MATHEMATIC MODELLING OF THE OPTIMAL CARGO HANDLING MANAGEMENT PROCESS AT THE LARGE TRANSPORT HUBS

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Task of the optimal unloading management process of the multi-nomenclature divisible cargo at the transport terminals belongs to the class of tasks, which includes specifics found in both, distributive and the optimal management tasks.

Goal of this paper is to construct a qualitative model for the task of managing the unloading process of the multi-nomenclature cargos at the large transport hub.

The paper deals with the following tasks:
1. Setting general formulation of the problem.
2. Defining special cases of the problem.
As an example, mathematical model of one of the special cases is being reviewed in more detail.

Keywords: distribution logistics, optimal cargo management, divisible goods unloading

1. Introduction

This paper examines the problem of the optimal unloading management process of the multi-nomenclature divisible cargo at the interconnected transport terminals. Such problems relate to the class of tasks which includes challenges associated with both, distributive and the optimal management tasks. Normally, the efficiency of the combinatorial algorithms, applied for the solution of the similar classes of tasks, is diminishing substantially with the increase of the task size and the complexity of its initial formulation.

This paper examines a general formulation of the problem under study, while formulating several specific cases of the given general task, constructing a mathematical model for one of these cases, and later, proposing an analytical method for finding its optimal solution.

It is important to note that neither the qualitative model of the general formulation of the given task, nor the mathematical model of one of its subtasks, being evaluated in this paper, nor the analytical method of its solution developed in this paper have been previously encountered by the authors in any of the scientific publications devoted to the optimal logistics management of processing the multi-nomenclature divisible cargo at the transport terminals.

Authors have reviewed a substantial number of articles on this topic published by renowned European scientific journals; in those articles, tasks associated with processing such type of cargo are generally being solved by applying methods of partial enumeration, using either the unclear logics, or the Boolean algebra. In some of those papers, the multi-criterion optimisation methods are being applied, with a condition that the objective function, that is - the criterion of the decision-making person (DMP), has been already pre-defined. This is also relevant for the classic study literature on the optimisation models and methods in economy.

However, one must note, that the topic being researched in this paper belongs to the problem of use and distribution of the various types of resources, most of which are usually researched with the application of the Leontiev’s intersectoral balance theory. Mathematical model of one of the subtasks constructed and solved in this paper is closely linked with the well known “costs-output” model from the Leontiev’s intersectoral balance theory, and specifically - dual model of the subtask’s mathematical model is, up to the detail of specific notations, identical with the “costs-output” model.
Previously mentioned link between the Leontiev’s model “costs-output” and the subtask’s model under review in this article is thoroughly explained in paper [1]. Aside from that, it is important to note that in the study material [2], along with specific tasks associated with the optimisation of production processes, there are also present some specific logistics tasks, which bear similarities with the general formulation of the problem reviewed in this article.

2. Qualitative Model of the Problem under Study

First of all, we lay out the problem in its general formulation:
- there are \( n \in \mathbb{N} \) interconnected large transport terminals;
- to the \( m \in \mathbb{N} \) (\( m \leq n \)) of these terminals \( k \in \mathbb{N} \) trainloads have arrived, each of those delivering \( l \in \mathbb{N} \) \((i = I,k)\) types of divisible cargos (number of various divisible cargos is \( n \in \mathbb{N} \)), while the amount of the \( i\)-type divisible cargo, delivered by the \( j\)-trainload is equal to \( G_{i,j} \) \((i = I,k, j = I,k)\) tons;
- capacity of the \( i\)-type terminal is \( C_{i,n} \) while any other terminal may receive any type of cargo from any another terminal under the following conditions:
  - the capacity of each terminal may not be exceeded;
  - transportation of one ton of the \( i\)-type cargo \((j = I,r)\) cargo from the terminal \( j\) to the terminal \( j\) \((j_1, j_2 = I,n: j_1 \neq j_2)\) is conducted by the transport of the terminal \( j_1\) and requires from the terminal \( j_1\) total expenditure in amount of \( T_{j_1,j_2} \) \((j_1, j_2 = I,n: j_1 \neq j_2)\) monetary units (m.u.);
- there are \( l \in \mathbb{N} \) companies available for the unloading of the cargo from the arriving trainloads under the following conditions: company \( i \) \((i = I,I)\) undertakes the unloading of cargo \( j \) \((j = I,r)\) at the rate of \( g_{i,j} \) \((i = I,I; j = I,r)\) tons of cargo per day with the price of \( c_{i,j} \) \((i = I,I; j = I,r)\) m.u. per ton.

