RANKING METHOD BASED ON THE DIFFERENCE BETWEEN WEIGHTED OUTPUT AND INPUT

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In this paper we developed a new method to establish common weights for measuring the efficiency score of Decision-Making Units (DMUs), based on multiple inputs and multiple outputs. In the new method, these common weights are estimated according to the difference between a weighted sum of outputs and a weighted sum of inputs of the DMUs. We suggest two approaches to rank the DMUs: the first method ranks the DMUs according to the absolute "net profit" (the difference between a weighted sum of outputs and a weighted sum of inputs), while the second method ranks the DMUs according to the relative efficiency score of the ratio between a weighted sum of outputs and a weighted sum of inputs. In addition we present a method to fix objective bounds for the weights of the variables in the Data Envelopment Analysis (DEA), which is based on the above ranking method. We proved that these bounds are feasible solutions for the DEA methodology. The ranking methods are illustrated on a case study of 24 hospitals in Israel.

Keywords: Data Envelopment Analysis (DEA); Common Weights; Hospitals

1. Introduction

The purpose of the paper is to rank units (as cites, hospitals, schools, banks, etc.) according to multiple inputs and multiple outputs in the Data Envelopment Analysis (DEA) context.

The Data Envelopment Analysis (DEA) was first introduced by Charnes, Cooper and Rhodes (CCR) in 1978. The DEA is a non-parametric method to evaluate the relative efficiency of Decision-Making Units (DMU) based on multiple inputs and multiple outputs. The efficiency score is measured as a ratio between a weighted sum of outputs and a weighted sum of inputs, even if the production function is unknown. The weights are chosen so as to find the best advantage for each unit to maximize its relative efficiency, under the restriction that this score is bound by 100% efficiency. If a unit with its optimal weights receives the score efficiency of 100%, it is efficient, and for a score smaller than 100% it is inefficient. These optimal weights differ from unit to unit. There are DEA researchers that emphasize the difficulty to rank all the units on one scale, claiming that DEA provides only a dichotomy classification into two groups: efficient and inefficient. If the number of units is small relative to the number of inputs and outputs, most of the units will be efficient.

Sometimes there is a need to fix common weights to the inputs and outputs for all the units, in contrast to the DEA. The idea of Common Set of Weights (CSW) was first published by Cook, Roll and Kazakov (1989), and later tested and reformulated by Roll, Cook and Golany (1991). There are several aims for the use of these common weights; the first one boils down to the fact that for the normalized data, the common weights indicate the importance of each factor (input/output) to determine the efficiency of the units. One can utilize these common weights to disregard factors (inputs/outputs) with very small...
importance. The idea of the importance of the factors has been discussed by Sengupta (1990), Arnold et al. (1996). Especially in the cases when weights are negative, one can examine if this factor was chosen in the right set, namely if it is output (input) instead of input (output). The second aim of common weights is to help us to define the bounds for the weights of inputs/outputs for the DEA method. The common weights may be the mid point of the range of the bounds of each weight. The third aim is to rank all the units with the common weights on one scale and not as in DEA where weights vary from unit to unit.

There are several methods in the literature for establishing the common weights while each method implements another objective function for this purpose. We shall present a few methods:

1) The Canonical Correlation Analysis (CCA) method, {Friedman and Sinuany-Stern (1997)}. In this method, the objective function for finding the common weights for the inputs and outputs is to maximize the correlation between a weighted sum of outputs and a weighted sum of inputs.

2) The Discriminant Analysis of Ratio (DR/DEA) method, {Sinuany-Stern and Friedman (1998)}. In this method, the discrimination into two groups: efficient and inefficient sets from the DEA, is first carried out. The objective function in this method is to maximize the discrimination between the means of these two groups.

3) The Global Efficiency (GE) method, {Ganley and Cubbin (1992)}; this method focuses on the measurement of aggregate technical efficiency by using common weights for all units. The objective function is to maximize the sum of the efficiency scores of all the units, where each efficiency score is the ratio between a weighted sum of outputs and a weighted sum of inputs. This is done by obtaining the optimal solution by one programming. It can be well-recognized that the latter comes in contrast to the DEA which advocates the solution of the linear programming $n$ times, generating a separate set of optimal weight for each unit. The disadvantage of the GE method boils down to the fact that its objective function is nonlinear; therefore the solution is not obviously optimal. Some more methods for determining common weights are given in the literature review of Adler et al. (2002).

