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DETERMINED MODEL OF COORDINATION PLAN CALCULATION FOR AN ARTERIAL HIGHWAY

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The paper deals with research of the influence of link lengths to the possibility of constructing an acceptable coordination plan for a highway. A simplified mathematical model of the highway is created. It stresses on the question of the phase shift that must be tuned on the i-th traffic light relative to basic traffic lights to realize free platoon passage through the traffic light. Numerical characteristics of the quality of TL location is introduced. A matrix of losses helps to estimate the magnitude of possible losses of the cycle time. An algorithm for calculating of coordination plan is given for the model.

Keywords: coordination, traffic light, platoon, cycle, phase, nomogram of residues

1. Introduction

In the past decade, the number of vehicles in urban areas has increased sharply, while organization of the movement and transport routes remains previous. Traffic jams on arterial highways of cities have become commonplace. Meanwhile, optimal regulation of traffic on city streets can lead to increase of the capacity of existing roads by 20–25%. The following two basic approaches are used to solve the problem:

- implementation of adaptive intellectual control, where a traffic light chooses by itself switching mode depending on the number of cars on the crossroad [1];
- implementation of coordinated regulation [2];

It is necessary to solve a series of practical, scientific and methodological problems to implement coordinated regulation on the road network of cities of the Republic of Belarus. These problems are related to the development of special methods of calculation of coordinated regulation effectiveness for different objects and configurations, specific of conflict interaction, etc. [2].

The task of this research is to find out the influence of length ratio of highway links to the possibility of coordination plan constructing; or what we can expect of the highway with a given set of link lengths $L = \{l_1, l_2, \ldots, l_n\}$?

It is shown in the literature [3] that approximate equality of the distances between traffic lights is necessary for a good coordination plan. This is achievable in practice, only in rare cases. It will be shown below that a good coordination plan can be obtained in the case if these distances are multiples of one another. Also it is an interesting question: how should a highway of length L look like with the worst coordination plan? The last question relates to the so-called problems of constructive mathematics. That is finding or building an object with specific, predetermined properties.

2. Mathematical Model of the Object

The object of study is an urban highway of length *L*, starting and ending with a T-junction intersections [4]. This is the central street of the city with the most intensive traffic. The highway consists of *n+1* intersections, where the zero and the *n*-th intersections are start and end points of the highway (see Figure 1). Each intersection is equipped with traffic lights (TL), running on 2-phase cycle.

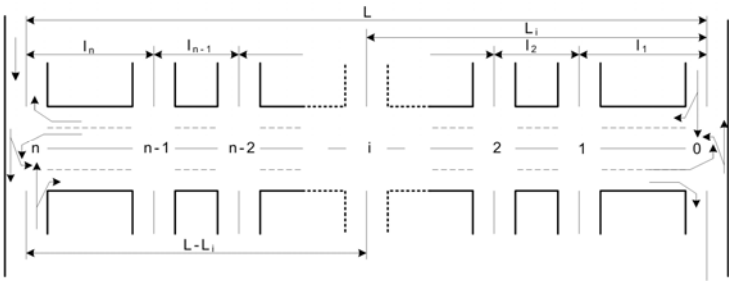

Figure 1. The scheme of a highway with n+1 intersections

The model is imposed with the following restrictions:

1. Platoons of vehicles with equal size enter the highway and leave it at T-junction intersections (the ends of the highway). Size of a platoon (number of cars in it) is regulated by the duration of green phase of the cycle time. It is considered that there are enough cars waiting to enter the highway at the T-junction intersection for the formation of platoons of a given length. The number of platoons entering and exiting both ends of the highway is equal;

2. All cars in a platoon move with equal velocity v. There is no platoon diffusion effect, i.e. platoon dithering [5]. Platoon size is not changing during the whole time of movement along the highway, i.e. it is considered that the number of cars turning from the highway and leaving the platoon balances the number of cars coming to the highway from crossing roads and joining the platoon;

3. The duration of the cycle time T_c is assumed to be the time required for the passage of the leading edge of the platoon with length of l_{plat} (see Figure 2) along the shortest link of the highway $l_k = \min_{i=1}^{n}\{l_1, l_2, ..., l_n\}$. Thus $T_c = l_k / v$;

4. All traffic lights TL_0, TL_1, ..., TL_n of the highway works with the same cycle duration T_c. Traffic lights at the beginning and end of the highway TL_0 and TL_n, respectively, are basic and work synchronously. All other traffic lights have a phase shift relative to the basic traffic lights, if it is necessary;

5. The length l_{plat} of platoons formed at the inputs (TL_0, TL_n) of the highway (Figure 2), is defined as $l_{plat} = v * t_g$, where t_g is the time of green signal of traffic lights TL_0 and TL_n.

