The property verification methods of complex stochastic system based on directed graph

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Abstract

With the increasing complexity of computer hardware and software systems, how to ensure system accuracy and reliability becomes an increasingly pressing issue. The quantitative verification of multiple until formula property has important practical significance in the field of biology. In this paper, for particular probability reward model, we give the detailed analysis of properties verification methods of the multiple until formula with transition step and transition reward constraints based on the weighted directed graph. At last, the example analysis is given. The theoretical analysis and example result show that the feasibility and validity of the method.

Keywords: Probabilistic system; Model checking; Multiple until formula; Directed graph

1 Introduction

Model checking is a formal verification technique with higher degree of automation and is an efficient automatic detecting mean as it can provide the counterexample information when the properties are violated. Model checking has obvious advantages compared with other formal verification means.

In recent years, along with the application range of traditional model checking technology used for qualitative verification continues to be expanded, the properties quantitative validation technology based on complex parametric probabilistic model become one of the research topics that many experts and scholars interested, and gradually become new hotspot in trusted system verification field.

In the process of model checking, probabilistic computation tree logic formula and continuous stochastic logic formula are generally used to describe the property of system. On these logic formula, multiple until formula can describe the periodic oscillations changes of biological species and other important characteristics in the field of systems biology, which makes high-performance verification and counterexample analysis about this kind of property become an open research topic [1, 2].

In the literature [3], stratified continuous time Markov chain model is used to explain the verification process of multiple until formula property with time boundaries, the corresponding verification algorithm and example analysis process are proposed by constructing synchronous automata model of formula property automatam and continuous time Markov chain model.

In the literature [4], proposes an algorithm on counterexample generation for model checking probabilistic timed automata based on the weighted directed graph.

In the literature [5], the semantic representation of the probabilistic timed automata was gave by Markov decision processes, and the until formula counterexample generation algorithm and representation method were proposed.

Although domestic and foreign scholars have made a lot of works in the field of until formula properties verification, however most of the existing research are oriented to the until formula with time constraint and rarely study until formula properties verification with the reward constraints. In view of this, the paper will give the detailed analysis of properties verification methods of multiple until formula with transition step and transition reward constraints in Markov reward model.

The rest of paper is organized as follows: Section 2 presents the necessary background on probability model with reward parameter. In Section 3 introduces the syntax and semantics of multiple until constraint formula. Section 4 explains how to verify multiple until constraint formula based on the digraph. Section 5 gives the corresponding example description. Section 6 gives the basic algorithm description. Finally, in Section 7 we give the conclusions and directions of future research.
2 Probabilistic reward model

In this paper, we study properties verification method of multiple until constraints formula with transition step constraint and transition reward constraint for Markov Reward Model (MRM). Below the definition of MRM is described.

**Definition 1.** MRM is six tuple that is divided into discrete time Markov reward model (DTMRM) and continuous time Markov reward model (CTMRM). DTMRM and CTMRM are expressed as \( M = (S, P, L, AP, \nu) \) and \( M = (S, R, L, AP, \nu) \) respectively, where \( S \) is a finite set of states, \( AP \) is a set of atomic propositions, \( L: S \to 2^{AP} \) is a labelling function that assigns to each \( s \in S \) a set \( L(s) \) of atomic propositions, and \( R: S \times S \to R_{\geq 0} \) is a rate matrix, \( \nu \in \text{Distr}(S) \) is the initial distribution set and \( N: S \times S \to R_{\geq 0} \) is transition reward function.

The transition reward of MRM is often used to represent different characteristics of the system model, such as storage space, network bandwidth, message number of successfully transmitted, power consumption, negotiation procedure and so on.

In the CTMRM, the transition probability from state \( s \) to its some successor state \( s' \) within \( t \) meet the exponential distribution and the value is \( \frac{R(s, s')}{E(s)} \left(1-e^{-E(s)t}\right) \), where \( E(s) \) indicates sum of transition rate of the state \( s \) and \( E(s) = \sum_{s' \in S} R(s, s') \).

Because of the multiple until constraint formula studied in this paper is unrelated to the transition time, so before we verify the properties, the transition probability in CTMRM need to be pretreated, then the transition rate matrix of CTMRM is transformed into the discrete transition probability distribution of DTMRM, denoted by \( P_u \). This pretreatment process named as discretization for continuous model.

