# MODEL OF THE INFLUENCE OF SURFACE FACTORS ON ROAD NETWORK TRAFFIC CONDITIONS[1]

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## Abstract

A model of the influence of surface factors to network traffic conditions is considered. Road surface quality is assumed to be weather dependent, and may change quickly for the worse as a result of snowfall, for example. In turn, road service cleaning activity may change these results to the better. Changing surface factors may cause redistribution of the flows on the links to some new equilibrium. In general, total travel time over the network will increase. This is a measure of disutility. The corresponding loss function should be minimized by optimal road service cleaning activity. The optimization problem is formulated and some approaches to decision-making are suggested. Traffic safety condition is taken into account as additional restriction for optimization process. Particular problems include the number of cleaning teams required, allocation of the links and identification of the best sequence of the links for cleaning.

## 1. Introduction

Surface quality factors affect the road network traffic conditions in many aspects: decision to make a trip, choice of destination, travel mode, route, disutility measure and safety. This article deals with one of those possible aspects - the influence of quick changes of the road surface quality on the generalized travel cost as disutility measure.

Quick changes of the surface factors are assumed to be weather connected. The road quality depends on snow amount, slipperiness, smoothness ratio, temperature and some other factors. Traffic conditions may change quickly for the worse as a result of snowfall, for example. In turn, road service recovery activity (snow cleaning and salt or sand throwing) may change traffic conditions for the better.

We suppose that network flows on the links tend to equilibrium. The principal question is, if quick changes in surface quality are accompanied by corresponding quick changes in the distribution of equilibrium flows. Although this question is of great importance for practical inferences, the theoretical formulation of the problem is quite similar for both cases.

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Equilibrium flows on the links change near some average values. The variance depends on the weather behavior on one hand, and on the road service quality on the other hand. It is reasonable to assume that drivers will keep their usual routes, if the troubles connected with surface factors are not too hard and durable. It depends also on road service cleaning activity. Not only changes for the worse should be quick, but changes to the better should be quick as well.

Thus, if equilibrium is stable in relation to the quick and short changes of surface quality, the only effect of such changes will be the increase or decrease of the total travel time over the network. This is a measure of disutility. More exactly, additional travel time is our cost per time unit, which we pay for each separate link as long as it is not cleaned. Therefore additional total travel time over the network should be decreased as much as possible and as quickly as possible, i.e. the corresponding loss function should be minimized.

Safety level is of grate importance as well, because accident rate may increase dramatically as the result of surface slippery. Traffic security requirement is represented as additional restriction for recovery process.

The optimization problem is formulated in this paper and some approaches to decision-making are suggested. Particular problems include the number of cleaning teams required, allocation of the links and identification of the best sequence of the links for cleaning for each team.

## 2. Problem representation

### 2.1. Basic assumptions

The main suggestion is, that the travel time on the link depends not only on congestion factors, but also on the link surface factor. It is attractive from the mathematical point of view to assume, that the surface factor increases travel time linearly. This seems to be quite reasonable, because the surface factor affects the average speed on the link. So, let us suppose that the travel time on the link is

$$t_i = \varphi_i\left(x_i, h_i\right) = t_i\left(x_i\right)s_i\left(h_i\right) \;,\;\; i = 1,\dots,n \tag{2.1}$$

where

- $x_i$ = flow on link $i$,
- $t_i(x_i)$ = travel time on link $i$ under normal surface conditions (influence of congestion factor),
- $h_i$ = surface quality variable,
- $s_i(h_i)\geq 1$ = increase of travel time under $h_i$ in comparison with the normal surface condition.

We may regard $s_i\geq 1$ as independent variables (without specifying $h_i$ and $s_i(\cdot)$ ), which are changing in some way in connection with weather conditions and recovery efforts. The corresponding total travel time may thus be written as:

$$T(s) = \sum_i T_i(s) = \sum_i t_i(x_i)s_i x_i \tag{2.2}$$

and additional total travel time is

$$\Delta T(s) = \sum_i \Delta T_i(s) = \sum_i t_i(x_i)(s_i - 1)x_i \tag{2.3}$$

Additional total travel time $\Delta T(s)$ is cost per time unit, which we pay for worse surface conditions, and $\Delta T_i(s)$ is cost per time unit for each link. The corresponding loss function is

$$L = \int_{t_0}^{t_1} \Delta T(s)dt = \sum_i \int_{t_0}^{t_1} \Delta T_i(s)dt \tag{2.4}$$

where $t$ is usual clock time, measured from some starting point $t_0$ and up to point $t_1$, when we believe that recovery procedure is complete. The $s_i$ factors in their turn depend on $t$ , i.e. $s_i = s_i(t)$ as a result of weather and recovery. So, the problem is to decrease $\Delta T$ as much as and as quickly as possible, to minimize $L$ by mean of specifying $s_i(t)$ as control functions (see figure 1). The initial point $s_i = 1, \forall i$, corresponds to normal surface conditions. Notice, that functions $T_i(s)$, $\Delta T_i(s)$ in equations (2.2),(2.3) depend only on $s_i$.

Particular problems include the number of cleaning teams required, allocation of the links and identification of the best sequence of the links for cleaning.

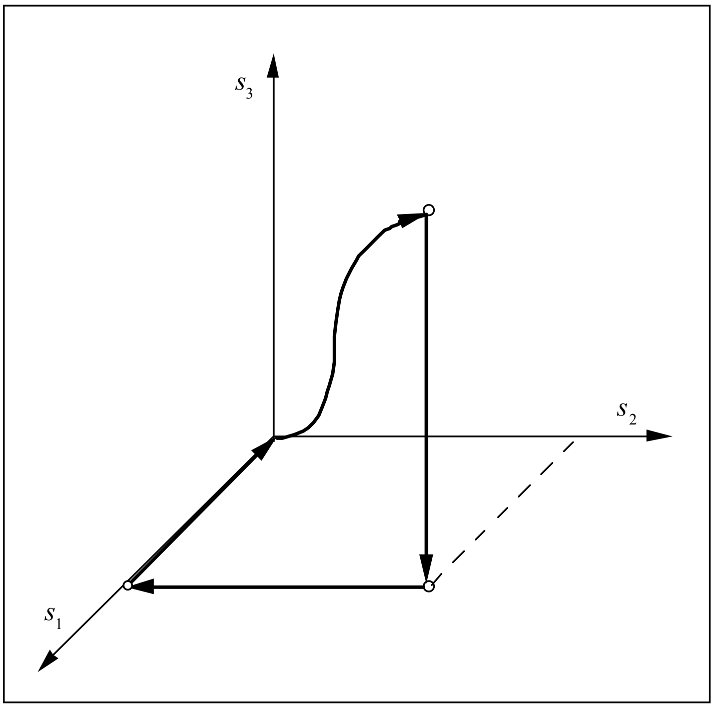


**Figure 1**. *Change of surface factor s(t) during snowfall and subsequent cleaning*

The problem however may have different levels of complexity.

