\}}\left(x,t\right),\\ x=\left(x_1,x_2,x_3\right):\ 0<x_i<l_i,\ t>0,\ n=\overline{1,\tilde{N}},\end{gathered}\tag{5}$$

should be found;

where the eddy diffusion coefficient and the molecular diffusion coefficient $D^2\left(x\right)$ is the piecewise constant function of the number of stratified mediums parallel to the plane X_1OX_2 along the vertical axis OX_3, the initial condition $C^{\{n\}}\left(x,t\right)\Big|_{t=0}=C_0^{\{n\}}\left(x\right)$, $x=\left(x_1,x_2,x_3\right):\ 0\le x_i\le l_i$, the boundary conditions at each fixed $j=\overline{0,M-1}$ [10,11].

It follows from the formula (5) that, basically, there is a possibility of finding the desired concentration of each harmful substance at each point of the confined space, at each moment of time. It should be noted that the solution according to the formula (5) is not trivial.

8. Conclusions

This work, by making specific references to investigation of ecological problems shows the legitimacy of two approaches: the apparatus of the theory of chance and mathematical statistics and the conceptually alternative approach connected with using the apparatus of the theory of differential equations in partial derivatives and equations of mathematical physics. The investigation comes to the conclusion that the second approach manages to penetrate deeper into the physical essence of the examined phenomena and yielding analytical results directly connected with the physical aspect of a particular process. Investigator faces the prospects of receiving precise or approximate results with respect to the process in its dynamic development; he also get the possibility of forecasting the course of a particular physical phenomenon without the necessity of collecting a huge volume of statistical data. The second approach is deeper and more comprehensive – for example, if problems of motor traffic ecology are to be solved. On the other hand, the expediency of using approaches of the theory of chance and mathematical statistics is not argued. The two approaches, not conflicting with each other, are useful and mutually complementary. The task of the author is to show the legitimacy and the advantages of using a powerful apparatus of mathematical physics in cases where only the apparatus of probabilistic approaches is currently used as a rule.

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