There is also a condition stating that every day of downtime of the trainload \( s \) \((s = I,k)\) at the terminal \( q \) \((q = I,m)\) costs that particular terminal some financial losses in amount of \( f_{s,q}(t) \) \((s = I,k; q = I,m)\) m.u.

One needs to find:
(A) What agreements regarding the unloading of the trainloads should terminals make in order to minimise their total expenditure?
(B) How many and which kinds of the companies should be involved in the process?
(C) Taking this all into account, how many days are necessary for the full unloading?

Paper [1] indicates three possible cases of the given task, while describing one of these options in detail (sub-task III).

3. Analytical Solution for the Model of the Subtask III

In order to construct the mathematical model of the given subtask, let us introduce the following designations and assumptions:
- \( t \in \mathbb{N} \) is the number of days needed to unload the entire trainload \((t \text{ is the value being sought for});
- \( n \in \mathbb{N} \) \((n \leq l)\) is a number of companies, engaged in a process of unloading a given trainload \((n \text{ is also a sought-for number});
- \( S^{\text{def}} = \{\sigma(1),...,\sigma(n)\} \) is a multitude of the indexes/numbers of companies engaged in the unloading process: \( \sigma(i) \in \{1,1,...,l\} \forall i = I,I; \sigma(i) \neq \sigma(j) \forall i, j = I,I: i \neq j \) \((\text{where } \sigma(i) \ (i \in S) \text{ are the sought-for indexes});
- \( x_i \in \mathbb{R}^{|S|} \ (i \in S) \) where \(|S|\) denotes the capacity of the multitude \( S\), is an optimal amount of the divisible cargo in tons, which in course of the \( t \) days may be unloaded by the company \( i \) \((i \in S)\) \((\text{where } x_i \ (i \in S) \text{ are the sought-for exogenous variables});
- let us also presume, without the loss of the generality, that sequence of the unloading prices of all companies \( \{c_{i,j} \}_j^{\text{def}} \) and not only those involved in the unloading process) is arranged in a non-descending order, therefore, presuming that \( c_{i \leq i,j} \forall i = I,(1-l) \) .
Now, using the previously introduced designations, we may formulate the mathematical model of the given subtask III:

\[
\begin{align*}
\text{min} & \quad f \cdot t + \sum_{i \in S} c_i \cdot x_i \\
\text{s.t.} & \quad x_i - g_i \cdot t \leq 0 \quad \forall i \in S, \\
& \quad \sum_{i \in S} x_i = G.
\end{align*}
\]  

(1)

Here and further in the text, \( R^k_+ \) denotes the non-negative orthant in a \( k \)-dimensional \( \forall k \in N \) material Euclidean space. In the model (1) there is also provided a size of the penalty \( f \) in m.u.; unloading costs \( \{c_i\}_{i=1}^l \) in m.u.; volumes of the unloading operations in tons \( \{g_i\}_{i=1}^l \) per day; amount of the divisible cargo \( G \) in tons.

