Goldratt’s Game for the simulation purpose in the management course is presented in two different visual forms (Vensim and Excel). The first model is clearly controllable and attractive for classroom. The second one is carefully investigated under the several control rules.

**Keywords:** simulation, fluctuations, control, chain

1. Fluctuations and Control in Management

The role of fluctuations and its counterintuitive character may be understood by simulation in the appropriate way some supply chain (Goldratt’s matchsticks game [1], Beer distribution game [2]). The first model is created in Vensim (Ventana. Systems, Inc.) simulation environment (see Fig. 1) and it clearly shows the nature and role of fluctuations in the simplest matchsticks case. By changing separate dice rules one can model a drum – buffer-rope effects. The picture becomes more complicated if another feedback is included.

![Figure 1. Vensim model](image)

In the second model we introduce a control into the impact and interaction of statistical fluctuations and dependent events on the stylised process flow which consists of 3 separate, defect-free operations with queues (inventory buffers) between them. We use the model described by Johnson [3]. The aim of the article is comparing statistical characteristics of the system, described by Johnson (2002) to those of the system after applying control variable. Both systems are modelled by imitation modelling; each consisting of three operations with number of runs 180 with the time steps up to 32000 for each of the systems. For the second system there is a minor control applied \( U_k(t) = U = \text{const} = 1 \) with a fixed constant threshold \( P_k = P = \text{const} = 30 \) for all the operations of the system at any moment in time \( 1 \leq t \leq 350 \).
2. Properties of Initial Model

The initial model has the following block structure (Fig. 2).

This model can be used to illustrate many concepts in production management. We use it to reinforce the notation that reduction of variation in processes is more efficient and economical than increasing inventory or work-in-progress in this case. The model simulates a production line with 3 work stations. The product must pass through each of the 3 work stations in order from 1 to 3. We can use pennies to simulate the process. The potential amount of work completed at each station in the given time period is a uniform distribution of 1, 2, 3, 4, 5, and 6. This random potential work is determined each period by the tossing of a die. The actual work forwarded to the next station can be represented by equation (1) below.

The model might be formalized as the following equations

\[ Q_k(i) = Q_k(i-1) - Y_k(i-1) + Y_{k-1}(i), \]

\[ Y_k(i) = \min \{Q_k(i), R_k(i)\}, \]

\[ R_k(i) = RG_k(i), \]  

(1)
where $Q_k(i)$ represents the input of $k$th operation block ($k$th device),
$Y_k(i)$ represents the output of $k$th operation block ($k$th device),
$Y_0(i)$ is the independent input of the system ($Y_0(i)$ generates at every step a uniform distributed integer numbers between 1 and 6),
$R_k(i)$ is the output of random generator $RG_k(i)$, which produces an independent uniform distributed integer numbers between 1 and 6, i.e. $1 <= R_k(i) <= 6$.

The value $R_k(i)$ is the number of resource units available for use at time $i$ for completing the $k$th operation according to the equations (1).

$Q_{K_{max}}(i)$ represents the output of the system (number of units produced) during time cycle $i$,
$k$ is the ordinal number of the operation, $1 <= k <= K_{max}$, where $K_{max}$ is the maximum number of the operations inside the system,
$i$ represents the operation step. It is a single time period, $1 <= i <= T$, where $T$ is the maximum time step of observations. $Q_k(0) = 0$, $1 <= k <= K_{max}$.

It is assumed that prior to start the system is at null values, i.e. there are no shortages or excess of uncompleted production in progress.

In papers [1–3] the model is investigated only on some dozens of time intervals and with insufficiently explicitly. To estimate influence of control on behaviour of system (1), the detailed and multi-fold study of model dynamics is required.

For the model on Fig. 2 consisting of 3 devices, step by step monitoring all variables entering the equation of model is carried out. In statistical experiments it was revealed, that realizations of queues $Q_k(i)$, $1 <= k <= 3$ at model have no trend of stabilization, even on rather long intervals of time up to 50'000 steps. Fig. 4 portrays modification of values of queues $Q_1(t)$, $Q_2(t)$, $Q_3(t)$ on an interval in length of 350 steps for 10 realizations.

For deriving statistical conclusions there are performed 180 independent runs of the model. All the varying values of the model are measured and brought in a database on each time step. Thus, for each moment there are available 180 independent measurements of each of the parameters. Fig. 5 illustrates the change of average values of inputs $Q_1(t)$ - $Q_3(t)$ on an interval in length of 350 steps (averaged 180 runs of the model).

