

OBJECTIVE AND SUBJECTIVE APPROACHES TO DETERMINING THE CRITERION WEIGHT IN MULTICRITERIA MODELS

R. Ginevičius, V. Podvezko

*Vilnius Gediminas Technical University
Saulėtekio al. 11, Vilnius, LT-10223. Vilnius-40, Lithuania
Tel: (+370 5) 2 74 50 00; fax: (+370 5) 2 70 01 14
E-mail: romualdas.ginevicius@adm.vtu.lt*

1. INTRODUCTION

In practice, a decision-maker (DM) has to rate the available alternatives depending on their qualitative characteristics and significance for an object being evaluated. All decision-making approaches can be conventionally subdivided into quantitative and qualitative ones. A qualitative approach represented by verbal analysis [1, 2, 3] allows the DM to choose 'the best' alternative from a number of options or at least to rank them in the descending order with respect to their significance. However, in many cases, it is often necessary to determine the extent to which the considered alternatives differ, taking into account the overall influence of a sufficiently great amount of subcriteria on a particular characteristic. Their significance for each of the evaluated alternatives may vary to a great extent as well as being not clearly defined, thus making the rating of the alternative a complicated problem. The above difficulties may be eliminated by the employment of quantitative methods for determining the efficiency of particular activities.

Recently, quantitative methods have been widely used for this purpose. They consist of some relatively simple approaches based on the total ranking of all the criteria, a geometrical mean of the criteria weights of every alternative [4–6], a method of additive weighting SAW (Simple Additive Weighting), as well as on more complicated approaches (TOPSIS [7, 8], VICOR [8]), which are sensitive to variation of the initial data, and complex proportional evaluation [9–11]. Simple approaches rely on uniform maximized criteria, when the maximum value is the best, while more advanced methods use both maximized and minimized criteria for which the minimum value is considered to be the best.

All quantitative approaches are based on the matrix $\mathbf{R} = \|r_{ij}\|$ ($i=1, \dots, m; j=1, \dots, n$) of the criteria significances R_1, \dots, R_m , characterizing the compared alternatives A_1, \dots, A_n . These significances r_{ij} may be statistical data or the estimates provided by experts. The above values may be combined into an aggregate estimate if they are dimensionless or normalized. One of the normalization procedures is as follows:

$$\tilde{r}_{ij} = r_{ij} / \sum_{j=1}^n r_{ij},$$

where \tilde{r}_{ij} means the normalized values of the criteria.

The influence of the particular criteria R_i ($i=1, \dots, m$) on the final decision differs, therefore, when using the quantitative approaches to decision-making, it is necessary to determine the weights of the criteria ω_i . Usually, the quantitative methods of evaluating the alternatives are modified versions of simple additive weighting (SAW), when a resulting value S_j of the j -th alternative ($j = 1, \dots, n$) is calculated by the formula:

$$S_j = \sum_{i=1}^m \omega_i \tilde{r}_{ij} \tag{1}$$

The evaluation of the criteria weights ω_i may be subjective, objective and integrated [12, 13]. In any case, the values of the weights should be normalized, i.e. the total value for all the criteria should be equal to one: $\sum_{i=1}^m \omega_i = 1$.

The present paper considers the problem of determining the weights of the criteria characterizing the efficiency of the investigated processes based on subjective, objective and integrated approaches.

2. METHODS OF DETERMINING THE CRITERIA WEIGHTS

Subjective methods of weight determination are based on expert evaluation. His/her experience and knowledge allows for providing the most valuable information about the compared objects.

There are numerous techniques for determining the criteria weights (significances), including ranking, i. e. giving the minimum value 1 to the best and most significant criterion and the highest possible value m to the least significant criterion. Pairwise comparison of the criteria that is based on prescribing the value 1 to more important criterion and 0 to less important factor with subsequent mathematical (statistical) treatment of the obtained values is also used [14]. When expert systems are preferred, various scales of measurement are employed.

The present paper uses a popular method of pairwise comparison of criteria (analytic hierarchy process) developed by T. Saaty [15, 16] and having a sound mathematical basis. This approach uses the scale „1–3–5–7–9” to transform the qualitative expert estimates into the quantitative ones. The data obtained in pairwise comparison of all the criteria R_1, \dots, R_m are written down in the square inversely symmetrical matrix $\mathbf{P} = \|p_{ij}\|$ ($i, j=1, \dots, m$), $p_{ij} = 1/p_{ji}$, with the values of p_{ij} changing from 1 to 9, depending on whether the significance of the criteria R_i and R_j is the same or if R_i considerably exceeds R_j . Apparently, it is much easier to compare any two criteria rather than determine a relative significance of all m criteria, especially when (in a general case) m may obtain a large value.