### 2.2. Cleaning during the snowfall

Cleaning activity during the snowfall will create somewhat different trajectory than shown in figure 1. In this case $s_i$ values will decrease for links, which are currently cleaned, and $s_i$ values will increase for other links, which have already been cleaned, as well as for the links, which are not cleaned yet. The same is truth for the link which is currently cleaned. As the result some links may require multiple cleaning.

On the other hand road service standards assume different maintenance requirements for the different roads classes. Class A road standard, for example, declares that snow cleaning starts when the snow depth is more than 2 cm and recovery time is at most 2 hours. The similar requirements are developed for other roads classes.

### 2.3. Redistribution of the flows

Changing surface factors may cause redistribution of the flows on the links to some new user equilibrium (UE). Thus, for some given surface factors $s=(s_1,...,s_n)$ we have UE flows $x=(x_1,...,x_n)$, which in general depend on $s$, $x=x(s)$. As was mentioned in the introduction, it is not obvious whether we should take into account this possible dependence for quick changes in $s$. So, we shall use the simple notation $x_i$, when assuming flows to be some fixed average values independent of $s$, and $x_i(s)$, when assuming the flows to be UE flows depending on current surface factors. The corresponding total travel time may thus be written as:

$$T(s)=\sum_i T_i(s)=\sum_i t_i(x_i(s))s_i x_i(s) \tag{2.5}$$

and additional total travel time is

$$\Delta T(s)=\sum_i \Delta T_i(s)=\sum_i \left[t_i(x_i(s))s_i x_i(s)-t_i(x_i^0)x_i^0\right] \tag{2.6}$$

where $x_i^0$ are flows on links with normal surface factors $(s_i=1,\forall i)$, $x_i^0=x_i(1,\ldots,1)$.

If redistribution of flows on links is assumed, then extraordinary effects of the behavior of function $T(s)$ in (2.5) may be observed. For example, it may occur, that $T(s)$ decreases with $s_i$ for some links, and therefore it is better not to clean them at all (see also Braess's paradox, Sheffi (1985)). Some links may have optimal values of snow amount, which provides the best minimal value of $T(s)$ (see figure 2).

The best sequence of the links for cleaning may be quite different in cases of fixed flows $x_i$, and surface dependent flows $x_i(s)$. So, cleaning of the links with high priority under fixed flows may cause very small gains after flow redistribution. And, in turn, the best sequence for the surface dependent flows may not be attractive in case of fixed flows.

**Example 1**

Let us suppose, that we have 3 different links from node 1 to node 2, corresponding total trip rate from node 1 to node 2 is equal 10 and surface factors $s_i=2,\forall i$. System is in equilibrium and the travel times over links are the same for all links. Assume that exactly one link can be cleaned. Which one should be preferred. The effect of different choices before the flows redistribution and after flows redistribution is shown in Table 1.

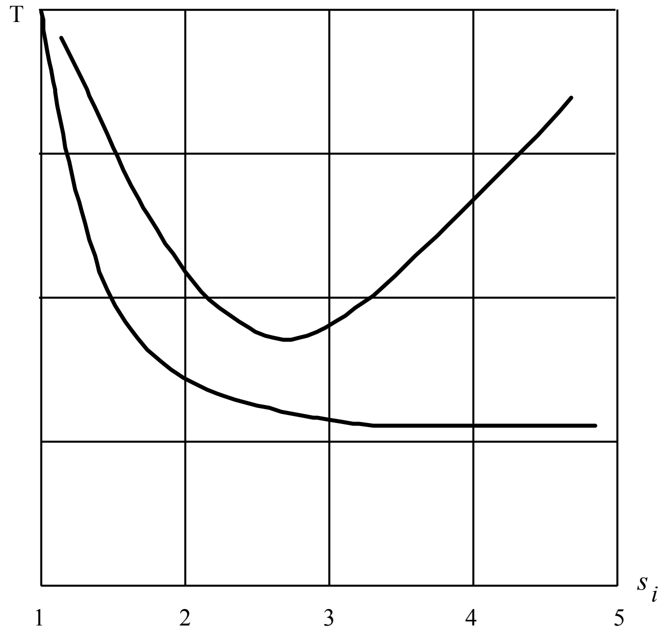


**Figure 2**. *Extraordinary behavior of function T in case of surface factor dependent flows*

| Number of the link $i$ | Surface factors $s_i$ | Equilibr. flows on links $x_i$ | Trav. times on links $s_i t_i(x_i)$ | Tot. travel times on links $s_i x_i t_i(x_i)$ | Tot. travel time before flows redistribution | Tot. travel time after flows redistribution |
|---|---|---|---|---|---|---|
| 1 | 2 | 5 | 25 | 125 | | |
| 2 | 2 | 3 | 25 | 75 | 250 | 250 |
| 3 | 2 | 2 | 25 | 50 | | |
| 1 | 1 (cleaned) | 7 | 25 | 175 | | |
| 2 | 2 | 3 | 25 | 75 | 187.5 | 250 |
| 3 | 2 | 0 | --- | ---- | | |
| 1 | 2 | 5 | 25 | 125 | | |
| 2 | 1 (cleaned) | 5 | 25 | 125 | 212.5 | 250 |
| 3 | 2 | 0 | --- | ---- | | |
| 1 | 2 | 4 | 20 | 80 | | |
| 2 | 2 | 2 | 20 | 40 | 225 | 200 |
| 3 | 1 (cleaned) | 4 | 20 | 80 | | |

**Table 1.** Total travel time over the network before and after redistribution of the flows

So the optimal choice is to clean the link number 1 (the most congested one) if there is now flows redistribution, and it is better to clean link number 3 (the less congested one) if flow redistribution will take place. It is interesting to notice, that in first two cases redistribution of the flows after cleaning changes situation for the worse. This is in accordance with well known difference between user equilibrium flows distribution and system optimum flows assignment.

