EFFECT OF THE ROAD SURFACE FACTORS ON THE URBAN TRAFFIC CONDITIONS

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Abstract

Surface quality factors affect the road network traffic conditions in many aspects: decision to make a trip, choice of destination, travel mode, route, disutility measure and safety. This article deals with one of those possible aspects - the influence of quick changes of the road surface quality on the generalized travel cost as disutility measure.

A model of the influence of surface factors to network traffic conditions is considered. Road surface quality is assumed to be weather dependent, and may change quickly for the worse as a result of snowfall, for example. In turn, road service cleaning activity may change these results to the better. The similar effect may have long lasting changes of the road surface quality as the result of surface degradation or surface recovery and renovation. We suppose that network flows on the links tend to equilibrium. Changing surface factors may cause redistribution of the flows on the links to some new equilibrium. In general, total travel time over the network also will change. This is a measure of utility/disutility. The corresponding loss function may be considered as the criteria for optimal road service cleaning (recovery) strategy.

Key words: Transportation Networks, Surface factors, Traffic conditions, Equilibrium Analysis

Introduction

The main suggestion is that the travel time on the link depends not only on congestion factors, but also on the link surface factor. It is attractive from the mathematical point of view to assume, that the surface factor increases travel time linearly. This seems to be reasonable, because the surface factor affects the average speed on the link. So, let us suppose that the travel time on the link is

$$t_i = \varphi_i(x_i, h_i) = t_i(x_i)s_i(h_i), \ i = 1,...,n$$

where

x_i = flow on link i,

$t_i(x_i)$ = travel time on link i under normal surface conditions (influence of congestion factor),

h_i = surface quality variable,

$s_i(h_i) \geq 1$ = increase of travel time under h_i in comparison with the normal surface condition.

We may regard $s_i \geq 1$ as independent variables (without specifying h_i and $s_i(\cdot)$ exactly), which are changing in some way in connection with weather conditions and recovery efforts. The corresponding total travel time may thus be written as:

$$T(s) = \sum_i T_i(s) = \sum_i t_i(x_i)s_i x_i$$

and additional total travel time is

$$\Delta T(s) = \sum_i \Delta T_i(s) = \sum_i t_i(x_i)(s_i - 1)x_i$$

Additional total travel time $\Delta T(s)$ is cost per time unit, which we pay for worse surface conditions, and $\Delta T_i(s)$ is cost per time unit for each link. The corresponding loss function is

$$L = \int_{t_0}^{t_1} \Delta T(s)dt = \sum_i \int_{t_0}^{t_1} \Delta T_i(s)dt$$

where t is usual clock time, measured from some starting point t_0 and up to point t_1, when we believe that recovery procedure is complete. The s_i factors in their turn depend on t, i.e. $s_i = s_i(t)$ as a result of weather and recovery. So, the problem is to decrease ΔT as much as and as quickly as possible, to minimize L by mean of specifying $s_i(t)$ as control functions of recovery strategy (see Fig. 1). Particular problems include the number of cleaning teams required, allocation of the links and identification of the best sequence of the links for cleaning.

Note, that variables s_i may became also <= 1 for renovation scenario for example. However the loss function for this case will have quite different nature.

The problem may have different levels of complexity.

Cleaning activity during the snowfall will create somewhat different trajectory than shown in Fig. 1. In this case s_i values will decrease for links, which are currently cleaned, and s_i values will increase for other links, which have already been cleaned, as well as for the links, which are not cleaned yet. The same is truth for the link which is currently cleaned. As the result some links may require multiple cleaning.