The weights of the criteria ω_i are the normalized values of the proper vector of the pair wise comparison matrix \mathbf{P} , correlating with the highest Eigen value λ_{\max} of the matrix. Saaty's approach allows the compatibility of experts' estimates to be determined. The agreement of the estimates p_{ij} is used to establish the concordance index $S_I = (\lambda_{\max} - m)/(m - 1)$ and the concordance relationship S . In the case of absolutely compatible matrix \mathbf{P} , when the elements in all columns are in the same proportional relationship, $\lambda_{\max} = m$ и $S_I = 0$. The concordance relationship S is the relation between the calculated concordance index S_I and a random concordance index A_I , obtained by generating a considerable number of pairwise comparison random matrices of the same order m . Matrix \mathbf{P} is considered to be in concordance if $S < 0,1$ [15].

The objective approaches to calculating the criteria weights evaluate the structure of matrix \mathbf{R} representing the values r_{ij} or their normalized values \tilde{r}_{ij} , while the values of the weights may change together with the values themselves. An objective approach may comprise multiple regression analysis, a modified weighted least square method [12] and entropy. In this paper, a method based on entropy is used. For the i -th criterion entropy is determined by the formula [7, 9]:

$$E_i = -\frac{1}{\ln n} \sum_{j=1}^n \tilde{r}_{ij} \ln \tilde{r}_{ij} \quad (i=1, \dots, m). \quad (2)$$

Not normalized entropy weights are found from the expression:

$$d_i = 1 - E_i, \quad (3)$$

while the respective normalized weights α_i are obtained from the formula:

$$\alpha_i = \frac{d_i}{\sum_{i=1}^m d_i} \quad (i=1, \dots, m). \tag{4}$$

Subjective and objective weights ω_i and α_i are combined into the aggregate weights c_i using the formula [7, 9]:

$$c_i = \frac{\alpha_i \omega_i}{\sum_{i=1}^m \alpha_i \omega_i} \quad (i=1, \dots, m). \tag{5}$$

There are also some other methods of calculating the criteria weights [12–14].

3. A CASE STUDY OF THE CRITERION WEIGHT CALCULATION

We have used multiple criteria analysis for determining the criteria weights in evaluating regional social and economic development in Lithuania [5, 11], as well as for estimating the strategic potential of enterprises [6], and for determining the efficiency of economic development and commercial activities of various enterprises [4].

This paper will provide the quantitative evaluation of the performance of various university departments. The departments were compared against 12 criteria, 6 of which characterize research, 4 – teaching and 2 refer to other kinds of activities. The university experts considered only 12 key criteria because of the difficulties arising in pairwise comparison of a larger number of criteria in determining their weights, and because some statistical data were available on particular criteria. Since the number of teachers varied from department to department, the average values for every department were calculated against all the criteria for a so-called „statistical teacher”.

The criteria evaluating the performance and their average values calculated for 5 departments of the same faculty of the university are given in Table 1.

Table 1. Average data on the performance of five university departments

Criterion №	Field of activities	Type of activities	Unit of measure	Department № and criterion value				
				1	2	3	4	5
1.	Research	Scientific publications	In units	4.2	1.9	2.4	2.8	3.7
2.		Participation in international programmes	Thous. Lt	9.6	2.5	6.7	8.2	9.1
3.		Participation in applied research	Thous. Lt	5.1	3.8	4.4	6.7	5.8
4.		Participation in scientific conferences	In units	2.3	1.8	0.9	1.4	3.0
5.		Membership of journal editorial boards, reviewing of papers	In units	3.0	2.2	1.8	2.9	2.5
6.		Training doctoral students	In units	2.2	1.4	0.8	1.9	2.7
7.	Studies	Delivering lectures to BSc degree students	Hours per week	3.3	8.7	9.4	2.5	1.9
8.		Delivering lectures to MSc degree students	Hours per week	5.2	3.1	4.5	4.0	5.5
9.		Advising graduates in preparing graduation papers	In units	6.8	4.7	8.4	4.3	7.1
10.		Publishing text-books and teaching materials	Number of sheets	3.5	6.8	8.7	1.1	4.4
11.	Other activities	Social and political publications	In units	1.2	0.7	0.4	1.4	0.2
12.		Membership of university bodies (Senate, councils, committees, commissions, etc.)	In units	0.3	0.1	0.3	0.2	0.5

A commission of 27 experts representing high-rank university staff members evaluated the subjective criteria weights. First, the experts ranked all 12 criteria, describing the efficiency of teachers, to determine the concordance of expert judgements. The concordance coefficient W was calculated [9, 14] and the obtained value ($W = 0.568$) showed good agreement of opinions. Pairwise comparison of 12 criteria based on Saaty’s approach revealed that the concordance relationship S was below critical (0.1) only for 13 experts (out of 27). This means that the expert estimates were in concordance. The criteria weights correlating with the highest Eigen values of the matrices and arithmetical means of the weights for all experts were calculated for all matrices, which were in concordance.

The subjective weight values of the criteria computed by T. Saaty’s approach and objective and aggregate weights calculated by the formulas (2–5) are given in Table 2.