Let us construct a task which is dual to the task (1):

\[
\begin{align*}
\text{max} & \quad G \cdot y_0 \\
\text{s.t.} & \quad y_i \geq y_0 - c_j \quad \forall i \in S, \\
& \quad \sum_{i \in S} g_i \cdot y_i = f.
\end{align*}
\]  

(2)

Task (2) accounting for the substitution of the variables

\[
y_j = \begin{cases} 
  z_j, & \text{if } i \in S \cup \{0\} \\
  0, & \text{if } i \in S \quad \equiv \{1, \ldots, l\} / S
\end{cases}
\]  

(3)

takes the following form:

\[
\begin{align*}
\text{max} & \quad G \cdot z_0 \\
\text{s.t.} & \quad z_i \geq z_0 - c_j \quad \forall i = I, I, \\
& \quad \sum_{i \in I} g_i \cdot z_i = f, \\
& \quad z = (z_0, z_1, \ldots, z_l)^T \in R^l_+,
\end{align*}
\]  

(4)

which is a dual problem to the constructed initial model [1]. As it may be seen from the paper [1], the given model is completely identical with the Leontiev’s “Costs-Output” model [3-5]. In addition to that, papers [6-7] researched the task of the divisible product output maximisation with the limited material and financial resources, the concluding mathematical model of which was based on the Leontiev’s model. Considering the fact that the given model is also identical to the source model constructed on the basis of the Leontiev’s model, which was researched in detail in the papers [6-7], the approach similar to the one applied within the papers [6-7] is chosen in this section as a research methodology for the model (1).

The solution, if there is one, of the model (4) as a \((l+1)\)-dimensional vector \((z^*_0, z^*_1, \ldots, z^*_l)\), which satisfies all the substantial constraints, as well as the sign restrictions of the model (4), and provides for the maximal \( z^*_0 \) value among all possible solutions of the objective function \( \tilde{w}(z), z \in R^{l+1}_+ \). The existence of the solution \((z^*_0, z^*_1, \ldots, z^*_l)\) implies finding the solution for the source model (4), which will be a \((n+2)\)-dimensional vector \((n; t^*, x^*_1, \ldots, x^*_n)\), acquired from \((z^*_0, z^*_1, \ldots, z^*_l)\) сначала с first with the transformation (3), and then with help of the complementary slackness theorem [8, 9].

Therefore, we should find out whether model (4) has a solution, and if the answer is positive - to find one. In other terms, we should answer the following three questions:
1. Whether the set of the feasible solutions is non-empty, that is, whether
   it is possible that
   \[ \{ z \in R^{l+1}_{+} : z_i \geq z_0 - c_i \ \forall i = \{1, \ldots, l\} ; \sum_{i=1}^{l} g_i \cdot z_i = f \} \neq \emptyset. \]

2. Whether the objective function \( w(z) \) is constrained at the top, within the
   set of feasible solutions \( Z \)?

3. In case of positive responses to both previous questions – how can one find
   vector \( (z_0^*, z_1^*, \ldots, z_l^*) \)?

First of all, in order to answer these questions, let us notice from the model (4)
that if there is a solution \( (z_0^*, z_1^*, \ldots, z_l^*) \), then the following
conditions must be fulfilled:

\[
\begin{align*}
z_i^* & \geq z_0^* - c_i \ \forall i = \{1, \ldots, l\}, \\
z^* & \in R^{l+1}_{+}.
\end{align*}
\]

Let us also show that under these mandatory conditions (5), there is no place for
strict inequalities. In order to be sure about this, let us assume the contrary, that
is, presume that there is at least one number \( i_0 \in \{1, \ldots, l\} \) in existence,
which satisfies condition:

\[ 0 < z_{i_0}^* > z_0^* - c_{i_0}. \]  \hspace{1cm} (6)

From the duality theorem it is obvious that, the inter-dual models (1) and (4) may
either both have a solution or both not have one. That is why, assuming that vector
\( (z_0^*, z_1^*, \ldots, z_l^*) \) is a solution for the dual model, one should by default,
also assume that there is vector \( (n; \mathbf{x}_1^*, \ldots, \mathbf{x}_n^*) \), which is a solution of
the source/primary model (1). Further, we will need to apply the following useful
theorem arising from the complementary slackness and the duality theories [8, 9]:

\begin{enumerate}
  \item If only one of the multitudes of the permitted solutions of the inter-dual
tasks is non-empty, then:
    \begin{align*}
    & \text{if } \overline{X} \neq 0, \overline{Y} = 0 \Rightarrow \sup_{\overline{x}} c_\overline{x} >= \infty ; \\
    & \text{if } \overline{X} = 0, \overline{Y} \neq 0 \Rightarrow \sup_{\overline{y}} b_\overline{y} >= -\infty ,
    \end{align*}
\end{enumerate}

where \( \overline{X} \) and \( \overline{Y} \) accordingly denote multitudes of the permitted
solutions of the direct and the dual tasks.