Changing surface factors may cause redistribution of the flows on the links to some new user equilibrium (UE). Thus, for some given surface factors $s = (s_1,...,s_n)$ we have UE flows $x = (x_1,...,x_n)$, which in general depend on s, $x = x(s)$. As was mentioned in the introduction, it is not obvious whether we should take into account this possible dependence for quick changes in s. So, we shall use the simple notation x_i, when assuming flows to be some fixed average values independent of s, and $x_i(s)$, when assuming the flows to be UE flows depending on current surface factors. The corresponding total travel time may thus be written as:

$$T(s) = \sum_i T_i(s) = \sum_i t_i(x_i(s)) s_i x_i(s)$$

and additional total travel time is

$$\Delta T(s) = \sum_i \Delta T_i(s) = \sum_i \left[t_i(x_i(s)) s_i x_i(s) - t_i(x_i^0) x_i^0 \right]$$

where x_i^0 are flows on links with normal surface factors $(s_i = 1, \forall i)$, $x_i^0 = x_i(1,\dots,1)$.

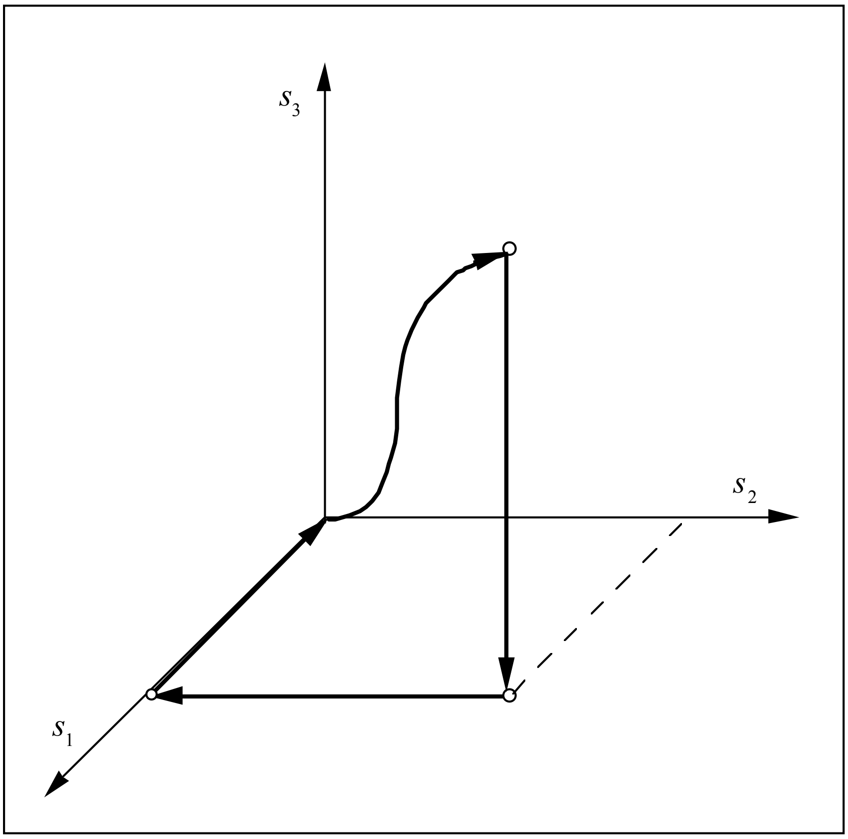

Fig. 1.*Change of surface factor s(t) during snowfall and subsequent cleaning*

If redistribution of flows on links is assumed, then extraordinary effects of the behavior of function $T(s)$ in may be observed. For example, it may occur, that $T(s)$ decreases with s_i for some links, and therefore it is better not to clean them at all (see also Braess's paradox, Sheffi (1985)). Some links may have optimal values of snow amount, which provides the best minimal value of $T(s)$.

The best sequence of the links for cleaning may be quite different in cases of fixed flows x_i, and surface dependent flows $x_i(s)$. So, cleaning of the links with high priority under fixed flows may cause very small gains after flow redistribution. And, in turn, the best sequence for the surface dependent flows may not be attractive in case of fixed flows.

Basic Optimization Problem Formulation

The following idea was suggested by Lars-Göran Mattsson.

