Proceedings of the 11th International Conference "Reliability and Statistics in Transportation and Communication" (RelStat'11), 19–22 October 2011, Riga, Latvia, p. 398-407. ISBN 978-9984-818-46-7
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RELIABILITY ANALYSIS BASED ON MARKOV MODELS ADJUSTED TO VARIOUS MAINTENANCE POLICIES

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Equipment deterioration can be analysed with discrete state-transition models, which are used as a foundation for numerical evaluation of various reliability and operational parameters. In this paper we discuss an approach in which a system with scheduled inspections and possible repair activities is described by the discrete state-transition deterioration model. The model can be translated into a semi-Markov process, which after solving, yields numerous reliability characteristics including average equipment life, its deterioration rate represented by the so-called life curve, probability of failure within given time horizon, etc. The analysis is performed for particular maintenance policy that has been integrated in the model, and by adjusting the model to specific changes in this policy (e.g., modifications of repair frequencies) their consequences for system reliability can be evaluated. In the text we present briefly the methodology behind model creation and concentrate on its one specific aspect: automatic adaptation of the model to the adjusted maintenance policy with modified frequencies of repairs. In particular, it is shown that such adaptation can be reduced to the adjustment of transition probabilities that are found in the model, and such adjustment can be numerically implemented with one of the standard root-finding algorithms.

Keywords: state-transition deterioration model, semi-Markov process, reliability analysis, root-finding algorithm

1. Introduction

Efficient and, at the same time, cost-effective maintenance is an important element of reliable operation in contemporary complex technical systems. Selecting the optimal maintenance strategy must take numerous issues into account and among them reliability and economic factors are often of equal importance. On one side, it is obvious that for successful system operation failures must be avoided and this opts for extensive and frequent maintenance activities. On the other, superfluous maintenance may result in very large and unnecessary cost. Finding a reasonable balance between these two is a key point in reliable system operation.

In order to be able to plan such maintenance appropriate models it is necessary that equipment deterioration process would represent and, at the same time, would take into account various maintenance operations. In this paper we present a methodology that assists a person who decides about maintenance activities by evaluating risks and costs associated with choosing different maintenance strategies. Instead of searching for a globally optimal solution to a problem: "what maintenance strategy would lead to the best reliability and dependability parameters of system operation", in this approach different maintenance scenarios can be examined in "what-if" studies and their reliability and economic effects can be compared so that a person managing the maintenance is assisted in making informed decisions [1–3].

The method has been presented initially in [1] and its specific extensions are further described in [4–8]. In this work, we summarize the current state of development and concentrate on one important aspect of the methodology: fully automatic adjustment of the model to possible modifications of the maintenance policy which are often required for studies requested by the user.

The main contents of the paper is divided into two parts. The first one (section 2) presents general description of the methodology, which is based on state-transition deterioration models, semi-Markov processes and the concept of a life curve used for visualisation of equipment ageing. The problem of automatic model adjustment is presented in the second part (section 3), which includes detailed discussion of possible numerical algorithms that can be implemented for probability approximation.

2. Modelling the Ageing Process

There are numerous factors that have an effect on the ageing process of equipment that undergoes scheduled inspections and maintenance actions. Among them there are various aspects related to its physical characteristics, operating practices, and the maintenance policy. The method described in this paper uses a generic model that assumes that the equipment will deteriorate in time and, if not maintained, will eventually fail. To counteract, the scheduled inspections are performed and if the deterioration

process is discovered, preventive maintenance is applied that can restore the condition of the equipment. Such a maintenance activity will return the system to a specific state of deterioration, whereas repair after failure will restore to "as new" condition [9–10]. With these assumptions, the maintenance policy components that must be recognized in the model are: monitoring or inspection (how the equipment state is determined), the decision process (which determines the outcome of the decision), and, finally, the maintenance actions – the repairs (or possible decision outcomes).

2.1. The state-transition model

One of the approaches that can properly incorporate all the above suppositions about the aging process and maintenance activities is based on state-space (Markov) model [11–16]. The model consists of the states the equipment can assume in the process, and the possible transitions between them.

The method described in this paper uses a model of the Asset Maintenance Planner (AMP) that was initially developed and implemented by George J. Anders and Henryk Maciejewski [17–18]. The AMP model is designed for equipment exposed to deterioration but undergoing maintenance at prescribed times. It computes the probabilities, frequencies and mean durations of the states of such equipment. The basic ideas in this approach are the probabilistic representation of the deterioration process through discrete stages, and the provision of a link between deterioration and maintenance.

