CORRECTIVE MAINTENANCE AND RELIABILITY ASSOCIATED COST ESTIMATION OF AGING MULTI-STATE SYSTEMS

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This paper considers corrective maintenance contracts for aging air conditioning systems, operating under varying weather conditions. Aging is treated as an increasing failure rate. The system can fall into unacceptable states for two reasons: through performance degradation because of failures or through an increase in demand of cold. Each residence in acceptable state, each repair and each entrance to an unacceptable state are associated with a corresponding cost. A procedure for computing this reliability associated cost is based on the Markov reward model for a non-homogeneous Poisson process. By using this model an optimal maintenance contract that maximizes the total expected cost may be found. A numerical example for a real world air conditioning system is presented to illustrate the approach.

Keywords: corrective maintenance, reliability associated cost, aging, multi-state system, Markov reward model

1. Introduction

Many technical systems are subjected to aging and degradation during their lifetime. Most of these systems are repairable. Maintenance and repair problems have been widely investigated in the literature. Barlow and Proshan [3], Gertsbakh [4], Pham and Wang [16], Wang [19] survey and summarize theoretical developments and practical applications of maintenance models.

Maintenance strategies addressed in most previous research works have been developed for improving reliability of aging binary-state systems or products. There is relatively limited number of research works focused on the maintenance strategies for aging Multi-state System (MSS). A heuristic approach has been proposed by Nourelfath and Ait-Kadi [14] for the optimisation of series–parallel MSSs when the maintenance resources are limited. Hsieh and Chiu [6] have proposed an optimal maintenance policy for a multi-state deteriorating standby system, by determining the optimal number of standby components and the optimal state in which deteriorating components shall be replaced. Nourelfath and Dutuit [15] have proposed a combined approach to solve the redundancy optimisation problem for multi-state systems under repair policies. The problem of imperfect preventive maintenance optimisation was considered for ageing MSS by Levitin and Lisnianski [9].

With the increasing complexity of the systems, only specially trained staff with specialized equipment can provide system service. In this case, maintenance service is provided by an external agent and the owner is considered as a customer of the agent for maintenance service (Pongpech and Murthy [17]). The maintenance outsourcing has been discussed by Jaturonnatee et al. [8] and Huang et al. [7]. Murthy and Asgharizadeh [13] have proposed a model to determine the optimal pricing strategy, the number of customers to service and the number of service channels for a monopolist service agent providing the maintenance service. The maintenance contract models have been studied by Bai and Pham [2], which discussed discounted warranty cost models for repairable series systems. The maintenance contract selection and spares provisioning planning based on the multi-criteria decision models has been discussed by Almeida [1]. Tarakci at al. [18] considered a manufacturer who has a process with an increasing failure rate over time. In order to improve the process performance, preventive maintenance and corrective maintenance are outsourced to an external contractor. Authors recommend to use the incentive contracts to induce the contractor to select the maintenance policy that optimises the total profit of the manufacturer and contractor.
Most of papers, that consider the maintenance contract selection, assume that the manufacturing process is binary-state. However the methods for the optimal maintenance contract planning for a multi-state aging system in a life cycle period till now have not been comprehensively developed.

This paper presents a case study where an aging air conditioning system with minimal repair is considered. Aging is considered a process which results in an age-related increase of the failure rate. The Markov reward model is built for computing the reliability associated cost, accumulated during the system’s life span. By using the model a corrective maintenance contract with maximum reliability associated cost can be defined from the set of different contracts available in the market. The approach is based on the Non-Homogeneous Markov Reward Model. The main advantage of the suggested approach is that it can be easily implemented in practice by reliability engineers.