![Figure 4. Visualization of 10 realizations for input $Q_1(t)$ (the left-hand side graph), for input $Q_2(t)$ (the middle graph) and for entrance $Q_3(t)$ (the right graph). The trend to increase of amplitudes of queues on entrances of model also is accompanied by increased variance](image1)

![Figure 5. The changing in average values of outputs $Q_1(t)$ - $Q_3(t)$ on an interval of 350 steps (averaged 180 runs of the model). The amount of objects in queue is depicted on the vertical axis. Histograms of the corresponding inputs are arranged in the right side of the figure. Data for histograms are collected on an interval of time from 300 up to 350 steps for 180 realizations, i.e. each of the histograms represents a set of 50*180 = 9000 observations](image2)
From the presented graphs follows one important observation: before each operation the amount of unprocessed objects is constantly growing. The sum of inputs \( Q_1(t) + Q_2(t) + Q_3(t) \) makes the total number of the objects which have been accumulated by the system at present time “t”.

In a running model not all resources are used rationally. From the second equation of system (1) it follows that at \( R_k(i) > Q_k(i) \), a part of resources, which we shall designate as \( R_{kw}(i) = R_k(i) - Q_k(i) \), is not used on a step “i” and consequently is irrevocably lost. It is obvious, that with the increasing queues \( Q_1(t), Q_2(t), Q_3(t) \) the losses of resources should decrease, as probability of occurrence of queues with number of objects in each of them decreases (which is below 6). Since the use of resources is limited on each time step by an inequality \( 1 \leq R_k(i) \leq 6 \) at length of queue \( Q_i(t) > 6 \) the model should function more effectively as all of the resources should be used.

Let’s consider, how the accumulated sum of losses of resources eventually varies (see Fig. 6).

From graphs on Fig. 6 there can be seen constant increase of average cumsum, the lost resources are visible. Eventually the tendency of increase of resource losses at the subsequent stages of processing in relation to the previous operations is kept. Histograms of distribution of resource losses presented in section \( t = 350 \) show very wide and asymmetrical spread of values. The model shows unexpected behaviour as with increase of average size of queues (Fig. 5) average losses of resources (Fig. 6) also increase. As in reality losses of resources grow at increment of average sizes of queues it can mean growth in fluctuations of number of objects in queues. The observed effect will remain, if the probability of occurrence of small queues (up to 5 details) does not aspire to zero with growth of time. Increase expenditures by storage of not completed production \( Q_1(t) + Q_2(t) + Q_3(t) \), and expenditures grow on payment of increasing losses of resources \( R_{kw}(i) \), \( 1 \leq k \leq 3 \) from the point of view of economy.

It is obvious that the behaviour of model described by a set of equations (1) is non-rational for modelling productions. The possibility appears to make correcting control with the purpose of possibilities of the best use on resource management. If immediate interference in “process of manufacture” is not possible to use of additional resources can begin one of possibilities change on behaviour of the system.

3. System with Control of Additional Resources

So, there is a problem how “to improve” functioning model by means of introduction of additional resources in such a way, to increase profit (or other criterion), not rendering influence on statistical properties of initial generators of resources. On Fig. 7 the block diagram of admissible updating of system is presented. We investigate influence of “weak” management, which can be characterized as enough in the big size of a threshold \( P_k >> 6 \) in comparison with a dispersion of a source of the basic resource, and as when it is supposed to use external sources of resources \( UG_k(i) \), which on each step can add no more than one unit \( U_k(i) = 0 \) or 1 for each of “k” resources \( 1 \leq k < 3 \).
The formal objective is to inset into the system (1) additional number of resource units in accordance to several rules, which we later refer to as control variables. The control $U_k(i)$ is given separately for each process through a feedback mechanism. The value of control variable $U_k(i)$ for $k^{th}$ operation is determined by the threshold parameter $P_k$, which in the context of the present article is considered constant for all the operations. If the queue length $Q_k(i)$ on intake of a $k^{th}$ operation at a time $i$ exceeds the value of $P_k$ then the resource of $k^{th}$ operation is added a predetermined non-random resource $U_k(i) > 0$ in the general case. Thus, $k^{th}$ operation on $i^{th}$ step will be completed with an increased resource $R_k(i) = RG_k(i) + U_k(i)$. The system with control variable will take the following form:

$$Q_k(i) = Q_k(i-1) - Y_k(i-1) + Y_k(i), Y_k(i) = \min\{Q_k(i), R_k(i)\}, R_k(i) = RG_k(i) + U_k(i),$$

$$U_k(i) = \begin{cases} U_k & \text{if } (Q_k(i-1) - Y_k(i-1)) > P_k, \\ 0 & \text{else} \end{cases}$$

(2)

To the equations (2) there corresponds the block diagram on Fig. 8.

