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LOADING OF HETEROGENEOUS TRUCK FLEET BY HEAVY OBJECTS

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The article deals with transporting heavy objects weighing several tons with different trucks. Their disposable fleet is assumed heterogeneous in the sense of load capacity, for example from 3 tons to 12 tons.

The basic decision problem is formulated as follows: Given two points *A* and *B*; given a truck fleet consisting of several vehicles that do not have the same load capacity; given a linear function expressing dependence of the costs of passing from *A* to *B* on the loaded mass for each truck; given several heavy objects with known weights. The problem is to decide, which trucks will be used and what loading plan will be chosen.

It is mentioned that the problem is a generalization of “classic” Dump of Stones Problem where the fleet consists of two equal trucks and therefore, if the solution exists, the sums of weights of both loads should be as close as possible.

An integer linear programming model is presented as a tool for solution of the basic problem.

Keywords: truck loading, heterogeneous fleet, heavy objects, cost, linear programming.

JEL Classification: C61, L92, R42

1 Introduction

Motivation of the authors to write this paper is based on the typical feature of Czech trucking industry – many small carriers having fleets consisting of several trucks with different loading capacity, e.g. 3-22 tons. From time to time they get a contract to transport a group of heavy objects, such as boxes weighing several tons, from point A to point B. Carriers are then faced the question of which cars to use and how those objects load into them optimally. However, they decide intuitively, without any optimisation technique. In other words, the (capacitive) heterogeneity brings more difficulties for decision makers than the homogeneity.

From the macro point of view, on the national level, it is quite natural to consider the system of freight transport heterogeneous since the various modes of transport (road, railway, maritime etc.) are not equally suitable for transportation of different goods. Therefore, it can be expected that the theoretical background of modal split on the macro level is studied – see e.g. [1].

The main issue of managerial decision making on micro level in road transport of goods is maximum capacity utilization of trucks, both spatial (cargo space) and weight (tonnage). Of course, they also make sure that the vehicle is in motion as much as possible and choose optimal routes for them. Therefore, in the paper, the first objective function expresses the loading percentage.

Another indicator, influencing the freight transport economy, is fuel consumption. In the professional bibliography, various factors influencing it are studied. E.g. in [8] it is the altitude over see, in [10] it is the payload, i.e. the weight of loaded objects and the dependence of fuel consumption on payload is linear for the given vehicle on the same track. Since the authors have not found any paper with non-linear dependency, this linear dependency is used as the second objective function in the paper.

The topic of the paper belongs to the family of problems that were originally formulated for homogeneous fleet and later reformulated for heterogeneous fleet. Probably the best known example is the "classic" *vehicle routing problem* presented first in [6] as "The Truck Dispatching Problem" in 1959. On the other hand, the *heterogeneous fleet vehicle routing problem* seems to be very topical as one can see e.g. from [2], [9] or [11].

The second example is truck and loader matching, e.g. in mining industry [3], first formulated and solved for homogeneous fleets and later for heterogeneous ones.

The present paper is focused on transporting heavy objects weighing several tons. As said in [12], the “classis” version has been first formulated for two equal trucks as Dump-of-Stones Problem. In Russian it is called “Kucha kamney” (куча камней) and until now one can find it in quite new bibliography, e.g. in [13]. However, they do not write about it as a managerial decision-making problem, but as a task for PC programmers.

It is necessary to add, that the Dump-of-Stones problem is NP-hard. It is easy to show that the well known Partition Problem, which is NP-complete [7], can be transformed to Dump-of-Stones Problem.

An extension to more than two equal trucks can be found in [4], paragraph 14.1.2 or in [5], 3.2.2. The authors of the present paper have not met any bibliographical item dealing with extension to a fleet, which is heterogeneous as concerns tonnage. The following text is an attempt to fill this gap.

2 Loading of Trucks by Heavy Objects

Loading capacity utilization of trucks to transport heavy objects has its own specificities. Suppose that the carrier possess two trucks with tonnage 5t, i.e. 10t in total. It is necessary to transport 3 heavy packing cases with equal weight 2.7t, i.e. 8.1t in total, which is much less than the available capacity 10t. Nevertheless, these two trucks are not sufficient for shipping of these objects since two of them are together heavier than 5t.

2.1 Basic Problem

Assume that the carrier has n trucks with load capacities $q_1 \geq q_2 \geq \ldots \geq q_n$. He wants use them, or some of them, to transport m heavy objects (e.g. boulders, crates, etc.) weighing $h_1 \geq h_2 \geq \ldots \geq h_m$ from point A to point B. He wants to prepare a plan of transport, i.e. to determine, which vehicles take place in transport and how the objects divide among them so that the vehicles are loaded as much as possible, but not overloaded.

As shown above, such a plan need not always exist, even if the sum of the capacities is significantly greater than the sum of the weights, even if $q_n \geq h_1$, i.e. when no object itself overloads any vehicle.

