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POTENTIAL ACCURACY OF ESTIMATION OF EARTH-IONOSPHERE WAVEGUIDE PARAMETERS THROUGH DELAYS OF REFLECTED WAVES

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The problem of electromagnetic pulse radiator location from single station observation is considered, based on the hop model of pulse propagation in spherical waveguide 'Earth - ionosphere'. The waveguide parameters are the distance from the radiator and the effective reflecting heights of the ionosphere. They can be evaluated through the delays of the waves reflected by the ionosphere with respect to the ground wave. The errors in values of delays give rise to the errors in the waveguide parameters. These connections are investigated in this paper. It is demonstrated how the waveguide parameter errors depend on the distances and heights of reflections. A method to reinterpret the hop model by using some equivalent antenna array conception is proposed.

Keywords: *electromagnetic radiation, ionosphere, waveguide, hop model, distance, effective heights, delays, lightning discharge, atmospherics*

1. Introduction

Ionospheric layers and the Earth's surface create a spherical waveguide which should have a significant influence on features of electromagnetic (EM) fields propagating in it. At relatively low frequencies (ELF, VLF and LF bands) the ground and the lower ionospheric layer (D in daylight or E in night time) are good conductors. It is known that EM fields cannot penetrate deeply into those media. Consequently, geometric dimensions, namely the distance between the radiating and receiving points and the effective heights of the waveguide, become the main factors which determine the characteristics of propagation.

There are some methods for evaluating these dimensions from a receiving signal only. If the distance is not larger than 1500–1700 km, a hop model of propagation could be considered. It is supposed that the received signal consists of a ground wave and some waves reflected from the D or E ionospheric layer (Fig. 1).

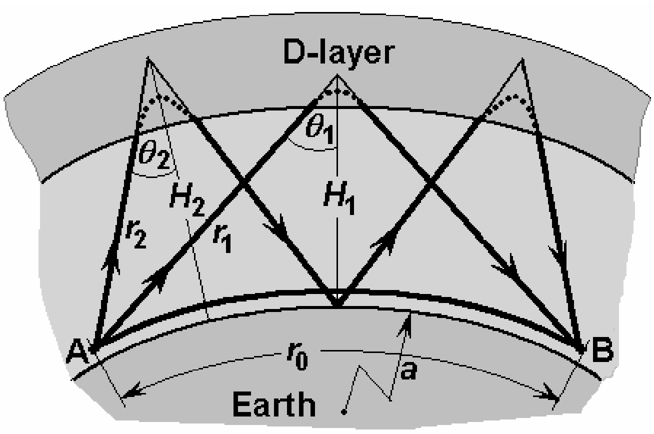

Figure 1. The hop model of ionospheric waveguide (in daytime)

EM radiation initiated from point A propagates to point B. The receiver in B registers the process of interaction of the ground wave $E_g(t)$ passed the distance r_0, and a few ionospheric waves. Only two waves from them are shown in Fig. 1. The single-hop wave $E_{i1}(t)$ and the two-hop wave $E_{i2}(t)$ pass the ways signed as r_1 and r_2 being reflected from the effective heights H_1 and H_2 with angles θ_1 and θ_2 respectively.

The input signal of receiver can be written as

$$u(t) = E_{\mathrm{g}}(t) + \sum_{\mathrm{n=1}}^{\mathrm{N}} E_{in}(t) = I(0,t) * [h_{\mathrm{c}}(0,t) * h_{\mathrm{g}}(t) + \sum_{\mathrm{n=1}}^{\mathrm{N}} h_{\mathrm{c}}(\theta_{\mathrm{n}},t) * h_{in}(t)], \tag{1}$$

where sign $*$ is symbol of convolution, $I(0, t)$ is the temporal form of the current pulse at antenna input points; $h_c(\theta, t)$ is the pulse function of antenna radiation in the direction set by angle θ; $h_g(t)$ and $h_{in}(t)$ are pulse functions of traces for ground wave and n-th ionospheric wave respectively. Evidently, the reflected waves $E_1(t)$ and $E_2(t)$ are delayed in relation to ground wave by the times indicated as τ_1 and τ_2 respectively.

