

## USE OF STATISTICAL TECHNIQUES FOR CRITICAL GAPS ESTIMATION

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The critical gap is very important parameter for capacity calculation of unsignalized intersections. This parameter is a stochastically distributed value and it cannot be measured directly on the field. Only rejected gaps and accepted gaps of each minor stream vehicle can be measured at the intersection. Consequently, some statistical model or procedure for estimating critical gaps is needed. There are many different methods or procedures for critical gap estimation and many studies about estimating the critical gap have been carried out. But in Slovakia we have only a little practical experiences because similar studies have rarely been done in our country. This paper presents a short overview of three methods for the estimation of critical gaps at unsignalized intersections: method of Raff (1950), Maximum Likelihood Method (MLM) of Troutbeck (1992) and method of Wu (2006). These methods are presented on a practical example. Input data – observed accepted and maximum rejected gaps of each minor stream vehicle – were determined on the basis of video survey at the unsignalized intersection in Žilina. Critical gaps are estimated by using these three methods and results are compared each other.

**Keywords:** critical gap estimation, rejected and accepted gap, maximum likelihood method, probability, unsignalized intersection

### 1. Introduction

Methods of capacity calculation for unsignalized intersections are mostly based on Gap acceptance procedure (GAP). In this procedure the critical gap is an important parameter for capacity calculation. There are different values of critical gaps for drivers in different geometry and traffic conditions. The problem is the critical gaps cannot be measured directly. Only reject gaps and accepted gaps of single vehicles in the minor stream can be measured at the intersection. Consequently, some statistical model or procedure for estimating critical gaps at unsignalized intersections is needed. There are many different methods or procedures for critical gap estimation (more than 20 methods), such as method of Raff (1950), Harders (1968), Aworth (1970), Sieglöch (1973), Troutbeck (1992) and so on.

The most commonly used methods for estimating the critical gap are the method of Raff (1950) and Troutbeck (1992). The method of Raff (1950) is based on macroscopic model and it is the earliest method for estimating the critical gap which is used in many countries because of its simplicity. In Troutbeck's microscopic model the probability of the critical gap is calculated through the Maximum Likelihood Method (MLM). An iteration process is necessary. The outcome of Brilon et al. comprehensive analyses [1] is MLM of Troutbeck (1992) gives the best results. This method is recommended for estimating the critical gaps in the best-known standard manuals for traffic engineering HCM 2000 and HBS 2001. Unfortunately, the model of Troutbeck (1992) is very complicated to common use for traffic engineers. Besides the mentioned methods, Wu (2006) developed a macroscopic method based on equilibrium of probabilities between the rejected and accepted gaps [2]. Using this method is simple; it can be carried out easy by spreadsheet programs without iterations.

This paper briefly introduced method of Raff (1950), MLM of Troutbeck (1992) and method of Wu (2006). In Slovakia we have only a little practical experiences about estimating the critical gap because similar studies have rarely been done in our country. This paper presents practical example of critical gaps estimation by using these three methods. The critical gaps were calculated and compared each other. Input data (accepted gaps and maximum rejected gaps) were determined on the basis of video survey at the unsignalized intersection in Žilina.

## 2. Methods for Estimating the Critical Gap

The critical gap  $t_c$  can be defined as the minimum time interval between the major stream vehicles that is necessary for one minor stream vehicle to make a maneuver (see Figure 1). Values of critical gaps are different for different drivers (some of them are too fast or risky, some of them are slow or careful) and there are dependent on types of movements, geometry parameters of intersections, traffic situation. Due to this variability gap acceptance process is considered as a stochastic process and the critical gaps are random variables. The estimation of critical gaps tries to find out values for the variables and as well as for the parameters of their distributions, which represent typical driver behavior at the investigated intersection.