### 2.4. Different kinds of cleaning and number of cleaning machines

Different kinds of cleaning activities may cause different effects in $s_i$ values. For example, cleaning, smoothing and salt or sand throwing provide decreasing $s_i$, not necessary to 1. Besides that, different kinds of cleaning may require different time of operation and may have also different cost.

Number of cleaning machines is the question of opposite kind. For the purposes of the best cleaning performance it is necessary to invoke as many cleaning machines as possible. Reasonable restriction may be the cost restrictions and road service resources available.

### 2.5. Feasible increasing of the travel time

Road service standards assume certain restrictions for snow amount for different roads classes. Those standards are formulated as maximal feasible thickness of snow measured in centimeters. Alternative approach could be corresponding feasible travel time increasing represented in the form of inequality

$$t_i = t_i(x_i)s_i(h_i) \le K^{\max} t_i(x_i)$$

where $K^{\max}$ is quotient of maximal feasible time increasing. This is equivalent to

$$s_i(h_i) \le S_i^{\max} \quad \text{or} \quad h_i \le H_i^{\max}$$

### 2.6. "Just in time"

Many processes in the world requires very regular and exact traffic schedule. Time delays may cause very expensive looses. So, it is reasonable to introduce a set of the time restrictions in the following way:

$$t_i = t_i(x_i)s_i(h_i) \le T_i^{\max}$$

Consequently this mean, that

$$s_i(h_i) \le T_i^{\max} / t_i(x_i) \quad \text{or} \quad h_i \le h_i^{\max}(x_i)$$

where $h_i^{\max}(x_i)$ is decreasing function.

### 2.7. Safety level

Let us assume, that $\lambda_i = \lambda_i(x_i, h_i)$ is any safety measure. It may be, for example, accident rate, or probability of accident per car or per kilometer or something else. Then safety level requirements may be represented in the form of inequality

$$\lambda_i = \lambda_i(x_i, h_i) \le \Lambda_i^{\max}$$

if $\lambda_i(x_i, h_i)$ increase by $x_i, h_i$ then safety restriction may be written as

$$h_i \le h_i^{\lambda}(x_i) \quad \text{or} \quad s_i \le s_i^{\lambda}(x_i)$$

where $h_i^{\lambda}(x_i), s_i^{\lambda}(x_i)$ are decreasing functions.

### 2.8. Stochastic approach

Snowfall intensity and duration are the random variables. Road service has in it's disposal numerous observations. This circumstance allow us to estimate statistical distribution of the intensity and duration variables and so, consider the snowfall as a stochastic process. In this case recovery process also may be considered as stochastic integral optimization problem, and the solution of this problem will be than a stochastic control strategy.

## 3. Basic optimization problem formulation

### 3.1. Single cleaning machine, zero moving time[2]

Suppose, that

$\tau_j$ = the time it takes to clean link $j$ and
$c_j$ = the cost per time unit of not cleaning link $j$.

Define the sequence of links by means of variables

$$y_{ij} = \begin{cases} 1, \text{ if link j is the ith link to be cleaned} \\ 0, \text{ otherwise} \end{cases}$$

Then, the optimization problem is

$$\min L = \sum_i \sum_l y_{il} c_l (\sum_{k \le i} \sum_j y_{kj} \tau_j) \tag{3.1}$$

subject to

$$\sum_j y_{ij} = 1, \ \forall i; \ \sum_i y_{ij} = 1, \ \forall j \tag{3.2}$$

The solution to the problem is to rank the sequence of links in order of decreasing of $c_j / \tau_j$.
Let us proof it formally.

Let $(c_1 / \tau_1), \ldots, (c_n / \tau_n)$ be the optimal sequence of links, which provides the minimal value of the function $L$ which is now

$$L = \sum_{i=1}^{n} \left( \int_0^{\sum_{j=1}^{i} \tau_j} c_i dt \right)$$

[2] The following two ideas were suggested by Lars-Göran Mattsson.

Then $\frac{c_1}{\tau_1} \geq, \ldots, \geq \frac{c_n}{\tau_n}$ .

Suppose otherwise, that $i, j$ are two consecutive links $(j = i+1)$ and $\frac{c_i}{\tau_i} < \frac{c_j}{\tau_j}$.

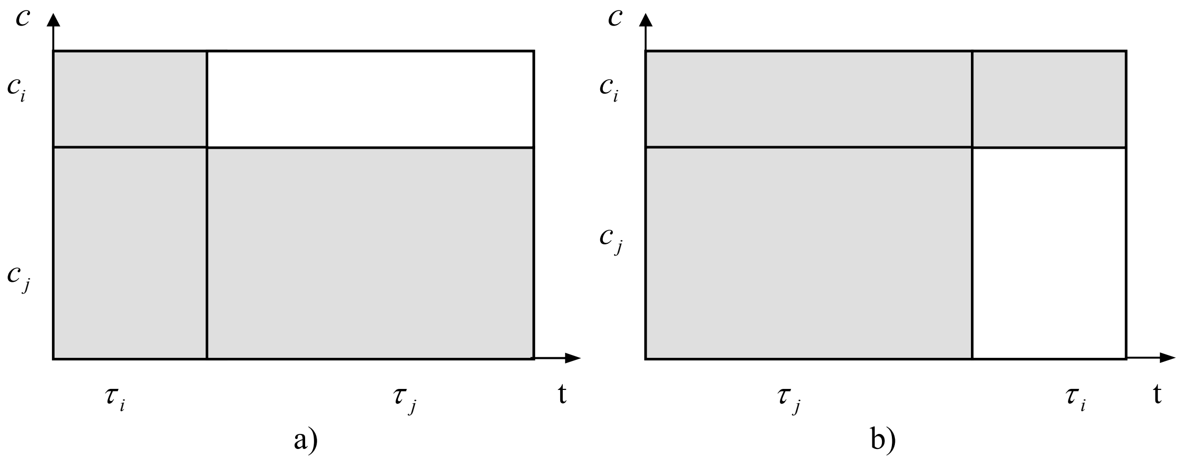


**Figure 3**. *Loss function: a) cleaning of link $j$ after link $i$ ; b) cleaning of link $i$ after link $j$*

Denote $Q = (c_i + c_j)(\tau_i + \tau_j)$ as the loss function contribution for not cleaning links $i,j$ during time $\tau_i + \tau_j$. Then the decrease $Q$ due to cleaning of links $j$ immediately after link $i$ is $\Delta Q_{ij} = c_i \tau_j$. In turn, cleaning of the link $i$ immediately after link $j$ provides the corresponding decrease $\Delta Q_{ji} = c_j \tau_i$. In accordance with our suggestion $\Delta Q_{ij} < \Delta Q_{ji}$. This means, that the sequence is not optimal, because the loss function may be decreased by changing the order of links $i,j$. So, the statement is proved.