2. Problem Formulation and Model Description

2.1. Reliability Associated Cost and Corrective Maintenance Contracts

We will define the Reliability Associated Cost (RAC) as the difference between total income from system using and total cost incurred by the user in operations and maintenance of the system during its lifetime. Therefore,

\[ RAC = US - OC - RC - PC, \]  

where
- US is income (reward) from system using,
- OC is the system operating cost accumulated during the system lifetime;
- RC is the repair cost incurred by the user in operating and maintaining the system during its lifetime;
- PC is a penalty cost, accumulated during system life time, which was paid when the system failed.

Let \( T \) be the system lifetime. During this time the system may be in acceptable states (system functioning) or in unacceptable ones (system failure). After any failure, a corresponding repair action is performed and the system returns to one of the previously acceptable states.

A Maintenance Contract is an agreement between the repair team and the system’s owner. The Maintenance Contract defines possible Maintenance Contract level, mean repair time and repair rate. Repair cost depends on repair time and, so, it corresponds to a maintenance contract level.

The problem is to find the expected reliability associated cost corresponding to each maintenance contract and choose the contract, maximizing this cost. According to the suggested approach, this cost is represented by the total expected reward, calculated via a specially developed Markov reward model.

2.2. Markov Reward Model for Aging System

Markov reward model was first introduced by Howard [5], and applied to multi-state system (MSS) reliability analysis by Lisnianski and Levitin [12].

We suppose that the Markov model for the system has \( K \) states that may be represented by a state space diagram as well as transitions between states. Intensities \( a_{ij}, i, j = 1, \ldots, K \) of transitions from state \( i \) to state \( j \) are defined by corresponding failure and repair rates.

It is assumed that while the system is in any state \( i \) during any time unit, some payment \( r_i \) will be made. It is also assumed that if there is a transition from state \( i \) to state \( j \) the amount \( r_{ij} \) will by paid for each transition. The amounts \( r_i \) and \( r_{ij} \) are called rewards. The objective is to compute the total expected reward accumulated from \( t = 0 \), when the system begins its evolution in the state space, up to the time \( t = T \) under specified initial conditions.

Let \( V_j(t) \) be the total expected reward accumulated up to time \( t \), if the system begins its evolution at time \( t = 0 \) from state \( j \). According to Howard [5], the following system of differential equations must be solved in order to find this reward:

\[ \frac{dV_j(t)}{dt} = r_{jj} + \sum_{i=1}^{K} a_{ij} r_{ij} + \sum_{i=1}^{K} a_{ij} V_i(t), \quad j = 1, 2, \ldots, K \]  

(2)
The system (2) should be solved under specified initial conditions: \( V_j(0) = 0, j = 1, 2, \ldots, K \).

For an aging system, its failure rate \( \lambda(t) \) increases with age. In the case of minimal repair, the intensities \( a_{ij}, i, j = 1, \ldots, K \) of transitions from state \( i \) to state \( j \) corresponding to failures are dependent on time. The total expected reward can be found from differential equations (2), by substitution of formula for \( \lambda(t) \) instead of corresponding \( a_{ij} \) values.

3. Numerical Example

3.1. The System Description

Consider an air conditioning system, used around the clock in varying temperature conditions and consists of two air conditioners, main and reserved. The main air conditioner is aging multi-state cold generating unit with 4 different levels: the first level of perfect functioning, the second level with reduced capacity (partial filter obstruction), the third level with complete failure (full filter obstruction) and the fourth level with complete failure (another reasons). The reserved air conditioner may be only in two states: perfect functioning or full failure.

The work schedule of the system is as follows. For regular temperature conditions the main air conditioner must be on-line. For peak temperature conditions if the main unit is in the level with reduced capacity, in addition to main air conditioner the reserved one must be on-line.

The state-space diagram for this system is presented on Figure 1. States 1–4 correspond to regular conditions and states 5–9 correspond to peak conditions. States 4 and 8 correspond to perfect functioning of the main air conditioner. States 3 and 7 corresponds to the reduced capacity level of the main conditioner. If the system enters to the state 7 the reserved air conditioner starts immediately his functioning. States 2 and 6 corresponds to complete failure of the main air conditioner because full filter obstruction and states 1 and 5 corresponds to complete failure because another reasons. State 9 corresponds to failure of the reserved air conditioner.