If multitudes of the permitted solutions of these inter-dual tasks are not empty, then
both tasks have solutions.

Now let us return back to the assumption (6). From the (6), as well as from the
complementary slackness theorem and the theorem stated above, it follows that:

\[
\begin{align*}
x_{j^*}^* & = g_{j^*} \cdot t^* , \\
x_{j^*}^* & = 0.
\end{align*}
\]

Therefore, and, as it is seen from the substantial constraints-inequalities of the
source model (1), there is \( x_j^* = x_j^* = \ldots = x_n^* = t^* = 0 \). And this is impossible,
because in this case, from the substantial constraint-equality of the model (1) it
would follow that \( G = 0 \) that is - amount of the divisible cargo delivered
to the terminal by the trainload is equal to zero, which contradicts the formulation
of the given problem. Therefore, our assumption (6 is false; meaning that under the
conditions (5) there is no such number for which the inequalities would be strictly
fulfilled. As the result, only the option of the fulfillment of all the conditions from
(5) in form of equalities is left. This, in turn, means that

\[ z_i^* = \max\{ 0 ; z_0^* - c_i \} \ \forall i = \{1, \ldots, l\}. \]  \hspace{1cm} (7)

And so the formula (7) finds the optimal values of the vector \( (z_0^*, z_1^*, \ldots, z_l^*)^T \) of
the dual model (2), and therefore, the only thing left is to find the optimal value of
the variable \( z_0 \). In order to find \( z_0^* \), let us
take into account formula (7) under the substantial constraint-equality \( \sum_{i=1}^{l} g_i \cdot z_i = f \) of the dual model (4). As the result of this substitution we get:

\[
\sum_{i=1}^{l} g_i \cdot \max \{0; z_0^* - c_i\} = f,
\]

(8)

which is the equation of the unknown variable \( z_0^* \). It follows, that the optimal value \( z_0^* \), we are looking for, is the root of the equation (8), if such root is existent and unique. In other terms, we need to study the equation (8) in order to determine existence and uniqueness of its root. For this, let us return to our assumption about the arrangement of the elements of the sequence \( \{c_i\}_{i=1}^{l} \) by non-descension (see beginning of this section). Employing this assumption, which does not limit the generality of the researched task in any way, it is easy to notice that left part of the equation (8) is a continuous function in relation to the variable \( z_0^* \), while this function (that is the left part of the equation (8)), as the function of the argument \( z_0^* \), is strictly growing within the semi-interval \( (c_n; +\infty) \), using values from 0 to \( +\infty \)

Therefore the sought-for root \( z_0^* \) of the equation (8) is both existent and unique. In order to find this root, which as it was just proven – is existent and unique, we should first find the number \( n \in \mathbb{N} \quad (n \leq l) \) of the companies involved in the unloading process. For this purpose let us introduce the number

\[
\max \left\{ i : \sum_{j=1}^{i} g_i \cdot \max \{0; c_i - c_j\} < f \quad a \leq i \leq l \right\},
\]

(9)

which in its essence, obviously, is the sought-for number of the companies engaged in the unloading process, that is:

\[
n_{\text{def}} = \max \left\{ i : \sum_{j=1}^{i} g_i \cdot \max \{0; c_i - c_j\} < f \quad a \leq i \leq l \right\}
\]

Because:

\[
\sum_{j=1}^{i} g_i \cdot \max \{0; c_i - c_j\} \geq f,
\]

it follows that:

* if \( n < l \), then \( z_0^* \in (c_n; c_{n+1}] \);
* if \( n = l \), then \( z_0^* = +\infty \) and \( z_0^* \in (c_n; +\infty) \).