The transformation method that transforms \( R \) of CTMRM into transition probability distribution \( P_u \) of the discrete model is divided into two cases:

1. If \( E(s') = \sum_{s' \in S} R(s', s'') \neq 0 \) for some state \( s' \) in CTMRM, then in DTMRM we have \( P_u(s', s') = \frac{R(s', s')}{E(s')} \), and \( P_u(s', s') = 1 - \sum_{s'' \in S - s'} P_u(s', s'') \).

2. If \( E(s') = \sum_{s' \in S} R(s', s'') = 0 \) for some state \( s' \) in CTMRM, then in DTMRM we have \( P(s', s'') = 0 \) for all states \( s'' \in S \).

Below the properties, quantitative validation process of DTMRM will be shown.

3 The description of multiple until constraint formula

In the process of probabilistic model checking, the temporal logic applied in discrete probability model mainly includes PCTL and its various variants logic, in which the path formula supported transition reward and transition step is defined as:

\[ \varphi := \Phi[\varphi_1 \land \varphi_2] - \nu \bigcup_{n=1}^{\infty} \varphi_1 \bigcup_{n=1}^{\infty} \varphi_2. \]

where \( n \in [0, \infty] \) and \( r \in [0, \infty] \) respectively represent transition step interval and transition reward interval constraint, \( \Delta \in \{>, <, \geq, \leq\} \) be arithmetic comparison operator, \( \bigcup \) be until operator, \( O \) be next operator and \( \Phi \) indicates the corresponding state formula.

The path formula \( \varphi_1 \bigcup_{n_1}^{n_2} \varphi_2 \) is called single until constraint formula, and \( \varphi_1 \bigcup_{n_1}^{n_2} \varphi_2 \bigcup_{n_2}^{n_3} \varphi_3 \) is called double until constraint formula and \( \varphi_1 \bigcup_{n_1}^{n_2} \varphi_2 \bigcup_{n_2}^{n_3} \varphi_3 \bigcup_{n_3}^{n_4} \varphi_4 \) \( (k \geq 3) \) are collectively known as multiple until constraint formula.

Let \( \sigma \) be a path, then \( \sigma = [\varphi_1 \bigcup_{n_1}^{n_2} \varphi_2 \bigcup_{n_2}^{n_3} \varphi_3 \bigcup_{n_3}^{n_4} \varphi_4] \) established if and only if there are a series of transition processes so that: All states in the transition processes at most \( \text{Sup}(n_1) \) steps satisfy the property \( \varphi_1 \) and the cumulative sum of transition reward during these transition steps satisfies condition \( r_1 \), and then the path migrate to one state that satisfies \( \varphi_2 \). On the Next transition processes, all states at most \( \text{Sup}(n_2) \) steps satisfy the property \( \varphi_2 \) and the cumulative sum of transition reward during these transition steps satisfies condition \( r_2 \), and then the path migrate to one state that satisfies \( \varphi_3 \), and so on. Final, the transition processes end in the last state of \( \sigma \), and the state satisfies \( \varphi_k \) \( (\text{Sup}(n)) \) denotes the upper bound of interval \( n \).

As we can be seen from the semantic analysis of \( \varphi_1 \bigcup_{n_1}^{n_2} \varphi_2 \bigcup_{n_2}^{n_3} \varphi_3 \bigcup_{n_3}^{n_4} \varphi_4 \) that multiple until constraint formula \( \varphi_1 \bigcup_{n_1}^{n_2} \varphi_2 \bigcup_{n_2}^{n_3} \varphi_3 \bigcup_{n_3}^{n_4} \varphi_4 \) is the superposition of single until constraint formulas and the number of superposition depends on the value of \( k \).

For the calculation of the satisfying probability of \( \varphi_1 \bigcup_{n_1}^{n_2} \varphi_2 \bigcup_{n_2}^{n_3} \varphi_3 \bigcup_{n_3}^{n_4} \varphi_4 \), if path \( \sigma \) is starting at the state that need to be verified of the system model and can be segmented into several sub paths and each sub path
The properties verification method based on directed graph

Because of the basic expression, form of the complex stochastic system model is state transition system of directed graph form, so we can verify the properties directly based on the state transition graph of the system model aiming at the characteristics of the formula to be verified.

In this paper, we consider the until formula property verification problem with the transition step constraint and transition reward constraint. In literature [6], the problem of generating counterexample single of until formula with time constraint for CTMC is described in detail and an approximation algorithm of minimal counterexample path set is given based on the structure of UDTMRM.