Figure 2. Platoons enter the highway

3. Coordination Plan Calculation

The main task of coordinated regulation is to ensure non-stop movement of vehicles along a highway. Solution of this problem is directly related to the economic side of the road traffic, because non-stop movement leads to fuel economy, reduction of harmful emissions and noise level. As objective function in the effective management of traffic in urban road network, it has the highest rating in the scale of other objective functions used in traffic optimisation.

The totality of phase shift, phase duration and cycle time for all intersections in the system of coordinated regulation, forms coordination plan. Thus, this three traffic light control parameters determine fully the nature of traffic on the highway. One of them, namely, cycle time T_c, has been defined above. It remains to determine phase shift, as well as phase duration for each traffic light.

There are 3 cases of highway topology:

- all distances between intersections are equal;
- distances between intersections are multiple of certain distance (it is clear that this distance is the minimum of all distances);
- there is not such a segment of the highway, i.e. distances between intersections are not multiple of certain distance.

The first two cases are trivial. It is possible to organize coordinated regulation without phase shift at that rate, and then this is a case of simultaneous and synchronous system. All traffic light signals have the same indication at intersections of coordinated direction at any given time, and phase shifts are zero in such a system.

The shortest highway link l_k is considered as base (as cycle duration T_c has been calculated with it). Let us divide l_k into 10 equal parts $\Delta l = l_k / 10$. The second (time) discrete $\Delta t = T_c / 10$ corresponds to Δl.

The question is: how large should phase shift be installed at the i-th TL relative to the basic TL to make a platoon driving freely from the left to the right (see Figure2) pass the i-th TL without stopping? This means that a platoon should pass the distance L_i from TL_0 to TL_i without stopping.

Let us find out the remainder of division L_i by l_k:

$$L_i = p * l_k + r_i, \tag{1}$$

where p is quotient, and r_i is remainder, the distance less than $2l_k$, i.e. $0<r_i< l_k$.

Thus, the segment l_k goes integer number of times into distance L_i from the right end of the highway. There is TL_i in the (p+1)-th segment, as shown in Figure 3. In this case, it locates in the interval 7–8. It is rounded to the nearest integer number (7). Figure 3 shows that the beginning of the green signal phase for TL_i must be late on the value of r_i, which is $0{,}7T_c$, or ahead on the value of 10-r_i, which is $0{,}3T_c$. Thus, r_i points the number of time discrete Δt, during which the delay of green signal must be done on the highway relative to the basic TL in right to left direction of motion.

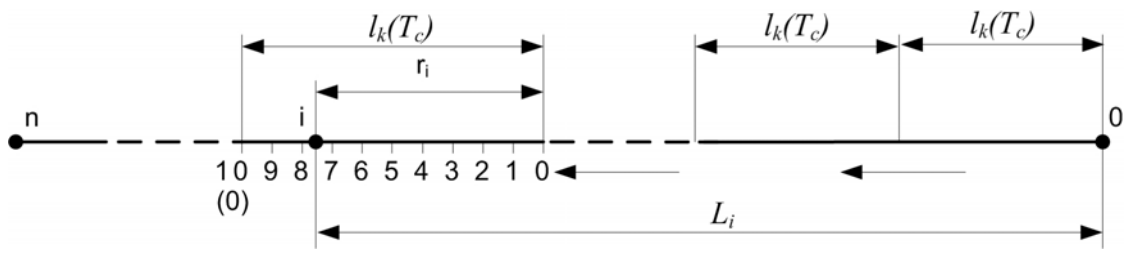

Figure 3. Position of segments l_k in the right to left direction on the highway and formation of remainder r_i

A similar situation is observed with oncoming traffic that is from TL_n to TL_i:

$$L - L_i = m * l_k + c_i. \tag{2}$$

Figure 4 shows that the residue c_i equals to 2 after rounding, i.e., lighting of green signal for TL_i should be delayed in 2 times discrete Δt relative to the basic signal to let opposite platoon pass. Summing up three times discrete of lead and two of lag, we get 5 Δt out of 10 Δt, which forms the cycle time T_c. That is half of the cycle must already be occupied with green signal for the highway. Let us choose platoon length l_{plat} as 2 Δl. This platoon passes TL in 2 Δt. So, 5 Δt +2 Δt = 7 Δt. Thus, 3 Δt of green signal are remained for side direction at the i-th intersection.