Therefore, the left part of the equation (8) within the semi-interval \( (c_n; c_{n+1}] \) has the following form:

\[
\sum_{j=1}^{l} g_i \cdot (z_0^* - c_i)
\]

meaning that within the semi-interval \( (c_n; c_{n+1}] \) equation (8) takes a more simple form:

\[
\sum_{j=1}^{l} g_i \cdot (z_0^* - c_i) = f.
\]

From this, one may, obviously, elaborate a closed formula:

\[
z_0^* = \frac{f + \sum_{i=1}^{n} g_i \cdot c_i}{\sum_{i=1}^{n} g_i}.
\]

(10)
Taking into account expression (10) in the formula (7), we get a closed formula:

\[
\begin{align*}
\frac{f + \sum_{j=1}^{n} g_j \cdot (c_j - c_i)}{\sum_{j=1}^{n} g_j} & \quad \text{if } i = \overline{1,n}; \\
0 & \quad \text{if } i = (n+1), \overline{1,l}.
\end{align*}
\]

(11)

As the result, optimal solution of the dual model (4) is fully defined by the closed formulas (10), (11). In order to find the optimal solution \((n; t^*, x_1^*, ..., x_n^*)\) of the source model (1), we will use, as it was previously mentioned, transformation (3) and the conditions of complementary slackness (4). And in particular, the fact that according to the complementary slackness theory, vector \((t^*, x_1^*, ..., x_n^*)\) is the solution of the model (1) only when the following \((2l+1)\) conditions-equalities and \(2l\) conditions-inequalities are satisfied:

\[
\begin{align*}
\sum_{i=1}^{l} x_i^* &= G; \\
x_j^* &\geq 0; \ x_j^* \cdot (z_j^* - z_j - c_j) = 0 \ \forall j = \overline{1,l}; \\
x_j^* &\leq g_j \cdot t^*; \ z_j^* \cdot (x_j^* - g_j \cdot t^*) = 0 \ \forall j = \overline{1,l}.
\end{align*}
\]

(12) (13) (14)

Further, from the closed formulas (10) and (11) we see, that conditions (13) and (14) are satisfied only in case if:

\[
\begin{align*}
t^* > 0; \ x_i^* &\equiv 0 \ \forall i = (n+1), \overline{1,l}; \ x_i^* = g_i \cdot t^* \ \forall i = \overline{1,n}.
\end{align*}
\]

(15)

Considering these new facts, from (12) we get the following closed formula for finding the optimal number of days \(t^*>0\) over the course of which all divisible cargo will be unloaded from the train arriving to the transport terminal:

\[
\begin{align*}
t^* &= \begin{cases} 
\frac{G}{\sum_{i=1}^{n} g_i}, & \text{if } \frac{G}{\sum_{i=1}^{n} g_i} \text{ is integer;} \\
\frac{G}{\sum_{i=1}^{n} g_i} + 1, & \text{if } \frac{G}{\sum_{i=1}^{n} g_i} \text{ is fraction.}
\end{cases}
\end{align*}
\]

(16)

Finally, by substituting (16) into the last identical equalities from (15), we get following closed formula for determining the optimal amounts of the divisible cargo in tons, which will be unloaded by company \(i \ (i = \overline{1,n})\) in \(t^*\) days:

\[
x_i^* = \frac{g_i \cdot G}{\sum_{i=1}^{n} g_i} \ \forall i = \overline{1,n},
\]

(17)

So, the optimal solution \((n; t^*, x_1^*, ..., x_n^*)\) may be found in the closed form by formulas (9), (16) и (17). Let us briefly interpret these closed formulas. In order to do this, we need to return to the definition of the non-descending arrangement of the elements within sequence \(\{ c_i \}_{i=1}^{n} : \ c_i \leq c_{i+1} \ \forall i = \overline{1,(l-1)}\). It is obvious that given assumption means that companies involved in the unloading process have to be previously ranked by their price rates in the non-descending order, which may be done easily. With this assumption, a short interpretation of the formulas (9), (16) and (17) may be given as follows:
- first \( n \) of the ranked companies should be involved, as these are the cheapest ones in terms of the unloading prices per ton (see formula (9));