Thus, the waveguide inherent features should appear as delays, or arrival times of those reflected waves. From the geometrical properties of the waveguide, one can obtain the system of equations

$$T_{\mathrm{n}} = (1 - 2Z_{\mathrm{n}} \cos R_{\mathrm{n}} + Z_{\mathrm{n}}^2)^{1/2} - R_{\mathrm{n}}, \quad n = 1, 2, \ldots \tag{2}$$

where $T_n = c\tau_n/2na$, $R_n = r_0/2na$, $Z_n = 1+H_n/a$ are normalized delays, distances and effective reflection heights respectively; $c = 2{,}998 \cdot 10^8$ *m/sec* is light velocity in a vacuum, $a = 6378$ *km* is the radius of the Earth. Previously we have proposed to estimate these delays from the received signal with a pseudocepstral method using Huang – Hilbert decomposition [1].

As the two-hop model is considered, the system (2) consists of two equations, but three unknown quantities are in it, namely, the distance r_0 and the ionospheric waves' effective heights H_1 and H_2. Therefore, this system is undetermined, since the number of equations is less in 1 than the number of unknown quantities. The algorithms have been examined in [2] to expand the system based on some common physical considerations; Fermat‘s theorem, in particular.

Errors in the solving of the undetermined system have decisive influences on precision in the evaluation of waveguide parameters. The errors in estimation of waveguide length and effective heights are considered below. If digital processing of receiving signals is used these errors depend strongly on the sampling rate.

2. Inaccuracy in the Estimation of Ionospheric Waveguide Length

It is important to find specific relations for inaccuracy in the estimation of waveguide length r_0 with effective heights H_n and delays τ_n in (2). Differentiating the Eq. (2) by τ_n as an implicit function which relates the length with the delay, one can obtain the relative inaccuracy of the waveguide length estimation

$$\delta r_0 = \frac{\Delta r_0}{r_0} = -\frac{c\Delta\tau_{\mathrm{n}}}{2nar_0}\left(1 - \frac{Z_{\mathrm{n}} \sin R_{\mathrm{n}}}{\mathrm{T_n} + R_{\mathrm{n}}}\right)^{-1}, \tag{3}$$

where $\Delta\tau_n$ is the absolute error of the estimation of the delay τ_n. It is obvious that the potential accuracy in the τ_n estimation by digital processing has to correspond to the considered signal sampling rate.

The graphs of Eq. (3) for different reflection heights are represented in Fig. 2. One can establish that the relative inaccuracy δr_0 depends on the evaluated distance r_0 non-monotonously. There are minima in middle distances (100–700 km) with positions depending on reflecting heights. If ionospheric conditions are unperturbed (i.e. lower ionosphere edge $H_0 > 50$ km) then the value $\delta r_0 \leq 0.05$, as the sampling interval is no more than 1 microsecond. The errors increase as the height of reflecting layer is reduced. Thus, they would be larger in the daylight than in the night time. It would be expected in the last case that the value δr_0 does not exceed 0.02.

The area painted over in Fig. 2 corresponds to the ordinary locations of reflecting D and E layers in the ionosphere.

Extremely great errors of distance estimation should be observed when ionospheric perturbations are influenced by strong ionization factors, and reflecting layers can rise to more smaller heights than in unperturbed conditions. The hypothetic case for $H_0 = 20$ km is also illustrated in Fig. 2. It is found in this situation that the error for middle distances increases up to four times in contrast to errors for ordinary conditions.

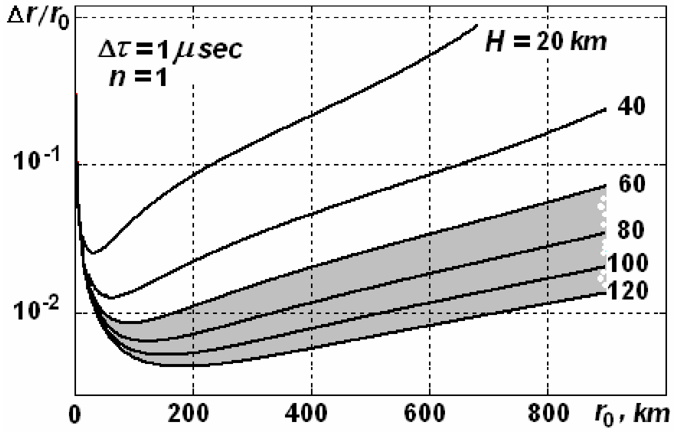

Figure 2. Relative inaccuracy of waveguide length estimation

As the value δr_0 depends on the evaluated distance r_0 non-monotonously, it can find the minimum position for every reflection height. Examining Eq. (3) for extreme points, one can obtain the transcendental equation for r_{0min}

$$[(Z_n\cos(r_{0min}/2na)-1](r_{0min}+c\tau_n)r_{0min}+[2naZ_n\sin(r_{0min}/2na)-r_{0min}-\tau_n]c\tau_n=0. \qquad (4)$$