In ref. [7], the solving method of state satisfying probability of single until formula \( \varphi \cup \mathcal{J}_k \psi \) with transition step constraint for DTMC is introduced in detail.

The basic idea is as follows: First, the original system model is appropriate pre-processed and is transformed into a weighted directed graph in which weight information indicates the transition probability of the original model and it is inversely proportional to the transition probability. In directed graph, the path that transition weight is least called the strongest evidence path and the process of finding the strongest evidence path is called shortest path (SP) problem. Then the solve problem of counterexample path of \( \varphi \cup \mathcal{J}_k \psi \) formula in DTMC model is transformed into the solve problem of \( k \) shortest path set with transition step constraint for corresponding vertices based on weighted directed graph.

In this paper, the solving method of state satisfying probability for multiple until constraint formula \( \varphi_1 \cup_{i=1}^k \varphi_2 \cup_{i=1}^k \varphi_k \) will use the method similar to the above.

**Definition 2.** Weighted directed graph for DTMC

For DTMC \( M = (S, AP, L, P) \), the weighted directed graph \( D = (V, E, w) \), where vertex set \( V = S \), edge set \( E = \{(v, v') | v, v' \in S \land P(v, v') > 0\} \) and transition weight \( w = -\log(v, v') \).

**Definition 3.** SP problem

Given a weighted directed graph \( D = (V, E, w) \) and \( s, t \in V \), the SP problem is to determine a path \( \sigma \) from \( s \) to \( t \) such that \( w(\sigma) \leq w(\sigma') \) for any path \( \sigma' \) from \( s \) to \( t \) in \( D \). If the length of path \( \sigma \) and \( \sigma' \) satisfy \( |\sigma| \leq h \) and \( |\sigma'| \leq h \), then corresponding problem is known as the hop constrained shortest path (HCSP), which is a special case of the constrained shortest path (CSP) problem.

For the solution of SP problem, Bellman and other scholars proposed various efficient algorithms, such as Bellman-Ford algorithm and so on [8, 9]. The strongest evidence path for single until formula \( \varphi \cup \mathcal{J}_k \psi \) can be solved in time \( O(m + n \log(n)) \) where \( m \) is the number of the edges and \( n \) is the number of the vertexes. For \( \varphi \cup \mathcal{J}_k \psi \), the single constraint SP problem can be solved in time \( O(hm) \) when \( h < n - 1 \).

**Definition 4.** K shortest path (KSP) problem

Given a weighted directed graph \( D = (V, E, w) \) and \( k \in \mathbb{N}_0 \), the KSP problem is to find \( k \) distinct paths \( \sigma_1, \sigma_2, \ldots, \sigma_k \) between \( s \) and \( t \) in \( D \) such that \( w(\sigma_i) \leq w(\varphi_j) \) for \( 1 \leq i \leq j \leq k \) and for every path \( \sigma \) between \( s \) and \( t \), if \( \sigma \in \{\sigma_1, \sigma_2, \ldots, \sigma_k\} \), then \( w(\sigma) \leq w(\sigma_i) \).

Thus, the solution of smallest counterexample path set for formula \( \varphi \cup \mathcal{J}_k \psi \) can be converted to the KSP problem and the solution of smallest counterexample path set for formula \( \varphi \cup \mathcal{J}_k \psi \) can be converted to the HKSP problem. In literature [10], the solution algorithm of HKSP for formula \( \varphi \cup \mathcal{J}_k \psi \) of DTMC model can be solved in time \( O(hm + k \log(m/n)) \).

The properties validation method of multiple until constraint formula in this paper will use the above algorithm and has obvious differences with the existing research works in the following two aspects: On the one hand the object model is processed is different, on the other hand, the property need to be checked is different.

Therefore, we must solve the following two problems:

First, how to transform the DTMRM model for multiple until constraint formula \( \varphi_1 \cup_{s_1} \varphi_2 \cup_{s_2} \ldots \cup_{s_{n-1}} \varphi_k \) into weighted directed graph and how to represent the transition probability and transition reward.

Second, how to describe verification algorithm of multiple until constraint formula based on the weighted directed graph.