- unloading tasks should be distributed among these companies in proportion to their productivity (see formula (17));

- the unloading time is expressed by the ratio of the total amount of the unloading work to the aggregate productivity of the companies involved in the unloading (see formula (16));

- company which offers the lowest unloading price per ton will always be involved, irrespectively of its productivity (see formulas (9) and (17));

- the bigger is the number of companies involved in the unloading process (that is value of the parameter \( n \) according to formula (9)) the bigger are the losses of the terminal due to the trainload’s downtime (see formula (9));

- if the losses of the terminal due to the trainload’s downtime are high enough, meaning that the strict inequality \( \sum_{j=1}^{k} \max\{0; c_j - c_j / f(t) \} \) is fulfilled, then all \( l \) companies should be involved (see formula (9));

- the following reverse economically-logical task may be elaborated and solved: one need to determine the amount of the penalty \( f(t) \) (where \( f(t) \) is not necessarily a constant or piecewise constant!) at which, firstly, maximal number of companies will get involved; and secondly, total costs of the terminal (or terminals) will be minimal (see model (1) and formula (9)).

**Remark.** The approach offered in this paper for the solution of the subtask III may be directly (that is without the according modifications) applied for the study of neither the general problem, qualitative model of which is given in the beginning of the section 2, nor the substasks I and II, if the losses of the terminal due to trainload’s downtime in these substasks (as well as other possible substasks) are not constant values. Even in case one views subtask II under the assumption that losses of the terminal due to trainload’s downtime \( f(t) \) have a piecewise constant value (for instance – losses differ on a daily basis, remaining constant over the course of one day), then its mathematical model, unlike the model (1) of the subtask III, will be a quasilinear extreme problem:

\[
\begin{align*}
\min_{x \in \mathbb{R}^n} & \quad w(x, t) \\
\text{subject to} & \quad x_i - g_i \cdot t \leq 0 \quad \forall i \in S, \\
& \quad \sum_{i \in S} x_i = G.
\end{align*}
\]

It is obvious that under the condition of non-linearity of the function \( f(t) \), mathematical models of the general problem and its different variations/subtasks will be non-linear (some even – nonconvex) extreme tasks.

### 4. Computational Experiment

In order to illustrate the results (9), (16) and (17) obtained in the previous section, let us assume that the total amount of cargo, which has arrived on the trainload to the transport terminal equals to 1125 t; each day of downtime of the trainload costs terminal 400 LVL; there are only 5 companies specialising on this particular type of cargo available to the terminal, while the first company agrees to unload 85 t of cargo per day and charges 4.50 LVL per ton, second company – 100 t of cargo per day at the rate of 6.00 LVL per ton, third company – 65 t of cargo per day at 4.00 LVL per ton, fourth company – 120 t of cargo per day at 7.30 LVL per ton, and the fifth company – 80 t of cargo per day at 5.10 LVL per ton. It is necessary to answer the following questions:

- What agreements regarding the unloading of the trainload should the terminal make in order to minimise its total expenditure?

- How many and which companies should be involved in the process?

- Taking this into account, how many days are necessary for the full unloading of the arriving divisible cargo?
It is obvious that the aforementioned simple problem may, over some time, be solved using the full enumeration. However, the elaborated analytical solution in the closed form allows answering all the outlined questions without any difficulties.

As a result, in the denotations of the constructed mathematical model we have (previously arranging the prices by non-descension):

\[ G = 1125; \]
\[ f = 400; \]
\[ l = 5; \]
\[ g_1 = g'_1 = 65; \]
\[ g_2 = g'_2 = 65; \]
\[ g_3 = g'_3 = 90; \]
\[ g_4 = g'_4 = 100; \]
\[ g_5 = g'_5 = 120; \]
\[ c_1 = c'_1 = 4.00; \]
\[ c_2 = c'_2 = 4.50; \]
\[ c_3 = c'_3 = 5.10; \]
\[ c_4 = c'_4 = 6.00; \]
\[ c_5 = c'_5 = 7.30, \]

where index of the parameter \( i \) (or \( g'_i \)) denotes the number of a company in a series.

