Study on Xiangyang’s population and aging trend prediction based on discrete population development equation model

Qiong Gu¹,², Wu Zheng¹, Xianming Wang³*

¹ College of Mathematics and Computer Science, Hubei University of Arts and Science, No.296 Long Zhong Road Xiangyang, Hubei 441053, China
² Centre for the Study of Logic and Intelligence, Southwest University, No.2 Tiansheng Road, Beibei District, Chongqing 400715, China
³ Oujiang College, Wenzhou University, Chashan University Town, Wenzhou 325035, China

Abstract

Population problem is an important factor that influences economy and social development of China. This paper takes the statistic data of 6th census in 2010 in Xiangyang as the accordance to establish a discrete model of population development equation, to analyse the population aging trend in the future in Xiangyang from a short period, and further to predict the long-term population development trend and aging population change condition in Xiangyang in the case of different total fertility rate to provide reference accordance for the government to make relevant social and economic decisions.

Keywords: population aging, population development equation, discrete model, total fertility rate

1 Introduction

Population problem is one of the important factors that influence social and economic development, and correct forecast on the development trend of future population has important guiding significance to the overall planning of the country and local government. Many foreign and domestic experts and scholars have been paying more and more attention to population prediction and control issue and propose many different prediction methods and models, such as population index growth model [1] established by British demographer T.Malthus, logistic model raised by Netherlands mathematician Verhulst [2], and also time-space regression model [3], grey model[4], ARMA model [5], neural network model [6] and gene expression model [7] and so on. In order to predict and analyse the population development of China, Scholars like Song Jian and Yu Jingyuan and others [8] raised population development equation model, and Jiang Yingyuan and others [9] applied this model to predict the population of China.

According to the international convention, the population of 65 years and above people exceeds 7% of the total population or the population of 60 years and above exceeds 10% means that the population aging is coming. From the 6th census data of Xiangyang, we can see that the population of 65 years and above is 467,159 in 2010 in Xiangyang, accounting for 8.49% of the total population, and have exceeded the definition boundary of international population aging proportion as 7%. At present, the population of 40 years to 59 years accounts for 16.8% of the total population, so it can be predicted that the aged people in Xiangyang will become more after 10 to 20 years. It is found from the population structure pyramid of Xiangyang that the population aging speed of Xiangyang will still accelerate and various social problems brought by aging will become more serious. Facing to the inevitable population aging problems, what kind of policy should the government to take so as to be able to reduce the influences from population aging and shorten the continuing time of irrational population structure? What kind of population and family planning policy is rational and can make the population of Xiangyang develop stably and sustainably? This paper takes the 6th census data in Xiangyang as the accordance, uses the discrete model of population development equation to predict the population development in Xiangyang, to analyse the population aging trend in the future in Xiangyang from a short period and further predict the long-term population development trend and aging population change condition in Xiangyang in the case of different total fertility rate.

2 Discrete Model of Population Development Equation

In the condition of not considering social factors like population migration and so on, the factors that influence population structure are birth rate and mortality rate. Suppose that \( x_t \) expresses the population of \( r \) years (within \( r \) years but not within \( r + 1 \) years) in the year of \( t \), the maximum age is \( m \), and \( m \), \( r = 0,1...m \). In order to study the population of different ages at any time, suppose \( b_t \) expresses the fertility rate of childbearing age women at years old in the year of \( t \), namely, the
number of babies born by each r years old female on average in the year of t. Suppose \( r_1, r_2 \) is the reproductive age interval and \( k_r(t) \) expresses the ratio of the population of r years old female in the total population in the year of t, then the population of people born in the year of t is:

\[
b(t) = \sum_{r=r_1}^{r_2} b_r(t)k_r(t)x_r(t),
\]

(1)

\( d_r(t) \) expresses the mortality rate of the r years old population in the year of t, and let \( s_r(t) = 1 - d_r(t) \), then \( s_r(t) \) is the survival rate of r years old in the year of \( r \) years old population in the year of t. \( d_r(t) \) is infant mortality rate, then the infant survival rate is \( s_r(t) = 1 - d_r(t) \). We can get:

\[
\begin{align*}
x_r(t) &= s_r(t)b(t) \\
x_r(t+1) &= s_r(t)x_r(t), \quad r = 0, 1, 2, \ldots, m - 1.
\end{align*}
\]

(2)