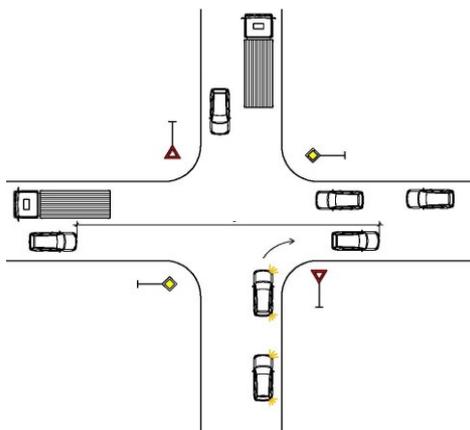


Figure 1. The critical gap for minor stream right-turn vehicle

In unsignalized intersection theory it is generally assumed that drivers are both consistent and homogeneous. Consistent drivers are expected to behave the same way every time in all similar situations. This means: a driver with a specific  $t_c$ -value will never accept a gap less than  $t_c$  and he will accept each major stream gap larger than  $t_c$ . However, within a population of several drivers, each of which behaves consistent, different drivers could have their own  $t_c$ -value. These  $t_c$ -values are then treated as a random variable with a special statistical density function  $f_{tc}(t)$  and cumulative distribution function  $F_{tc}(t)$ . The population of drivers is homogeneous if each sub-group of drivers out of the population has the same functions  $f_{tc}(t)$  and  $F_{tc}(t)$ .

The problem is that the critical gaps cannot be measured directly. Only rejected gaps and accepted gaps of each minor stream vehicle can be measured at the intersection. The critical gaps can be estimated from these input data using some statistical method or procedures.

For the estimation of critical gaps from observations three methods are briefly described in this article and applied in a practical example – method of Raff (1950), Maximum Likelihood Method (MLM) of Troutbeck (1992) and method of Wu (2006).

### 2.1. The Raff's Method

The method of Raff (1950) is based on macroscopic model and it is the earliest method for estimating the critical gap which is used in many countries because of its simplicity. This method involves the empirical distribution functions of accepted gaps  $F_a(t)$  and rejected gaps  $F_r(t)$ . When the sum of cumulative probabilities of accepted gaps and rejected gaps is equal to 1 then a gap of length  $t$  is equal to critical gap  $t_c$ . It means the number of rejected gaps larger than critical gap is equal to the number of accepted gaps smaller than critical gap.

$$F_a(t) = 1 - F_r(t). \quad (1)$$

### 2.2. The Maximum Likelihood Method

The model of Troutbeck (1992) for estimating critical gaps is based on the Maximum Likelihood Method (MLM). This microscopic model assumes the log-normal distribution of accepted and maximum rejected gaps. In this model, only the accepted gap and the maximum rejected gap of each vehicle are

treated pair wise. For one individual minor street driver  $i$  we have observed: one accepted gap  $a_i$  and one corresponding the maximum rejected gap  $r_i$ . The maximum rejected gap is the largest value of all rejected gaps for one minor street driver.

The MLM is based on the assumption that minor stream drivers behave consistently. It means that each driver will reject every gap smaller than his critical gap and will accept the first gap larger than the critical gap. Under this assumption, the distribution of the critical gaps lies between distributions of largest rejected and accepted gaps (see Figure 2). The parameters of distribution function of the critical gaps, the mean  $\mu$  and variance  $\sigma^2$ , are obtained by maximizing the likelihood function. The likelihood function is defined as the probability that the critical gap distribution lies between the observed distribution of the maximum rejected gaps and the accepted gaps:

$$\prod_{i=1}^n [F(a_i) - F(r_i)], \quad (2)$$

where

- $L$  – maximum likelihood function,
- $a_i$  – logarithm of the accepted gap of driver  $i$ ,
- $r_i$  – logarithm of the maximum rejected gap of driver  $i$ ,
- $F(a_i), F(r_i)$  – cumulative distribution functions for the normal distribution.