Suppose, that

τ_j = the time it takes to clean link j and

c_j = the cost per time unit of not cleaning link j.

Define the sequence of links by means of variables

$$y_{ij} = \begin{cases} 1, \text{ if link j is the ith link to be cleaned} \\ 0, \text{ otherwise} \end{cases}$$

Then, the optimization problem is

$$\min L = \sum_i \sum_l y_{il} c_l (\sum_{k \le i} \sum_j y_{kj} \tau_j) \tag{1}$$

subject to

$$\sum_j y_{ij} = 1, \ \forall i; \ \sum_i y_{ij} = 1, \ \forall j \tag{2}$$

The solution to the problem is to rank the sequence of links in order of decreasing of c_j / τ_j.

Let us proof it formally.

Let $(c_1 / \tau_1), \ldots, (c_n / \tau_n)$ be the optimal sequence of links, which provides the minimal value of the function L which is now

$$L = \sum_{i=1}^{n} \left(\int_0^{\sum_{j=1}^{i} \tau_j} c_i dt \right)$$

Then $\dfrac{c_1}{\tau_1} \ge, \ldots, \ge \dfrac{c_n}{\tau_n}$.

Suppose otherwise, that i, j are two consecutive links $(j = i + 1)$ and $\dfrac{c_i}{\tau_i} < \dfrac{c_j}{\tau_j}$.

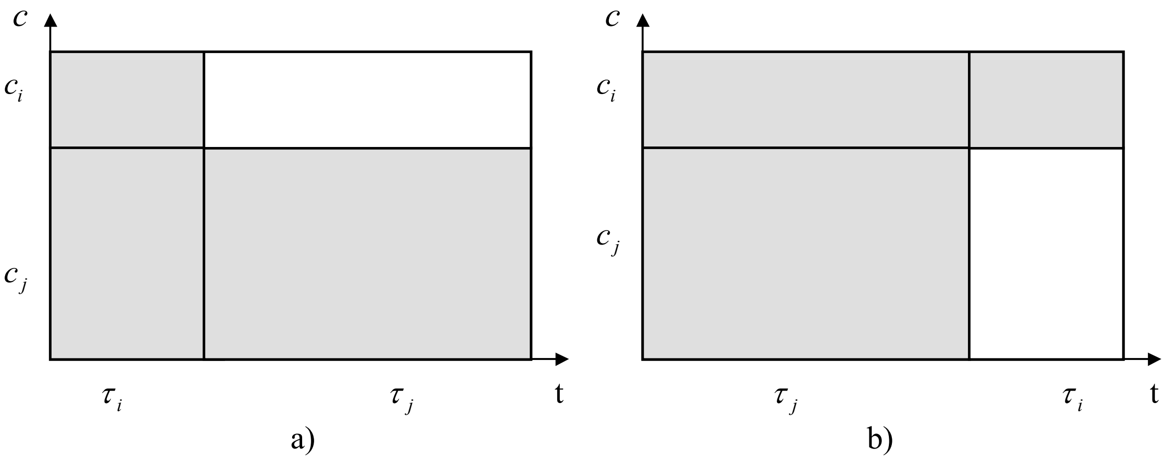

Fig. 2. *Loss function: a) cleaning of link j after link i b) cleaning of link i after link j*

Denote $Q = (c_i + c_j)(\tau_i + \tau_j)$ as the loss function contribution for not cleaning links i, j during time $\tau_i + \tau_j$. Then the decrease Q due to cleaning of links j immediately after link i is $\Delta Q_{ij} = c_i \tau_j$. In turn, cleaning of the link i immediately after link j provides the corresponding decrease $\Delta Q_{ji} = c_j \tau_i$. In accordance with our suggestion $\Delta Q_{ij} < \Delta Q_{ji}$. This means, that the sequence is not optimal, because the loss function may be decreased by changing the order of links i, j. So, the statement is proved.