For structure of a typical AMP model see Figure 1. In the model, the deterioration progress is represented by a chain of deterioration states $D1 \ldots DK$ which then leads to the failure state F. In most situations, it is sufficient to represent deterioration by three stages: an initial ($D1$), a minor ($D2$), and a major ($D3$) stage of deterioration ($K = 3$). This last is followed, in due time, by equipment failure (F) which requires extensive repair or replacement.

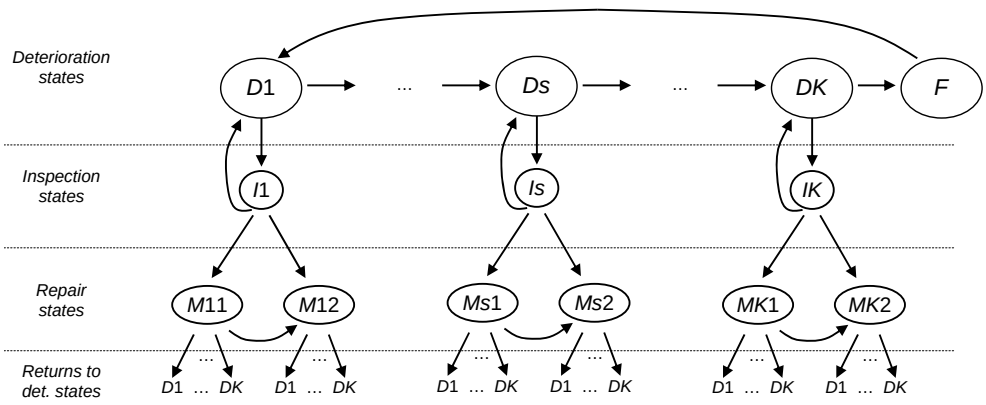

Figure 1. The state-transition model representing the deterioration chain with inspection and repair states (an example with two types of repairs is shown)

In order to slow deterioration and thereby extend equipment lifetime, the operator will carry out maintenance according to some pre-defined policy. In the model shown on Figure 1, regular inspections (I*s* states) are performed, which result in decisions to continue with, e.g., minor ($Ms1$) or major ($Ms2$) repair (more than two types of repairs can be modelled), or to return to the deterioration state Ds without any repair. The expected result of all maintenance activities is a single-step improvement in the deterioration chain; however, it is possible to take into account also cases where no improvement is acquired or even where some damage is done through human error in carrying out the maintenance, which results in returning to the stage of more advanced deterioration.

The choice probabilities (at transitions from inspection states) and the probabilities associated with the various possible outcomes are based on user input and can be estimated, e.g., from historical records or operator expertise. Therefore, creation of the model and then its fine-tuning to some real historical data of equipment operation and maintenance records is a complex task that requires expert intervention.

Mathematically, the state-transition model of Figure 1 can be translated into a semi-Markov process, and then solved by the well-known procedures. The solution will yield all the state probabilities, frequencies and mean durations. Another technique, employed for computing the so-called first passage times (FPT) between states, will provide the average times for first reaching any state from any other state. If the end state of the passage is F, the FPTs are the mean remaining lifetimes from any of the initiating states. For state $D1$ this estimates the expected equipment life for the maintenance policy that has been incorporated in the model.

2.2. Using the model in reliability analysis

A convenient way to represent the deterioration process is by the life curve of the equipment [9]. Such a curve (see the first graph on Figure 2) shows the relationship between asset condition, expressed in either engineering or financial terms, and time. This concept is easy to comprehend for a non-expert end user (for example, a manager analysing various options of the maintenance policies) who does not need to know all the intrinsic details of state-transition models and Markov processes.

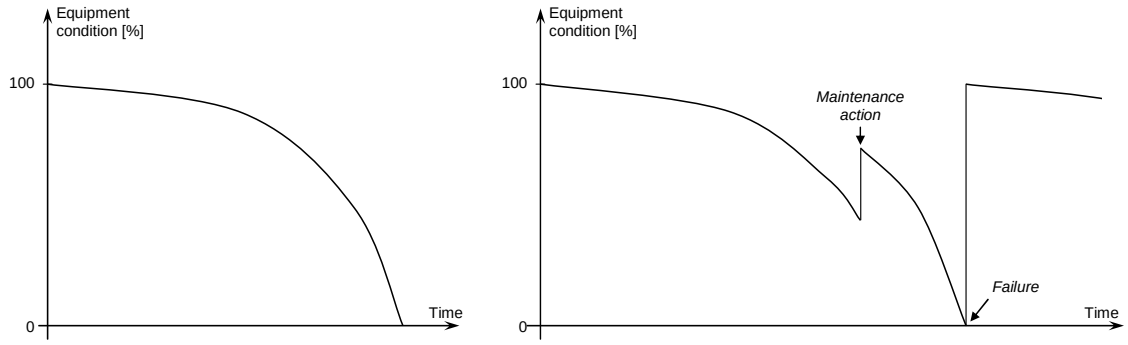

Figure 2. A simple life curve computed from the semi-Markov model (left) and a complex life curve representing some maintenance scenario (right)

A life curve that corresponds to some given Markov model can be created as follows. As pointed out above, computing the average first passage time (FPT) from the first deterioration state (D1) to the failure state (F) yields an average lifetime of the equipment, i.e., the length of the curve. On the other hand, solving the model for the state probabilities makes possible computing the expected state durations, which are then used to determine the shape of the curve, i.e. the rate of deterioration over different phases of equipment wear (some additional decisions are required as to how the deterioration states are mapped to ranges of the asset condition values).

