A NEW LEARNING STRUCTURE OF ADAPTIVE ALGORITHM FOR DIGITAL PREDISTORTION

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Adaptive digital predistortion is a dominant technique for the linearization of power amplifier. The learning structure of adaptive algorithm is a major determinant of linearization performance. Indirect structure is widely used in due to its easy implementation and relatively good performance. In this paper, a new learning structure combined with both indirect and direct is proposed. Simulations show that this joint method can avoid feedback and system noise, hence achieve a significant improvement on linearization performance over indirect structure.

Keywords: Power Amplifier; Predistortion; Learning Structure; Linearization performance.

1. Introduction

It is well known that power amplifier (PA) is one of the key parts for wireless communication system. Signals, especially with high peak-to-average power ratio (PAPR), will always produce nonlinearity, thus causing spectral broadening as well as in-band distortion when passing through PA. Linearization technology, therefore, is critical to suppress spectral regrowth, especially in-band distortion.

Among all the linearization techniques, digital predistortion is one of the most cost effective due to its stability, adaptivity and ease of implementation. On the other hand, to adapt to the varying nonlinear characteristics of power amplifiers, adaptive algorithm is indispensable and plays the core role in digital predistortion system.

Adaptive algorithm is deployed to calculate coefficients of PA model with feedback signals from PA and original signals i.e. parameter estimation and then adjust the so-called predistorter’s parameters to cancel the nonlinearity of PA. Least Mean Square (LMS) and Recursive Least Square (RLS) are conventional adaptive algorithm. Both of them can be used as adaptive algorithm and have the approximate performance. Therefore, the learning structure is a major factor that determines the linearization performance. There are two conventional learning structures of adaptive algorithm currently, named direct and indirect learning structure. The former structure has the advantage of simplicity and anti-noise, but the classical adaptive learning algorithm cannot be used directly for that the power model is always unknown. Indirect learning structure is easy for implementation, while it is vulnerable to noise.

Therefore, this paper concentrates on the new learning structure of adaptive algorithm [1] that is based on these two learning structures. Simulations show that compared with the indirect learning structure, this proposed joint structure can not only reduce the noise from feedback path, but also achieve a desirable linearization performance.

2. Predistortion Theory

The nonlinear characteristics of PA can be presented by their amplitude modulation to amplitude modulation conversion (AM/AM) and amplitude modulation to phase modulation conversion (AM/PM). The predistorter, whose coefficients are adaptively derived from the adaptive algorithm, is in charge of producing anti-phase and anti-amplitude curves to the PA model, i.e. inverse to the AM/AM and AM/PM characteristics of PA, thus compensating the nonlinearity [2].

As seen in Figure 1, in a transmitting system, predistorter with characteristic function $F(|v_e|)$ is placed before the nonlinear power amplifier. The characteristic function $G(|v_d|)$ represents the nonlinear characteristics of power amplifier. Observe that both $F(|v_e|)$ and $G(|v_d|)$ depend only on the power (or
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amplitude) of input signals, but not on their input phases. By appropriately choosing the coefficients of $F(V_i)$ through adaptive algorithm, making $F(V_i)$ and $G(V_d)$ mutually inverse, then an equitant linear amplification will be achieved as follows:

$$F(V_i) \cdot G(V_d) = K.$$  \hspace{1cm} (1)

where $K$ is constant of system gain. The output signal $V_o = KV_i$ is then be linearly amplified when passing both predistorter and the power.

3. Predistortion Learning Structure

The architectures of adaptive algorithm for digital predistortion are divided into two types. One is the direct learning architecture [3], which must identify nonlinear characteristics of PA and then find the inverse characteristics of PA, which is of high computational complexity. Another type is the indirect learning architecture [4], which does not need to find inverse nonlinear characteristics of PA. Therefore, the indirect learning architecture is more attractive in the adaptive algorithm.

3.1. Direct Learning Structure

A direct adaptive predistortion block diagram is shown in Figure 2. In this structure, $x(n)$ and $y(n)$ represent input and output signal, respectively. At the same time, supposed the desired response of system is $d(n)$ and linear magnification equals to $G$. If $e(n) = d(n) - y(n)$ in the algorithm converges, i.e. $e(n) = 0$, then the output of the amplifier is linear to input signal, and $y(n) = G \cdot x(n)$.