The purpose of our paper is to develop a new ranking method designated as SDEA that is based on multiple inputs and multiple outputs, with common weights for all units. Our method focuses on the difference between a weighted sum of outputs and a weighted sum of inputs, namely the “net profit”. The objective function of this method is to maximize the sum of the “net profits” of all the units. The idea of maximizing the difference between the weighted outputs to the weighted inputs, for each unit, with weights that vary from unit to unit has been suggested in the past as the Additive model {see Charnes et al. (1985)}, and some of its ideas were further developed by Ali and Lerme (1997).

In addition we present a method to fix objective bounds for the weights of the variables in the DEA, which is based on the above ranking method. We proved that these bounds are feasible solutions for the DEA methodology. The ranking methods are illustrated on a case study of 24 hospitals in Israel, in order to single out the most efficient one. We choose 2 inputs: the number of standardized beds in the end of 2003, the number of standardized beds in day care (ambulatory), as well as 3 outputs: the number of total discharges in 2003, the number of hospitalisation days during 2003 and the same in ambulatory care.

Our paper is presented as follows: Section 2 introduces the SDEA method; a numerical example is outlined in Section 3. Section 4 contains the summary and conclusions.

2. Essentials of SDEA

Our new ranking method finds the common set of weights of the inputs/outputs where the objective function is to maximize the sum of all the “net profits” of the DMUs under two types of constraints. The first one is that for each DMU, its net profit can’t be positive as the DEA methodology and the Global Efficiency (GE) method (see Appendix). The second one is that the common weights are bonded from below by some value $\varepsilon$ suggested by Sueyoshi (1999).

Consider $n$ Decision-Making Units (DMUs), when each DMU $j (j=1,...,n)$ utilizes $m$ inputs $x_j=\left(x_{j1}, x_{j2}, \ldots, x_{jm}\right)^T > 0$ for producing $s$ outputs $Y_j=\left(Y_{j1}, Y_{j2}, \ldots, Y_{js}\right)^T > 0$. In this case, $\sum_{r=1}^s U_j Y_{jr}$ represents the weighted sum of outputs of DMU $j$, $\sum_{r=1}^m V_j X_{jr}$ stands for the weighted sum of inputs of DMU $j$. The net profit of DMU $j$ may be therefore calculated as $S_j = \sum_{r=1}^s U_j Y_{jr} - \sum_{r=1}^m V_j X_{jr}$. 
The objective function is: \( Z = \max \sum_{j=1}^{n} \left( \sum_{r=1}^{s} U_{rj} Y_{rj} - \sum_{i=1}^{m} V_{ij} X_{ij} \right) = \max \sum_{j=1}^{n} S_j \). The constraints are:

the net profit is less or equal to zero, namely \( S_j = \sum_{r=1}^{s} U_{rj} Y_{rj} - \sum_{i=1}^{m} V_{ij} X_{ij} \leq 0 \).

We obtain thus the linear problem

\[
Z = \max \sum_{j=1}^{n} \left( \sum_{r=1}^{s} U_{rj} Y_{rj} - \sum_{i=1}^{m} V_{ij} X_{ij} \right) = \max \sum_{j=1}^{n} S_j
\]

s.t.

\( S_j = \sum_{r=1}^{s} U_{rj} Y_{rj} - \sum_{i=1}^{m} V_{ij} X_{ij} \leq 0 \quad j=1,\ldots,n \quad (1) \)

\( U_{rj} \geq \varepsilon_r^+ > 0 \quad r=1,2,\ldots,s \)

\( V_{ij} \geq \varepsilon_i^- > 0 \quad i=1,2,\ldots,m \)

Our method differs from GE in two characteristics: first, our method can be solved by means of linear programming so that the optimal solution may be obtained, and second, our method bases on the difference between the weighted sum of outputs and weighted sum of inputs and not on the ratio between the weighted sum of outputs and the weighted sum of inputs.