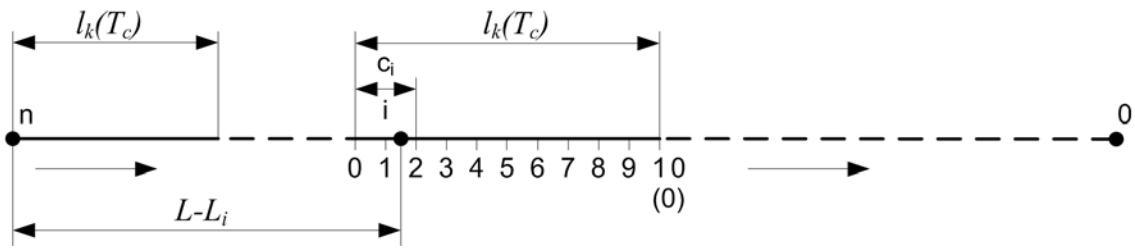

Figure 4. Position of segments l_k in the left to right direction on the highway and formation of remainder c_i

4. Construction of Matrix-Nomogram of Losses (Remainders)

Transport losses in this case mean irretrievable loss of some part of green phase t_g of the cycle time T_c.

As it is shown above, remainders r_i and c_i for TL_i definitely characterize the TL, its well-defined place in the relations of all link lengths of the highway. Traffic lights (crossing) can be positioned more or less successfully on the highway. Following quantity is numerical characteristic of quality of TL location i:

$$Q_i = \begin{bmatrix} |c_i - r_i| \; if \; |c_i - r_i| \le 5 \\ 10 - |c_i - r_i| \; if \; |c_i - r_i| \ge 5 \end{bmatrix}, \tag{3}$$

which characterizes use degree of t_g (green signal of TL_i) on the highway.

The quantity Q_i shows the time difference between the arrivals of platoons from two opposing directions that are registered at the *i*-th TL. The best option is platoon simultaneous approach from the left and the right to TL_i. Then $Q_i = 0$ and t_g is used with maximum efficiency, i.e. opposite traffic flows are crossing the *i*-th TL simultaneously. Formula (3) shows that $Qi \le 5$ for any c_i and r_i, i.e. $Qi \le 0.5T_c$. This means that losses of the cycle time can not exceed half of the cycle duration.

Let us introduce two types of losses of green phase t_g of the cycle time T_c:

- there is traffic flow passes TL only in one direction while green signal is burning on the highway;
- there is not any vehicles while green signal is burning on the highway.

Definition 1. Semiproductive part of green phase of traffic light is called the time during which traffic flow through TL_i is observed and only in one direction on the highway.

This is the first type of losses of green phase of the cycle. Let us call the second type of losses as non-productive, i.e. there are not vehicles from both directions at TL_i. Such losses of the active phase of the cycle time T_c are harder.

The magnitude of possible losses of T_c on the whole highway can be estimated by using a matrix-nomogram of losses (Table 1). The matrix has dimension of 10x10 according with the number of possible values of $c_{i.}$ and r_i . There is a value of loss of the green phase Q_i at the intersection of the selected remainders $c_{i.}$ and r_i for the *i*-th TL. Remaining time of T_c-Q_i is given for platoon (with length of l_{plat}) passing through the cross, as well as for crossing direction.

Values c (lines) and r (columns) are with the minus sign-matrix, which means a delay in $c*\Delta t$ ($r*\Delta t$) relative to basic TL, located at the inputs of the highway. *c* and *r* have additional value (in brackets) starting from the fifth row and fifth column. This means that the delay of green phase at the pointed negative value can be replaced by advancing to the value indicated in parentheses. The absolute values of these values are complementary to 10, i.e. to the cycle time $Tc = 10\Delta t$.