As was written above, technical requirements demand that the main on-line air conditioner are needed under regular conditions and additional reserved one in peak condition, so that there are four acceptable states – states 3–4 and states 7–8, and 5 unacceptable states: states 1–2, states 5–6 and state 9.

Aging is indicated as increasing failure rate functions: \( \lambda_{31}(t) = 1 + 0.9t \text{ year}^{-1} \), \( \lambda_{35}(t) = 1 + 0.9t \text{ year}^{-1} \) and \( \lambda_{75}(t) = 1 + 0.9t \text{ year}^{-1} \). Other failure rates are constant: \( \lambda_{32} = \lambda_{36} = 4 \text{ year}^{-1} \) and \( \lambda_{76} = 1 \text{ year}^{-1} \).

Repair rates are the following: \( \mu_4 = \mu_8 = 700 \text{ year}^{-1} \), \( \mu_{67} = 365 \text{ year}^{-1} \), \( \mu_{34} = 2000 + 200 \text{ year}^{-1} \) (see Table 1).

Following Lisnianski [10], the variable demand, representing variable weather conditions, may be described as a continuous time Markov chain with 2 levels. The first level represents a regular temperature conditions and the second level represents peak temperature conditions. The cycle time is \( T_c = 24 \text{ hours} \) and the mean duration of the peak is \( t_d = 9 \text{ hours} \). The transition intensities of the model can be obtained as

\[
\lambda_d = \frac{1}{T_c - t_d} = 0.066 \text{ hours}^{-1} = 584 \text{ year}^{-1}, \quad \lambda_{d1} = \frac{1}{t_d} = 0.111 \text{ hours}^{-1} = 972 \text{ year}^{-1}.
\]

We denote:

- \( C_{us} \) – is the income (reward) from system using.
- \( C_{op} \) – is the system operations cost accumulated during the system lifetime.
- \( C_{rm} \) – is the repair cost paid for every order of the external maintenance team;
- \( C_r \) – is the repair cost incurred by the user maintaining the reserved air conditioner;
- \( C_p \) – is a penalty cost, which is paid when system failed.
Service agents can suggest 10 different Corrective Maintenance Contracts, available in the market. Each contract $m$ is characterized by repair rate and corresponding repair cost (per repair) $C^m_r$ as presented in the Table 1.

**Table 1. Maintenance Contract Characteristics**

<table>
<thead>
<tr>
<th>Maintenance Contract</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Repair rate (year$^{-1}$)</td>
<td>2000</td>
<td>1800</td>
<td>1600</td>
<td>1400</td>
<td>1200</td>
<td>1000</td>
<td>800</td>
<td>600</td>
<td>400</td>
<td>200</td>
</tr>
<tr>
<td>Mean Repair Time (days)</td>
<td>0.18</td>
<td>0.21</td>
<td>0.23</td>
<td>0.26</td>
<td>0.30</td>
<td>0.37</td>
<td>0.46</td>
<td>0.61</td>
<td>0.91</td>
<td>1.83</td>
</tr>
<tr>
<td>Repair cost ($ per repair)</td>
<td>1453</td>
<td>1143</td>
<td>899</td>
<td>708</td>
<td>557</td>
<td>438</td>
<td>344</td>
<td>270</td>
<td>213</td>
<td>167</td>
</tr>
</tbody>
</table>

The income (reward) from system using $C^m_r$ is equal $180000 per year. The repair cost incurred by the user $C^r_u$, is equal to $50 per repair. The operation cost $C_{op}$, is equal to $7200 per year. The penalty cost $C_p$, which is paid when the system fails, is equal to $13140 per failure.
3.2. The Markov Reward Model for the System

The transition intensity matrix for the system is as shown in (3).