Simple life curves obtained for specific maintenance policies (i.e. specific models) can later be combined in composite life curves which describe possible complex maintenance scenarios [1], [8]. The second graph on Figure 2 shows an example of such a scenario when a preventive maintenance is performed at some moment in time (restoring the asset condition to approx. 80%), after that the failure occurs and leads to equipment replacement. This curve is composed of three segments of simple life curves and they do not need to represent the same models, i.e. a situation can be modelled when a change in maintenance policy takes place, e.g. after the repair.

Furthermore, having the model and its (simple) life curve, one can compute the probability of failure (PoF) within given time period T for the equipment which currently is in some specific deterioration condition AC. The procedure is as follows:

(1) for the current value of asset condition AC, find from the life curve the corresponding deterioration state Dc and then compute a state progress SP (%), i.e. estimate how long the equipment has already been in the Dc state;
(2) running FPT analysis on the model, find the probability distribution functions d_c(t) and d_{c+1}(t) of First Passage Time from the current state Dc and the subsequent deterioration state $D(c+1)$, to the failure state F;
(3) interpreting the state progress SP as a weight which balances the current equipment condition between Dc and $D(c+1)$, estimate the final value of the probability as:

$$\mathrm{PoF}(T) = d_c(T) \cdot (1 - \mathrm{SP}) + d_{c+1}(T) \cdot SP . \quad (1)$$

When equipment deterioration over the time period T is represented by a complex life curve, then the above procedure must be applied to the every simple curve segment that is included in T.

3. Adjusting the Model to Different Repair Frequencies

In practical applications when different maintenance policies need to be investigated, one of the most important aspect of system operation is appropriate representation of various types and frequencies of repairs that may be included in possible maintenance policies under consideration. In the approach presented in this paper it is the task of the expert to properly incorporate these characteristics of the original

("continue as before") policy in the model, but once such model is available the non-expert end user may wish to analyse various hypothetical policies that are created by simple modifications of the original repair frequencies. For example, the end user may wish to consider an option "what if the minor repair rate is reduced by half" or "what if the minor and major repairs are removed completely but the medium repair is performed twice as often". Such studies require a mechanism that, having the expert-created model representing some original repair policy, would be able to generate a derived model that would correspond to the same equipment (i.e. the same deterioration chain as presented in Figure 1) but submitted to a modified repair policy with different repair frequencies. It should be noted that the same mechanism would be also helpful during construction of the model by the expert when fine-tuning of repair frequencies is needed in order to achieve compliance of the model with some real-world repair policy stored in historical records of equipment operation.

3.1. The adjustment procedure

Let's assume that the deterioration model under consideration consists of K deterioration states and R repairs. Also, let P^{sr} denotes probability of selecting maintenance r in state s (assigned to the decision after inspection state Is) and P^{s0} represents probability of returning to state Ds from inspection Is which corresponds to a situation when no maintenance is scheduled as a result of the inspection. The foremost condition that must be met at all times is that in all deterioration states $s = 1 \ldots K$:

$$\mathrm{P}^{s0} + \sum_r \mathrm{P}^{sr} = 1 . \tag{2}$$

Let F^r represents the frequency of some repair r as it is generated by the model. The problem of model adjustment can be formulated with various assumptions and with different goals in mind but in this approach it is defined as follows:

> Given an initial semi-Markov model M_0, with internal structure representing deterioration, inspection and repair states as described above and producing the initial vector of repair frequencies $\mathbf{F}_0 = [\,\mathrm{F}_0^1,\ \mathrm{F}_0^2\ \ldots\ \mathrm{F}_0^R\,]$, modify the probabilities P^{sr} assigned to transitions from inspections states Is so that the resulting model generates some requested vector of goal frequencies $\mathbf{F}_\mathrm{G}$.

Usually, the vector $\mathbf{F}_\mathrm{G}$ may represent the observed historical values of the repair frequencies (when an expert user is working on fine-tuning of the initial model) or some hypothetical frequencies (when an end user investigates possible modifications of the repair policy).