### 3.2. Multiple cleaning machines, nonzero moving time

For $M$ cleaning machines denote by $\tau_j^m$ the time it takes to clean link $j$ by machine $m$ and $d_{jl}^m$ the time to move machine $m$ from link $j$ to $l$ . Also, let

$$y_{ij}^m = \begin{cases} 1, \text{ if link j is the ith link to be cleaned by m, } m = 1, \ldots, M \\ 0, \text{ otherwise} \end{cases}$$

Then, the optimization problem is

$$\min L = \sum_m \sum_i \sum_l y_{il}^m c_l \Big( \sum_{k \leq i} \sum_j y_{kj}^m \big( \tau_j^m + \sum_r y_{k-1,r}^m d_{rj}^m \big) \Big) \tag{3.3}$$

subject to

$$\sum_j y_{ij}^m \leq 1, \forall i, m; \quad \sum_i \sum_m y_{ij}^m = 1, \forall j; \quad y_{0j}^m = 0, \forall m, j \tag{3.4}$$

We assume that a machine cannot be cleaning more than one link at a time, each link should be cleaned by some machine at some point of time and it takes no time to move a machine to the first link.

### 3.3. Discussion

The first approach is the special case of the second one for $M = 1$, $d_{ij} = 0, \forall i,j.$ It provides a very simple and beautiful solution, which may be applied if the links ordered by decreasing $c_j / \tau_j$ are adjacent, i.e. the end of each link in the sequence is the beginning of the next link. In this case such a sequence is the optimal solution to the problem. Assuming $c_j = \Delta T_j = t_j(x_j)\ (s-1)x_j$ and $t_j = \gamma t_j(x_j)s_j$ where $\gamma$ is a fixed multiplier, we can obtain the criteria for a cleaning sequence in decreasing order of $x_j(s_j - 1)/s_j$ Besides that, if all surface factors $s_j$ are equal for all links, then only decreasing order of flows $x_j$ is sufficient.

However, in general those links are not adjacent, but are located in jumping order. So, we should take into account the time of moving the cleaning machine from link to link. It should not be successful to include moving times into cleaning times, because all moving times will change after each step, and the corresponding best order of the links also will change after each step. It may be shown, that recalculation of the new optimal order after each step may provide a non-optimal solution. In other words, optimization of each separate step doesn't certainly led to the global optimization.

**Example 2**
Suppose that road network shown on figure 4 consists of 6 links, which have approximately the same priorities, but still $c_1/\tau_1 > ... > c_6/\tau_6$. Choosing the links according to priorities decreasing order we shall obtain the cleaning sequence 1,2,...,6. It is not difficult to see that the same result will be obtained in case of recalculation of priorities of links after each step with consideration of moving times from link to link. However this cleaning sequence includes at least two extra moving of cleaning machine without cleaning. Contiguous cleaning sequence 1,3,5,6,4,2 seems to be optimal in this case.

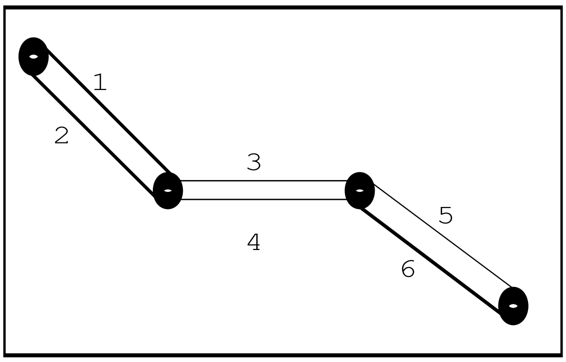


**Figure 4.** *Optimization of each particular step produce not optimal cleaning sequence*

Nevertheless the optimal solution to the problem (3.1) is quite important because it defines theoretical priorities among links, and allows to calculate minimal and maximal possible loss function values, which may serve as a yardstick for measuring the performance of other solutions.

The second approach represents a discrete optimization problem with linear constraints. It is necessary to notice here, that the moving time $d_{ij}$ from link to link is not fixed. It may change many times in the process of cleaning, as will the corresponding shortest paths between links. The proper value of $d_{ij} = d_{ij}(s)$ should be calculated with consideration of the time movement when it is used. Thus optimization problem solution will require shortest path recalculations in each step.

In a more general case we should take into account also that continuation of the snowfall may take place during the cleaning activity. Therefore some links may require multiple cleaning. In this case it is not evident how long the cleaning process may be, and when it will be finished.

## 4. Restrictions for basic optimization problem

### 4.1. Cycles

It is not easy to foresee if jumping will take place for an optimal solution or not. Probably it depends on homogeneity of the transport network. If link priorities differ drastically then the jumping order is quite possible. It is desirable to avoid this for at least two reasons. The first is that road service resources should also be taken into account to minimize cleaning machine path lengths. The second is, that there are may more jumping trajectories of cleaning than contiguous ones, and we may probably need simplification of the problem.

Let us introduce an additional restriction, that the cleaning route is a cycle in case of one cleaning machine and a contiguous path in case of multiple cleaning machines. This means, that only adjacent links should be cleaned, no one link should be passed without cleaning, and all links should be cleaned at least once. It is not difficult to prove by mathematical induction, that cycles always exist for networks with two-way links. For more general results see Evans and Minieka (1992). Notice, that the cycle of links is not the same as the cycle of vertices.

The problem of cycle identification may be represented as a Traveling Salesman Problem (see Lawler, Lenstra et al (1985)) with the matrix of distances $q_{ij} = 0$ , if links $i, j$ are adjacent, and $q_{ij} = 1$ otherwise. It may also be formulated as a minimization problem for $y_{ij}^m$ variables:

$$\min \sum_m \sum_k \sum_j y_{kj}^m (\sum_r y_{k-1,r}^m d_{rj}^m) \tag{4.1}$$

subject to (3.4) to find cycles if they exist, or to find the best approximation to cycles otherwise. If an optimal solution is found, and the minimum value of the objective function calculated is equal to zero, then it becomes possible to modify the initial optimization problem (3.3) to

$$\min \sum_m \sum_i \sum_l y_{il}^m c_l (\sum_{k \le i} \sum_j y_{kj}^m \tau_j^m) \tag{4.2}$$

with the additional restriction

$$\sum_m \sum_i \sum_l y_{kj}^m (\sum_r y_{k-1,r}^m d_{rj}^m) = 0 \tag{4.3}$$

This provides a decomposition of the initial problem objective function. However, the main interest is not to find one cycle, but to find all cycles, because there may be many of them.

