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THE REFLECTION OF ELECTROMAGNETIC WAVES FROM A RANDOMLY INHOMOGENEOUS MEDIUM

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The problem of reflection of a horizontally polarized plane electromagnetic wave from an infinitely thick dielectric layer with randomly inhomogeneous dielectric permittivity is regarded. The complex dielectric permittivity together with correlation lengths and dispersions of the random fluctuations of its imaginary and real part are presumed to be constant through the layer. The fluctuations are presumed to be delta-correlated. Under these assumptions, the complex Fresnel coefficient is considered, the Fokker-Planck equation for probability density of its module is obtained from the Riccati equation and its stationary solution is then found, which is shown to be either a trigonometric or a power function, which depends on the dielectric parameters of the medium.

Keywords: *Fresnel coefficient, the Fokker-Planck equation*

The problem of reflection of a horizontally polarized plane electromagnetic wave from a dielectric layer with randomly inhomogeneous dielectric permittivity, situated in free space and stretching from $z = 0$ down to $z = -L$ (z is the vertical coordinate), is considered. Namely, the dielectric permittivity is presumed to be complex and to consist of a constant background part and a fluctuating part, depending on z:

$$\varepsilon = \beta + \tilde{\beta}(z) + i\alpha + i\tilde{\alpha}(z). \qquad (1)$$

In contrast to some previous work on such problems [1, 2], fluctuations of both the real and the imaginary part of the dielectric permittivity are taken into account. They are presumed to be delta-correlated and mutually independent (both statistically and functionally) random functions.

Considering the incident wave as $E(x,z) = w(z)\exp(ikx\sin(\theta))$ and taking into account (1), one can obtain from the wave equation the following equation for w(z):

$$\frac{d^2 w}{dz^2} + k^2 w(-\sin^2\theta + \beta + i\alpha + \tilde{\beta}(z) + i\tilde{\alpha}(z)) = 0, \qquad (2)$$

where k is the wave number of a vacuum and θ is the incidence angle. We consider then the incident wave to be of unit amplitude, that is $\exp(-ikz\cos(\theta))$, and presume the solution of (2) to look like $w(z) = A(z)\exp(-ikqz) + B(z)\exp(ikqz)$, where $q \equiv \sqrt{-\sin^2\theta + \beta + i\alpha}$. The new unknown functions A and B are related by the additional condition $A'(z)\exp(-ikqz) + B'(z)\exp(ikqz) = 0$.

Introducing the reflectivity coefficient as

$$R(z) \equiv B(z) * A^{-1}(z) * \exp(2ikqz), \tag{3}$$

one easily obtains the Riccati equation for R:

$$\frac{dR}{dz} = 2(i\eta - \gamma)R + \frac{(1+R)^2}{2|q|^2}[(i\gamma - \eta)\tilde{\alpha} + (i\eta + \gamma)\tilde{\beta}], \tag{4}$$

where $kq \equiv \eta + i\gamma$.

The continuity conditions for the tangential components of the electric and magnetic fields at z=-L lead to the continuity of w(z) and its derivative at z=-L, which in turn gives the initial condition for the Riccati equation:

$$R(-L) = (q - \cos(\theta))/(q + \cos(\theta)). \tag{5}$$

Introducing two real functions U(z) and φ(z) so, that

$$R \equiv \frac{\sqrt{U-1}}{\sqrt{U+1}} e^{i\varphi(z)}, \tag{6}$$

one obtains a system of two real differential equations instead of the Riccati equation:

$$\frac{dU}{dz} = -2\gamma(U^2-1) - 2b\tilde{\alpha}(U^2-1) - 2\tilde{\alpha}\sqrt{U^2-1}(bU\cos(\varphi) - a\sin(\varphi)) +$$
$$+ 2a\tilde{\beta}(U^2-1) + 2\tilde{\beta}\sqrt{U^2-1}(b\sin(\varphi) + Ua\cos(\varphi)). \tag{7}$$

$$\frac{d\varphi}{dz} = 2\eta + 2a\tilde{\alpha} + 2\tilde{\alpha}(aU\cos(\varphi) + b\sin(\varphi))\frac{1}{\sqrt{U^2-1}} +$$
$$+ 2b\tilde{\beta} + 2\tilde{\beta}(bU\cos(\varphi) - a\sin(\varphi))\frac{1}{\sqrt{U^2-1}}, \tag{8}$$

where $a \equiv \frac{2\gamma}{|q|^2}$, $b \equiv \frac{2\eta}{|q|^2}$.