There are numerous approaches that can be used in order to accomplish such model adjustment. In the proposed solution, an iterative approximation approach has been chosen in order to preserve an original construction of the model M_0 as mush as possible. In this method a sequence of tuned models M_0, M_1, M_2,… M_N is evaluated in N steps with each consecutive model approximating desired goal with a better accuracy. Starting with $i = 0$, the procedure consists in the following steps:

1° for the current model M_i compute its vector of repair frequencies $\mathbf{F}_i$

2° evaluate an error of M_i as a distance between vectors $\mathbf{F}_\mathrm{G}$ and $\mathbf{F}_i$

3° if the error is within the user-defined limit ε, consider M_i as the final model and stop the procedure ($N = i$); otherwise proceed to the next step

4° create a new model M_{i+1} by *tuning* values of P_i^{sr}, then correct P_i^{s0} according to condition (2)

5° return to the step 1° for the next iteration.

The error computed in step 2° can be expressed in many ways. As the absolute values of repair frequencies may vary in a broad range within one vector $\mathbf{F}_i$, yet the values of all are significant for model evaluation, the relative measures work best in practice:

$$\left\|\mathbf{F}_\mathrm{G} - \mathbf{F}_i\right\| = \frac{1}{R}\sum_{r=1}^{R}\left|\mathrm{F}_i^r / \mathrm{F}_\mathrm{G}^r - 1\right| \tag{3}$$

or

$$\left\|\mathbf{F}_\mathrm{G} - \mathbf{F}_i\right\| = \max_r\left|\mathrm{F}_i^r / \mathrm{F}_\mathrm{G}^r - 1\right| . \tag{4}$$

The latter formula is more restrictive: it ensures that any repair does not differ from the goal more than the imposed limit and therefore this version has been used in the numerical implementation of the method that is considered in this paper.

3.2. Generation of the adjusted model

Of all the steps that compose the iteration run, it is clear that adjusting probabilities P_i^{sr} in step 4° is the heart of the whole method. This is accomplished with the following two assumptions that are introduced not only to simplify the task, but also to improve quality of the result.

The first assumption is related to the fact that although, in general, the P^{sr} probabilities represent $K \cdot R$ free parameters that could be freely modified in order to arrive at the requested goal, their uncontrolled modification can lead to serious deformation of the model and this should be avoided. To this point a restrictive condition is adopted: if the probability of some particular repair must be modified, it is modified *proportionally* in *all* deterioration states, so that during the adjustment the proportion between this repair probabilities over all states remains unchanged and are the same as in the initial model M_0:

$$\forall\, i, r \quad \mathrm{P}_0^{1r} : \mathrm{P}_0^{2r} : \ldots : \mathrm{P}_0^{Kr} \;\sim\; \mathrm{P}_i^{1r} : \mathrm{P}_i^{2r} : \ldots : \mathrm{P}_i^{Kr}\,. \tag{5}$$

This assumption also significantly reduces dimensionality of the problem, because now only R scaling factors, denoted as the vector $\mathbf{X}_{i+1}=[\,\mathrm{X}_{i+1}^1,\ \mathrm{X}_{i+1}^2,\ \ldots\ \mathrm{X}_{i+1}^R\,]$, must be found to compute all new probabilities required to create the model M_{i+1}:

$$\mathrm{P}_{i+1}^{sr} = \mathrm{X}_{i+1}^r \cdot \mathrm{P}_0^{sr}, \quad r = 1\ldots R,\ s = 1\ldots K. \tag{6}$$

Moreover, and this observation leads to the second assumption, although the frequency of a repair r depends on the probabilities of all repairs (modifying probability of one repair changes, among others, state durations in the whole model; thus, it changes the frequency of all states) it can be assumed that, in a case of a single-step small adjustment, its dependence on repairs other than r can be considered negligible and

$$\mathrm{F}_i^r = \mathrm{F}_i^r\left(\mathrm{X}_i^1, \mathrm{X}_i^2 \ldots \mathrm{X}_i^R\right) \approx \mathrm{F}_i^r\left(\mathrm{X}_i^r\right). \tag{7}$$

With these two assumptions, generation of a new model in step 4° of the above described procedure is reduced to the problem of solving R non-linear equations in the form of

$$\mathrm{F}_i^r\left(\mathrm{X}_i^r\right) = \mathrm{F}_G^r \quad . \tag{8}$$

This task can be accomplished with one of the standard root-finding algorithms that will be presented in the next point.