### 4.2. Cleaning time limits

Some links may be of special significance for the traffic, and so, require special restrictions. This is in accordance with the road service standards, which usually are formulated as "to clean no later than". The corresponding restrictions for an arbitrary link $l$ is the following:

$$\sum_m \sum_i y_{il}^m \left( \sum_{k \le i} \sum_j y_{kj}^m \left( \tau_j^m + \sum_r y_{k-1,r}^m d_{rj}^m \right) \right) \le \Theta_l \tag{4.4}$$

### 4.3. Relative weights of links

The significance of the links may be connected with other factors besides total travel time. Road service standards, for example, include road importance classification by some criteria. To combine significance factors of different nature it is reasonable to introduce a weight function for the generalized travel cost $c_j$. It may be $c_j = w_j \Delta T_j$, for example. This approach may be used instead of additional restrictions to avoid unnecessary implementation complexity.

### 4.4. Safety and "Just in time" restrictions

Cleaning optimal strategy is a sequence of links to be cleaned. This process is time connected process, i.e. each step occupy some definite time interval. Thus, surface factors changes may be considered as time process $s_i = s_i(t)$. Corresponding flows on links distribution may be calculated for each time point, according with the model accepted.

Safety and "Just in time" restrictions in general are formulated as $s_i \le s_i^{\max}(x_i)$ inequalities. Those inequalities may be checked for each cleaning strategy during the time of it's implementation. In case of stochastic approach we may consider the probability of safety or "Just in time" requirements violation. Of course, if corresponding restrictions are too strong, the optimization problem may have no solution or, alternatively, optimization may led to the significant increasing of the number of cleaning machines.

### 4.5. Number of cleaning machines and different kinds of cleaning

Suppose, that there exists a fixed set of possible cleaning activities, and $\omega_m$ is the index number of a particular kind of cleaning, performed by machine $m$. Let $b_0(\omega_m)$ be the cost of employment, and $b(\omega_m)$ the cost per time unit of using of machine $m$. The time of cleaning $\tau_i^m$ and time of moving $d_{ij}^m$ may also depend on $\omega_m$. As the result, the total time $\Theta_m$ of using machine $m$ depends on $\omega_m$ and the corresponding trajectory.

Define the cost of recovery as

$$B(\omega, y) = \sum_m (\Theta_m b(\omega_m) + b_0(\omega_m)) \tag{4.5}$$

This implies that the loss function (2.4) will also be $L = L(\omega, y)$. If any cost restrictions for (4.5) exist, then it is possible to include these restrictions in the optimization problem, with possible optimization on $M$ and $\omega_1, \dots, \omega_M$. Otherwise we may compare different optimal pairs *(L*,B*)* obtained for some fixed $M$ and $\omega_1, \dots, \omega_M$, see Brivers (1994).

# 5. Alternative optimization problem formulation

## 5.1. Integral sum optimization

Let us consider the case of one cleaning machine. Denote by $y_1, y_2, ..., y_k$ the sequence of links to be cleaned, where $y_i \in \{1,...,n\}$ is the number of the link which is to be cleaned at step $i$ . The objective function may be represented in the form

$$L = \sum_i \Delta T(s)\Delta t_i = \sum_i \Delta T(s)(d_{y_{i-1},y_i} + \tau_{y_i}) \quad (5.1)$$

as the integral sum in (2.4). $\Delta t_i$ are time intervals inside which the integrand is assumed to be fixed. Of course this interval may be split in a number of smaller intervals, to make the approximation more accurate. In general $s = s(t)$ may increase or decrease at each step. The time interval $\Delta t_i$ also depends on current $s_i$ values, and finally it is possible that $x_i = x_i(s)$.

This equation should illustrate the essential properties of the algorithm for the loss function calculation. Thus the optimization problem becomes

$$\min_{y_1,...,y_k} \sum_i \Delta T_i(s)(d_{y_{i-1},y_i} + \tau_{y_i}) \quad (5.2)$$

without additional restrictions besides $y_i \in \{1,...,n\}$. The feasible region is a *k*-dimensional square of discrete points. Values $d_{y_0,y_1}$ are initial moving times connected with some starting point of location. They may also be set to zero. If there is no snowfall during the cleaning activity, we may require that $k = n$ and $y_i \neq y_j$ if $i \neq j$ (i.e. $y_1,...,y_n$ is a permutation). However, this is not necessary, because the optimal solution automatically will be a permutation in this case. For multiple cleaning machines similar notation may be used with the additional possibility $y_i^m = 0$ when not cleaning.

## 5.2. Road network modernization problem

There is however a problem which is very similar to the cleaning problem but is contiguous one. Let us represent travel time on links as

$$t_i = t_i(x_i / w_i)s_i$$

where

$w_i$ = capacity factor of the link as a scale parameter of the model,
$s_i$ = surface quality variable, not connected with the snow amount, but representing road quality itself.

Total travel time over the network is a disutility measure. It should be decreased during the network modernization activity. The best solution is to make capacity parameters as large as possible and to make the road quality multipliers as small as possible. Of course some cost restrictions should be taken into account. Cost function may consist of two components - the cost of capacity increasing and cost of road quality improving:

$$\cos t = q_1 \sum \Delta w_i + q_2 \sum \Delta s_i \leq COST$$

Under the cost restriction optimization problem may be formulated. Loss function however is not an integral of additional total travel time over the network, but the total travel time itself. The solution of the problem assumes only identification of the set of links for modernization and corresponding capacity-quality improving balance, without specifying of the exact sequence of links modernization.

This model allows to estimate the effect of new links introducing as well. The link which does not exist currently may be represented as link with zero capacity and/or infinite surface factor quotient.

## 6. Numeric solution of the problem

### 6.1. Theoretical solution

Analytical solution of the problem (3.1), (3.2), as was mentioned above, defines an optimal sequence of links to be cleaned in decreasing order of $x_i(s_i - 1)s_i$. This solution usually can not be applied in practice by at least two reasons: it assumes that there is only one cleaning machine and optimal sequence of links is a cycle. Taking into account moving times from link to link, number of cleaning teams and other factors makes the problem much more complex and it is not likely that analytical solution could exist.