![Figure 2. Direct learning structure of predistortion](image)

Direct learning structure has the advantages of simple structure, after the algorithm, converging can achieve better predistortion effects, and predistortion coefficients are not affected without noise of amplifier nonlinear system. However, its drawback is cannot directly use the classical adaptive learning algorithm.

3.2. Indirect Learning Structure

If the coefficients of the predistortion are adaptive recursive estimation directly, the amplifier model is obtained first, so in order to avoid getting the power amplifier model, the indirect learning structure predistorter is used as shown in Figure 3. The benefits of indirect learning structure are we need not to identify the amplifier model first, predistorter model can be identified directly, and the structure is simple. $G$ is the linear magnification of amplifier in Figure 3, the amplifier output $y(n)$ is the input to the predistorter training network after attenuated $G$ times, the actual predistortion module is copy the

![Figure 3. Simplified predistorter schematic](image)
parameters of training network module. In the ideal case, \( y(n) = G \cdot x(n) \) is desirable, which \( z(n) = \hat{z}(n) \) is required, the target of estimation algorithm is to calculate the parameters of estimation module and then pass to the predistortion module.

\[
\begin{align*}
    y(n) &= G \cdot x(n) \\
    z(n) &= \hat{z}(n) \\
    e(n) &= y(n) - z(n)
\end{align*}
\]

\[
\begin{align*}
    \text{Training Network} \\
    \text{PD} \\
    \text{PA} \\
    \text{U/G}
\end{align*}
\]

\[\text{Figure 3. Indirect learning structure of predistortion}\]

Indirect learning structure can use the classical adaptive learning algorithm and is easy to implement in engineering. However, its drawback is the addition noise of amplifier nonlinear system can make the parameters deviate from the optimum value [5], so that the predistortion linearization effects become poor.

3.3. New Predistortion Learning Structure

Indirect learning structure is simple and easy for implementation, but its drawback is susceptible to additive noise. It is natural to suppose that if indirect structure is combined with direct, this new joint structure could contribute to improvement on linearization performance.

This new learning structure proposed is shown as Figure 4. Firstly, the switch \( S \) is set to the A-side, and then indirect learning structure of adaptive algorithm 1 in the dotted line frame will work. Since indirect learning structure is susceptible to additive noise, adaptive algorithm in this structure cannot achieve optimal parameters of PA model, although it is possible close to the best parameters. Since direct learning structure has the advantage of anti-noise, it can be used to amend the parameters based on indirect learning structure. When the adaptive algorithm 1 converges, switch \( S \) is set to the B-side and the algorithm 2 takes effect using convergent parameters of algorithm 2 as the initial value. Because the parameters derived from indirect learning structure is not within the optimal parameter area, the adaptive algorithm 2 can directly use the classical adaptive algorithm. The parameters will be more close to the optimal parameters of PA model through algorithm 2.

\[\text{Figure 4. New predistortion learning structure}\]

4. Adaptive Predistortion Algorithm

The adaptive algorithm is a key technology of predistortion. As for adaptive linearization technique, the core issue is how to get and automatic adjustment function parameters. The study of
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adaptive part, this can be adaptive abstracting and using numerical analysis method to solve the problem in different ways to achieve the three elements of adaptive algorithm: the minimization algorithm, the objective function form and the error function, the final purpose is to make the objective function minimize which is based on the error function $e(n)$, in a manner such that the output signal approximates the expected output signal.

Matrix least square (LS) linear equation is given by as follows:

$$Z = UA,$$  \hspace{1cm} (2)

where $Z$ and $A$ represents $[z(n), z(n-1), \ldots, z(n-L+1)]^T$ and $[a_{10}, \ldots, a_{(2k+1)0}, a_{11}, \ldots, a_{(2k+1)1}]^T$ respectively.

For memory polynomial model, $U$ is a coefficient matrix $A$ with corresponding equations as follows:

$$U = (Y_{10}, \ldots, Y_{(2k+1)0}Y_{11}, \ldots, Y_{(2k+1)1})$$

$$Y_{ij} = [r_{ij}(n), r_{ij}(n-1), \ldots, r_{ij}(n-L+1)]^T.$$  \hspace{1cm} (3)

The solution of Equation (3) can be obtained by the following steps:

$$U^H A = U^H Z, W = U^H U, \text{ then }$$

$$V = WA.$$  \hspace{1cm} (5)

$$A = W^{-1} V.$$  \hspace{1cm} (6)

So $A$ is the best estimate LS under the length $L$ of the sample data in Equation (6).