One aspect of the procedure requires additional attention, though: applying equation (6) with $X_{i+1} > 1$ may violate condition

$$\sum_r \mathrm{P}_{i+1}^{sr} \le 1 \tag{9}$$

in some deterioration state s. This situation needs special tests that would detect such illegal probability values and then reduce them proportionally so that their sum does not exceed 1: a so called *scale-down transformation* needs to be applied. As the case studies show such situations do occur during model tuning towards repair frequencies that are remarkably higher than in the initial model M_0. In its simplest form, the scale-down operation consists in dividing each probability P^{sr} in the offending state s by the sum of all repair probabilities in this state, as they are computed with equation (6) without scaling:

$$\mathrm{P}^{sr} = \mathrm{P}^{sr} \big/ S_{Ds}\,, \quad S_{Ds} = \sum_{r=1}^{R} \mathrm{P}^{sr}\,. \tag{10}$$

This will also lead to $\mathrm{P}^{s0} = 0$, which means that every inspection ends with some repair and there are no direct returns from inspection state *Is* to deterioration state *Ds*. Moreover, this obligatory correction mechanism can result in violation of the proportionality rule (5) as an unavoidable side effect. In such cases modification of the model is more serious than the initial assumptions allow but this must be tolerated if the goal repair frequencies requested by the end user are to be achieved.

3.3. Numerical methods used for probability estimation

With all the assumptions regarding scaling factors and their influence on repair frequencies, generation of a new model is now reduced to the problem of solving *R* equations in the form of (8). This can be accomplished with one of the standard numerical algorithms for finding roots of a non-linear function. The method described in this paper has been tested with implementation of the following three algorithms: the Newton method working on linear approximation of $\mathrm{F}_i^r()$, the secant method and the false position (*falsi*) method.

3.3.1. Newton method On Linear Approximation (NOLA)

In this solution it is assumed that $\mathrm{F}_i^r()$ is a linear function defined by points $\mathrm{F}_i^r(\mathrm{X}_i)$ (obtained after solving the model in step 1°) and $\mathrm{F}_i^r(0)$ (which can be assumed to be equal zero). Then simply

$$\mathrm{X}_{i+1}^r = \mathrm{F}_\mathrm{G}^r / \mathrm{F}_i^r . \tag{11}$$

Noteworthy advantage of this approach lies in the fact that no other solution than the current frequency $\mathrm{F}_i^r(\mathrm{X}_i)$ is required to compute the next approximation, so errors of previous steps do not accumulate and convergence is good from the first iteration (the method has no memory effects).

Table 1 and Figure 3 present the details of exemplary model adjustment with this method implemented. The sample model consisted of three deterioration states and three types of repairs: minor, medium and major (*K* = *R* = 3). The values of goal frequencies has been selected to be ≈50% of that in the original repair policy ($\mathbf{F}_\mathrm{G}$ = ½$\mathbf{F}_0$), which corresponds to some hypothetical repair policy "what if repair frequencies of this piece of equipment are reduced by half".

Table 1. Sample model adjusted to $\mathbf{F}_\mathrm{G}$ = ½$\mathbf{F}_0$ with the NOLA method, ε = 1E-4

i	**Relative freq. of repairs** $\mathrm{F}_i^r/\mathrm{F}_\mathrm{G}^r$			**Error**	**Scaling factors** X_{i+1}^r		
	***r* = 1**	**2**	**3**		***r* = 1**	**2**	**3**
0	2.007762	2.000581	2.049800	1.049814	0.49810	0.49990	0.48780
1	1.127838	1.230000	1.254300	0.254331	0.88670	0.81300	0.79720
2	1.018676	1.039839	1.044400	0.044398	0.98170	0.96170	0.95750
3	1.002895	1.006516	1.007300	0.007255	0.99710	0.99350	0.99280
4	1.000467	1.001065	1.001200	0.001179	0.99950	0.99900	0.99880
5	1.000076	1.000161	1.000200	0.000183	0.99990	0.99980	0.99980
6	1.000010	1.000032	1.000000	0.000040	1.00000	1.00000	1.00000

As it can be seen from the submitted data, the adjustment process goes smoothly and without any perturbations: the average convergence ratio is nearly constant and the model firmly approaches the required goals for all three frequencies. As the goal vector has been assumed to be 50% of the frequencies in the original (initial) model, the relative error in the first iteration equals to approx. 100% (the frequencies are twice as large as required) and in each subsequent iteration it is reduced by a factor of 4 to even 6. Finally, the imposed accuracy of 1E-4 (0.01%) is reached after 6 steps. It should be noted that such high precision in model tuning has been selected for illustrative purposes in this paper, while in practical engineering cases accuracy of 1% is more than adequate; in the discussed example such level is obtained after just three steps.