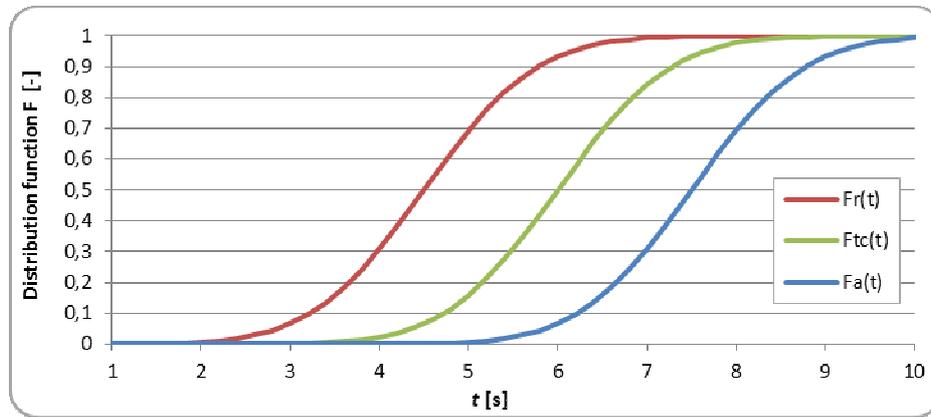


Figure 2. Theoretical distribution function of accepted gaps  $Fa(t)$ , maximum rejected gaps  $Fr(t)$ , and critical gaps  $Ftc(t)$

The logarithm of function (3) is as follows:

$$L = \sum_{i=1}^n \ln[F(a_i) - F(r_i)]. \quad (3)$$

Likelihood parameters  $\mu$  and  $\sigma^2$  are solutions when the partial derivative of equation (4) is equal to 0. They can be simplified as follows:

$$\begin{cases} -\sum_{i=1}^n \frac{f(a_i) - f(r_i)}{F(a_i) - F(r_i)} = 0 \\ -\frac{1}{2\sigma^2} \sum_{i=1}^n \frac{(a_i - \mu) \cdot f(a_i) - (r_i - \mu) \cdot f(r_i)}{F(a_i) - F(r_i)} = 0 \end{cases}, \quad (4)$$

where

$f(a_i), f(r_i)$  – probability density functions for the normal distribution with parameters  $\mu$  and  $\sigma^2$ .

Parameters  $\mu$  and  $\sigma^2$  can be calculated by numerical and iteration techniques [5]. Subsequently the mean  $E(t_c)$  and variance  $D(t_c)$  of critical gap can be derived by equations:

$$E(t_c) = e^{\mu+0,5\sigma^2}. \quad (5)$$

$$D(t_c) = E(t_c)^2 \cdot (e^{\sigma^2} - 1). \quad (6)$$

### 2.3. The Model Based on the Macroscopic Probability Equilibrium

The theoretical background of Wu's model [2] is the probability equilibrium between the rejected and the accepted gaps. The equilibrium is established macroscopically from the cumulative distribution of the rejected and accepted gaps. The observed probability that a gap of length  $t$  is accepted is  $F_a(t)$  and that it is not-accepted is  $1-F_a(t)$ . The observed probability that a gap of length  $t$  is rejected is  $F_r(t)$  and that it is not-rejected is  $1-F_r(t)$ . Generally  $F_r(t) \neq 1-F_a(t)$  and  $1-F_r(t) \neq F_a(t)$ .

Considering the observed probability of both – acceptance and rejection, Wu's method is based on the following probability equilibrium:

$$P_{r,ic}(t) = F_r(t) \cdot P_{r,ic}(t) + F_a(t) \cdot P_{a,ic}(t). \quad (7)$$

Denote the distribution function of the critical gaps to be estimated by  $F_{ic}(t)$ , then the probability  $P_{r,ic}(t)$  that a gap of length  $t$  in the major stream would be rejected is  $F_{ic}(t)$ , and the probability that it would be accepted is  $1-F_{ic}(t)$ . Substituting  $P_{r,ic}(t)=F_{ic}(t)$  and  $P_{a,ic}(t)=1-F_{ic}(t)$ , for Wu's model we have the following distribution function  $F_{ic}(t)$  of the critical gaps:

$$F_{ic}(t) = \frac{F_a(t)}{F_a(t) + 1 - F_r(t)} = 1 - \frac{1 - F_r(t)}{F_a(t) + 1 - F_r(t)}. \quad (8)$$

For implementing the proposed macroscopic model of Wu is detailed calculation procedure step by step given in [2]. This procedure is easily implemented into Excel spreadsheet.