Table 1. Matrix-nomogram of losses

RTL movement

RTL movement

c\r	0	-1	-2	-3	-4	-5(+5)	-6(+4)	-7(+3)	-8(+2)	-9(+1)
0	0	1	2	3	4	5	4	3	2	1
-1	1	0	1	2	3	4	5	4	3	2
-2	2	1	0	1	2	3	4	5	4	3
-3	3	2	1	0	1	2	3	4	5	4
-4	4	3	2	1	0	1	2	3	4	5
-5(+5)	5	4	3	2	1	0	1	2	3	4
-6(+4)	4	5	4	3	2	1	0	1	2	3
-7(+3)	3	4	5	4	3	2	1	0	1	2
-8(+2)	2	3	4	5	4	3	2	1	0	1
-9(+1)	1	2	3	4	5	4	3	2	1	0

Control process consists of the distribution of the cycle time T_c at the *i*-th TL. TL_i lights up green phase on the highway after time r_i (if $|r_i|< |c_i|$), or c_i. (if $|c_i| <| r_i|$) from the beginning of work of basic TL. Let's consider the first case ($|r_i|< |c_i|$). This means that a platoon with size l_{plat} comes the first to TL_i from the right. It passes TL_i for the time $2\Delta t$, because its length is $2\Delta l$.

The platoon on the right may be still is passing TL_i, while a platoon on the left comes to the intersection. Combined pass of both platoons is a zone of the highest efficiency. If r_i is large, you might have the opposite situation, i.e. unproductive zone, where the platoon on the right has already left TL_i and the platoon on the left has not yet appeared.

Thus green time t_g on the highway should be:

$$t_g = Q_i + l_{plat} / v. \tag{4}$$

Time for crossing direction is as follows:

$$T_{gcross} = T_c - (Q_i + l_{plat}/v). \quad (5)$$

Formulas (4), (5) are sufficient to calculate traffic light phases. The following algorithm allows calculating a coordination plan for TL.

5. Algorithm of Coordination Plan Calculation

Distances between crosses are input data for the algorithm of coordination plan calculation:

1. The shortest link is found out $l_k = \min_{i=1}^{n}\{l_1, l_2, ..., l_n\}$.
2. Cycle time duration is calculated $T_c = l_k / v$ (sec).
3. Reminders r_i and c_i are calculated for each TL ($i = \overline{1, n-1}$) with (1), (2).
4. The minimal necessary duration of green time for TL_i on the highway Q_i is chosen from the matrix-nomogram of losses (Table 1) at column and row cross.
5. Green time t_g on the highway is the minimal necessary time Q_i plus time of platoon travel through TL_i l_{plat}/v . It is received green time on the highway t_g as a result
6. The rest of cycle time is given for the crossing direction t_{g_s}. See formula (5).

6. Conclusions

Traffic has two sides: determine and stochastic. In the paper a method of coordinated plan calculation is given, which consider to determine the aspect of traffic. It is planned to make the determined model more accurate in further research. This will be achieved by revision of model restrictions. They should become more realistic. For instance, platoon diffusion and uncertainty of number of cars should be considered. This will allow taking into account stochastic aspect. Then it will become possible to calculate coordinated plans for urban streets with this methodology.

References

1. Anfilets, S., Shuts, V. Evaluating the Effectiveness of the Adaptive Control System in Brest Region, *International Congress Of Heavy Vehicles, Road Trains And Urban Transport, Minsk, Belarus, October 06–09, 2010, Belarussian National Technical University.* Minsk: BNTU, 2010, pp. 222–226.
2. Rozhanski, D. V., Vrubel, J. A., Kapsky, D. V., Navoi, D. V, Mozalevsky, D. V., Korzhova, A. V., Polhovskaya, A. S., Kostyukovich, E. N. Methods of effectiveness assessing of the coordinated regulation of transport and pedestrian flows, *Traffic management, pedestrian and freight transportation improvement, Minsk, Belarus, October 23–24, 2009, Belarusian National Technical University.* Minsk, BNTU, 2010, pp. 159–165.
3. Vrubel, J. A. *Traffic management. Part2*. Minsk: Byelorussia Fund of Traffic Safety, 1996, pp. 283–287.
4. Shuts, V. N. Determined model for coordinated regulation of motor transport movement along an arterial highway with T-junction intersections, *Vestnik BSTU*, Vol. 4, 2009, pp.45–48.
5. Kukharyonak, G. M., Cape, D. V., Navoi, D. V., Rozhanski, D. V., Shuts, V. N. Investigation of the mechanism of the coordinated platoon of vehicles break-up while driving on the link of arterial highway, *Vestnik BSTU,* Vol. 5, 2010, pp.63–68.
6. Li, Wei and Tarko, Andrew P. Effective and Robust Coordination of Traffic Signals on Arterial Streets, *Research Report*, Vol. 1, 2007.
7. Gartner, N. H., Stamatiadis, C. Arterial-based control of traffic flow in urban grid networks, *Mathematical and Computer Modeling*, Vol. 35, 2002.