Table 2. Weight values of criteria characterizing the performance of departments

Criterion№	1	2	3	4	5	6	7	8	9	10	11	12
Weights												
Subjective (Saaty)	0.196	0.150	0.099	0.071	0.035	0.104	0.061	0.090	0.071	0.088	0.018	0.019
Objective (entropy)	0.039	0.076	0.020	0.076	0.017	0.071	0.193	0.019	0.031	0.160	0.185	0.115
Aggregate	0.111	0.163	0.028	0.077	0.083	0.106	0.170	0.025	0.031	0.202	0.048	0.032

As shown in the table, objective entropy weights and, consequently, the aggregate weights, depending on the structure of the initial data, are biased towards the criteria of minor importance (i.e. publications on social and political problems, delivering lectures to BScs, publishing of text-books and teaching materials, participation in social activities of the university), while the experts emphasized research as a key criterion for evaluating university teachers’ work. It is clear that the actual weight values of the criteria largely affect the total estimate of particular departments and their rankings. Apparently, the objective and, consequently, the aggregate weights of the criteria may be recommended to assess the performance of particular university teachers and departments, if there is a correlation between the subjective and objective weights.

Table 3 provides the values computed for the method of simple additive weighting (SAW) and ranking of the departments based on various criteria weights. The last row of the table contains the calculations for the same weights (for comparison). It should be noted that all 12 criteria of work efficiency (see Table 1) are maximized, i.e. the highest criterion value correlates with the best performance of a teacher. Therefore, the application of more complicated methods of total ranking can hardly be effective.

Table 3. Evaluation of department performance by simple additive weighting (SAW)

Department Type of weight	1		2		3		4		5	
	Value	Rank								
Subjective	0.232	2	0.158	5	0.193	3	0.181	4	0.235	1
Objective	0.215	2	0.200	3	0.223	1	0.176	5	0.187	4
Aggregate	0.211	2	0.194	4	0.229	1	0.157	5	0.209	3
Equal	0.226	1	0.168	5	0.195	3	0.185	4	0.222	2

The inherent conflict between subjective and objective approaches to weight calculations of the criteria is shown to place the emphasis on different aspects of performance evaluation.

CONCLUSION

The results of various activities are closely associated with a great number of criteria the contribution of which varies to a large extent. Therefore, the determination of the criteria weights is of

paramount importance. The subjective weights elicited from highly skilled experts are of great value for assessing the efficiency of work from various perspectives.

The availability of objective and aggregate weights allows us to assess the actual performance of a particular organisation identifying the difference between the present situation (which is considered to be ideal) and the level of performance intended to be achieved in the future. If there is a correlation between the subjective and objective weights, the latter may be used in decision making along with the aggregate weights.

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Session 3

Transport and General Management

FINANCIAL RISK OF PROVIDING THE UNIVERSAL TELECOMMUNICATIONS SERVICE IN LATVIA

Irina Klevecka¹, Janis Lelis²

*¹Riga Technical University, Institute of Telecommunications
12 Azenes Str., Riga, LV-1048, Latvia
Ph.: (+371) 6006970, e-mail: klevecka@iseco.lv*

*²Telecommunication Association of Latvia
1 Akademijas Laukums, office 702, Riga, LV-1050, Latvia
Ph.: (+371) 9556340, e-mail: jlelis@com.latnet.lv*

1. INTRODUCTION

In accordance with the requirements of the European Union, the Latvian government is obliged to establish the Universal telecommunications service – a minimum set of services of a definite quality which will be available to all users at an affordable price, irrespectively to their geographical location.

Under the conditions of a liberalized market an operator would not offer its services in the territory of economically risky regions or to unprofitable customers, if it did not have the corresponding obligations to do so, because provision of these services would produce incommensurably large costs. In this case, the task of the Latvian government is to find ways how to compensate the additional expenses of the Universal service provider.

International experience shows that there are several sources of compensation of the costs of Universal service obligations, one of them being the state budget. In Latvia the net costs¹ of the operator, which provides the Universal service under the corresponding obligations, are expected to be rather high. It means that if a Universal service provider covered all its expenses by itself, its competitive ability would decrease dramatically. On the other hand, implementation of a financial scheme in line with the Universal service obligations is hindered by the inability to evaluate the operator's costs that are caused by the provision of telecommunication services to everyone who demands it for a reasonable price.

2. MINIMUM SET OF THE UNIVERSAL SERVICE OBLIGATIONS

The main aim of the EU Universal service policy and legislation in telecommunications sector is to ensure that a defined set of services is available to all residents of a country, independently of their location and at an affordable price, especially considering the interests of disabled users and users with special needs.

The Universal Service Directive [1] contains rules and principles designed to protect the interests of consumers under the conditions of a liberalized market of telecommunications services. The main obligation of the Universal service remains the provision on request of a connection to the public telephone network at a fixed location and access to publicly available telephone services, in the whole territory of the European Union.

In accordance with the requirements of the EU, a minimum set of services, which must be ensured to every resident of a country independently of his/her geographical location, includes the following:

- ◆ Provision of connection at a fixed location to the public telephone network and access to publicly available services of local, national and international voice telephony;

¹ Net costs are defined approximately as the difference between the operational results of the Universal service provider (revenues and costs) that would be obtained with and without the Universal service obligations.