For multiple cleaning machines M denote by τ_j^m the time it takes to clean link j by machine m and d_{jl}^m the time to move machine m from link j to l. Also, let

$$y_{ij}^m = \begin{cases} 1, \text{ if link j is the ith link to be cleaned by m, } m=1,\dots,M \\ 0, \text{ otherwise} \end{cases}$$

Then, the optimization problem is

$$\min L = \sum_m \sum_i \sum_l y_{il}^m c_l (\sum_{k \le i} \sum_j y_{kj}^m (\tau_j^m + \sum_r y_{k-1,r}^m d_{rj}^m)) \quad (3)$$

subject to

$$\sum_j y_{ij}^m \le 1, \forall i, m; \quad \sum_i \sum_m y_{ij}^m = 1, \forall j; \quad y_{0j}^m = 0, \forall m, j \quad (4)$$

We assume that a machine cannot be cleaning more than one link at a time, each link should be cleaned by some machine at some point of time and it takes no time to move a machine to the first link.

The first approach is the special case of the second one for $M = 1$, $d_{ij} = 0, \forall i,j$. It provides a very simple and beautiful solution, which may be applied if the links ordered by decreasing c_j / τ_j are adjacent, i.e. the end of each link in the sequence is the beginning of the next link. In this case such a sequence is the optimal solution to the problem. Assuming $c_j = \Delta T_j = t_j(x_j)\ (s-1)x_j$ and $t_j = \gamma t_j(x_j) s_j$ where γ is a fixed multiplier, we can obtain the criteria for a cleaning sequence in decreasing order of $x_j(s_j - 1)/s_j$ Besides that, if all surface factors s_j are equal for all links, then only decreasing order of flows x_j is sufficient.

However, in general those links are not adjacent, but are located in jumping order. So, we should take into account the time of moving the cleaning machine from link to link. It should not be successful to include moving times into cleaning times, because all moving times will change after each step, and the corresponding best order of the links also will change after each step. It may be shown, that recalculation of the new optimal order after each step may provide a non-optimal solution. In other words, optimization of each separate step doesn't certainly led to the global optimization.

Nevertheless the optimal solution to the problem (1) is quite important because it defines theoretical priorities among links, and allows to calculate minimal and maximal possible loss function values, which may serve as a yardstick for measuring the performance of other solutions.

The second approach represents a discrete optimization problem with linear constraints. It is necessary to notice here, that the moving time d_{ij} from link to link is not fixed. It may change many times in the process of cleaning, as will the corresponding shortest paths between links. The proper value of $d_{ij} = d_{ij}(s)$ should be calculated with consideration of the time movement when it is used. Thus optimization problem solution will require shortest path recalculations in each step.

In a more general case we should take into account also that continuation of the snowfall may take place during the cleaning activity. Therefore some links may require multiple cleaning. In this case it is not evident how long the cleaning process may be, and when it will be finished.

Restrictions for Basic Optimization Problem

It is not easy to foresee if jumping will take place for an optimal solution or not. Probably it depends on homogeneity of the transport network. If link priorities differ drastically then the jumping order is quite possible. It is desirable to avoid this for at least two reasons. The first is that road service resources should also be taken into account to minimize cleaning machine path lengths. The second is, that there are may more jumping trajectories of cleaning than contiguous ones, and we may probably need simplification of the problem.

Let us introduce an additional restriction, that the cleaning route is a cycle in case of one cleaning machine and a contiguous path in case of multiple cleaning machines. This means, that only adjacent links should be cleaned, no one link should be passed without cleaning, and all links should be cleaned at least once. It is not difficult to prove by mathematical induction, that cycles always exist for networks with two-way links. For more general results see Evans and Minieka (1992). Notice, that the cycle of links is not the same as the cycle of vertices.