\[ a = \begin{bmatrix}
  a_{11} & 0 & 0 & \mu_{44} & \lambda_d & 0 & 0 & 0 & 0 \\
  0 & a_{22} & 0 & \mu_{24} & 0 & \lambda_d & 0 & 0 & 0 \\
  \lambda_{31}(t) & \lambda_{32} & a_{33} & 0 & 0 & 0 & \lambda_d & 0 & 0 \\
  \lambda_{41}(t) & 0 & \lambda_{43} & a_{44} & 0 & 0 & 0 & \lambda_d & 0 \\
  \lambda_d N & 0 & 0 & a_{55} & 0 & 0 & \mu_{58} & 0 & 0 \\
  0 & \lambda_d N & 0 & 0 & a_{66} & 0 & \mu_{68} & 0 & 0 \\
  0 & 0 & \lambda_d N & 0 & \lambda_{75}(t) & \lambda_{76} & a_{77} & 0 & \lambda_{79} \\
  0 & 0 & 0 & \lambda_d N & \lambda_{85}(t) & \lambda_{87} & a_{88} & 0 & 0 \\
  0 & 0 & 0 & \lambda_d N & 0 & 0 & \mu_{97} & 0 & a_{99}
\end{bmatrix}, \quad (3)

where

\[ a_{11} = -\left( \mu_{44} + \lambda_d \right), \quad a_{44} = -\left( \lambda_{41}(t) + \lambda_{43} + \lambda_d \right), \quad a_{77} = -\left( \lambda_{75}(t) + \lambda_{76} + \lambda_{79} + \lambda_d \right), \]
\[ a_{22} = -\left( \mu_{24} + \lambda_d \right), \quad a_{55} = -\left( \mu_{58} + \lambda_d \right), \quad a_{88} = -\left( \lambda_{85}(t) + \lambda_{87} + \lambda_d \right), \]
\[ a_{33} = -\left( \lambda_{31}(t) + \lambda_{32} + \lambda_d \right), \quad a_{66} = -\left( \mu_{68} + \lambda_d \right), \quad a_{99} = -\left( \mu_{97} + \lambda_d \right). \]

To calculate the total expected reward, the reward matrix for the system is built in the following manner (see Lisnianski and Levitin [12] and Lisnianski et al. [11]).

If the system is in states 3, 4, 7 and 8, the income (reward) from system using minus the operation cost associated with use of air conditioners, should be received during any time unit.

The transitions 3 → 1, 4 → 1, 7 → 5, 8 → 5 and 7 → 9 are associated with the entrance to unacceptable states and rewards associated with this transitions are the penalty.

The transitions 1 → 4, 5 → 8, 9 → 3 and 9 → 7 are associated with the repair of the air conditioner, provided by external team, and the reward associated with this transition is the repair cost paid for every order of the external maintenance team. The transitions 2 → 3 and 6 → 7 are associated with the filter repair, and the reward is the cost of this repair. The reward matrix for the system of air conditioners is as shown in (4).

\[ r = \begin{bmatrix}
  0 & 0 & 0 & -C_r^m & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & -C_r & 0 & 0 & 0 & 0 & 0 \\
  C_p & -C_p & C_{as} - C_{op} & 0 & 0 & 0 & 0 & 0 & 0 \\
  -C_p & 0 & 0 & C_{as} - C_{op} & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & -C_r^m & 0 \\
  0 & 0 & 0 & 0 & 0 & -C_r & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & C_{as} - C_{op} & 0 & -C_p \\
  0 & 0 & 0 & -C_p & 0 & 0 & C_{as} - C_{op} & 0 & 0 \\
  0 & 0 & -C_r^m & 0 & 0 & 0 & -C_r^m & 0 & 0
\end{bmatrix}, \quad (4)