In general case we have a constrained discreet optimization problem. Particular kind of objective function and restrictions set depends on complexity level accepted. Unfortunately numeric solution of optimization problems formulated above is probably very difficult problem even in simplest cases. The main reason is, that the loss function has no any regular structure. It looks like a random function defined in discreet points (one and zero) of multidimensional space and there is no correlation between neighbors (neighborhood in usual metrics). This may be explained as the result of numbering of links and paths. Corresponding integer numbers and integer coordinates of points does not reflect real distances or loss function values.

### 6.2. Heuristic approaches

Instead of direct optimization let us consider the possibility of a heuristic approach. In general, the problem is quite similar to chess playing algorithms. Numbers $y_1, y_2, \ldots$ are our moves, and the weather is our opponent, increasing $s_i$ factors. The usual technique is sample tree analysis. The sample tree consists of different sequences of links for cleaning. That means we have number of possibilities for the first move, than, for each possible first move choice we have number of possibilities for the second move, and so on. The total number of moves is not necessary predetermined value (in case of snowfall for example). But if it is, than we can obtain in the principle the exact solution of the problem by means of full sampling through all possibilities.

Such approach however may be applied only for very small road networks, because sample tree grows up very quickly with the increasing of number of links and number of moves. As the result, it becomes impossible to sample through all possibilities and to find in so way an exact optimal solution. So, it is reasonable to cut some branches at each step by some criteria, to make the tree more narrow and longer. In this case we take into account not all possible choices for the current move to evaluate the next one, but only part of them. The same reduction is fulfilled for each step. This process must be terminated at some point when calculations becomes too durable. The subtree of sequences obtained may hopefully include the branch which is starting part of the best optimal sequence. Otherwise the best branch of the subtree may serve as approximation to the optimal trajectory.

Notice that all this process is intended to determine the first move only or few consecutive moves. After that the procedure must be repeated from the new starting point. Of course it is possible to use for further prolongation the branches which are already evaluated if there is no changes in external weather influence. However all those questions are concerned with implementation details.

The possible criteria for subset of moves selection may be very different. Let us consider some of them. Recall that $n$ is the number of links and $k$ is the number of moves evaluated or the length of the sample tree.

**All possibilities and permutations.** All possibilities for each move mean that there is no any reduction of the sample tree branches and we have to evaluate $n$ choices of the next move for all $n$ choices of the previous one. This include the possible jumping trajectories as well as the multiple cleaning of the links. The total number of different sequences is equal to $n^k$.

If there is no snowfall, than it is reasonable to avoid multiple cleaning and to take into account only links which are not cleaned yet. This mean that we have $n$ possible choices for the first move, than $n-1$ possible choices for the second move and so on. In other words we consider only set of permutations as the feasible set of links sequences. The total number of different possibilities is equal to $n(n-1)\ldots(n-k+1)$.

**The best branches.** Though we have $n$ possible choices for each current move, not all of them are equally effective for the loss function minimization. We can order them by the ratio of loss function decreasing and to take only $r$ best possibilities ($r$ is the fixed number) for the further analysis. In this case we will have to sample through $r^k$ different links sequences.

Value of $r$ should be determined heuristically as the compromise between the desire to include in consideration more trajectories on the one hand and to reduce the calculation time on the other hand. The two opposite opportunities are: $r = n$ (all possible sequences) and $r = 1$ (the single sequence). The last one has a small chance to be successful due to example 2. The reason is that the best choice for each local step may produce not a good global sequence.

The slightly different approach is to fix not the value of $r$ itself, but the quotient $0 < q < 1$, which determines a boundary for the loss function decreasing relatively to the largest possible decreasing for the current move. More exactly, let $\Delta L^{\max} = \max\{|\Delta L_1|, |\Delta L_2|, \ldots, |\Delta L_n|\}$ where $\Delta L_i$ are values of loss function decreasing for all $n$ possible moves for the current step. Than the subset of current moves may be defined as $\{\Delta L_i : \Delta L_i > q\Delta L^{\max}\}$. In other words, we take into account for the further analysis not all possible current moves and not only predetermined fixed number of the current best moves, but all nearly best moves. As the result, the total number of different trajectories to be evaluated is unknown in advance.

**Adjacent links and cycles.** It was mentioned above, that the attractive rational approach is contiguous cleaning, i.e. only adjacent links are to be cleaned. If multiple cleaning of the links is not necessary, than it is quite probable to expect that the optimal cleaning path may be a cycle. Notice that cycle is a special case of permutation and it always exist for the road networks with two-way links. However it is not difficult to construct an example when it is better to include some redundant moves without cleaning into cleaning sequence than to follow the cycle requirement strictly. It depends also on the balance of moving time and cleaning time for particular links.

So, making a choice for the current move we should take into account only adjacent links which are not cleaned yet if possible. If all adjacent links are already cleaned than we move the cleaning machine to the new starting point without cleaning.

Adjacent links strategy is the special case of all possibilities strategy and may be considered as the most general approach at the same time, because moving of cleaning machine is contiguous by the nature and only difference is particular kind of cleaning of current link including moving without cleaning as one of possibilities. Loss function increases in time and each move corresponds to additional term in integral sum (5.1). The problem of heuristic approach is to choose the branch of moving trajectory in order to minimize corresponding partial sum for specified time interval.

Since the typical intersection includes four links in average, the approximate estimation of the number of different moving sequences is equal to $4^k$.

**Heuristic strategies summary.** Thus, summarizing the mentioned above strategies, the possible criteria with estimates of corresponding computational efforts required, are the following:

| | |
|---|---|
| all possibilities | - $n^k$ |
| permutations only | - $n(n-1)\ldots(n-k+1)$ |
| $r$ best | - $r^k$ |
| 1 best | - 1 |
| all links nearly best | - ? |
| adjacent links, cycles | - $\approx 4^k$ |

The last approach seems to be the most attractive one, because it provides a reasonable reduction of computational efforts and assumes continuous cleaning, if possible.

## 7. Numeric example

### 7.1. Results

Let us consider some numeric examples. Three road networks with different number of directed links ($n$ =22,28,32) with symmetrical origin-destination matrixes were generated. All links were supposed to be two-way links with symmetrical properties (travel times and snow factors). Single cleaning machine, no flows redistribution and no snowfall continuation assumed.