The problem of cycle identification may be represented as a Traveling Salesman Problem (see Lawler, Lenstra et al (1985)) with the matrix of distances $q_{ij} = 0$, if links i,j are adjacent, and $q_{ij} = 1$ otherwise. It may also be formulated as a minimization problem for y_{ij}^m variables:

$$\min \sum_m \sum_k \sum_j y_{kj}^m \left(\sum_r y_{k-1,r}^m d_{rj}^m\right)$$

subject to (4) to find cycles if they exist, or to find the best approximation to cycles otherwise. If an optimal solution is found, and the minimum value of the objective function calculated is equal to zero, then it becomes possible to modify the initial optimization problem (3) to

$$\min \sum_m \sum_i \sum_l y_{il}^m c_l \left(\sum_{k \le i} \sum_j y_{kj}^m \tau_j^m\right)$$

with the additional restriction

$$\sum_m \sum_i \sum_l y_{kj}^m \left(\sum_r y_{k-1,r}^m d_{rj}^m\right) = 0$$

This provides a decomposition of the initial problem objective function. However, the main interest is not to find one cycle, but to find all cycles, because there may be many of them.

Alternative Optimization Problem Formulation

Let us consider the case of one cleaning machine. Denote by $y_1, y_2, \ldots, y_k$ the sequence of links to be cleaned, where $y_i \in \{1,\ldots,n\}$ is the number of the link which is to be cleaned at step i. The objective function may be represented in the form

$$L = \sum_i \Delta T(s) \Delta t_i = \sum_i \Delta T(s)(d_{y_{i-1}, y_i} + \tau_{y_i})$$

as the integral sum. Δt_i are time intervals inside which the integrand is assumed to be fixed. Of course this interval may be split in a number of smaller intervals, to make the approximation more accurate. In general $s = s(t)$ may increase or decrease at each step. The time interval Δt_i also depends on current s_i values, and finally it is possible that $x_i = x_i(s)$.

This equation should illustrate the essential properties of the algorithm for the loss function calculation. Thus the optimization problem becomes

$$\min_{y_1,\ldots,y_k} \sum_i \Delta T_i(s)(d_{y_{i-1},y_i} + \tau_{y_i})$$

without additional restrictions besides $y_i \in \{1,\ldots,n\}$. The feasible region is a k-dimensional square of discrete points. Values d_{y_0,y_1} are initial moving times connected with some starting point of location. They may also be set to zero. If there is no snowfall during the cleaning activity, we may require that $k = n$ and $y_i \neq y_j$ if $i \neq j$ (i.e. $y_1,\ldots,y_n$ is a permutation). However, this is not necessary, because the optimal solution automatically will be a permutation in this case. For multiple cleaning machines similar notation may be used with the additional possibility $y_i^m = 0$ when not cleaning.

Heuristic Approaches

Instead of direct optimization let us consider the possibility of a heuristic approach. In general, the problem is quite similar to chess playing algorithms. Numbers $y_1, y_2,\ldots$ are our moves, and the weather is our opponent, increasing s_i factors. The usual technique is sample tree analysis. The sample tree consists of different sequences of links for cleaning. That means we have number of possibilities for the first move, than, for each possible first move choice we have number of possibilities for the second move, and so on. The total number of moves is not necessary predetermined value (in case of snowfall for example). But if it is, than we can obtain in the principle the exact solution of the problem by means of full sampling through all possibilities.

Such approach however may be applied only for very small road networks, because sample tree grows up very quickly with the increasing of number of links and number of moves. As the result, it becomes impossible to sample through all possibilities and to find in so way an exact optimal solution. So, it is reasonable to cut some branches at each step by some criteria, to make the tree more narrow and longer. In this case we take into account not all possible choices for the current move to evaluate the next one, but only part of them. The same reduction is fulfilled for each step. This process must be terminated at some point when calculations becomes too durable. The subtree of sequences obtained may hopefully include the branch which is starting part of the best optimal sequence. Otherwise the best branch of the subtree may serve as approximation to the optimal trajectory.