According to Raff's definition for the critical gap (Equation (1)) it can be written [2]:

$$F_{ic}(t) = \frac{F_a(t)}{F_a(t) + 1 - F_r(t)} = 1 - \frac{1 - F_r(t)}{F_a(t) + 1 - F_r(t)}. \quad (9)$$

It means that the critical gap estimated from Raff's method is the median value but not the mean value of the critical gap.

## 3. Practical Example of Estimating the Critical Gaps

### 3.1. Results of Measurements

Measurements were carried out at the unsignalized intersections in Žilina. All traffic streams at this intersection were video-recorded. Subsequently video-recordings were processed by own specific software (using MATLAB). Time position of each vehicle on a certain point (designed line across the road) was recorded in a spread sheet MS Excel with accuracy 0,04 s (frame rate of camera: 25 frames per sec.). For each minor stream vehicle was recorded: arrival time, vehicle type, direction and departure time. For each major stream vehicle was recorded: passing time a designed line across the road, vehicle type and direction. All data were processed. The accepted and largest rejected gaps for individual minor stream vehicle were determined (for each minor stream separately). If a driver from minor stream accepted the lag and did not reject any time gap: a largest rejected time gap was equal 0 s.

Example of estimating critical gaps of a minor left-turn stream at unsignalized intersection in Žilina (see Figure 3) by three estimating methods using statistical techniques (Raff, MLM and Wu) is given in this article. Traffic survey was carried out by video recording in duration one hour (peak hour). All observed input data – accepted and the maximum rejected gaps for each minor stream left-turn vehicle are presented on Figure 4. Two simples for estimating critical gaps have been given. In simple 1 there are not considered vehicles which had the maximum rejected gap equal to zero, so 80 couples of accepted and maximum rejected gaps were derived for estimating critical gap by all three methods. In simple 2 there are also considered vehicles which immediately entered to intersection without waiting and for estimating critical gap were derived 80 rejected gaps and 103 relevant accepted gaps. Data from simple 2 were used only for two macroscopic methods of Raff and Wu, because of requirement of MLM to have couples of accepted and maximum rejected gaps for estimation of critical gaps.