The probability density P of the random function U is then formally introduced via delta-function [2] and the Fokker-Planck equation for this probability density is then obtained, taking into account the above mentioned equations for U and φ. To get rid of the trigonometric functions in this equation, this is averaged over the period of the fast variations of these functions, which ultimately yields the Fokker-Planck equation for *P*:

$$\frac{\partial P}{\partial z} = 2\gamma\frac{\partial}{\partial U}\left[P\left(U^2-1\right)\right] + 4(b^2\sigma^2 l + a^2 s^2 r)\frac{\partial}{\partial U}\left[(U^2-1)\frac{\partial}{\partial U}\left(P(U^2-1)\right)\right] +$$
$$+ 2(b^2\sigma^2 l + a^2 s^2 r)\frac{\partial}{\partial U}\left[(U^2-1)\frac{\partial}{\partial U}(U^2 P)\right] + 2(a^2\sigma^2 l + b^2 s^2 r)\frac{\partial}{\partial U}\left[(U^2-1)\frac{\partial P}{\partial U}\right], \tag{9}$$

where σ^2, l are the dispersion and the correlation length of $\tilde{\alpha}$; s^2, r are the dispersion and the correlation length of $\tilde{\beta}$.

The method scheme of the above is presented below:

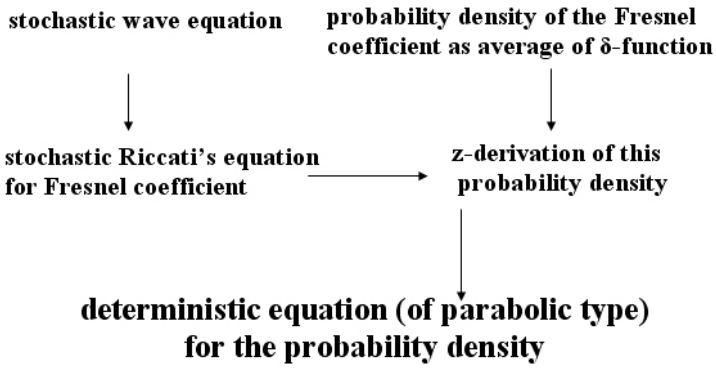

Figure 1. The method's scheme

The stationary solution, which corresponds to the case of an infinitely thick dielectric layer, of the 'averaged' Fokker-Planck equation (9) is then found (assuming the absence of probability density flux across the boundary) and looks like either

$$P = N \cdot \exp(-2\gamma \cdot \frac{1}{\sqrt{3km - 6k^2}} \cdot arctg \frac{\sqrt{3k}U}{\sqrt{m - 2k}}) \cdot \frac{1}{3kU^2 + m - 2k} \tag{10}$$

or $P = N(U - w)^{-n-1} \cdot (U + w)^{n-1}$ (11)

which depends on the relationship between the medium's parameters (see table 1 below). Here

$$k \equiv 2(b^2\sigma^2 l + a^2 s^2 r),\ m \equiv 2(a^2\sigma^2 l + b^2 s^2 r),\ w \equiv \frac{\sqrt{6k^2 - 3km}}{3k},\ n \equiv \frac{\gamma}{\sqrt{6k^2 - 3km}}.$$

The normalization factor N is to be found from the condition $1 = \int_1^\infty PdU$, and for solution (11)

$$N = 2wn \frac{1}{\frac{(1+w)^n}{(1-w)^n} - 1}.$$

Table1. Types of the stationary solution of the Fokker-Planck equation and the relationship between the parameters of the medium.

What is fluctuating:	Solution (10) (with arctangent)	Solution (11)
Both the imaginary and the real part of the permittivity	$\gamma^2(\sigma^2 l - 2s^2 r) > \eta^2(2\sigma^2 l - s^2 r)$	$\gamma^2(\sigma^2 l - 2s^2 r) < \eta^2(2\sigma^2 l - s^2 r)$
Only the real part	$2\gamma^2 < \eta^2$	$2\gamma^2 > \eta^2$
Only the imaginary part	$\gamma^2 > 2\eta^2$	$\gamma^2 < 2\eta^2$

The result obtained takes into account random inhomogeneities within the irradiated medium and is therefore of value from the viewpoint of remote sensing of media with strong volume scattering, such as multi-year sea ice and some others.

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