Since the Equation (6) involves complex matrix inversion problem, large amount of calculation and difficulty to implementation, the matrix $U$ must be decomposed into the matrix composition, which are relatively simple.

Singular value decomposition (SVD) of the matrix is used to complete the least square algorithm. SVD is an important matrix factorization of linear algebra and is widely deployed in many fields such as signal processing and statistical. Eigen value decomposition is an efficient method to extract features of matrix, but its drawback is applicable only for the square. SVD method, however, can be applied to an arbitrary matrix, regardless of the value of the matrix being a real number or a complex number.

It is obvious that if the order of a matrix $U$ is $M \times N$, then $U$ can be derived from two unitary matrices $W$ and $V$ which are orthogonal matrices with $\Sigma$ which is a diagonal matrix and has the same dimensions of matrix $U$, i.e., $U$ can be written as:

$$U = W \Sigma V^T,$$  \hspace{1cm} (7)

which leads to:

$$\|Ua - z\| = \|W \Sigma V^T a - z\|,$$

$$= \|W(\Sigma V^T a) - W(\Sigma V^T c)\|,$$

$$= \|W(\Sigma V^T(V^T a - c))\|. \hspace{1cm} (8)$$

For $W$ is an orthogonal matrix, we have that $\|Ua - z\| = \|\Sigma y - c\|$. Next step is to seek the smallest $y$, which makes $\|\Sigma y - c\|$ minimum. It can be simply achieved in that $\Sigma$ is a diagonal matrix [6].

Assuming the rank of matrix $U$ is $r$, then the corresponding equation is yielded as follows:

$$\Sigma y = \begin{bmatrix} \sigma_1 y_1 - c_1 \\ \vdots \\ \sigma_r y_r - c_r \end{bmatrix}, \Sigma y - c = \begin{bmatrix} -c_{r+1} \\ \vdots \\ -c_n \end{bmatrix}. \hspace{1cm} (9)$$
Thus, \( y_i = \frac{c_i}{\sigma_i}, \) \((i = 1, 2, \ldots, r)\) makes \( \Sigma y - c \) reaching its minimum length \( \left[ \sum_{i=1}^{n} c_i^2 \right]^{1/2} \). When \( r = m \) and the column of the matrix \( U \) is \( m \), the least squares problem can be solved without error.

The matrix transposing \( \Sigma \) and getting the inverse of the nonzero diagonal elements of matrix is defined as \( \Sigma^{-1} \), then the \( r \) elements of \( y = \Sigma^{-1} c \) will be equal to \( \frac{c_i}{\sigma_i}, \) \((i = 1, 2, \ldots, r)\), and the rest of elements to zero. At the same time, since \( y = V^T x, c = W^T z \), \( x \) can be computed from the following Equation:

\[
x = V\Sigma^{-1}W^T z.
\]

As a result, the minimum norm of LS algorithm can be obtained using Equation 10.

5. Predistorter Framework

Adaptive digital predistortion structure has two main categories based on lookup table LUT [7] or polynomial digital adaptive predistortion method.

Predistorter that is based on lookup table structure is using one or more lookup tables to represent the nonlinear relationship of predistorter. The basic principle is that the appropriate predistortion coefficients are stored in terms of table. Input signals are processed according to the index of the table to find the corresponding predistortion coefficients for computing, and the results are output to the successor circuits to achieve the purpose of correction of nonlinear power amplifier. At the same time, through a feedback path, the contents of the lookup table are updated dynamically by adaptive algorithm.

The method based on LUT is easy to implementation, but the algorithm converges relatively slowly, and a lot of memory cells are required, which also increases the resource cost. Polynomial linearization method is better than the lookup table. It has many advantages: fast convergence, requires less storage unit. The design of the predistorter structure is also use the method of the memory polynomial. Predistortion based on polynomial model mainly deploys some form of function, calculating the numerical, which is the original input signal through predistorter. It needs only to adjust the number of the polynomial coefficients, and bears a faster convergence rate. This method concentrates more on the issue of saving RAM resources in a lookup table.

Predistortion has three common memory polynomial models: Memory Polynomial/ Wiener model (MP/W), Memory Polynomial (MP) and Generalized Memory Polynomial (GMP).