Monitoring is carried out on value of threshold $P_k$ for length of queue $Q_k(i)$. Let us consider, as the system will be functioning at threshold $P_k = 30$, which has been chosen heuristically.
We would like to discover by means of a simulation modelling as in system with the control described above to receive properties the best, than they are in initial system. Let us consider, as dynamics of queues will vary at threshold $P_k = 30$, $U_k(i)$ can accept values $0$ or $1$, for each of "$k$" resources $1 \leq k \leq 3$

$U_k(i) = \begin{cases} 1 \text{, if } \{Q_k(i-1) - Y_k(i-1)\} > 30, \\ 0 \text{, else} \end{cases}$

On Fig. 9 average values of queues and histograms, corresponding them in an interval of time from 300 up to 350 time steps are represented at 180 realizations of model with control.

Dynamics of queues in model with control essentially differs from dynamics of queues in initial model. Let us note convergence of all average lengths of queues to one value $Q(t) = 20$. As all thresholds are equal to one number $P_1 = P_2 = P_3 = 30$ the average lengths of queues come to the same value. We shall note also very big similarity of all histograms of queues $Q_k(t)$.

For system 2 the equation of balance for products is true:

$$
\sum_{t=1}^{t} Q_0(t) - \sum_{t=1}^{t} Y_0(t) = \sum_{t=1}^{t} Q_1(t) + \sum_{t=1}^{t} Q_2(t) + \sum_{t=1}^{t} Q_3(t). \tag{3}
$$

The difference between number of products CumSum of $Y_0$ which have arrived on the system input and number of products CumSum of $Y_3(t)$, which have gone out the system, shows common number of the products, which have remained inside the system. $Y_0(i)$ – is the independent input of the system.

For system with control the number of products inside the system approximately equally 60, thus the number of not completed products is in equally allocated between operations approximately on 20 units.

Also holds the following equation of balance for resources

$$
\sum_{t=1}^{t} R_1(t) + \sum_{t=1}^{t} R_2(t) + \sum_{t=1}^{t} R_3(t) = \\
= \sum_{t=1}^{t} R_{used}(t) + \sum_{t=1}^{t} R_{used}(t) + \sum_{t=1}^{t} R_{used}(t) + \sum_{t=1}^{t} R_{used}(t) + \sum_{t=1}^{t} R_{used}(t) + \sum_{t=1}^{t} R_{used}(t) = \tag{4}
$$

$$
= \sum_{t=1}^{t} Q_2(t) + \sum_{t=1}^{t} Q_3(t) + \sum_{t=1}^{t} Y_3(t) + \left\{ \sum_{t=1}^{t} R_1(t) + \sum_{t=1}^{t} R_2(t) + \sum_{t=1}^{t} R_3(t) \right\}.
$$
The sum in parenthesis in expression (4) represents all the resources lost by system at moment “t”. Let us see how the cumulative sum of the lost resources varies at increasing of functioning time of the system (Fig. 10).

![Graph showing cumulative sum of resources lost over time](image)

**Figure 10.** Dependence of average cumulated sums losses of resources on time (the left graph) and histogram of resources wasted at time = 350 (180 observations) for model with control.

Constant increase of average CumSum the lost resources are visible from graphs on Fig. 10. Common character of change of processes is the same, as well as for model without control. We shall note distinctions in a velocity of increase of losses of resources for model with control.

For example, average growth rate of losses of resources at last stage of handling “CumSum[R3w(t)]” for model with control (mean = 66, Fig. 10) noticeably below, than for model without control (mean = 79.1, Fig. 6).

![Graph showing control resource usage](image)

**Figure 11.** The graph of use of controllable resources. A change of averages cumulated sums (at the left). Histograms of controllable resources cumulated sums at time = 350 (180 observations) for model with control (at the right).

### 4. The Comparative Analysis of Models

Both models show rather significant dispersion for all observable characteristics. In order for it to be possible to determine more precisely distinctions in behaviour of models, the combined system schematically presented on Fig. 12 has been created. We compared the two models under identical common inputs and identical common recourses flows. The aim of such modelling is quantitative estimation of effect control of system behaviour efficiency.
The combined model allows using identical inputs and generators of all resources for both submodels; therefore distinctions will be shown only due to management. At separate studying models substantial growth of number of experiments would be required to reduce influence of a selective statistical error of supervision. We shall notice, that performance of conditions $U_{Gk}(t) = 0$ or $P_k = \text{[big enough number]}$, allow to supervise identity of functioning both submodels.

By means of control the additional resources are introduced into model, which in a considered case with 100% probability are used only for manufacture of products. It is interesting to compare an amount of incomplete products, which have been accumulated in system with control those accumulated in system without control.

