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ANALYSIS OF A SYSTEM PULSE RESPONSE BASED ON EMPIRICAL MODE DECOMPOSITION

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The aim of this paper is to work out an approach to filter non-minimal phase transients before their complex spectra processing used empirical mode decomposition. The algorithm is based on zero-pole analysis of the signal with elimination of extraneous zeroes. Subsequent decomposition permits to expand the logarithmic complex spectrum of minimum phase part into a little amount of complex intrinsic mode functions (IMFs). Deconvolution based on that IMFs allows finding the components of an empirical convolutional model of the signal considered. On the other hand, analysis of these IMFs is a way to produce a complex quasicepstrum. The latter points to an inherent structure of the signal reflected not only the amplitudes but phases relations in the signal, too. The set of Matlab codes to process the transient pulses of diverse physical nature is worked out.

Keywords: *non-stationary signal, minimum phase approximation, logarithmic spectrum, Hilbert transforms, pseudocepstrum*

1. Introduction

The expansion of non-stationary signals in different bases is commonly used in signal processing techniques. In addition to mathematically well-founded methods the new approaches have been worked out in nowadays. The empirical mode decomposition (EMD) has a significant place among them. The EMD expands a real valued signal data set into a number IMFs which do not belong to a predetermined signal basis. The current EMD basis is derived directly from the signal itself. However, for complex valued data some problems take place since well-defined relations should be maintained among real and imaginary parts of the signal. The sounding example is a data set in the form of a complex spectrum, which needs to be treated delicately prior to further processing.

A special interest in non-stationary transient signals (or video pulses) processing has observed in that areas as broadband radars, sonar, exploratory geophysics, or natural electromagnetic phenomena. A linear model is generally introduced as a description of these signals. It is supposed that the signal is formed by linear combination of some delayed pulses arrived to the observation point along non-coincided paths. As a rule, such a signal belongs to the non-minimum phase class. The frequency spectrum of it is as follows

$$\dot{S}(i\omega) = \dot{S}_{\mathrm{mph}}(i\omega)\dot{S}_{\mathrm{ap}}(i\omega), \tag{1}$$

arising as the product of minimum $S_{\mathrm{mph}}(i\omega)$ and non-minimum (or all-pass) $S_{\mathrm{ap}}(i\omega)$ phase parts. (1)

Many algorithms in digital signal processing are based on assumption that the general information about the signal features is contained in the first multiplier of (1). Therefore, the problem of finding the minimum phase part from spectrum of the signal has to be resolved prior to subsequent processing.

The organization of the paper is as follows. In Section 2 the filtering of minimum phase analogue (MPA) from the processed signal is considered based on zero-pole modelling of it with elimination of extraneous zeroes. In Section 3 the aims is to compare the EMD expansions of the MPA and the initial signal itself. In Section 4 the complex logarithmic spectrum of the MPA is analysed by EMD to find the IMFs of it. An approach is worked out to recover the presumed convolutional components of the signal in temporal domain based on above IMFs. A concept of complex pseudocepstrum is introduced, too.

2. Zero-Pole Modelling of the Processed Signal and Minimum Phase Analogue

Any real signal can be considered as an impulse response of a system excited by the Dirac's δ-pulse. The Z-transform as applied to it allows describing the appropriated system function as a ratio

$$H(z) = \frac{\sum_{k=0}^{M} b(k) z^{-k}}{1 + \sum_{k=1}^{N} a(k) z^{-k}} = b(0)\frac{\prod_{k=1}^{M}(1 - z_k z^{-1})}{\prod_{k=1}^{N}(1 - p_k z^{-1})} \tag{2}$$

of two polynomials in z^{-1} where the roots of the numerator polynomial, z_k, are the zeroes of *H(z)* and the roots of the denominator polynomial, p_k, are called the poles.

There are many algorithms to calculate the coefficients *a(k)* and *b(k)* from the signal in frames of zero-pole (Z-P) modelling methods. Perhaps, the mostly used of them are Steiglitz-McBride or Prony filters [1]. The first one performs a least squares minimization of the error

$$E(z) = X(z) - H(z) \tag{3}$$

where *X(z)* is the Z-transform of the investigated signal *x(t)*. With Prony's method the coefficients *a(k)* of the so-called all-pole model

$$H(z) = \frac{b(0)}{1 + \sum_{k=1}^{N} a(k) z^{-k}} \tag{4}$$

are found by minimizing the L_2-norm energy error in temporal domain

$$\varepsilon_p = \sum_{l=0}^{L} \left| x(l) - x_p(l) \right|^2 \tag{5}$$

where x_p is a time series recovered from the model, L – the number of samples in original signal *x(t)*.