Particular strategies applied were the following:

- Best theoretical. The sequence of links to be cleaned according to theoretical priorities- decreasing order of $c_i/\tau_i$ which is equivalent in this particular case to decreasing order of flows $x_i$. Possible jumping order of links was ignored, i.e. moving time from link to link if they are not adjacent is equal to zero.
- Best cycle. Only contiguous trajectories of cleaning were taken into account, i.e. feasible set of cleaning sequences was restricted by cycles. The Best cycle was found by means of full sampling through all possibilities. This approach however, require significant computational efforts.
- First cycle. The first variant in previous full sampling strategy may be considered as the random cycle selected stochastically. This provides the First cycle result.
- Best heuristic. Heuristic approaches described above as adjacent links cleaning strategy, were applied with slightly different properties concerned with the lengths of subtrees evaluated and

number of moves accepted for the current best branch selected. The most successful one was proposed as Best heuristic.
- Worst theoretical. This strategy is the opposite to the Best theoretical approach, i.e. links are to be cleaned by increasing order of $c_i/\tau_i$, equivalent to increasing order of flows $x_i$. Jumping order was ignored as well.

Notice that the Best theoretical result can not be achieved in general. It may be only a lucky random case, that all links ordered by decreasing of ratio $c_i/\tau_i$ will form a continuous cycle. Unlike this, Worst theoretical result is a lower bound only for cycles, but not for jumping trajectories.

Table 2 contains some numeric results.

| Number of links | n=22 | | n=28 | | n=32 | |
|---|---|---|---|---|---|---|
| | Loss function value | % relatively to the best theoretical | Loss function value | % relatively to the best theoretical | Loss function value | % relatively to the best theoretical |
| Best theoretical | 400 | 1.00 | 1682 | 1.00 | 1135 | 1.00 |
| Best cycle | 414 | 1.03 | 1740 | 1.03 | 1208 | 1.06 |
| Best heuristic | 423 | 1.05 | 1666 | 1.04 | 1255 | 1.10 |
| First cycle | 500 | 1.25 | 2064 | 1.22 | 1487 | 1.31 |
| Worst theoretical | 555 | 1.38 | 2300 | 1.36 | 1733 | 1.52 |

**Table 2.** Loss function values for different cleaning strategies

The road network which was generated for the third case ($n$=32) is depicted in figure 5.

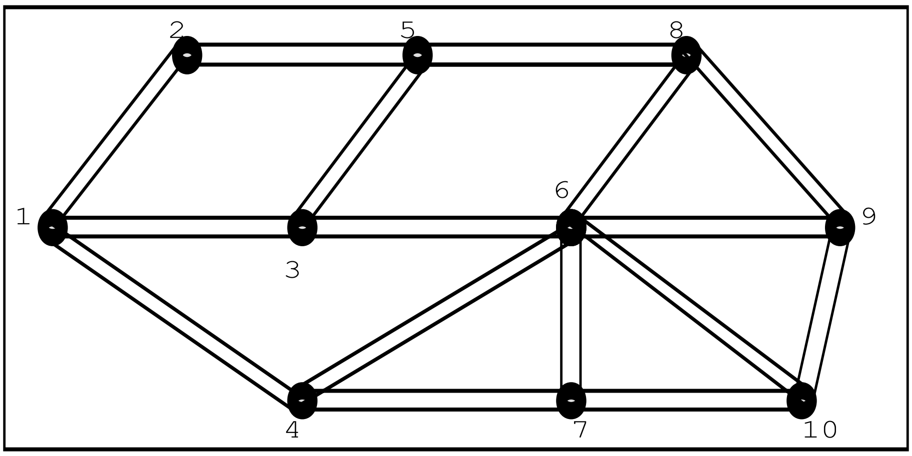


**Figure 5.** *Road network to be cleaned (32 links)*

Corresponding numeric characteristics of the links are represented in table 3. Number $i$ is a number of pair of symmetrical links there, which are also denoted by two numbers - starting node and end node. For example, link $k-m$ is link from node $k$ to node $m$ and is quite similar to link $m-k$ from node $m$ to node $k$. Travel time on link when there is no congestion (free-flow travel time) is denoted by $t_i^0$. Congestion factor $c_i(x_i)$ is defined as ratio $c_i(x_i) = t_i(x_i)s_i / t_i^0$.

| Number of pair of links $i$ | Symmetrical links $k-m$ , $m-k$ | Free-flow travel times $t_i^0$ | Flows on links $x_i$ | Travel times $t_i(x_i)s_i$ | Congestion factors $t_i(x_i)s_i/t_i^0$ |
|---|---|---|---|---|---|
| 1 | 1-2 , 2-1 | 12 | 113 | 295 | 26 |
| 2 | 1-3 , 3-1 | 20 | 48 | 234 | 12 |
| 3 | 1-4 , 4-1 | 10 | 144 | 309 | 31 |
| 4 | 2-5 , 5-2 | 6 | 149 | 191 | 32 |
| 5 | 3-5 , 5-3 | 15 | 83 | 281 | 19 |
| 6 | 3-6 , 6-3 | 12 | 120 | 312 | 26 |
| 7 | 4-6 , 6-4 | 17 | 59 | 234 | 14 |
| 8 | 4-7 , 7-4 | 9 | 102 | 202 | 22 |
| 9 | 5-8 , 8-5 | 8 | 162 | 276 | 35 |
| 10 | 6-7 , 7-6 | 12 | 69 | 189 | 16 |
| 11 | 6-8 , 8-6 | 14 | 98 | 305 | 22 |
| 12 | 6-9 , 9-6 | 12 | 65 | 182 | 15 |
| 13 | 6-10 , 10-6 | 15 | 44 | 162 | 11 |
| 14 | 7-10 , 10-7 | 11 | 60 | 156 | 14 |
| 15 | 8-9 , 9-8 | 14 | 82 | 260 | 19 |
| 16 | 9-10 , 10-9 | 16 | 55 | 208 | 13 |

**Table 3.** Characteristics of the road network to be cleaned (32 links)

Cleaning strategies which were implemented for this example and corresponding links sequences are shown in table 4.