The Dual Problem

Let us define a dual variable \( \lambda_j \) that is fitted to the constraint of each unit \( j \) and the dual variables \( L_r^+ \) and \( L_i^- \) that are fitted to the constraints of each output/input, respectively. The dual problem will be defined as follows:

\[
V = \min \left\{ \sum_{r=1}^{s} \varepsilon_r^+ L_r^+ + \sum_{i=1}^{m} \varepsilon_i^- L_i^- \right\}
\]

s.t.

\[
\sum_{r=1}^{s} \lambda_{rj} Y_{rj} + L_r^+ \geq \sum_{r=1}^{s} Y_{rj} \quad r=1,2,\ldots,s
\]

\[-\sum_{j=1}^{n} \lambda_{rj} X_{rj} + L_i^- \geq -\sum_{j=1}^{n} X_{rj} \quad i=1,2,\ldots,m \quad (2)\]

\( \lambda_j \geq 0 \quad j=1,2,\ldots,n \)

\( L_r^+ \leq 0 \quad r=1,2,\ldots,s \)

\( L_i^- \leq 0 \quad i=1,2,\ldots,m \)

Lemma 1. If \( S_j^* = \sum_{r=1}^{s} U_{rj} Y_{rj} - \sum_{i=1}^{m} V_{ij} X_{ij} = 0 \) \( \forall j=1,2,\ldots,n \) then \( \frac{1}{n} \sum_{j=1}^{n} \lambda_{rj} = 1 \).

Proof: If \( S_j^* = \sum_{r=1}^{s} U_{rj} Y_{rj} - \sum_{i=1}^{m} V_{ij} X_{ij} = 0 \) \( \forall j=1,2,\ldots,n \) then \( Z^* = 0 \).

According to the strong duality property [23] there exist \( Z^* = V^* = 0 \) therefore

\[\text{Min} \left\{ \sum_{r=1}^{s} \varepsilon_r^+ L_r^+ + \sum_{i=1}^{m} \varepsilon_i^- L_i^- \right\} = 0 \] and because \( \varepsilon_r^+ > 0 \quad r=1,2,\ldots,s \) \( \varepsilon_i^- > 0 \quad i=1,2,\ldots,m \) and \( L_r^+ \leq 0 \quad r=1,2,\ldots,s \) \( L_i^- \leq 0 \quad i=1,2,\ldots,m \) then \( L_r^+ = 0 \) \( \forall r=1,2,\ldots,s \) \( L_i^- = 0 \) \( \forall i=1,2,\ldots,m \).
In addition all the variables of the primal problem exist in the basis, therefore according to the complementary slackness theory \[ \sum_{j=1}^{n} \lambda^*_j Y_{nj} = \sum_{j=1}^{n} Y_{nj} \quad r = 1, 2, \ldots, s. \] This constraint can be written as: \[ \sum_{j=1}^{n} (\lambda^*_j - 1) Y_{nj} = 0 \quad r = 1, 2, \ldots, s \] the same for the constraints on the inputs. The solution of these constraints gives the following solution \[ \lambda^*_j = 1 \quad \forall j = 1, 2, \ldots, n, \] therefore \[ \frac{1}{n} \sum_{j=1}^{n} \lambda^*_j = 1. \]

**Ranking according to SDEA**

We suggest two methods to rank the DMUs, the first method ranking the DMUs according to the absolute "net profit" (the difference between a weighted sum of outputs and a weighted sum of inputs), and the second method ranking the DMUs according to the relative efficiency score of the ratio between the weighted sum of outputs and the weighted sum of inputs.

Let us define the “net profit” \( S^*_j \) of DMU \( j \) as: \[ S^*_j = \sum_{i=1}^{m} U^*_i Y_{ij} - \sum_{i=1}^{m} V^*_i X_{ij}, \]
where:

- \( U^*_i \) and \( V^*_i \) are the optimal common weights from the SDEA method, and they are the same for the two ranking approaches;
- if \( S^*_j = 0 \) the DMU \( j \) is efficient;
- if \( S^*_j < 0 \) the DMU \( j \) is inefficient.