Equation (1) and (2) describe the population development process. \( s_r(t) \), \( s_r(t) \) and \( k_r(t) \) can be got by the statistic data calculation of population census. We find that population policy can control the population development by controlling \( b_r(t) \). Let us make further analysis on \( b_r(t) \). Suppose \( \beta(t) = \sum_{r} b_r(t) \), and \( \beta(t) \) is the total fertility rate, namely, the number of babies born by each woman on average in the year of t. If let \( b_r(t) = \beta(t)h_r(t) \), thereinto, \( \sum_{r} h_r(t) = 1 \), \( h_r(t) \) is called women’s fertility mode, which is r years old female’ fertility weighted factor. The current population policy in China is to realize less birth by changing \( \beta(t) \) and get late childbirth by controlling \( h_r(t) \), and finally to slow down population growth.

Introduce the vector and matrix, let

\[
X(t) = \begin{bmatrix} x_{0}(t) \\ x_{1}(t) \\ \vdots \\ x_{m}(t) \end{bmatrix},
\]

\[
A(t) = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ s_{1}(t) & 0 & \cdots & 0 & 0 \\ 0 & s_{2}(t) & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & s_{m-1}(t) \end{bmatrix},
\]

\[
B(t) = \begin{bmatrix} 0 & \cdots & 0 & b_{1}^{*}(t) & \cdots & b_{m}^{*}(t) & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \end{bmatrix}_{m \times m+1}.
\]

Thereinto, \( b_r^{*}(t) = s_r(t)s_r(t)k_r(t)h_r(t) \), then the population development equation expressed by Equation (1) and (2) can be described as:

\[
X(t+1) = A(t)X(t) + \beta(t)B(t)X(t).
\]

(3)

\( A(t) \), \( B(t) \) and \( \beta(t) \) can be got by calculating the population census data, and then it is not hard to work out the equation (3).

3 Model Establishment and Solution

As Xiangyang has come into aging society, in order to make clearly the future population aging trend, we use population development equation to predict the population in a short period and analyse the change trend of future population aging speed. At the same time, suppose predicting the long-term population development process of Xiangyang in the case of different total fertility rate, and try to seek for a relatively good total fertility rate to make that the fertility rate can control too fast population growth but also ease the population aging condition.

3.1 SHORT-TERM POPULATION AGING TREND ANALYSIS

As the data information provided by the 6th census in Xiangyang is limited, and there are many factors that influence population development, it is relatively difficult to get the rule that mortality rate function and fertility mode changes as the time changes. The population fertility policy will not change in a relatively short period; in the condition of stable society, we usually suppose that mortality rate \( d_r(t) \), fertility mode \( h_r(t) \) and gender ratio \( k_r(t) \) relatively do not change. The data published from the 6th census in Xiangyang shows the total fertility rate of women \( \beta(t) = 1.452 \) at present. Under the supposition above, we use the data of the 6th census in 2010 in Xiangyang as the cardinal number to predict the population in the future five years in Xiangyang. The prediction result is shown as Table 1.
From the prediction result, we can see the population in Xiangyang is still growing. Although the data of the 6th census shows the population fertility rate in Xiangyang is relatively low, the total population still continuously grows because the population cardinal number is very big.

