APPLICATION OF CHI-SQUARE TEST OF INDEPENDENCE IN THE UTILIZATION OF POSTAL AND TELECOMMUNICATION SERVICES

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The introductory part of this paper concerns a marketing survey of the utilization of postal and telecommunication services in a regional entrepreneurial environment. The methodology of the chi-square test of independence between two qualitative statistical figure values is detailed in the next part of the paper. This methodology is applied to the results of the marketing survey in three groups; internal test of independence, cross test of independence and test of independence in the regional context. On the each identified pairs of qualitative statistic figure values (A and B) the chi-square test of independence was applied. The null hypothesis regarding their independence was defined. Then the assumptions concerning rejecting or not rejecting the null hypothesis were defined. In the case of rejecting the null hypothesis the degree of association was determined by one of the measures of association.

Keywords: marketing survey, chi-square test of independence, postal services, telecommunication services, regional disparities, measures of association

1. Introduction

The chi-square test of independence is a nonparametric statistical analysis method often used in experimental work where the data consist in frequencies or 'counts'. The most common use of the test is to assess the probability of association or independence of facts [1]. This paper deals with the application of a chi-square test to the results of a marketing survey focused on the utilization of postal and telecommunications services.

2. Characterization of the marketing survey

The survey was conducted in within the framework of the project No I16-07-140 titled 'Marketing Survey of the Utilization of Postal and Telecommunication Services in a Regional Entrepreneurial Environment' at the University of Zilina in 2008. It focused on the recovery of relevant information about the provided postal and telecommunication services and customer satisfaction with these services. Five districts in the Zilina region were selected for this survey: Bytca, Cadca, Kysucke Nove Mesto, Namestovo and Zilina. A map of the districts is shown in Figure 1.

The questionnaire comprised 25 questions. It was completed by a total of 250 small and medium enterprises across the five districts. The questions were divided into five sets, each referring to each other.

Set 1 – Postal services
✓ postal services providers
✓ frequency of selected postal services utilization
✓ knowledge of selected postal services
✓ method of postal fee payment
✓ method of mail delivery
✓ average monthly expenditure on postal services
✓ satisfaction with postal services
✓ strengths and weaknesses of the postal services
✓ trends in postal services development

Set 2 – Telecommunication services
✓ telecommunication services providers
✓ frequency of utilization of selected telecommunication services
Members of the sample were double classified (i.e. classified in two separate ways) and results were arranged in contingency tables. Contingency tables are the basis for the application of the chi-square test of independence.

*Figure 1. Map of the districts in the Žilina region in which the marketing survey of the utilization of postal and telecommunication services was conducted [2]*
3. Chi-Square Test of Independence

The chi-square test of independence is a nonparametric statistical test to determine if two or more classifications of the samples are independent or not. A common question with regards to a contingency table is whether it has independence. By independence, we mean that the row and column variables are unassociated (i.e. knowing the value of a row variable will not help us predict the value of a column variable, and likewise, knowing the value of a column variable will not help us predict the value of a row variable) [3].

The methodology of the chi-square test of independence between two qualitative statistic figure values is divided into four steps. The first step is the expression of the null and alternative hypothesis. The second step is to determine the significance level (\(\alpha\)). The third step is to calculate the chi-square test statistic (\(\chi^2\)). The fourth step is to compare the computed (\(\chi^2\)) with the critical value in the table for the significance level (\(\alpha\)) and then to make a statistical decision in regard to the null hypothesis.

3.1. Expression of Hypotheses

The null hypothesis \(H_0\) expresses the independence of variables. In contrast, the alternative hypothesis \(H_a\), which we want to prove to be true in the majority of cases, mostly expresses a statistical association of the variables. The truth of the alternative hypothesis is always shown only indirectly, in a way that will show that the null hypothesis is unlikely, and that the alternative hypothesis (the only remaining hypothesis) is therefore likely. Independence is tested by a chi-square test, which based on the chi-square distribution. The chi-square distribution has paramount importance in a dependency analysis in the association and contingency tables [3]. In a chi-square test of independence the null and alternative hypothesis are expressed:

\(H_0: \) The two qualitative statistic figure values A and B are independent

\(H_a: \) The two qualitative statistic figure values A and B are related

3.2. Determination of the Significance Level (\(\alpha\))

If we reject the null hypothesis when it is in fact valid, we make a 'type 1' error (i.e. we come to the conclusion that there is a relationship between the variables when in fact there is none). The significance level (\(\alpha\)) is the probability of committing a 'type 1' error. We can reduce the chances of making a 'type 1' error by selecting a smaller value for (\(\alpha\)). This makes it more likely that the null hypothesis will be accepted, but it also increases the risk of making a 'type 2' error (i.e. incorrectly concluding that there is no relationship between the variables). Determining the significance level is thus a sort of compromise between these two types of errors, and its choice depends on the type of tested facts, on the experience of the researcher, etc. [4].

Alpha (\(\alpha\)) is traditionally set at 5% (= 0.05), or 1% (= 0.01). Variations that occur with a probability less than the chosen significance level are called statistically significant at the selected significance level. In this paper the significance level \(\alpha = 0.05\) is used.

3.3. Calculation of the Chi-Square Test Statistic

The chi-square test statistic that asymptotically approaches a chi-square distribution was first introduced by the British statistician Karl Pearson in 1900. A chi-square distribution is mostly used in the testing of a compliance table with some theoretical model. This involves comparing observed and expected frequencies. Expected frequencies are those which should be observed if the statistic figure values A and B are independent [5]. In order to compare the observed and expected frequencies we produce the chi-square value (\(\chi^2\)) using the formula in equation 1:

\[
\chi^2 = \sum_{i=1}^{r} \sum_{j=1}^{c} \frac{(O_{ij} - E_{ij})^2}{E_{ij}},
\]

where:

\(\chi^2\) … the test statistic that asymptotically approaches a chi-square distribution,

\(O_{ij}\) … the observed frequency of the \(i^{th}\) row and \(j^{th}\) column,

\(E_{ij}\) … the expected (theoretical) frequency of the \(i^{th}\) row and \(j^{th}\) column,

\(r\) … the number of rows in the contingency table,

\(c\) … the number of columns in the contingency table.
A second important part of determining the chi-square test statistic is to define the degrees of freedom (df) of the test. The degree of freedom of a contingency table with r rows and c columns is computed using the following formula given in equation 2:

\[ \text{df} = (r - 1) \times (c - 1), \]  

(2)

When using the chi-square test in tables larger than 2 by 2 Cochran suggests that no more than 20% of the expected frequencies should be less than 5 and that all individual expected frequencies should be 1 or greater. [6, 7]. This suggestion, which can be written by using probability P (formula 3), is an assumption/restriction on the use of the chi-square test in contingency tables.

\[ P[E_{ij} < 5] \leq 0.2 \land E_{ij} > 1. \]  

(3)

If any expected frequencies in 2 by 2 tables are less than 10, but greater than or equal to 5, some authors suggest that Yates’ Correction of Continuity should be applied [8]. This is done by subtracting 0.5 from the absolute value of \( O_{ij} - E_{ij} \) before squaring (equation 4). However, the use of Yates’ Correction of Continuity is controversial, and is not recommended by all authors.

\[ \chi^2_{(\text{Yates})} = \sum_{i=1}^{r} \sum_{j=1}^{c} \frac{(O_{ij} - E_{ij} - 0.5)^2}{E_{ij}} \]  

(4)

3.4. Conclusion of the Null Hypothesis

The last step is comparing the calculated (\( \chi^2 \)) with the critical value in the table at the significance level (\( \alpha \)). The statistical decision in regard to the null hypothesis depends on validity of the inequality that is shown in formula 5.

\[ \chi^2 \geq \chi^2_{1-\alpha} [(r - 1) \times (c - 1)] \]  

(5)

There may be two variants [4]:

- formula (5) is valid – the computed (\( \chi^2 \)) is equal to or greater than the critical value. We reject the null hypothesis, and confirm the alternative hypothesis. The different between the observed and expected frequencies is statistically significant. Therefore we conclude that there is a relationship between the variables.

- formula (5) is invalid – the computed (\( \chi^2 \)) is less than the critical value. The null hypothesis cannot be rejected. The difference between the observed and expected frequencies is not statistically significant. However, this does not mean that the null hypothesis is true. It indicates that there is insufficient evidence of a relationship between the variables.