5.1. MP Model

Expression of MP model is given by:

\[
z_{MP}(n) = \sum_{k=0}^{K} \sum_{m=0}^{M-1} a_{km} x(n-m) |x(n-m)|^k.
\]

(11)

Memory polynomial predistortion is significantly effective for the actual power amplifier model [8]. The essence of the polynomial predistortion is the nonlinear approximation problems of power amplifier model, and the core of polynomial nonlinear approximation depends on the structure used. The existing structure is based on integer powers, without taking into consideration fractional power. Hence, an improved model of fractional order proposed to extend polynomial design and improve predistortion performance [9].

Fractional order memory polynomial can be expressed as:

\[
z_{FMP}(n) = \sum_{k=0}^{K} \sum_{m=0}^{M-1} a_{km} x(n-m) |x(n-m)|^{\frac{\beta(k+1)}{2}}.
\]

(12)

5.2. MP/W Model

MP/W model with cross terms is given below:

\[
z_{MPW}(n) = \sum_{k=0}^{K} \sum_{m=0}^{M-1} a_{km} x(n-m) |x(n-m)|^k
+ \sum_{k=1}^{K} b_k x(n) \sum_{m=0}^{M-1} c_{km} |x(n-m)|^k.
\]

(13)
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Coefficients $a_{km}$ and $b_k$ are estimated using the least squares method, and the coefficients $c_m$ obtained through the Newton iterative method [10]. Its convergence, however, is unstable in that it is susceptible to initial coefficient values. Therefore, the cross terms of polynomial introduction although theoretically can improve the predistortion effect, but the coefficients estimation is not robust enough.

5.3. GMP Model

Expression of GMP model is given by [9]:

$$z_{GMP}(n) = \sum_{k=0}^{K} \sum_{l=0}^{L} a_{kl} x(n-l)x(n-l)^{k-1} + \sum_{k=0}^{K} \sum_{l=0}^{L} b_{kl} x(n-l)x(n-l-m)^{k} + \sum_{k=0}^{K} \sum_{l=0}^{L} c_{kl} x(n-l)x(n-l+m)^{k}.$$  (14)

It can be seen that GMP model expression is so cumbersome that huge computation and complex algorithm are the limitations of coefficient estimation.

In summary, the MP model is most commonly applied due to its easy implementation and good approximation to practical PA model.

6. Simulation and Analysis

To verify the linearization performance of the new joint learning structure compared with the indirect, simulations are carried out in terms of the improvement on power spectrum density (PSD).

In this simulation, original signals use orthogonal frequency division multiplexing (OFDM) signals with bandwidth of 8MHz. The PA model is based on memory polynomial. Expression of a frequently-used memory polynomial is given by:

$$y(n) = \sum_{k=0}^{K} \sum_{l=0}^{L} a_{kl} x(n-l)x(n-l)^{k-1}.$$  (15)

Here:

$$a_{10} = 1.0513 + 0.0904j, \quad a_{50} = -0.0542 - 0.2900j, \quad a_{50} = -0.9657 - 0.7028j, \quad a_{11} = -0.0680 - 0.0023j, \quad a_{11} = -0.2234 + 0.2317j, \quad a_{51} = -0.2451 - 0.3735j, \quad a_{12} = 0.0289 - 0.0054j, \quad a_{12} = -0.0621 - 0.0932j, \quad a_{32} = 0.1229 + 0.1508j$$

The comparisons between joint and indirect structure on linearization performance of PSD are portrayed in Figure 5 and Figure 6.
It can be seen from Figure 5 and Figure 6 that both indirect learning structure and the new structure of the feedback signal have similar linearization performance on PSD in case of noise-free condition.

At the same time, in the case of a certain extent of noise, as seen in Figure 7 and Figure 8, the performance between joint and indirect structure show a difference on improvement on PSD. The former has a significant advantage over the latter. This suggests that the new joint learning structure bears a strong anti-noise characteristic, and hence, better linearization performance for PA than the indirect learning structure.
7. Conclusions

In this paper, a joint structure which uses indirect with direct for linearization of PA is presented. Simulations show that in the case of other condition being the same, this new structure of adaptive algorithm can avoid the noise interference, and therefore, obtaining a significant improvement over indirect learning structure.

Acknowledgements


References


Received on the 3rd of November 2013