<table>
<thead>
<tr>
<th>Age</th>
<th>2011</th>
<th>2012</th>
<th>2013</th>
<th>2014</th>
<th>2015</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-4</td>
<td>319562</td>
<td>323160.2</td>
<td>326806.4</td>
<td>329764</td>
<td>331130.2</td>
</tr>
<tr>
<td>5-9</td>
<td>250614</td>
<td>264907</td>
<td>279564.7</td>
<td>294580</td>
<td>306350.6</td>
</tr>
<tr>
<td>10-14</td>
<td>262343</td>
<td>246321</td>
<td>241095.2</td>
<td>241843.1</td>
<td>242771.8</td>
</tr>
<tr>
<td>15-19</td>
<td>438958</td>
<td>410846</td>
<td>378771.8</td>
<td>334236</td>
<td>293393.6</td>
</tr>
<tr>
<td>20-24</td>
<td>491697</td>
<td>493552</td>
<td>477170.5</td>
<td>461832.6</td>
<td>457817.9</td>
</tr>
<tr>
<td>25-29</td>
<td>395696</td>
<td>412710</td>
<td>437961.1</td>
<td>465298</td>
<td>475771.9</td>
</tr>
<tr>
<td>30-34</td>
<td>388379</td>
<td>381586</td>
<td>380444</td>
<td>387224.4</td>
<td>393617.3</td>
</tr>
<tr>
<td>35-39</td>
<td>485979</td>
<td>457145</td>
<td>422285.5</td>
<td>391049.8</td>
<td>382742.4</td>
</tr>
<tr>
<td>40-44</td>
<td>530538</td>
<td>536472</td>
<td>55052.8</td>
<td>551842.5</td>
<td>515717.6</td>
</tr>
<tr>
<td>45-49</td>
<td>489769</td>
<td>543013</td>
<td>536654.6</td>
<td>508658.2</td>
<td>514476.1</td>
</tr>
<tr>
<td>50-54</td>
<td>364486</td>
<td>336106</td>
<td>359195.7</td>
<td>397285.4</td>
<td>439060.3</td>
</tr>
<tr>
<td>55-59</td>
<td>364572</td>
<td>374832</td>
<td>380700.9</td>
<td>389189.1</td>
<td>384041.7</td>
</tr>
<tr>
<td>60-64</td>
<td>247700</td>
<td>263389</td>
<td>280934.4</td>
<td>305755.1</td>
<td>328583.5</td>
</tr>
<tr>
<td>65-69</td>
<td>177346</td>
<td>182812</td>
<td>190082.6</td>
<td>201436.9</td>
<td>214760.1</td>
</tr>
<tr>
<td>70-74</td>
<td>121284</td>
<td>126869</td>
<td>135469.9</td>
<td>142383.8</td>
<td>150189.2</td>
</tr>
<tr>
<td>75-79</td>
<td>89102</td>
<td>95630</td>
<td>102164.1</td>
<td>105792.7</td>
<td>101310.5</td>
</tr>
<tr>
<td>80-84</td>
<td>42087</td>
<td>46416</td>
<td>49371.2</td>
<td>52850.4</td>
<td>59597.86</td>
</tr>
<tr>
<td>85-89</td>
<td>19450</td>
<td>20682</td>
<td>21133.56</td>
<td>22086.06</td>
<td>23254.98</td>
</tr>
<tr>
<td>90-94</td>
<td>4380</td>
<td>5219</td>
<td>6098.094</td>
<td>7085.156</td>
<td>8162.724</td>
</tr>
<tr>
<td>95-99</td>
<td>1286</td>
<td>1428</td>
<td>1471.349</td>
<td>1499.016</td>
<td>1556.674</td>
</tr>
<tr>
<td>100 years and above</td>
<td>133</td>
<td>85</td>
<td>137.373</td>
<td>178.8721</td>
<td>21133.56</td>
</tr>
<tr>
<td>Total</td>
<td>5485361</td>
<td>5522256</td>
<td>5557866</td>
<td>5591871</td>
<td>5624487</td>
</tr>
</tbody>
</table>

We can see from the calculation result of population proportion at each age section that the proportions of aged people and children still continuously increase, but the proportion of adults continuously decreases. This is mainly because there were three times’ child-bearing peak after the establishment of the People’s Republic of China, the baby climax caused the population to grow rapidly and these persons born in the child-bearing peak period will come into aged period and thus to cause the growth of aging population; moreover, according to the baby climax occurrence time and statistic data of Xiangyang, we can deduce that the population aging speed in the future in Xiangyang will intensify further.

The condition of not considering population migration, in short time, it is very difficult for us to ease population aging trend through the fertility policy regulation. The population mortality rate is the key that decides the population aging development trend; by the comparison between the urban population and rural population mortality rate of Xiangyang, as shown in Figure 1, we can predict the change direction of population mortality rate of Xiangyang.

From Figure 1, we can see that the mortality rate of urban population is much lower than that of the rural population. Xiangyang is in “Four Xiangyang” construction at present, and the future urbanization degree, people’s living environment and living condition all will be changed a lot, these will further decrease the population mortality rate, and it will definitely cause the population aging further to accelerate. We can see from the prediction and analysis above that the population aging of Xiangyang is an inevitable population problem, and the population aging speed is still continuously intensifying.