| Step | Best theoretical links priorities | Best cycle | Best heuristic | First cycle | Worst theoretical links priorities |
|---|---|---|---|---|---|
| 1 | 8-5 | 1-2 | 1-2 | 1-2 | 6-10 |
| 2 | 5-8 | 2-5 | 2-5 | 2-1 | 10-6 |
| 3 | 5-2 | 5-8 | 5-2 | 1-3 | 3-1 |
| 4 | 2-5 | 8-5 | 2-1 | 3-1 | 1-3 |
| 5 | 1-4 | 5-2 | 1-4 | 1-4 | 9-10 |
| 6 | 4-1 | 2-1 | 4-7 | 4-6 | 10-9 |
| 7 | 3-6 | 1-4 | 7-4 | 6-3 | 4-6 |
| 8 | 6-3 | 4-7 | 4-6 | 3-5 | 6-4 |
| 9 | 2-1 | 7-4 | 6-8 | 5-3 | 7-10 |
| 10 | 1-2 | 4-1 | 8-5 | 3-6 | 10-7 |

| Step | Best theoretical links priorities | Best cycle | Best heuristic | First cycle | Worst theoretical links priorities |
|---|---|---|---|---|---|
| 11 | 4-7 | 1-3 | 5-8 | 6-4 | 6-9 |
| 12 | 7-4 | 3-5 | 8-6 | 4-7 | 9-6 |
| 13 | 6-8 | 5-3 | 6-3 | 7-6 | 6-7 |
| 14 | 8-6 | 3-6 | 3-6 | 6-7 | 7-6 |
| 15 | 3-5 | 6-8 | 6-9 | 7-10 | 8-9 |
| 16 | 5-3 | 8-6 | 9-8 | 10-6 | 9-8 |
| 17 | 9-8 | 6-9 | 8-9 | 6-8 | 5-3 |
| 18 | 8-9 | 9-8 | 9-6 | 8-6 | 3-5 |
| 19 | 7-6 | 8-9 | 6-7 | 6-9 | 8-6 |
| 20 | 6-7 | 9-6 | 7-10 | 9-6 | 6-8 |
| 21 | 9-6 | 6-7 | 10-7 | 6-10 | 7-4 |
| 22 | 6-9 | 7-10 | 7-6 | 10-9 | 4-7 |
| 23 | 10-7 | 10-7 | 6-10 | 9-8 | 1-2 |
| 24 | 7-10 | 7-6 | 10-9 | 8-5 | 2-1 |
| 25 | 6-4 | 6-4 | 9-10 | 5-2 | 6-3 |
| 26 | 4-6 | 4-6 | 10-6 | 2-5 | 3-6 |
| 27 | 10-9 | 6-10 | 6-4 | 5-8 | 4-1 |
| 28 | 9-10 | 10-9 | 4-1 | 8-9 | 1-4 |
| 29 | 1-3 | 9-10 | 1-3 | 9-10 | 2-5 |
| 30 | 3-1 | 10-6 | 3-5 | 10-7 | 5-2 |
| 31 | 10-6 | 6-3 | 5-3 | 7-4 | 5-8 |
| 32 | 6-10 | 3-1 | 3-1 | 4-1 | 8-5 |

**Table 4.** Links cleaning sequences for different strategies

### 7.2. Discussion

Best and Worst theoretical strategies are not real cleaning strategies. They represent only links priorities without taking into account moving times from link to link. Those priorities are always pairwise for symmetrical networks, i.e. links $k-m$ and $m-k$ are identical and should be cleaned just one after another.

The Best cycle loss function values, as it can be seen from table 2, are close to the Best theoretical loss function values. We can expect that the Worst cycles (if calculated) might be close to the Worst theoretical as well. At the same time, the First (random) cycles are closer to the lover bound of loss function values interval than to the upper one. The Best heuristic strategies are rather successful and are closer to the Best cycle result than to the First cycle.

It was already mentioned above that there may be many similar heuristic strategies of the same type which differ by some parameters. The Best heuristic is the most successful one, providing the relatively minimal loss function value. It is interesting to notice, that in all examples considered, the

Best heuristic strategies occurred to be exactly cycles. However it is not always necessary so. In general there may be two different situations.

First is the trap situation. This mean that heuristic strategy may fall into the trap (and usually some of them do), when all adjacent links are already cleaned and we must move cleaning machine to the next link without cleaning. This extra moving led to additional looses which we were unable to avoid because of heuristic oversight.

The second situation is the gain situation. It is not difficult to construct an example where the better result can be obtained by special including of extra moving without cleaning, though it was possible to choose the contiguous cleaning trajectory as well. However such situation may look like an artificial one and is not very typical.

### 7.3. New aspects

Numeric examples considered are of course the simplest one. Different levels of complexity were discussed in chapters 2 and 4. How can we implement them in cleaning strategy evaluation? Some of them are very essential and easy for implementation. So it is not difficult theoretically to take into account nature behavior - snow continuation and flows redistribution, though it may require much more computational efforts.

The more complex problem is to take into account different additional restrictions, such as the feasible increasing of the travel time, safety level, “just in time” and cost restrictions. The main reason is, that we may check those restrictions at each step while the cleaning trajectory grows up, but it is quite possible that we can encounter some restriction violation not in advance, but at the later stages of the process. In this case we must start the backward resampling to consider some new branches instead of discarded one (similar to the chess playing algorithmization again). It may require the longer sample tree. On the other hand, some of the restrictions may be verified easier in advance for the Best theoretical sequence, to check if they can be satisfied in principle. Otherwise the problem has no solution for the current resources. The possible outcome may be the increasing of number of cleaning machines.

Number of cleaning machines, initial starting points and different kinds of cleaning also may be included in sampling possibilities, but it will led to significant increasing of complexity level. However it is not a good idea to include all possible circumstances in one optimization problem. The rational decomposition of the problem may be more efficient. So, the necessary resources determining can be the separate problem, estimated by means of best theoretical links priorities.

The special technique may be applied to avoid traps, if it is really trap situation. The idea is to use Linear Programming approach for cycles identification (see chapter 4.1).

The networks constructed for numeric example are rather homogeneous because the difference between the Best and Worst theoretical loss function values are not grater than 30-50%. We may expect to observe some new interesting effects for unhomogeneous networks, when the difference between lower and upper bounds will be 200-300% and even more. If the road network is unhomogeneous and large enough, it may occur that heuristic strategy will be unable to find the most significant links in the beginning because local heuristic sampling tree is not long enough. So it is reasonable to increase if possible the length of local sampling tree to provide “visibility” of the links with the highest priorities. Alternative approach may be introducing of some kind of attraction field of links to combine the local and global perspectives and to improve thus the performance of heuristic approach.

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