3.3.2. The secant method

In this standard technique the function is approximated by the secant defined by the last two approximations computed for points X_{i-1}^r, X_i^r; thus the new solution is calculated as:

$$\mathrm{X}_{i+1}^r = \mathrm{X}_i^r - \frac{\mathrm{X}_i^r - \mathrm{X}_{i-1}^r}{\mathrm{F}_i^r - \mathrm{F}_{i-1}^r}\left(\mathrm{F}_i^r - \mathrm{F}_\mathrm{G}^r\right). \tag{12}$$

After that X_{i-1}^r is discarded and X_{i+1}^r and X_i^r are considered as the pair defining the secant for the next iteration.

To begin the procedure two initial points are needed. In this approach we propose to choose the first point equal to the initial frequency of the model M_0 (X_0^r=1), while the second point is computed as in the NOLA method: $X_1^r = F_G^r / F_0^r$. Having the pairs X_0^r, X_1^r (r = 1, 2, … R) the procedure starts according to equation (12) in iteration i = 1 and continues so in further steps.

In our exemplary case this method produced not-so constant convergence rate compared to the NOLA method presented in the previous point. Looking at the error values listed in table 2 for every iteration, there are some steps with very good improvement (e.g. error reduction from -0.1 to 0.008 for i = 3) but those are followed by mediocre progress in the next iteration (reduction from 0.008 to 0.006 for i = 4). Nevertheless, this does not exclude the method from application in the discussed system: the required accuracy is achieved in just one additional step (i = 7 compared to 6 for the NOLA method) and this is still an acceptable result.

Table 2. The same adjustment case (**F**$_G$ = ½**F**$_0$, ε = 1E-4) controlled by the secant method

i	**Relative freq. of repairs** F_i^r / F_G^r			**Error**	**Scaling factors** X_{i+1}^r		
	***r* = 1**	**2**	**3**		***r* = 1**	**2**	**3**
0	2.007762	2.000581	2.049800	1.049814	0.49810	0.49990	0.48780
1	1.127838	1.230000	1.254300	0.254331	0.85360	0.70140	0.66440
2	0.990467	0.918355	0.893500	-0.106504	1.01190	1.11150	1.14910
3	0.995514	1.004032	1.007800	0.007832	1.01050	0.99530	0.99110
4	1.006038	0.999935	0.999600	0.006037	0.99410	1.00010	1.00040
5	1.000219	1.000161	1.000200	0.000218	0.99980	0.99990	0.99990
6	1.000000	1.000129	1.000100	0.000120	1.00000	0.99980	0.99990
7	1.000010	0.999903	1.000000	-0.000084	1.00000	1.00010	1.00000

The graphs on Figure 3 provide an additional explanation of the problem for this particular case. It can be seen that for I = 3 the method proposed actually a very good approximation but with just a little overshoot and this caused problems in the next iteration. This is a known setback of the secant method, which is removed in the *falsi* method.

*3.3.3. The false position (*falsi*) method*

In this approach X_{i+1}^r is computed as in (12) but the difference lies in choosing points for the next iteration. While in the secant method always X_{i-1}^r is dropped, now X_{i+1}^r is paired with that one of X_i^r or X_{i-1}^r which lies on the opposite side of the root. In this way when (12) is applied the solution is bracketed between X_i^r and X_{i-1}^r (which is the essence of the *falsi* method).

As in 3.3.2, the two initial points are needed but now they must lie on both sides of the root, i.e.

$$\left(F_0^r - F_G^r\right)\cdot\left(F_1^r - F_G^r\right) < 0 . \tag{13}$$

Choosing such points may pose some difficulty. To avoid multiple sampling, as in the secant method it is proposed to select X_0^r=1 and to compute X_1^r like in NOLA method, but now with some "overshoot" that would guarantee (13):

$$X_1^r = \left(F_G^r / F_0^r\right)^{\alpha} \tag{14}$$

with a new parameter $\alpha > 1$ controlling the overshoot effect. The overshot must be sufficient to ensure condition (13) but, on the other hand, it should not produce too much of an error as this would deteriorate approximation process during initial steps and would produce extra iterations. If (13) is not met by initial value of X_1^r (14) can be re-applied with an increased value of α, although it should be noted that each such correction requires solving a new M_1 model and in effect this is the extra cost almost equal to that of the whole iteration.