Problem P0 (basic). Given integers $m > 1$, $n > 1$ and real numbers $q_1 \geq q_2 \geq \ldots \geq q_n > 0$, $h_1 \geq h_2 \geq \ldots \geq h_m > 0$. The problem is to find a subset $L \subset \{1, \ldots, n\}$ and a mapping g of the set $H = \{1, 2, \ldots, m\}$ to the set L meeting the basic capacity constraint:

$$\sum_{i:g(i)=j} h_i \leq q_j \tag{1}$$

and minimizing/maximizing the value of the given objective function

$$f(g, L) \to \min(\max) \tag{2}$$

where f may be expresses a "natural" characteristics, usually the relative filling of total loading capacity, or a financial indicator, e.g. the cost of fuel, or their combination.

2.2 Problem of Relative Filling Maximization

In this case the objective function compares the total weight of the transported objects with the total supplied capacity.

Problem 1. In the Problem 0 the condition (2) is formulated more precisely as

$$\frac{\sum_{i=1}^{m} h_i}{\sum_{j \in L} q_j} \to \max \tag{3}$$

2.3 Problem of Fuel Cost Minimization

It is supposed now that the j^{th} truck consumes fuel at the price $a_j + b_j h$ on the route between points A and B. Therefore, in this case, the objective function expresses the cost of the fuel consumed by all used vehicles.

Problem 2. In the Problem 0 the condition (2) is formulated more precisely as

$$\sum_{j \in L} \left(a_j + b_j \sum_{i:g(i)=j} h_i \right) \to \min \tag{4}$$

3 Solution by Linear Programming

In this section the use of integer linear programming (LP) to solution of Problem 1 and Problem 2 is presented. It is probably the best way of solution. To look for a polynomial exact method is inutile since both problems are obviously NP-hard due to the fact that they are generalizations of NP-hard Dump-of-Stones problem as said at the end of Section 1.

On the other hand, the size of the problems is workable by LP. The authors' experience shows that, usually, the number of vehicles n and number of objects m do not exceed 15.
LP models for both problems works with the following variables:

- x_j for all $j = 1, \ldots, n$, whereas $x_j = 1$ when the j^{th} vehicle is used, $x_j = 0$ otherwise;
- y_{ij} for all $i = 1, \ldots, m, j = 1, \ldots, n$, whereas $y_{ij} = 1$ when the i^{th} object is loaded on the j^{th} vehicle, $y_{ij} = 0$ otherwise.

3.1 Problem of Relative Filling Maximization – LP Solution

To solve Problem 1 by LP means:

Problem 1 LP. To find binary values x_j for all $j = 1, \ldots, n$, and y_{ij} for all $i = 1, \ldots, m, j = 1, \ldots, n$ meeting the following constraints:

$$\sum_{i=1}^{m} h_i y_{ij} \le q_j x_j \quad \text{for all } j = 1,\ldots,n \tag{5}$$

$$\sum_{j=1}^{n} y_{ij} = 1 \quad \text{for all } i = 1,\ldots,m \tag{6}$$

$$\sum_{j=1}^{n} q_j x_j \to \min \tag{7}$$

Comment. The constraint (5) ensures that no truck is overloaded, (6) ensures that all objects are loaded and transported. Following to the constraint (7) the denominator in (3) is minimized. Since the nominator is constant, the constraint (7) implies the constraint (3).

3.2 Problem of Fuel Cost Minimization – LP Solution

To solve Problem 2 by LP means:

Problem 2 LP. To find binary values x_j for all $j = 1, \ldots, n$, and y_{ij} for all $i = 1, \ldots, m, j = 1, \ldots, n$ meeting the following constraints:

$$\sum_{i=1}^{m} h_i y_{ij} \le q_j x_j \quad \text{for all } j = 1,\ldots,n \tag{8}$$

$$\sum_{j=1}^{n} y_{ij} = 1 \quad \text{for all } i = 1,\ldots,m \tag{9}$$

$$\sum_{j=1}^{n} \left(a_j x_j + b_j \sum_{i=1}^{m} h_i y_{ij} \right) \to \min \tag{10}$$

Comment. The constraint (8) ensures that no truck is overloaded, (9) ensures that all objects are loaded and transported. Following to the constraint (7) the total price of fuel is minimized.

4 Conclusion and Outline of Future Research

At the beginning of the paper it is emphasized that the most important decision problem the managers of carriers are confronted with is how to make their vehicles as busy as possible.

Further it is shown that several such problems were first solved for capacitively homogeneous fleet and only after many years the same problems were solved for heterogeneous fleet. The optimal loading plan for a set of heavy object the paper deals with underwent the same development.

The main part of the paper presents two variants of loading plan optimisation problems. They differ in objective function. The first maximizes the loading ratio, the second one minimizes the fuel costs. Since both problems are NP-hard, no effort for polynomial exact method of solution was made. A linear programming solution was proposed and the corresponding models were formulated.

The authors expect that the future research will take several directions. The first one may work with other objective functions e.g. taking into account the drivers' interests. The other direction may look for a more exact formula for dependence of fuel consumption on the load.

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