Below, we give the detailed description of how to transform DTMRM model into weighted directed graph, which is divided into two steps:

**Step 1:** Adapting the DTMRM

First, we make all states in the DTMRM that not satisfy \( \varphi_i \) (\( i = 1 \ldots k \)) absorbing. Then we remove all transitions that satisfy one of the following conditions:
The formal description of condition one is:
\[ \exists k \geq m > n \geq 1 \ s.t. s = Sat(\phi_n) \wedge s' \in Sat(\phi_k) \] and
\[ \neg \exists k - 1 \geq q \geq 1 \ s.t. s = Sat(\phi_q) \wedge (s' \in Sat(\phi_q) \vee s' \in Sat(\phi_{q+1})) \]

The formal description of condition two is:
\[ \forall k > n \geq 1 \ s.t. s, s' \in Sat(\phi_k) \wedge s, s' \in Sat(\phi_k) \]

Next, we add an extra state \( t_{i-1} \) for each \( \phi_i (i \neq k \wedge i > 1) \) state so that each outgoing transition from \( \phi_k \) state is replaced by the transition to \( t_{i-1} \) with probability 1, reward 0 and a numbered transition act and all transitions to \( t_{i-1} \) can be distinguished by the name of these transition acts. Meanwhile, we add the corresponding outgoing transitions for each \( t_{i-1} (i \neq k \wedge i > 1) \) state so that each outgoing transition from \( t_{i-1} \) is transmitted to one of the states that can be transmitted from previous \( \phi_i (i \neq k \wedge i > 1) \) state with the same probability, reward and a numbered transition act that is same with one specific transition to \( t_{i-1} \).

At last, we add an extra state \( t_{k-1} \) so that all outgoing transitions from a \( \phi_k \) state are replaced by a transition to \( t_{k-1} \) with probability 1 and reward 0.

The obtained DTMRM denoted as \( M' = (S', AP', L', P', N') \), where state space \( S' = S \cup \{ t_1, \ldots, t_{i-1} \} \), atomic propositions set \( AP' = AP \cup \{ a_{t_1}, \ldots, a_{t_{i-1}} \} \), labelling function \( L'(t_i) = \{ a_{t_i} \} \) and \( L'(s) = L(s) \) for \( s \in S' \wedge s \neq t_i \).

The probability matrix is divided into the following situations:
- If \( s \in Sat(\phi_i) \) then \( P'(s, t_{i-1}) = 1 \) and \( N'(s, t_{i-1}) = 0 \).
- If \( s \notin \bigcup_{i=1}^k Sat(\phi_i) \) then \( P'(s, s) = 1 \).

If \( s \in Sat(\phi_j) \) but \( i \neq j \) then \( P'(s, t_{i-1}) = 1 \wedge N'(s, t_{i-1}) = 0 \wedge P'(t_{i-1}, s') = P'(s, s') \) and \( N'(t_{i-1}, s') = N'(s, s') \), if \( s \in Sat(\phi_j) \) but \( i = j \) then the added orderly numbered transition acts can be denoted as \( act(s, t_{i-1}) = act(t_{i-1}, s') \) when \( P(s, s') > 0 \).

After adjusted, the path \( \sigma \) that satisfies \( \phi_1 \cup_{s_1} \phi_2 \cup_{s_2} \ldots \cup_{s_{i-1}} \phi_k \) in original model \( M \) is turned into the path \( \sigma' = \sigma_1 \cdot t_1 \cdot \sigma_2 \cdot t_2 \cdot \ldots \cdot \sigma_{k-1} \cdot t_{k-1} \) in the model \( M' \) and the corresponding multiple until constraint formula to be verified is turned into \( \phi_1 \cup_{s_1} a_{t_1} \cup_{s_2} a_{t_2} \ldots \cup_{s_{i-1}} a_{t_{i-1}} \) where \( \sigma = \sigma_1 \cdot \sigma_2 \cdot \ldots \cdot \sigma_{k-1} \).

**Step 2:** Converting model into a weighted directed graph

The DTMRM model \( M' \) obtained in the first step can be transformed into a weighted directed graph by definition 3. The only difference with the definition 3 is that the weight information in the weighted directed graph of multiple until constraint formula
\[ \phi_1 \cup_{s_1} \phi_2 \cup_{s_2} \ldots \cup_{s_{i-1}} \phi_k \] not only contains the transition probability weight, but also contains the transition reward weight.