The choice of the method depends on assumptions about inherent structure of the model since internal blocks in it can be connected in series or parallel or more complicated manner. It is conceived in a case when the signal is formed by linear combination of some delayed pulses arrived to the observation point along non-coincided paths the Prony method should be more adequate with presumably parallel structure of that model.

Both these filters considered above have been realised in diverse Matlab versions.

The orders M for numerator and N for denominator of the system function (2) should be assigned before zero-pole modelling. It is rather difficult problem which has no standard rules to solve it. One the ways is to calculate the error of approximation as in (5) enlarging value N gradually with fixed difference among N and M. The process should be stopped as soon as the relative error decreases below the predetermined level. In order to preserve stability of the model it is recommended to set maximum value of N no more then $(0.3 – 0.4)\cdot L$. Value of M usually is $N – 1$ or less.

It has to be noted that this procedure reduces noise in the recovered signal x_p in comparison with initial signal x owing to limitation of polynomial items by reasonable values of N and M. By choice the length of x_p it can correct the influence of rectangular window, which is applied with necessity to the initial data before digital signal processing.

Striking examples of transients are atmospherics or some electromagnetic pulses radiated by natural lightning during thunderstorms. A scientific interest associated with them is in an inverse problem to find geometry and electro physical features of lightning channel starting from inherent structural properties of the atmospherics. Figure 1*a* shows the atmospherics received at a distance of 30 km from a lightning discharge. It was selected as a trial signal in the subsequent text. The result of the zero-pole modelling with Prony's algorithm of the highest order is presented in the same place, too.

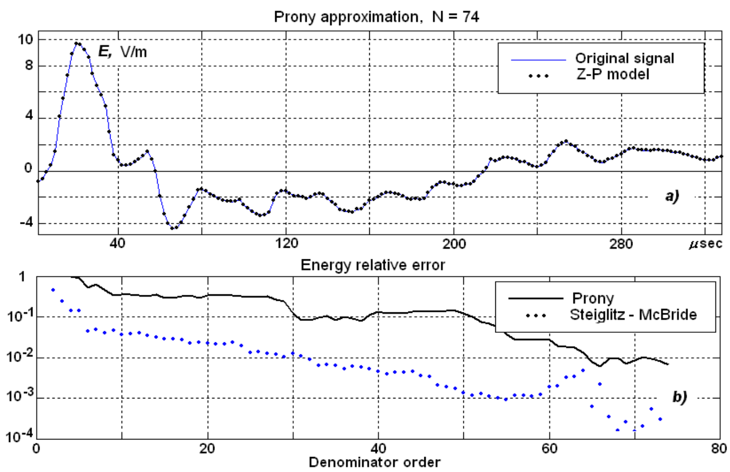

Figure 1. Analysed atmospherics (a); errors of zero-pole (Z-P) modelling versus model order (b)

Energy errors of Z-P modelling based on Prony's and Steiglitz-McBride's methods are compared in Figure 1*b*. It is seen that the same errors for the considered signal can be reached with such the order Steiglitz-McBride's algorithm which is less about two times than the proper order of Prony's algorithm. Nevertheless, as it is pointed out above the concrete choice of the method depends on assumed inherent structure of the model.

Replacement of the original signal by certain minimum phase analogue (MPA) is used in signal processing rather frequently. The MPA has to be discovered prior to principal processing. There are many ways to estimate it, for example, based on classic method of the real cepstrum [2]. However, in the Z-P model context, for some signals it can be observed the zeroes situated outside the unit circle in the zeroes-poles map. It has to be considered as the sign of non-minimum phase nature of the transient.

The same signal as pointed out above represents in Figure 2 (bottom left) with lesser Z-P model order than earlier. Its zeroes-poles map is shown in the left top corner.