The required value r_{0min} is determined by the positive root in it. As the argument of trigonometric functions is small, then Eq. (4) can be approximated by the quadratic equation

$$r_{0min}^2+2c\tau_n\, r_{0min}-(c\tau_n)^2 a/H_n\approx 0. \qquad (5)$$

The positive root of it is

$$r_{0min}=c\tau_n[(1+a/H_n)^{1/2}-1]\approx c\tau_n\,(a/H_n)^{1/2}\approx 0. \qquad (6)$$

where it is taken into account that a/H_n>>1.

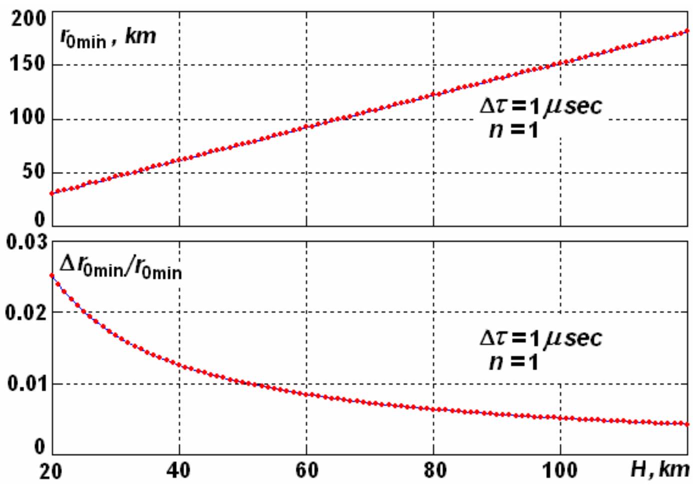

Figure 3. Solutions of Eq. (4)

Positions of the minima of Eq. (4) are displayed in Fig. 3 in relation to the reflection height H_1. It follows from it that the approximation of the function $r_{0min}(H)$ with a straight line is rather good. In that time, errors of distance estimation at minima points are very small.

It is seen from Fig. 2 that the errors of distance estimation become especially great if r_0 is very small (r_0<10 km) or if r_0 is near maximum distances (r_0~1800 km) still satisfied the hop model. In order to reduce these errors the usual method can be used, namely, to scale down the sampling interval in received signal digital processing. However, a necessary condition for this is the extension of the receiver frequency band. Complex upgrading of the receiver set may be necessary because of this. An alternative is simulated frequency band dilation using digital processing methods which leads to an evaluation of delays with a higher resolution than the sampling interval.

It is necessary to note that one can use Eq. (3) as an additional condition to redefine the undetermined system (2). Setting the maximum tolerable limit $\delta r_{0\mathrm{tol}}$ for distance estimation error, Eq. (3) may be apply as the inequality $\delta r_0 < \delta r_{0\mathrm{tol}}$ which restricts additionally the decision space of the system (2) by limiting r_0 and H_0 coupling.

3. Inaccuracy in Estimation of Ionospheric Waveguide Reflection Height

By analogy with (3) the error in the estimation of waveguide reflection height may be expressed via waveguide length r_0 and reflected wave delay τ_n. Supposing $H_\mathrm{n} = H_\mathrm{n}(\tau_\mathrm{n})$ and differentiating Eq. (2) by τ_n as an implicit function which relates the reflection height with the delay, one can obtain the relative inaccuracy of the estimation for H_n

$$\delta H_\mathrm{n} = \frac{\Delta H_\mathrm{n}}{H_\mathrm{n}} = \frac{c\Delta\tau_\mathrm{n}}{2nH_\mathrm{n}} \frac{T_\mathrm{n} + R_\mathrm{n}}{Z_\mathrm{n} - \cos R_\mathrm{n}}. \tag{7}$$

If r_0 is increased, the error (7) enlarges monotonically as it is seen in Fig. 4. The painted over area corresponds with the ordinary heights of reflecting D and E layers in ionosphere.