**First method to rank the DMUs**

The ranking is based on the “net profit” of each unit. The score is determined as \[ S^*_j = \sum_{i=1}^{m} U^*_i Y_{ij} - \sum_{i=1}^{m} V^*_i X_{ij}. \] The DMU that received the highest \( S^*_j \) is ranked in the first place.

In this method the ranking is dependent on the size of the DMU’s.

**Second method to rank the DMUs**

In this case, the DMUs are ranked according to the relative efficiency score of the ratio between the weighted sum of outputs and a weighted sum of inputs. The ranking score is determined by:

\[
T_k = 1 + \frac{S^*_k}{\sum_{i=1}^{m} V^*_i X_{ik}} = \frac{\sum_{i=1}^{m} V^*_i X_{ik}}{\sum_{i=1}^{m} V^*_i X_{ik}} = \frac{\sum_{i=1}^{m} U^*_i Y_{ik}}{\sum_{i=1}^{m} V^*_i X_{ik}}.
\]

**Bounded weights in SDEA**

In order to prevent the trivial solution where all weights are zero in the SDEA method \( U^*_r = 0 \quad \forall r = 1, 2, \ldots, S \) and \( V^*_i = 0 \quad \forall i = 1, 2, \ldots, m \) and therefore \( S^*_j = 0 \quad \forall j = 1, 2, \ldots, n, \) it is necessary to set a lower bound to the weights.

This issue has been extensively dealt with in scientific literature. The first research on bounds on the weights by a “Non Archimedean Quantity” (NAQ) was carried out by Charnes et al. (1979, 1984). Thompson et al. (1990) established the so-called assurance region (AR) in order to outline a region for the possible values of the weights. The information on the assurance region originates from experts’ opinions, common sense, or previous experience.

Sueyoshi (1999) suggested restricting the weights only by a lower bound which will be a function of the number of inputs and outputs. In the literature there are many papers on methods for determining the bounds on the weights: Cooper et al. (1999), Thompson et al. (1995), Roll et al. (1991), Dyson et al. (1988). The main purpose of the constraints on the weights is to reduce the number of efficient units and also to avoid the problems of extreme values of the weights.
Analytical Management

In our research, we adopt the Sueyoshi approach. It can be well-recognized that the primary and dual problems in the SDEA method will appear therefore as:

**Primary problem**

\[
Z = \max \sum_{j=1}^{n} \left( \sum_{r=1}^{s} U_r Y_{rj} - \sum_{i=1}^{m} V_i X_{ij} \right) = \max \sum_{j=1}^{n} S_j
\]

subject to

\[
S_j = \sum_{r=1}^{s} U_r Y_{rj} - \sum_{i=1}^{m} V_i X_{ij} \leq 0 \quad j = 1,2,\ldots,n
\]

\[
U_r \geq \frac{1}{(m+S) \max \{Y_{rj}\}} \quad r = 1,2,\ldots,s
\]

\[
V_i \geq \frac{1}{(m+S) \max \{X_{ij}\}} \quad i = 1,2,\ldots,m
\]

**Dual problem**

\[
V = \min \left\{ \frac{1}{\sum_{r=1}^{s} (m+S) \max \{Y_{rj}\}} L_r + \sum_{i=1}^{m} \frac{1}{(m+S) \max \{X_{ij}\}} L_i \right\}
\]

subject to

\[
\sum_{j=1}^{n} \lambda_j Y_{rj} + L_r \geq \sum_{j=1}^{n} Y_{rj} \quad r = 1,2,\ldots,s
\]

\[
- \sum_{j=1}^{n} \lambda_j X_{rj} + L_r \geq - \sum_{j=1}^{n} X_{rj} \quad i = 1,2,\ldots,m
\]

\[
\lambda_j \geq 0 \quad j = 1,2,\ldots,n
\]

\[
L_r \leq 0 \quad r = 1,2,\ldots,s
\]

\[
L_i \leq 0 \quad i = 1,2,\ldots,m
\]

*Lower bound on weights in SDEA*

The constraint that normalizes the weighted sum of inputs \( \sum_{r=1}^{s} V_r X_{rj} = 1 \) in the classical DEA approach may result in difficulties to attain a feasible solution, or, in certain cases, in creating non-realistic solutions. We suggest a new method for implementing bounds on the weights in the DEA.