Taking into consideration the transition intensity matrix (3), the system of differential equations that defines the Markov reward model for the air conditioning system for the calculation of the total expected reward, may be written as shown in (5).
The system is solved under initial conditions: $V_i(0) = 0, i = 1, 2, ..., 9$ using MATLAB®, the language of technical computing.

\[
\frac{dV_i(t)}{dt} = -C_{i4}^m \mu_{i4} - (\mu_{i4} + \lambda_{i_4}) V_i(t) + \mu_{i4} V_i(t) + \lambda_{i_5} V_5(t)
\]

\[
\frac{dV_5(t)}{dt} = -C_{54}^m \mu_{54} - (\mu_{54} + \lambda_{i_4}) V_5(t) + \mu_{54} V_4(t) + \lambda_{i_6} V_6(t)
\]

\[
\frac{dV_6(t)}{dt} = C_{6w} - C_{op} - C_p (\lambda_{i_1} + \lambda_{i_2}) + \lambda_{i_1} V_1(t) + \lambda_{i_2} V_2(t) - (\lambda_{i_1} + \lambda_{i_2} + \lambda_{i_6}) V_5(t) + \lambda_{i_3} V_3(t)
\]

\[
\frac{dV_3(t)}{dt} = C_{3w} - C_{op} - C_p (\lambda_{i_4} + \lambda_{i_5} V_1(t) + \lambda_{i_5} V_2(t)) - (\lambda_{i_4} + \lambda_{i_5} + \lambda_{i_8}) V_3(t) + \lambda_{i_6} V_6(t)
\]

\[
\frac{dV_6(t)}{dt} = -C_{68}^m + \lambda_{i_8} V_8(t) - (\mu_{i8} + \lambda_{i_8}) V_5(t) + \mu_{i8} V_8(t)
\]

\[
\frac{dV_8(t)}{dt} = -C_{8w}^m + \lambda_{i_8} V_8(t) - (\mu_{i8} + \lambda_{i_8}) V_8(t) + \mu_{i8} V_8(t)
\]

\[
\frac{dV_8(t)}{dt} = C_{8w} - C_{op} - C_p (\lambda_{i_7} + V_7(t) + \lambda_{i_7} V_7(t) + \lambda_{i_7} V_6(t) - (\lambda_{i_7} + \lambda_{i_7} + \lambda_{i_9}) V_7(t) + \lambda_{i_8} V_8(t)
\]

\[
\frac{dV_7(t)}{dt} = C_{7w} - C_{op} - C_p (\lambda_{i_7} + \lambda_{i_8} V_7(t) + \lambda_{i_8} V_6(t) + \lambda_{i_8} V_7(t)) - (\lambda_{i_7} + \lambda_{i_7} + \lambda_{i_9}) V_7(t)
\]

\[
\frac{dV_7(t)}{dt} = -C_{7w} + \lambda_{i_7} V_7(t) + \mu_{i7} V_7(t) - (\mu_{i7} + \lambda_{i_7}) V_7(t)
\]

4. Calculation Results

By using the suggested method one will find the best maintenance contract level $m$ that provides a maximum of Reliability Associated Cost during system lifetime. Figure 2 shows the expected Reliability Associated Cost for $T = 1$ years as a function of the Maintenance Contract Level ($m$). The eighth level ($m = 8$), which provides the maximum expected reliability associated cost ($\$140780$) for the system, corresponds to a mean repair time of 0.61 days. Choosing a more expensive Maintenance Contract Level, we pay an additional payment to the repair team. Choosing a less expensive one, we pay more for penalties because of transitions to unacceptable states.

![Figure 2. The expected Reliability Associated Cost vs. Maintenance Contract Level](image)

Conclusions

The case study for the estimation of expected reliability associated cost accumulated during system lifetime is considered for an aging system under minimal repair. The approach is based on application of a special Markov reward model, well formalized and suitable for practical application in reliability engineering. The optimal corrective maintenance contract ($m = 8$), which provides maximum expected reliability associated cost ($\$140780$), was found.
References


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