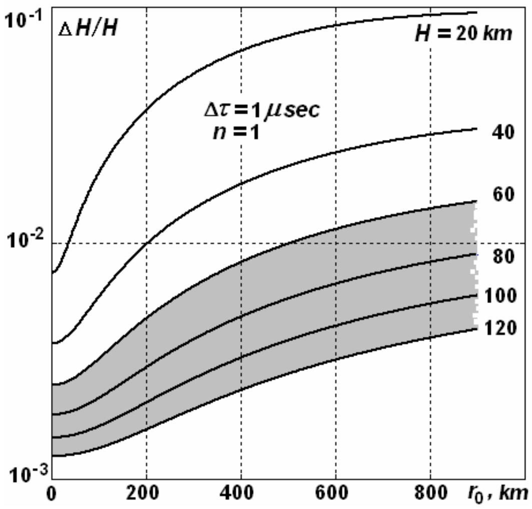

Figure 4. Relative inaccuracy of waveguide reflection height estimation

For the small $r_0 \rightarrow 0$ it may be discovered that τ_n tends to $2nH_\mathrm{n}/\mathrm{c}$. Thus, the error (7) achieves the minimum value

$$\delta H_{\mathrm{n}min} = \delta H_\mathrm{n}\Big|_{r_0 \rightarrow 0} = \frac{c\Delta\tau_\mathrm{n}}{2nH_\mathrm{n}}. \tag{8}$$

If sampling interval $\Delta\tau$ is no more than 2 microseconds and conditions in the ionosphere are unperturbed, error values (8) have not to exceed $0.5 \cdot 10^{-2}$. This is better by up to approximately one order than the errors (3) of waveguide length estimation.

4. Array Antenna Model of EM Waves Excitation in the Ionospheric Waveguide

The explanation for the fairly high accuracy of waveguide parameters estimation in hop model frames may be proposed as follows. Let us suppose that every wave which is a part of the sum in (1) is excited by a separate source S_0, S_1, S_2, as it is in Fig. 5 for two-hop model. These waves pass from the mentioned sources to receiving point *B*. The ways r_0, r_1, r_2 are different but they are the same as in the initial hop model (see Fig. 1). The sources form an discrete antenna array.

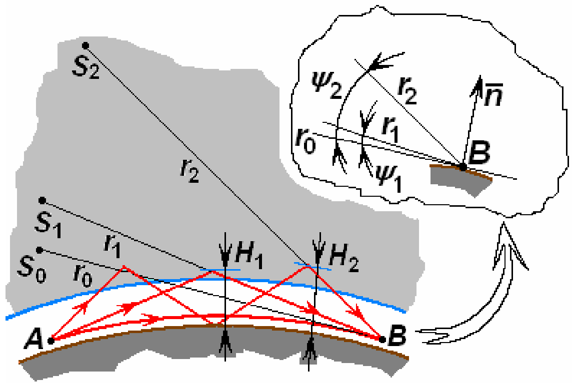

Figure 5. The equivalent antenna array included in the hop model

If the origin of the spherical coordinates system is selected at the point *B* then the positions of the sources have to be described by the equations:

$$R_n = (Z_n^2 - 2Z_n \cos(r_0/2na) + 1)^{1/2}, \tag{9}$$

$$\theta_n = \arccos \frac{Z_n \sin(r_0/2na)}{R_n}, \qquad n = 1, 2 \tag{10}$$

The distance r_0 is read off along the tangent line direction to the Earth's surface at the point *B*. The spacings d_{10} and d_{21} among radiators S_1, S_0, and S_2, S_1 are correspondingly

$$d_{10} = (r_0^2 + r_1^2 - 2r_0 r_1 \cos\theta_1)^{1/2}, \tag{11}$$

$$d_{21} = [r_1^2 + r_2^2 - 2r_1 r_2 \cos(\theta_2 - \theta_1)]^{1/2}. \tag{12}$$

In general, these spacings are not small in terms of wavelengths because radiation frequencies are relatively low (ELF, VLF or LF bands). Hence, the antenna array has to possess strongly pronounced directivity. Perhaps, that is the feature which allows us to explain the high potential accuracy in waveguide parameters estimation using the hop model.

This situation is typical for lightning discharge location when receiving an EM pulse of lightning (atmosperics) at a single point only.

5. Conclusions

In this research the hop model of the propagation of an electromagnetic pulse from a source allocated in the Earth-ionosphere spherical waveguide is considered. The pseudocepstral method is recommended for evaluating the delays of reflected waves. Specific dependencies are established for waveguide parameters estimation accuracy in the hop model propagation frames. If ionospheric conditions are not perturbed this accuracy is acceptable for many operational uses. A modification of the hop model is offered.

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