3.2 LONG-TERM POPULATION DEVELOPMENT TRENDS ANALYSIS

As the population in China comes into aging society, some experts and scholars take opposite attitudes towards the original only-child policy, and corresponding adjustments on family panning policy are also made by the country and each local government. For example, the couple are both rural residents, and one of the couple is in the family which is only-child family for two generations; one of the couple is a level B, grade two above disabled veteran; the man goes to the woman whose family has only one child to get married and settle; the couple who only has a girl; the couple both are ethnic minorities. The couple are both urban residents, whose first child is disabled and can’t grow up as a normal labour; the couple does not have child and requires to bear child after adopting a child legally; the couple both are from only-child families and other conditions; the couple meets the conditions above all can apply for bearing the second child. The report of the 18th CPC National Congress did not refer to stabilizing low child-bearing level but strengthened completing family planning policy step by step to promote population to develop in a long-term balanced mode; it shows that China has not only...
controlled the population quantity but also paid attention to the population quality and the sustainable population development.

From the short term, the family planning policy will not cause big problems to the population of Xiangyang temporarily, however, from the long term, it will have certain influences on the population structure. In order to seek for a kind of relatively good population fertility policy, we suppose that the other factors do not change in the dozens of years in the future, and predict the population development condition of Xiangyang in the case of different total fertility rate. With the improvement of people’s living quality, the baby cultivation cost in the future will increase gradually; even if in the condition of opening only-child bearing policy, a relative part of people who meet the conditions of bearing the second child do not choose to bear more children, so the total fertility rate at present in China basically is below 2. Following, we will suppose that the total fertility rate $\beta(t)$ is respectively 1.452, 2.0 and 2.5, and analyse and predict the total population and aging population proportion in the future 50 years in Xiangyang in these conditions; the result is shown as Figure 2.

From the analysis result, we can find that it is hard to get a win-win situation between population quantity control and population aging release. The relatively high fertility rate will make the population aging keep at a relatively low level, but will cause the rapid population growth; only when the total fertility rate keeps at two can a relatively good result appear.

4 Summary

This paper predicts the population development process in the future 5 years in Xiangyang by establishing the discrete model of population development equation; the total population in the future 5 years in Xiangyang will continuously increase and the aging speed will further accelerate. The population aging is an inevitable population problem, and several generations of us need to endure and face population aging; we can not only solve it through population policy but should release the economic pressure and social problems caused by population aging by making corresponding old-age supporting policy and social security policy, and regulating economic structure and other ways. In order to seek for a relatively rational population fertility policy, we predict the population development in the future 50 years in Xiangyang in different fertility rate conditions.
We find that the existing population policy will further speed up population aging and increase the pressure caused by population aging. When the total fertility rate of every woman is 2.0, the population development in the future in Xiangyang will be relatively stable, and the population-aging trend will get slow and finally keep at the level of 24%.

This paper predicts the population development process in Xiangyang and analyses the population aging trend in the future. It makes us clearly know the population situation in the face of Xiangyang, and it has important significance to achieving sustainable population development goal and making a rational population policy. In future, we will also analyse the change trend of factors influencing population development, such as population mortality rate, fertility mode and gender ratio and so on, so as to get more an accurate conclusion.

**Acknowledgments**

This work was supported by Educational Commission of Hubei Province of China (D20132601).

**References**


**Authors**

**Qiong Gu**, born on March 31, 1973, Jingmen, Hubei Province

Current position, grades: associate professor

University studies: received the B.Sc. in Accounting from Beijing Institute of Technology in 2002, She received her M.Sc. in Computer Science and Technology and PhD degree in Geosciences Information Engineering from China University of Geosciences, Scientific interest: She research interest fields include data mining, Machine Learning, and internet of things and nonparametric statistics

Experience: She has teaching experience of 5 years, has completed four scientific research projects.

**Wu Zheng**, born on March 14, 1990, Enshi, Hubei Province

Current position, grades: Student

University studies: received the B.Sc. in Mathematics from Hubei University of arts and Science

Scientific interest: His research interest fields include Statistics

Experience: He has teaching experience of 1 years.

**Xianming Wang**, born on November 11, 1979, Huanggang, Hubei Province

Current position, grades: the lecturer of School of Oujiang, Wenzhou University, China.

University studies: received his B.Sc. in Physics from Yunnan University in China. He received his M.Sc. from Chinese Academy of Sciences in China.

Scientific interest: His research interest fields include Web data processing, Web data mining

Publications: 5 papers published in various journals.

Experience: He has teaching experience of 5 years, has completed two scientific research projects.