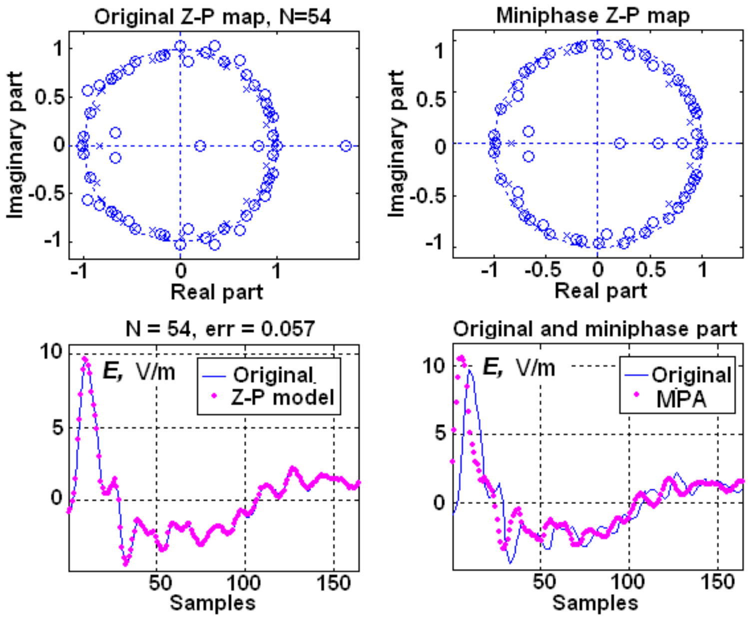

Figure 2. Minimum phase transformation (see explanations in the text above)

One can see the zeroes situated on the unit circle, therefore this signal is non-minimum phased. It can be transformed into MPA by mirror reflection of these extraneous zeroes inside the unit circle. The map after this transformation is shown at the top right picture on Figure 2. Proper MPA is displayed in the bottom right subplot. It demonstrates that delay of the first peak is noticeably lesser than in the original signal.

It is clear that any operation applied to a signal reduces information consisting in it. The correlation coefficient

$$C_{xy} = \frac{\sum_{l=1}^{L}(x_l - m_x)(y_l - m_y)}{\left[\sum_{l=1}^{L}(x_l - m_x)^2 \sum_{l=1}^{L}(y_l - m_y)^2\right]^{1/2}} \qquad (6)$$

can serve as a measure of proximity of time series x and y where m_x and m_y are the mean values of x and y, correspondingly.

Denoting original signal (Fig. 1) as u, zero-pole model as v, and MPA as w one can find from (6) using Matlab function **corrcoef**:

$\boldsymbol{C_{uv}}$ = corrcoef(u,v) = 0.99943, $\boldsymbol{C_{uw}}$ = 0.74623, $\boldsymbol{C_{vw}}$ = 0.74659.

Reduction of correlation coefficient in case of MPA may be explained by the evident shift along time axis (Fig. 2, at the bottom right subplot) as compared with the original signal.

3. Empirical Mode Decomposition in Temporal Domain

Empirical mode decomposition introduced by Huang et al. (see, for instance, [3]) deals with any signals both linear and nonlinear or non-stationary. It is a method to adaptively decompose a certain signal into a finite set of oscillatory items, called intrinsic mode functions. The IMFs are derived directly and adaptively from the signal itself forming a set of natural basis functions. The EMD process called the 'sifting' performs for a digital signal the mapping

$$x[l] = \sum_{q=1}^{Q} c_q[l] + r[l] \qquad (7)$$

where the $c_q[l]$, $q = 1,\dots Q$ denote the set of IMFs and $r[l]$ is the residual or the final member in this set also referred as a trend. The IMFs must obey two general assumptions:

a) have the same number of extreme and zero crossings or differ at most by one from the signal;

b) be symmetric with respect to the local zero mean.

Sifting algorithm has been described quite enough (see, e.g., [3-5]).