The procedure that is capable of preventing the above mentioned difficulties may be presented in the following steps:

**Step 1.** Solve the optimisation problem of SDEA of Eq. (1) and find out from it the common weights of all the inputs and outputs, \( V_i, i = 1,2,\ldots,m \quad U_r = 1,2,\ldots,s \).

**Step 2.** Calculate the maximal value over all the \( n \) units, of the weighted inputs from the SDEA model in Step 1, namely \( UL = \max_{j} \left\{ \sum_{r=1}^{s} V_r X_{rj} \right\} \).

**Step 3.** Calculate the ratio of the common weights \( V_i \) and \( U_r \) to the maximal value \( UL \) calculated in Step 2, that is \( \frac{V_i}{UL} \quad i = 1,2,\ldots,m \quad \frac{U_r}{UL} \quad r = 1,2,\ldots,s \). These are the fitted bounds for the weights that will solve all the problems and will give a suitable solution.
Lemma 2. The efficiency score based on common weights (SDEA method) is always less than the efficiency score in the DEA.

Proof: The DEA score is the maximal score that a unit can receive. If we restrict the problem to find only common weights, these weights produce a score smaller than the one with the varying weights, as defined in DEA methodology.

Lemma 3. Let us denote \( UL = \max_j \left( \sum_{i=1}^{n} V_{i(CW)} X_{j} \right) \) then \( V_{i(DEA)}^{*} = \frac{V_{i(CW)}}{UL} \) \( i = 1, 2, ..., m \) and

\( U_{r(DEA)} \geq \varepsilon_{r}^{*} = \frac{U_{r(CW)}}{UL} \) \( r = 1, 2, ..., s, \) are a feasible solution for unit \( j \) by the DEA model.

Proof:

\[
(1) \quad S_{j} = \sum_{r=1}^{s} U_{r(CW)} Y_{r} - \sum_{i=1}^{m} V_{i(CW)} X_{j} \leq 0 \quad \forall j = 1, 2, ..., n \Rightarrow E_{j} = \frac{\sum_{i=1}^{m} U_{i(CW)} Y_{r}}{\sum_{i=1}^{m} V_{i(CW)} X_{j}} \leq 1 \quad \forall j = 1, 2, ..., n
\]

\[
(2) \quad UL = \max_{j} \left\{ \sum_{i=1}^{n} V_{i(CW)} X_{j} \right\}
\]

from (2) : \( (3) \quad \frac{\sum_{i=1}^{m} V_{i(CW)} X_{j}}{UL} \leq 1 \quad \forall j = 1, 2, ..., n
\]

from (1) and (3) : \( (4) \quad \frac{\sum_{i=1}^{m} U_{i(CW)} Y_{r}}{UL} \leq \frac{\sum_{i=1}^{m} V_{i(CW)} X_{j}}{UL} \leq 1 \Rightarrow \frac{\sum_{i=1}^{m} U_{i(CW)} Y_{r}}{UL} \leq 1
\]

\[
(5) \quad \sum_{i=1}^{m} V_{i(DEA)} X_{j} = 1 \Rightarrow \sum_{i=1}^{n} V_{i(DEA)} X_{j} \geq \frac{\sum_{i=1}^{m} V_{i(CW)} X_{j}}{UL}
\]

\[
(6) \quad \sum_{i=1}^{s} U_{r(DEA)} Y_{r} \leq 1
\]

Therefore the solutions that were defined by the above bounds \( U_{r(DEA)} \geq \frac{U_{r(CW)}}{UL} \) \( r = 1, 2, ..., s, \) represent a feasible solution for the DEA model.

3. The Case Study ON Israeli Hospitals

Hospitals account for about 40% of Israel’s national health expenditure in 2003. This is the largest category of spending on health; community clinics, including those providing preventive medicine, account for about 38% of this expenditure [Central Bureau of Statistics (CBS) 2003].