Definition 6 Weighted directed graph for DTMRM
For DTMRM \( M' = (S', AP', L', P', N') \), the weighted directed graph \( D = (V, E, w) \), where vertex set \( V = S' \), edge set \( E = \{ v, v' | v, v' \in S' \wedge P'(v, v') > 0 \} \). The weight set of \( D \) denoted as \( W = \{ w_p, w_N \} \) \( w_p \in W_p \wedge w_N \in W_N \) where transition probability weight \( W_p = \{ w_p | w_p = -\log(P'(v, v')) \} \) and transition reward weight \( W_N = \{ w_N | w_N = N'(v, v') \} \).

In the weighted directed graph of DTMRM, the computation method of transition reward weight is
\[ w_N(\sigma) = \sum_{i=0}^{n-1} w_N(v_i, v_{i+1}) \] and the computation method of transition probability weight is:
\[ w_p(\sigma) = \sum_{i=0}^{n-1} w_p(v_i, v_{i+1}) = \sum_{i=0}^{n-1} -\log(P'(v_i, v_{i+1})) \]
\[ = -\sum_{i=0}^{n-1} \log(P'(v_i, v_{i+1})) = -\log(P'(-)) \]

**5 Case constructor**

Consider the DTMR which include six states and be shown in Fig. 1 and multiple until formula \( a \cup b \cup c \) the model after pre-processed be depicted in Fig. 2 and the corresponding weighted directed graph be depicted in Fig. 3.

The satisfaction sets of atomic propositions are \( Sat(a) = \{ s_0, s_1, s_2 \} \) , \( Sat(b) = \{ s_1, s_3, s_4 \} \) and \( Sat(c) = \{ s_2, s_3 \} \).

![Figure 1. The DTMR model to be verified](image-url)
6 Algorithm Description

In this paper, the satisfying path of formula \( \varphi_1 \cup \varphi_2 \cup \ldots \cup \varphi_k \) can be considered as the connection of paths, in which each path satisfies single until formula.

Then the satisfying path set and transition probability of formula \( \varphi_1 \cup \varphi_2 \cup \ldots \cup \varphi_k \) can be calculated by limiting satisfying path set of single until constraint formula with transition reward constraint on the basis of the HKSP algorithm.

Next, we roughly give satisfying path set algorithm of single until formula with transition step constraint and transition reward constraint. The algorithm named hop & reward constrained shortest paths (HRCSP), in which \( h \) denotes the upper bound of the transition step constraint, \( r \) denotes the upper bound of the transition reward constraint, \( \pi^r_s \) denotes the i-th shortest path from \( s \) to \( t \) so that the length of path is less than or equal to \( h \) and the sum of transition reward is less than or equal to \( r \).

Algorithm HRCSP(s,t,h,r)

- **Require**: weighted digraph \( D \), state \( s \) and state \( t \), and \( h \in N_{\geq 0}, r \in R_{\geq 0} \)
- **Ensure**: \( C = \{ \pi^r_{st} \} \) with all \( P(\pi^r_{st}) > 0 \)

1. compute \( \pi^r_{st} \) by BF:
2. \( k = 1 \);
3. \( pr = P(\pi^r_{st}) \);
4. While \( ( pr > 0 \& \& TRCSD(r)) \) do
5. \( \pi^r_{st} = \pi^r_{st} \)
6. \( k = k + 1 \);
7. \( \pi^r_{st} = NextPath(t,h,k) \);
8. **End While**
9. return \( \{ \pi^r_{st} \} \).

In HRCSP algorithm, function TRCSD denotes transition reward constrained satisfaction decision algorithm, which is mainly used for judge the path satisfaction for transition reward constraint.

The algorithm TRCSD is described as follows:

Algorithm TRCSD(\( r \))

- **Input**: finite path \( \rho \), reward intervals upper bound \( r \in R_{\geq 0} \)
- **Output**: true if the cumulate reward \( r \leq r \), false otherwise
1. \( s = first(\rho); total = 0 \);
2. While \( ((s') = Next(\rho,s)) \) do
3. \( totalr = totalr + N(s,s') \);
4. \( s = s' \);
5. If \( (totalr > r) \) return false;
6. **End While**
7. return true;

7 Conclusions and Future Research

In this paper, for property verification problem of multiple until constraint formulae, we put forward the construction method of weighted directed graph of MRM and the corresponding solution algorithms. The case constructor can show that the algorithms and the method are effective. In the further, we will further study the optimization problem of algorithms and extend the method in this paper to other probabilistic models.

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