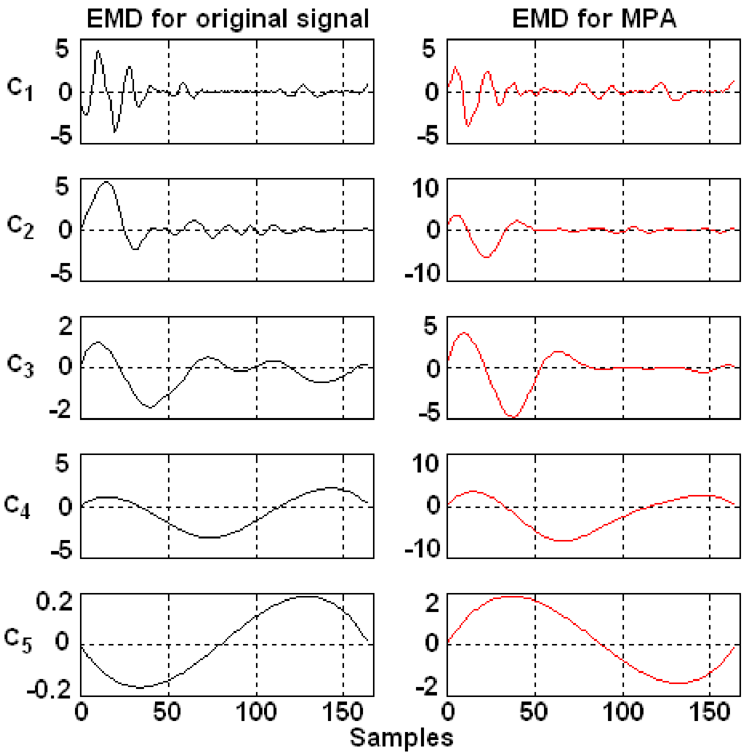

Figure 3. Intrinsic mode functions both the original signal and the minimum phase analogue

It seems that comparative analysis of the IMFs both a signal and its miniphase analogue is not yet performed. As a certain example, result of EMD in temporal domain is shown on Figure 3 for the same signals that are examined above. The IMFs are designed from c_1 to c_5.

In order to confronting corresponding IMFs one can calculate the vector of values of correlation coefficient (6) illustrated in the following Table:

$C_{1\text{-}1}$	$C_{2\text{-}2}$	$C_{3\text{-}3}$	$C_{4\text{-}4}$	$C_{5\text{-}5}$
[-0.49183	–0.17519	0.87089	0.9595	-0.99402]

It is seen that the correlation is increased as far as the IMFs become more and more inert functions in time.

As EMD suppose an additive model of signal by definition, then, using an analogue with wavelet analysis, one can confirm that these the most slowly oscillatory items of IMFs behave as approximating members in wavelet decomposition. Just they determine the basic energetics in the signal. In this sense, the signal by itself and MPA of it are interchangeable.

4. Empirical Mode Decomposition in Spectral Domain

However, additive model in temporal domain is not adequate for the linear systems, in particular, when response of the system is convolutional. In this case, being adjusted to the signal as it would be some function of time; the EMD is not complying with the convolutional model. To find the convolutional components it is necessary to perform the homomorphic transformation on the signal *u(t)* to transfer it from temporal domain to logarithmic spectral domain. The complex logarithmic spectrum is

$$\dot{S}_L(i\omega) = \mathrm{Log}\left[\mathrm{F}\left[u(t)\right]\right] = \mathrm{Log}(\dot{S}(i\omega)) = \log\left|\dot{S}(i\omega)\right| + i\phi(\omega) \qquad (8)$$

where *F* and *Log* are symbols of Fourier and complex logarithmic transformations accordingly, $\varphi(\omega)$ is phase of complex spectrum. On this domain convolutional components of the signal are projected additively, hence EMD is adequate at once.

Since spectrum (8) is a complex function the EMD procedures have to take into account the inherent relations of real and imaginary parts (or module and phase) of it. In the general case of arbitrary complex function one can make attempt to use the algorithms from [4] or [6] to find the EMD. For miniphase equivalent of a signal necessary conditions are maintained since the module and phase in logarithmic spectrum are joined by the Hilbert transform [7]:

$$\phi(\omega) = -\frac{1}{\pi} p \int_{-\infty}^{\infty} \frac{\ln|S(i\Omega)|}{\Omega - \omega} d\Omega; \qquad \ln|S(i\Omega)| = \frac{1}{\pi} p \int_{-\infty}^{\infty} \frac{\phi(\omega)}{\Omega - \omega} d\Omega \qquad (9)$$

where *p* denotes integration in the Cauchy principal value sense. In that way, if *u(t)* is minimum-phased function the items in (8) generate the canonical pair in terms of the analytic signal.

Applying EMD to the both items in (8) separately one can expand logarithmic spectrum into complex IMFs. This expansion would be considered as additive reflection of convolutional model which describes the original signal in time. Figure 4 shows real and imaginary components of the complex IMFs for the same MPA that has been examined above (see Fig. 3).