3.1. Output and Input Measures

The main problem that appears in hospitals research concerning the DEA is that there are many types of inputs and outputs. The choice of the inputs/outputs influences the results of the efficiency of the hospitals. Therefore a literature review on DEA efficiency of hospitals was done in our research. A total of 42 publications on the regarded issue was studied, out of which we shall outline the most important and relevant to the discussed subject: O’Neill (1988), reported on the efficiency of 27 hospitals in USA. Al-Shammari (1999) reported on the efficiency of 15 government hospitals in Jordan. Hofmarcher et al. (2002) reported on the efficiency of 15 hospitals in Austria. Kirigia et al. (2002) reported

The data reported in the references summarize the main inputs and outputs measures used in different DEA hospital studies. Evidently the most used input is no. of beds (52% of studies). Cost is used in 48% of the studies, supplies in 44% and employees in 41% of the studies.

Due to the lack of data on these additional inputs we did not include them in our study. However they all are reported in the literature as related to the number of beds, since the budget is largely derived by number of beds.

From the database available to us from the Health Ministry we used 2 inputs and 3 outputs. The inputs are: the number of standardized beds in the end of 2003 \(X_1\), the number of standard beds in day care (ambulatory) \(X_2\). The outputs are: the number of total discharges in 2003 \(Y_1\), the number of hospitalization days during 2003 \(Y_2\) and the same in ambulatory care \(Y_3\).

Day care (ambulatory) has been a venue for increasing the efficiency of hospitals in Israel. Thus, we included it in our input and output variables. Overall the input/output variables we have used are those common in the literature.

The list of hospitals in Israel includes 45 hospitals. We deleted hospitals that did not have internal care and outpatient clinics (day care) units. Consequently, 24 hospitals were left for our study. The data is presented in Table \(I\), which includes information on the 24 hospitals with 3 outputs and 2 inputs.

**Table 1. The numerical data**

<table>
<thead>
<tr>
<th>No.</th>
<th>DMU</th>
<th>INPUT</th>
<th>OUTPUT</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(X_1)</td>
<td>(X_2)</td>
</tr>
<tr>
<td>1</td>
<td>Asaf Harofe – Zrifin</td>
<td>675</td>
<td>72</td>
</tr>
<tr>
<td>2</td>
<td>Asuta – Tel Aviv</td>
<td>138</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>Bikur Holim – Jerusalem</td>
<td>193</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>Bney Zion – Haifa</td>
<td>366</td>
<td>16</td>
</tr>
<tr>
<td>5</td>
<td>Barzilai – Ashkelon</td>
<td>448</td>
<td>40</td>
</tr>
<tr>
<td>6</td>
<td>E.M.M.S – Nazaret</td>
<td>108</td>
<td>4</td>
</tr>
<tr>
<td>7</td>
<td>Emek – Afula</td>
<td>415</td>
<td>38</td>
</tr>
<tr>
<td>8</td>
<td>Hadassa (Ein Karem) – Jerusalem</td>
<td>618</td>
<td>70</td>
</tr>
<tr>
<td>9</td>
<td>Hadassa (Har Hatzofim) – Jerusalem</td>
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<td>23</td>
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<tr>
<td>10</td>
<td>Hagalil Hamaaravi – Naharia</td>
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<td>17</td>
</tr>
<tr>
<td>11</td>
<td>Hiel Yafe – Hadera</td>
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<td>75</td>
</tr>
<tr>
<td>12</td>
<td>Kaplan – Rehovot</td>
<td>403</td>
<td>23</td>
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<tr>
<td>13</td>
<td>Lady Davis – Haifa</td>
<td>378</td>
<td>32</td>
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<td>Meir – Kfar Saba</td>
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<td>15</td>
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<td>Rabin (Belinson) – Petah Tikva</td>
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<td>Rabin (Golda) – Petah Tikva</td>
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<td>23</td>
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<td>18</td>
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<td>10</td>
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<td>19</td>
<td>Rivka Ziv – Tzfat</td>
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<td>20</td>
<td>Shearay Tsedek – Jerusalem</td>
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<td>100</td>
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<td>21</td>
<td>Shiba – Tel Hashomer, Ramat Gan</td>
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<td>76</td>
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<td>22</td>
<td>Soraski – Tel Aviv, Jaffa</td>
<td>911</td>
<td>46</td>
</tr>
<tr>
<td>23</td>
<td>Soroka – Beer Sheva</td>
<td>618</td>
<td>30</td>
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Table 2. The scores and the ranking of the DMUs