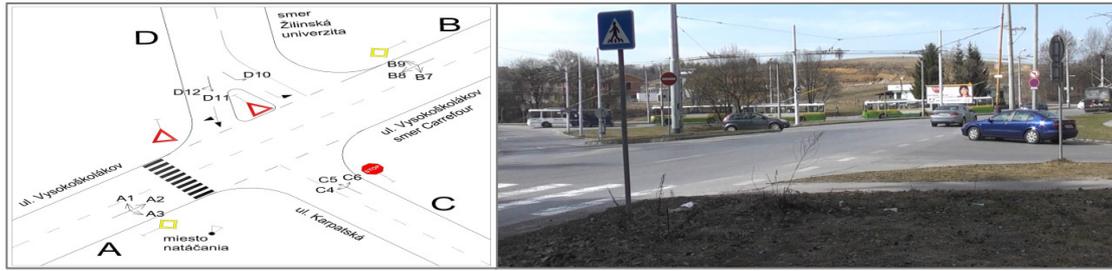


Figure 3. Monitored unsignalised intersection in Žilina

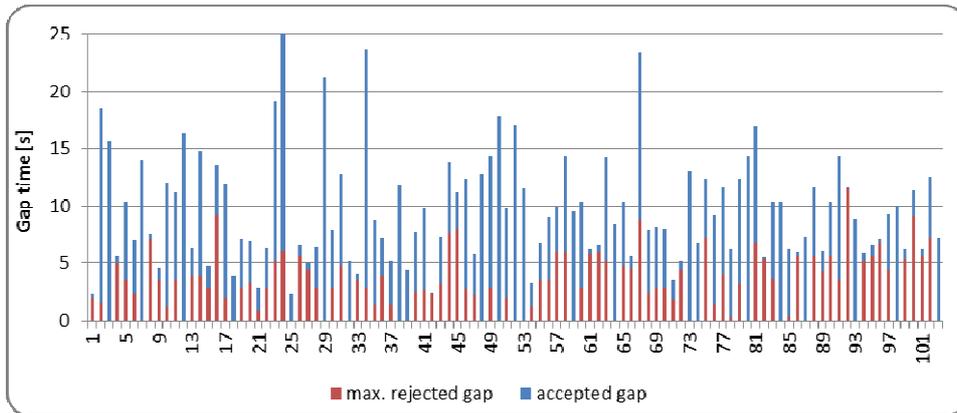


Figure 4. Measurements of the maximum rejected gaps and accepted gaps (Data: Žilina 2012, minor stream left-turn)

### 3.2. Estimation and Comparison of Critical Gaps

#### Results of Raff's method

The curves of accumulative probability of accepted gaps  $F_a(t)$  and maximum rejected gaps  $F_r(t)$  are shown on Figure 5. The critical gap  $t_c = 5,80 s$  is in cross point of  $F_a(t)$  and  $1-F_r(t)$ .

The critical gap was estimated by Raff's model also considering all accepted gaps from input data (simple 2). Although distribution function  $F_a(t)$  for simple 2 is slightly different to  $F_a(t)$  for simple 1, cross point is in the same gap time, the critical gap  $t_c = 5,80 s$ .

#### Results of MLM of Troutbeck

In accordance with MLM of Troutbeck the probability density functions of accepted gaps  $f(a_i)$  and maximum rejected gaps  $f(r_i)$ , distribution functions of accepted gaps  $F(a_i)$  and maximum rejected gaps  $F(r_i)$  were calculated for each logarithm of accepted gap  $a_i$  and logarithm of maximum rejected gap  $r_i$ . The likelihood parameters: the mean  $\mu$  and variance  $\sigma^2$ , were obtained as a solution of two equations (4) by iteration process. The iteration process was programmed in Excel program. Values  $\mu$  and  $\sigma^2$  were changed step by step until functions in equation (4) tend to zero. Finally the mean  $E(t_c) = 6,04 s$  and variance  $D(t_c) = 5,38 s^2$  of critical gap were derived by equations (5) and (6).

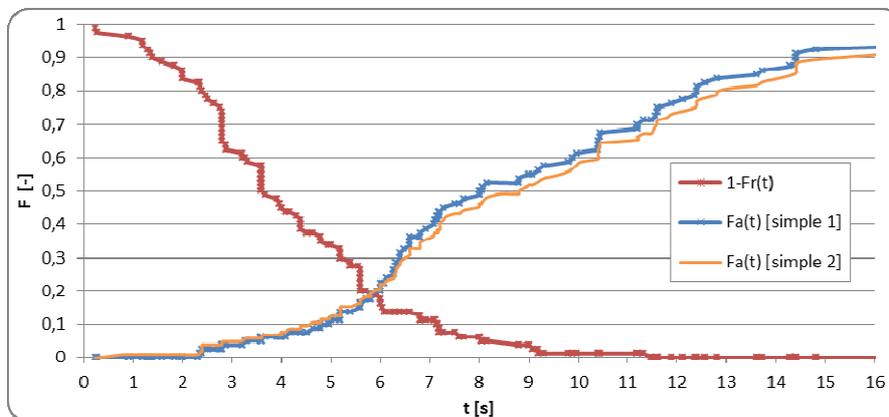


Figure 5. The critical gap estimation by Raff's method, distribution functions of the maximum rejected gaps –  $F_r(t)$  and accepted gaps –  $F_a(t)$  for simple 1 and simple 2