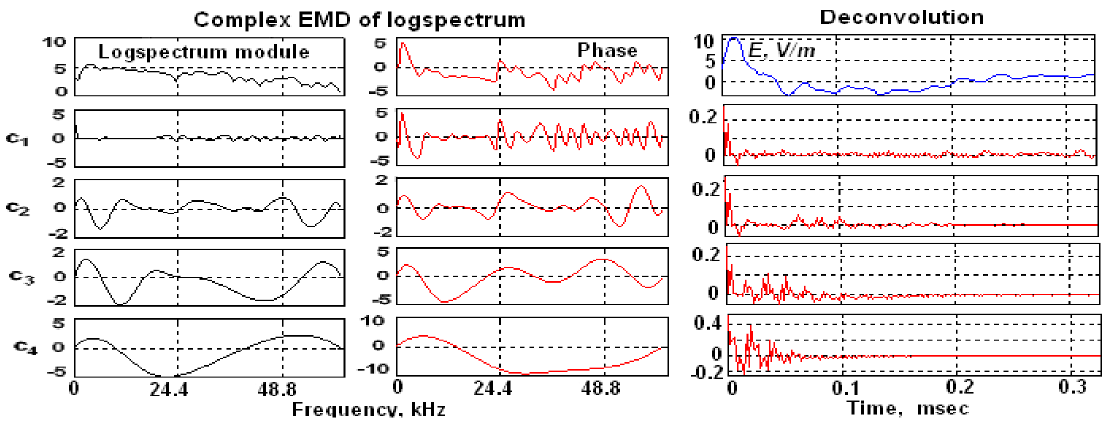

Figure 4. EMD of complex logarithmic spectrum and partial deconvolution results

Similarly to (7) the complex EMD performs the mapping

$$\dot{S}_L[n] = \sum_{k=1}^{K} \dot{c}_k[n] + \dot{r}[n] \qquad (10)$$

where the $\dot{c}_k[n]$ is one from *K* complex pairs, or IMFs from Fig. 4, $n = 1,\ldots N$, *N* is the number of samples taken into consideration in logspectrum. The last member from the right side in (10) is the complex residual.

Every complex IMF from (10) may be used as the starting material for inverse homomorphic transformation to return to the original temporal domain. In that way the proper partial convolutional member $U_k(t)$ of the processed signal would be recovered. To realize it the required sequence of operations is as follows

$$U_k(t) = \mathrm{FT}^{-1}[\exp(\dot{c}_k)], \qquad (11)$$

where FT^{-1} is symbol of the inverse Fourier transformation.

Applying the Eq. (11) to complex IMFs from (10) we achieve results illustrated in the right column of Fig. 4. These components are the items of an empirical convolutional model of the signal considered. The term "empirical" in this context implies that detailed structure of a system which has generated the signal investigated is unknown *a priori*.

One can consider these convolutional items (11) as some responses produced by successive elements pertained to an unknown structure. Apparently, that empirical model should be more complicated than it is supposed, as a rule, in wave propagation problems, for example. To set up conformity with appointed system items and to inspire reasonable physical meanings to the recovered partial members from (11) careful analysis of these results has to require as some original research. But it unfortunately is out of the context of the presented paper.

On the other hand, every complex IMF from (10) may be used too to produce an elementary complex pseudocepstrum in accordance with expression

$$C_k(t) = \mathrm{FT}^{-1}[\dot{c}_k]. \tag{12}$$

As is well known [2], a complex cepstrum, in fact, is a real function of time (or delays), just as the so-called real cepstrum. But in contrast to the latter the complex cepstrum holds the data about phase ratios in the signal analysed. Owing to that property inverse homomorphic transformation can be used to recover this signal starting from the complex cepstrum of it. One may suppose that the same correlations would be hold for partial complex pseudocepstrum (12) and the real pseudocepstrum introduced in [5].

5. Conclusions

In this paper, EMD is considered as applied to the miniphased analogue of a transient signal. It is displayed that the EMD in temporal domain is not adequate with convolutional model of the signal.

The new method for the analysis of inherent structure of transients has been proposed based on EMD in complex logarithmic spectrum domain.. The result of it may be presented as certain expansion of the signal in some empirical convolutional basis. The task is posed for future investigations to research features of this expansion in details including certain physical meaning of it. The appropriate computing procedures have been realized in Matlab. As an example, processing of an atmospherics is discussed.

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