<table>
<thead>
<tr>
<th>DMU</th>
<th>METHOD 1</th>
<th>METHOD 2</th>
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<td></td>
<td>SCORE</td>
<td>RANK</td>
</tr>
<tr>
<td>Asaf Harofe – Zrifin</td>
<td>-0.1759</td>
<td>22</td>
</tr>
<tr>
<td>Azota – Tel Aviv</td>
<td>0.0000</td>
<td>2</td>
</tr>
<tr>
<td>Bikur Holim – Jerusalem</td>
<td>-0.0390</td>
<td>7</td>
</tr>
<tr>
<td>Bnei Zion – Haifa</td>
<td>-0.0627</td>
<td>10</td>
</tr>
<tr>
<td>Barzilai – Ashkelon</td>
<td>-0.1142</td>
<td>17</td>
</tr>
<tr>
<td>E.M.M.S – Nazaret</td>
<td>-0.0069</td>
<td>4</td>
</tr>
<tr>
<td>Emek – Afula</td>
<td>-0.1128</td>
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</tr>
<tr>
<td>Hadassa (Ein Karem) – Jerusalem</td>
<td>-0.1145</td>
<td>18</td>
</tr>
<tr>
<td>Hadassa (Har Hatzofim) – Jerusalem</td>
<td>-0.0098</td>
<td>5</td>
</tr>
<tr>
<td>Hagalil Hamaaravi – Naharia</td>
<td>0.0000</td>
<td>2</td>
</tr>
<tr>
<td>Hilel Yafe – Hadera</td>
<td>-0.0751</td>
<td>14</td>
</tr>
<tr>
<td>Kaplan – Rehovot</td>
<td>-0.1436</td>
<td>19</td>
</tr>
<tr>
<td>Lady Davis – Haifa</td>
<td>-0.0650</td>
<td>11</td>
</tr>
<tr>
<td>Laniado – Natania</td>
<td>-0.0236</td>
<td>6</td>
</tr>
<tr>
<td>Meir – Kfar Saba</td>
<td>-0.1482</td>
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</tr>
<tr>
<td>Rabin (Belinson) – Petah Tikva</td>
<td>-0.1781</td>
<td>23</td>
</tr>
<tr>
<td>Rabin (Golda) – Petah Tikva</td>
<td>-0.0412</td>
<td>8</td>
</tr>
<tr>
<td>Rambam – Haifa</td>
<td>0.0000</td>
<td>2</td>
</tr>
<tr>
<td>Rivka Ziv – Tzfat</td>
<td>-0.0683</td>
<td>12</td>
</tr>
<tr>
<td>Shearay Tsedek – Jerusalem</td>
<td>-0.0701</td>
<td>13</td>
</tr>
<tr>
<td>Shiba – Tel Hashomer, Ramat Gan</td>
<td>-0.2868</td>
<td>24</td>
</tr>
<tr>
<td>Soraski – Tel Aviv, Jaffa</td>
<td>-0.1684</td>
<td>21</td>
</tr>
<tr>
<td>Soroka – Beer Sheva</td>
<td>-0.1059</td>
<td>15</td>
</tr>
<tr>
<td>Wolfson – Holon</td>
<td>-0.0563</td>
<td>9</td>
</tr>
</tbody>
</table>

Table 3. Common weights values

<table>
<thead>
<tr>
<th>FOR NORMAL VALUES</th>
<th>FOR REGULAR VALUES</th>
</tr>
</thead>
<tbody>
<tr>
<td>U1</td>
<td>0.327141</td>
</tr>
<tr>
<td>U2</td>
<td>0.207787</td>
</tr>
<tr>
<td>U3</td>
<td>0.2</td>
</tr>
<tr>
<td>V1</td>
<td>0.640326</td>
</tr>
<tr>
<td>V2</td>
<td>0.2</td>
</tr>
</tbody>
</table>