### Results of macroscopic method of Wu

The distribution functions of accepted gaps  $F_a(t)$  and maximum rejected gaps  $F_r(t)$  are shown on Figure 6. The distribution function of critical gap  $F_{ic}(t)$  was calculated in accordance with macroscopic probability equilibrium model of Wu by equation (8) and the curve of this function lies between  $F_a(t)$  and  $F_r(t)$ . The mean value of the critical gap  $t_c = 5,91$  s and the variance  $\sigma^2 = 2,89$  s<sup>2</sup> were calculated. The median value of the critical gap was also calculated and its value is equal to Raff critical gap (5,8 s).

For comparison, distribution function of the estimated critical gap from MLM of Troutbeck –  $F_{ic}(t)$  [MLM] – is also shown on Figure 6.

The critical gap was estimated by macroscopic model of Wu also considering all accepted gaps from input data (simple 2). The mean value of the critical gap  $t_c = 5,60$  s and the variance  $\sigma^2 = 3,34$  s<sup>2</sup> were calculated.

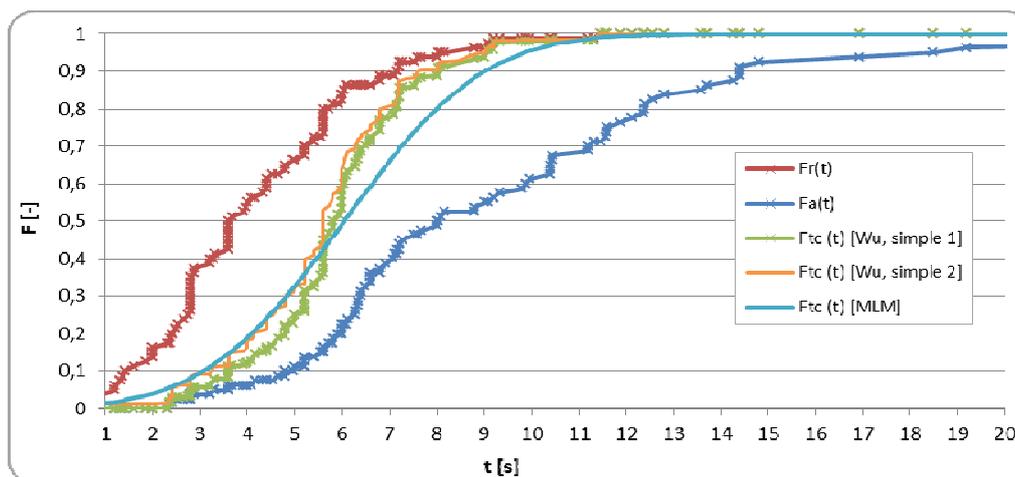


Figure 6. Distribution functions of the maximum rejected gaps –  $F_r(t)$ , accepted gaps –  $F_a(t)$ , critical gaps from the Wu's model –  $F_{ic}(t)$  [Wu] and distribution function of the estimated critical gap from MLM of Troutbeck –  $F_{ic}(t)$  [MLM]

## 4. Conclusions

Maximum likelihood method of Troutbeck (1992) belongs to the most important models for estimating the critical gaps and according to [1] gives the best results. The macroscopic method of Wu (2006) gives similar results for the mean critical gaps as that from recognized MLM of Troutbeck (1992). In addition, we can get not only the main value of critical gaps but also the median value, which is the same value as the result of Raff's method. Also we can directly get the probability distribution function of the critical gaps. Advantage of this method is simple calculation procedure without iteration as it is in the case of the Troutbeck model (1992). It can be carried out using spreadsheet programs, e.g. Excel, and therefore it is very useful for professionals of traffic engineering. In addition, not only couples of accepted and rejected gaps are required for estimation of critical gaps as in MLM of Troutbeck is required. It can be used all relevant accepted and maximum rejected gaps. In these cases the mean critical gap is shorter.

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