Pairs of observations on the basis of which there are histograms, are tested on inequality by nonparametric Wilcoxon Signed Ranks Test. If data are continuous, the sign test or the Wilcoxon signed-rank test can be used. The sign test computes the differences between the two variables for all cases and classifies the differences as either positive, negative, or tied. If the two variables are similarly distributed, the number of positive and negative differences will not differ significantly. The Wilcoxon signed-rank test considers information about both the sign of the differences and the magnitude of the differences between pairs. Because the Wilcoxon signed-rank test incorporates more information about the data, it is more powerful than the sign test. The test calculates negative ranks = 111, positive ranks = 59, ties = 10, $Z = -5.809$, asymptotical significance (2-tailed) = 0.000, therefore the difference is significant. Paired samples correlation is equal to 0.882.
Thus, we can draw an unexpected conclusion that in system with monitoring there are less lost resources than in the initial system. Accumulation of incomplete production in both systems happens for one common reason; namely, because of shortage of utilized resources. We shall note that an average amount of all resources acting in system, it is equal to average amount raw materials, acting on an input of initial system. If the amount of objects in queue on handling for some reason becomes less, than the resources currently available, then respective unutilised resources become lost irrevocably.

The comparative analysis of the accumulated losses of resources on right graph of Fig. 13 shows not strong, but explicitly notable shift of histograms which speaks about a drop of an average of losses of resources in system with control. As this distinction has appeared statistically significant in system with monitoring it is for some reason lost fewer resources though average sizes of turns for each of operations constantly increase in system without monitoring. If the turn increases, losses of resources fall. It would be logical to assume, that losses of resources will be less in that from systems, in which lengths of turns it is more. We have the outcome contradicting this logic. This rather strange outcome shows that with introduction in system of additional inspected resources we achieve also more effective use of resources than before introduction of monitoring.

The comparative analysis of statistical distributions of not ready products accumulated in systems shows an essential drop of an average store of products in system with monitoring up to 49 products after enough long functioning. The system without monitoring at the moment of time \( t = 350 \) had an average store of not ready production of 79 products, i.e. in one and a half time it is more. In a considered instant the system with control had in 2 times a smaller variance, than initial system. Thus the system with monitoring has more predicted behaviour, than system without monitoring that is illustrated on Fig. 14.

Let us compare outputs of two systems (Fig. 15)
As the difference between an output of initial system and the output of system with the control has statistically significant value size the system with the control is more productive. The reason is reduction of length of turns due to an expenditure of additional resources, which delivery is provided with a control system, and due to decrease in losses of normal resource amount.

Let us consider the estimation of additional profit, which can be received under certain conditions. As an example we shall take the following sizes. We shall admit that the prices of one unit are equal to the values presented in the Table 1.

<table>
<thead>
<tr>
<th>Cost, $ per a unit</th>
<th>Resource Wasted</th>
<th>Resource Used</th>
<th>Resource of control</th>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>5</td>
<td>7</td>
<td>10</td>
<td>30</td>
<td></td>
</tr>
</tbody>
</table>

Let us notice, that the cost of Resource of control is taken at 40% extra of cost of usually spent unit of a resource to compensate charges on management. Besides, expenses for storage of not ready objects, which essentially above at system without the control, here have not considered the graphs of dynamics of the received profit for two systems and the respective histograms, are illustrated on Fig. 16.

![Figure 16. Dynamics of resulting profit for two systems (on the left) and corresponding histograms (on the right)](image)

It is easy to see that the relative gain of profit for the system with the control over time $t = 350$ makes more than 10% in comparison with initial system, and thus the dispersion of the received profit decreases twice less than a dispersion of profit for initial system.

Thus, insignificant addition of the minimal resources (not exceeding on each step of one resource unit for each operation) at moments of time when the queue length exceeds threshold value leads to cardinal change of dynamics of initial system.

**Conclusions**

The comparative analysis of two systems shows, that introduction of the control
- leads to stabilization of number of not ready details in system,
- reduces the lost of resources,
- increases speed of an output of finished goods,
- allows at some level of prices for raw material and resources to increase profit 10% and more.

Besides the influence on security resources of each operation probably also influence length of turn before operation. The control system can be constructed so that to accumulate products in special buffers and to start products from buffers on operation on processing when queues become small.

Let us make some remarks on opportunities of control systems. We have considered a control system with allocation of additional resources for each operation on the basis of enough simple
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algorithms. Use of the expanded algorithms of resource management and buffers with functions of the forecast allows arriving at the solution for optimum control, common for the given class of models.

The problem appears complex enough because of presence of the big number of optimised parameters and is even more complicated by strong stochastic of model.

References


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