It turns out that \( \max \left\{ \sum_j V_j X_{ij} \right\} = 0.8403 \) and the bounds in the DEA method may be estimated therefore as listed in Table 4:

Table 4. Bounds in the DEA method

<table>
<thead>
<tr>
<th>FOR NORMAL VALUES</th>
<th>FOR REGULAR VALUES</th>
</tr>
</thead>
<tbody>
<tr>
<td>U1</td>
<td>0.00065074</td>
</tr>
<tr>
<td>U2</td>
<td>0.002</td>
</tr>
<tr>
<td>U3</td>
<td>3.964E-06</td>
</tr>
<tr>
<td>V1</td>
<td>5.8544E-07</td>
</tr>
<tr>
<td>V2</td>
<td>4.133E-06</td>
</tr>
</tbody>
</table>
4. Conclusions

In this paper we developed a new method to establish common weights for measuring the efficiency score of DMUs, based on multiple inputs and multiple outputs. In the new method, these common weights are estimated according to the difference between a weighted sum of outputs and a weighted sum of inputs of the DMUs. We suggest two approaches to rank the DMUs: the first method ranks the DMUs according to the absolute "net profit" (the difference between a weighted sum of outputs and a weighted sum of inputs), while the second method ranks the DMUs according to the relative efficiency score of the ratio between the a weighted sum of outputs and a weighted sum of inputs. If the DMUs have similar size of inputs and outputs, the ranking is similar. If not, the ranking is different. Which method is better? There is no ultimate answer, it depends on the context.

We have proved that weights obtained by the suggested procedure when divided by the weighted sum of inputs, can effectively serve as a lower bound for the DEA method. We have implemented this procedure for quality ranking of 24 hospitals in Israel. It appears that 3 hospitals (Asuta-Tel Aviv, Hagaill Hamaaravi-Naharia and Rambam-Haifa) out of the total amount proved to be efficient (their score was equal to 1), while the rest occupied positions 4–24. The ranking of the efficient hospitals proves to be the same for both suggested DEA approaches, while the inefficient ones tended to display significant differences for both types of ranking. One may appreciate the importance of the weights for normalized values of the scores as well.

Should we sub-divide the total amount of hospitals into 3 groups according to their accommodating capacity (small ones – up to 420 beds, intermediate – from 420 to 700 beds, and big ones – more than 700 beds), it can be well-recognized that small hospitals mostly tend to obtain higher scores by the first type of ranking, while the big ones, on the contrary, are ranked higher by the second DEA approach (e.g., the Bikur Holim-Jerusalem hospital which is a small one was ranked 7th by the first procedure and 17th by the second one, while the Rabin-Petach Tikva hospital which is a big one – 23rd and 14th, respectively, and the same for Soraski-Tel-Aviv hospital – 21th and 12th).

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References


Appendix. The global efficiency method (GE)

The global efficiency (GE) focuses on the measurement of aggregate technical efficiency by using the same set of weights (common weights) for the efficiency score for all the units. This is in contrast to the DEA efficiency score, which advocates the solution of the linear programming n times, generating a separate set of optimal weight for each unit. The GE involves an optimal solution by one programming for the common set of weights, which maximizes the sum of efficiency scores of all the units. Each efficiency score of the GE with the common weights has the same structure as the DEA efficiency score with the weights that vary from unit to unit; i.e. it is the ratio of total weighted output to total weighted input, bounded by 1. The formulation of the GE is as follows:

\[ Z = \max \sum_{j=1}^{n} E_j^* \]

s.t.

\[ E_j^* = \frac{\sum_{i=1}^{m} U_i Y_{ij}}{\sum_{i=1}^{m} V_i X_{ij}} \leq 1 \quad j = 1, 2, \ldots, n \]

\[ V_i, U_r \geq 0 \quad i = 1, 2, \ldots, m \quad r = 1, 2, \ldots, s \]

The objective function helps the efficiency score with the common weights to reach the DEA scores globally, because by the definition of the DEA, the sum of the DEA scores is the maximum value that any score can reach. The advantage of the GE is its simplicity, the ease with which the average manager may interpret results, and its ranking capabilities according to the common weights that are in the DEA context.

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