Table 3. The sample case ($\mathbf{F}_G = ½\mathbf{F}_0$, $\varepsilon = 1E\text{-}4$) adjusted by the *falsi* method

i	**Relative freq. of repairs** F_i^r / F_G^r			**Error**	**Scaling factors** X_{i+1}^r		
	***r* = 1**	**2**	**3**		***r* = 1**	**2**	**3**
0	2.007762	2.000581	2.049800	1.049814	0.24810	0.24990	0.23800
1	0.591533	0.686129	0.691200	-0.408464	1.87420	1.71690	1.72780
2	1.067305	1.086871	1.092200	0.092239	0.93400	0.90950	0.90310
3	1.004705	1.005032	1.004800	0.005045	0.99510	0.99430	0.99450
4	1.000238	1.000258	1.000300	0.000313	0.99980	0.99970	0.99960
5	1.000010	1.000000	1.000000	0.000014	1.00000	1.00000	1.00000

Theoretical characteristics of the *falsi* technique are well illustrated by the data shown in Table 3 and on Figure 3. Of all three methods presented here, this one generated the requested accuracy with the minimum number of iterations: 6 vs. 7 of NOLA and 8 of secant. The α parameter that controls the overshoot effect in the first approximation was equal 2 and this indeed generated an overestimation in the model M_1 but this was properly and quickly compensated in the steps that followed. As a result, the convergence flow, which can be seen on Figure 3, is close to the ideal case with two deviations from the target: an overestimation immediately followed by an underestimation after which the solution arrives at the final goal.

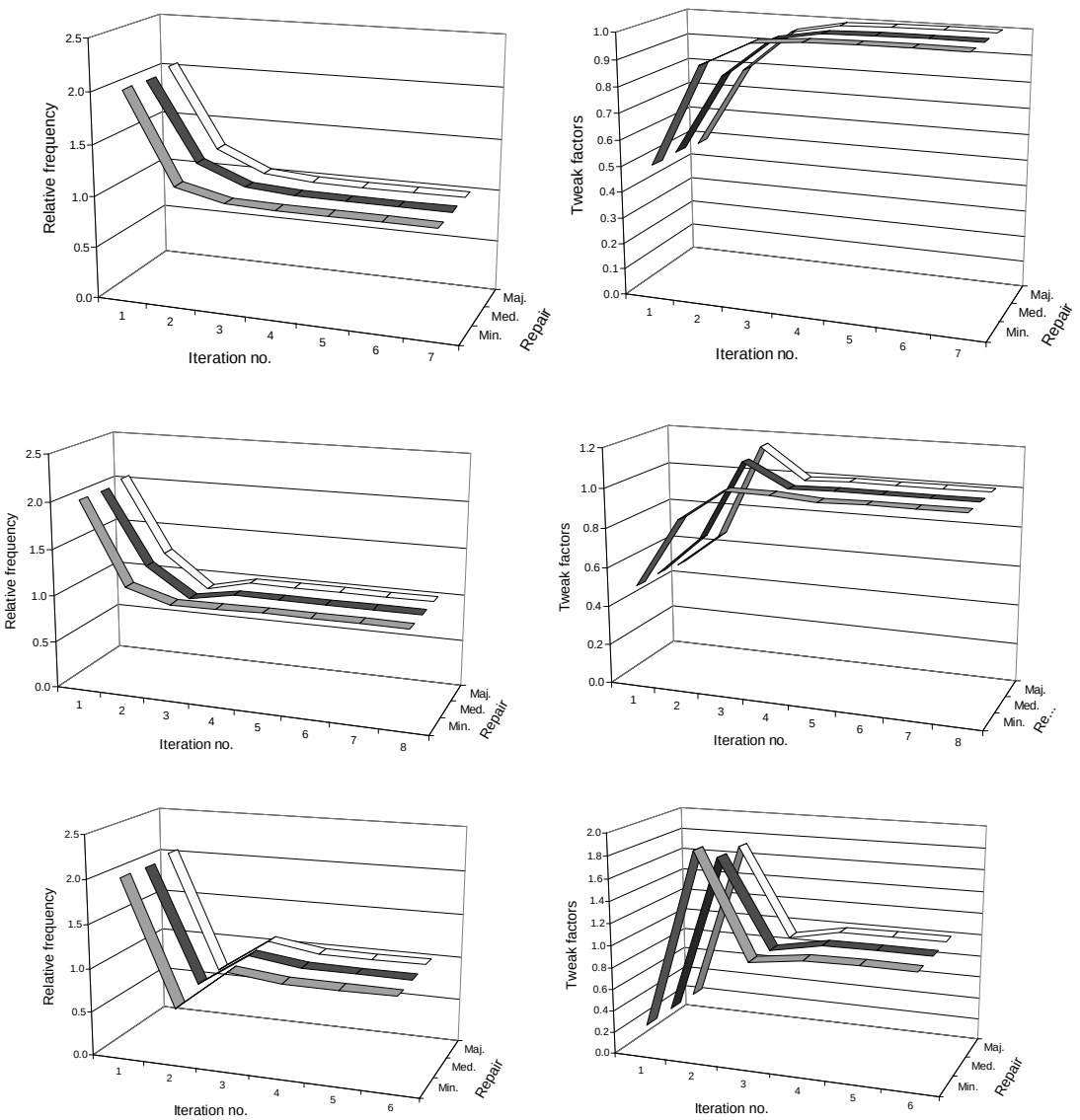

Figure 3. Adjusting the sample model to $\mathbf{F}_G = ½\mathbf{F}_0$ ($\varepsilon = 1E\text{-}4$) with three different numerical methods: NOLA (top), secant (middle) and *falsi* (bottom)