Notice that all this process is intended to determine the first move only or few consecutive moves. After that the procedure must be repeated from the new starting point. Of course it is possible to use for further prolongation the branches which are already evaluated if there is no changes in external weather influence. However all those questions are concerned with implementation details.

The possible criteria for subset of moves selection may be very different. Let us consider some of them. Recall that n is the number of links and k is the number of moves evaluated or the length of the sample tree.

All possibilities and permutations. All possibilities for each move mean that there is no any reduction of the sample tree branches and we have to evaluate n choices of the next move for all n choices of the previous one. This include the possible jumping trajectories as well as the multiple cleaning of the links. The total number of different sequences is equal to n^k.

If there is no snowfall, than it is reasonable to avoid multiple cleaning and to take into account only links which are not cleaned yet. This mean that we have n possible choices for the first move, than $n-1$ possible choices for the second move and so on. In other words we consider only set of permutations as the feasible set of links sequences. The total number of different possibilities is equal to $n(n-1)\ldots(n-k+1)$.

The best branches. Though we have n possible choices for each current move, not all of them are equally effective for the loss function minimization. We can order them by the ratio of loss function decreasing and to take only r best possibilities (r is the fixed number) for the further analysis. In this case we will have to sample through r^k different links sequences.

Value of r should be determined heuristically as the compromise between the desire to include in consideration more trajectories on the one hand and to reduce the calculation time on the other hand. The two opposite opportunities are: $r = n$ (all possible sequences) and $r = 1$ (the single sequence). The last one has a small chance to be successful. The reason is that the best choice for each local step may produce not a good global sequence.

The slightly different approach is to fix not the value of r itself, but the quotient $0 < q < 1$, which determines a boundary for the loss function decreasing relatively to the largest possible decreasing for the current move. More exactly, let $\Delta L^{\max} = \max\{|\Delta L_1|, |\Delta L_2|, \ldots, |\Delta L_n|\}$ where ΔL_i are values of loss function decreasing for all n possible moves for the current step. Than the subset of current moves may be defined as $\{\Delta L_i : \Delta L_i > q\Delta L^{\max}\}$. In other words, we take into account for the further analysis not all possible current moves and not only predetermined fixed number of the current best moves, but all nearly best moves. As the result, the total number of different trajectories to be evaluated is unknown in advance.

Adjacent links and cycles. It was mentioned above, that the attractive rational approach is contiguous cleaning, i.e. only adjacent links are to be cleaned. If multiple cleaning of the links is not necessary, than it is quite probable to expect that the optimal cleaning path may be a cycle. Notice that cycle is a special case of permutation and it always exist for the road networks with two-way links. However it is not difficult to construct an example when it is better to include some redundant moves without cleaning into cleaning sequence than to follow the cycle requirement strictly. It depends also on the balance of moving time and cleaning time for particular links.

So, making a choice for the current move we should take into account only adjacent links which are not cleaned yet if possible. If all adjacent links are already cleaned than we move the cleaning machine to the new starting point without cleaning.

Adjacent links strategy is the special case of all possibilities strategy and may be considered as the most general approach at the same time, because moving of cleaning machine is contiguous by the nature and only difference is particular kind of cleaning of current link including moving without cleaning as one of possibilities. Loss function increases in time and each move corresponds to additional term in integral sum. The problem of heuristic approach is to choose the branch of moving trajectory in order to minimize corresponding partial sum for specified time interval.

Since the typical intersection includes four links in average, the approximate estimation of the number of different moving sequences is equal to 4^k.

Heuristic strategies summary. Thus, summarizing the mentioned above strategies, the possible criteria with estimates of corresponding computational efforts required, are the following:

all possibilities	- n^k
permutations only	- $n(n-1)\ldots(n-k+1)$
r best	- r^k
1 best	- 1
all links nearly best	- ?
adjacent links, cycles	- $\approx 4^k$

The last approach seems to be the most attractive one, because it provides a reasonable reduction of computational efforts and assumes continuous cleaning, if possible.