Applying formula (9):

- if \( i = 1 \) we have
  \[ \sum_{j=1}^{l} g_j \cdot \max\{0; c_1 - c_j\} = \sum_{j=1}^{l} g_j \cdot \max\{0; 4.00 - c_j\} = \sum_{j=1}^{l} g_j \cdot 0 = 0; \]

- if \( i = 2 \) we have
  \[ \sum_{j=1}^{l} g_j \cdot \max\{0; c_2 - c_j\} = 65 \cdot \max\{0; 4.50 - 4.00\} + 0 = 65 \cdot 0.5 = 32.50; \]

- if \( i = 3 \) we have
  \[ \sum_{j=1}^{l} g_j \cdot \max\{0; c_3 - c_j\} = 65 \cdot \max\{0; 5.10 - 4.00\} + 85 \cdot \max\{0; 5.10 - 4.50\} + 0 = 65 \cdot 1.10 + 85 \cdot 0.60 = 122.50; \]

- if \( i = 4 \) we have
  \[ \sum_{j=1}^{l} g_j \cdot \max\{0; c_4 - c_j\} = 65 \cdot \max\{0; 6.00 - 4.00\} + 85 \cdot \max\{0; 6.00 - 4.50\} + 90 \cdot \max\{0; 6.00 - 5.10\} + 0 = 65 \cdot 2.00 + 85 \cdot 1.50 + 90 \cdot 0.90 = 338.50; \]

- if \( i = 5 \) we have
  \[ \sum_{j=1}^{l} g_j \cdot \max\{0; c_5 - c_j\} = 65 \cdot \max\{0; 7.30 - 4.00\} + 85 \cdot \max\{0; 7.30 - 4.50\} + 90 \cdot \max\{0; 7.30 - 5.10\} + 100 \cdot \max\{0; 7.30 - 6.00\} + 0 = 65 \cdot 3.30 + 85 \cdot 2.80 + 90 \cdot 2.20 + 100 \cdot 1.30 = 780.50; \]

then formula (9) provides:

\[ n = \max\{i \mid i = l(0 < 400); \}
\[ i = 2(32.50 < 400); \]
\[ i = 3(122.50 < 400); \]
\[ i = 4(338.50 < 400)\} = 4. \]

Applying formula (16):

\[ \frac{G}{\sum_{i=1}^{l} g_i} = \frac{1125}{65 + 85 + 90 + 100} \approx 3.309, \text{ then } \{3.309\} + 1 = 4. \]

Finally, applying formula (17), we get

\[ x_1^* = 85 \cdot \frac{1125}{4} \approx 281; \]
\[ x_2^* = 100 \cdot \frac{1125}{4} \approx 331; \]
\[ x_3^* = 65 \cdot \frac{1125}{4} \approx 215; \]
\[ x_4^* = 90 \cdot \frac{1125}{4} \approx 298. \]

So now it is possible to answer all of the given questions:

- Terminal should sign agreements with companies № 1, 2, 3 and 5 for 4 days (actually, the unloading process will be fully completed in approximately 3.3 days);
- the first company will unload approximately 281 t of cargo (the cost will be approximately 1265 LVL);
- the second company will unload approximately 331 t of cargo (the cost will be approximately 1985 LVL);
- the third company will unload approximately 215 t of cargo (this work will cost the terminal approximately 860 LVL);
- the fifth company will unload approximately 298 t of cargo (the cost will be approximately 1519 LVL);

Total expenditure of the transport terminal for the entire unloading process of the arriving cargo will be approximately 7230 LVL (including the penalty for the downtime of the trainload, which amounts to 1600 LVL).

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