3.3.4. Evaluation of the methods

On Figure 4 efficiency of the three approximation methods in another typical adjustment cases is illustrated. For these tests the model used in the previous examples was adjusted to the policies $\mathbf{F}_G = [0, 0, F_0^3]$ (only major repair is performed with the other two types removed) and $\mathbf{F}_G = ¼\mathbf{F}_0$ (frequencies of all repairs reduced to 25%). A short comment should be made about the first of these examples. With $\mathbf{F}_G = [0, 0, F_0^3]$, it is very easy to remove any repair from the model by simply assigning $P^{sr} = 0$ in all the states, effectively increasing the value of P^{s0} at the same time. In such case no adjustment is required as the requested goal ($F^r = 0$) is achieved immediately. Nevertheless, such modification does affect the frequencies of other repairs that are left in the model (F^3 in this case) and the adjustment mechanism is still necessary just to return them to their requested values.

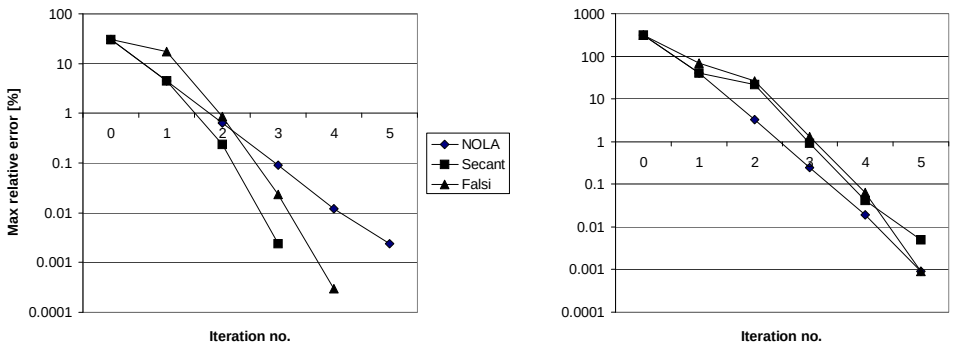

Figure 4. Convergence rate of the three approximation methods for two sample maintenance policies: $\mathbf{F}_G = [0, 0, F_0^3]$ (left) and $\mathbf{F}_G = ¼\mathbf{F}_0$ (right); $\varepsilon = 0.01\%$ in both cases

Comparing the effectiveness of the methods it should be noted that although simplifications of the NOLA solution may seem critical, in practice it works quite well. As it was noted before, due to its simplicity this method has one advantage over its more sophisticated rivals: since computation of the next solution does not depend on previous approximations, selection of the starting point is not so important and the accuracy during the first iterations is often better than in the secant or *falsi* cases. For example, in the case used in 3.3.1 ÷ 3.3.3 the NOLA method reached accuracy of 4.4% already after 2 iterations, while for secant and falsi methods the error after two iterations was, respectively, 11% and 9.2%. Superiority of the latter methods, especially of the *falsi* algorithm, manifests itself in the later stages of the process when the potential problems with an initial selection of the starting points have been diminished.

4. Conclusions

This paper described a modelling methodology that helps in choosing effective yet cost-efficient maintenance policy. Based on semi-Markov models representing the deterioration process, the equipment life curve and other reliability parameters can be evaluated. Once a database of equipment models is prepared, the end-user can perform various studies about different maintenance strategies and compare expected outcomes. As the results are visualized through the relatively simple concept of a life curve, no detailed expert knowledge about internal reliability parameters or configuration is required.

Additionally, the paper presented a method of model adaptation that allows automatic adjustment of the basic model to user-expected changes in maintenance policy. The numerical part of the method can be solved with common root-finding algorithms and it has proved its validity in numerous practical examples. Ability to adjust the models to such changes is crucial in practical studies of various possible maintenance scenarios.

Another important issue is how the adjustment modifies behaviour of the model in addition to reaching the desired repair frequencies and how the model should be constructed in order to accommodate the modifications without undesired side effects. For discussion of these problems please refer to [5–7]. In particular, while there is usually no problem with *reduction* of repair frequencies with the method described in this paper, special care must be taken when *increase* is requested because the probabilities cannot be enlarged indefinitely. In cases when the limit (2) is reached in all states even with $P^{s0} = 0$ and the requested frequencies are still not achieved, more substantial modifications of the model may be necessary. This also leads to specific directions as to how the model should be constructed in order to avoid such situations.

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