Numeric Example

Let us consider some numeric examples. Three road networks with different number of directed links ($n = 22,28,32$) with symmetrical origin-destination matrixes were generated. All links were supposed to be two-way links with symmetrical properties (travel times and snow factors). Single cleaning machine, no flows redistribution and no snowfall continuation assumed.

Particular strategies applied were the following:

- Best theoretical. The sequence of links to be cleaned according to theoretical priorities-decreasing order of c_i/τ_i which is equivalent in this particular case to decreasing order of flows x_i. Possible jumping order of links was ignored, i.e. moving time from link to link if they are not adjacent is equal to zero.
- Best cycle. Only contiguous trajectories of cleaning were taken into account, i.e. feasible set of cleaning sequences was restricted by cycles. The Best cycle was found by means of full sampling through all possibilities. This approach however, require significant computational efforts.
- First cycle. The first variant in previous full sampling strategy may be considered as the random cycle selected stochastically. This provides the First cycle result.
- Best heuristic. Heuristic approaches described above as adjacent links cleaning strategy, were applied with slightly different properties concerned with the lengths of subtrees evaluated and number of moves accepted for the current best branch selected. The most successful one was proposed as Best heuristic.
- Worst theoretical. This strategy is the opposite to the Best theoretical approach, i.e. links are to be cleaned by increasing order of c_i/τ_i, equivalent to increasing order of flows x_i. Jumping order was ignored as well.

Notice that the Best theoretical result can not be achieved in general. It may be only a lucky random case, that all links ordered by decreasing of ratio c_i/τ_i will form a continuous cycle. Unlike this, Worst theoretical result is a lower bound only for cycles, but not for jumping trajectories. Table 1 contains some numeric results.

Table 1. Loss function values for different cleaning strategies.

Number of links	n=22		n=28		n=32	
	Loss function value	% relatively to the best theoretical	Loss function value	% relatively to the best theoretical	Loss function value	% relatively to the best theoretical
Best theoretical	400	1.00	1682	1.00	1135	1.00
Best cycle	414	1.03	1740	1.03	1208	1.06
Best heuristic	423	1.05	1666	1.04	1255	1.10
First cycle	500	1.25	2064	1.22	1487	1.31
Worst theoretical	555	1.38	2300	1.36	1733	1.52

Discussion

Best and Worst theoretical strategies are not real cleaning strategies. They represent only links priorities without taking into account moving times from link to link. Those priorities are always pairwise for symmetrical networks, i.e. links $k-m$ and $m-k$ are identical and should be cleaned just one after another.

The Best cycle loss function values, as it can be seen from table 1, are close to the Best theoretical loss function values. We can expect that the Worst cycles (if calculated) might be close to the Worst theoretical as well. At the same time, the First (random) cycles are closer to the lover bound of loss function values interval than to the upper one. The Best heuristic strategies are rather successful and are closer to the Best cycle result than to the First cycle.

It was already mentioned above that there may be many similar heuristic strategies of the same type which differ by some parameters. The Best heuristic is the most successful one, providing the relatively minimal loss function value. It is interesting to notice, that in all examples considered, the Best heuristic strategies occurred to be exactly cycles. However it is not always necessary so. In general there may be two different situations.

First is the trap situation. This mean that heuristic strategy may fall into the trap (and usually some of them do), when all adjacent links are already cleaned and we must move cleaning machine to the next link without cleaning. This extra moving led to additional looses which we were unable to avoid because of heuristic oversight.

The second situation is the gain situation. It is not difficult to construct an example where the better result can be obtained by special including of extra moving without cleaning, though it was possible to choose the contiguous cleaning trajectory as well. However such situation may look like an artificial one and is not very typical.

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