4. Measures of Association

Use of the chi-square test of independence can provide information on whether the association between two qualitative statistic figure values A and B can be regarded as statistically significant or not. In the case of statistical significance of this association, it is possible to evaluate the degree of association only indirectly. Direct evaluation of the degree of association can be done using measures of association [3, 9], which are based on the computed chi-square value (\( \chi^2 \)). The measures of association are: the Phi coefficient, the Contingency coefficient C, Cramer’s V, and Tschuprow’s coefficient \( \tau \). The nearer the value of the measure of association is to 0, the greater the degree of independence between the qualitative statistic figure values A and B is confirmed (interpretation of the measure of association values is shown in table 1). There are some restrictions on the use of measures of association [9]:

- Phi coefficient is used in 2 by 2 tables,
- Contingency coefficient C (Pearson’s C) is only used for 5 by 5 tables or larger,
- Cramer’s V is the most popular measure of association regardless of table size (in this paper it was used in tables larger than 2 by 2 but smaller than 5 by 5),
- Tschuprow’s coefficient \( \tau \) is used only in square tables where row marginals are identical to column marginals (this coefficient is little used and is not supported by major statistical packages).
Table 1. The measures of association value and its interpretation [10]

<table>
<thead>
<tr>
<th>Measure of association value</th>
<th>Degree of association</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>perfect independence</td>
</tr>
<tr>
<td>0.0 – 0.1</td>
<td>trivial association</td>
</tr>
<tr>
<td>0.1 – 0.3</td>
<td>small association</td>
</tr>
<tr>
<td>0.3 – 0.5</td>
<td>moderate association</td>
</tr>
<tr>
<td>0.5 – 0.7</td>
<td>large association</td>
</tr>
<tr>
<td>0.7 – 0.9</td>
<td>very large association</td>
</tr>
<tr>
<td>0.9 – 1.0</td>
<td>nearly perfect association</td>
</tr>
<tr>
<td>1.0</td>
<td>perfect association</td>
</tr>
</tbody>
</table>

5. Application of the Chi-Square Test of Independence to the Results of the Marketing Survey

Several pairs of qualitative statistic figure values were identified within the framework of the marketing survey. For each of the identified pairs of qualitative statistic figure values (A and B) the chi-square test of independence was applied. It defined the null hypothesis concerning their independence. Then the criteria for rejecting or not rejecting the null hypothesis were defined. In the case of rejecting the null hypothesis, the degree of association was determined by one of the measures of association (according to the restrictions in chapter 4). The next three chapters include results, which are presented in three groups: internal test of independence, cross test of independence and test of independence in the regional context.

6. Internal Test of Independence

6.1. Internal Test of Independence inside Set 1 – Postal Services

In the internal test of independence inside set 1, both qualitative statistic figure values relate to postal services. Several pairs of qualitative statistic figure values were identified. The following hypotheses were devised to assess their independence:

H₀: Average monthly expenditure on postal services is independent of the postal services provider (assumption: H₀ will be rejected)

The assumption was confirmed. The average monthly expenditure on postal services depended on the postal services provider. A small association was confirmed.

H₀: Satisfaction with postal services is independent of method of mail delivery (assumption: H₀ can not be rejected)

The assumption was confirmed. The respondents’ satisfaction was independent of the method of mail delivery. The method of mail delivery depended on the respondents’ preferences.

H₀: Satisfaction with the postal services is independent of the method of postal fee payment (assumption: H₀ can not be rejected)

The assumption was confirmed. As in the previous case, there was a possibility to choose the method of postal fee payment, thus this factor did not affect the respondents’ satisfaction with the postal services.

H₀: Satisfaction with the postal services is independent of trends in postal services development (assumption: H₀ can not be rejected)

It was not possible to confirm or refute this assumption. The pair of qualitative statistic figure values was not suitable for the application of a chi-square test of independence. The clause (3) was not fulfilled.

H₀: Satisfaction with postal services is independent of factors which are considered strengths of the postal services. (assumption: H₀ will be rejected)

Respondents were asked to indicate in the questionnaire whether each factor was considered to be strength of the postal services, or not. In addition to the complex factor of quality, respondents also gave opinions on sub-factors that are part of quality. Satisfaction with postal services depended on the majority of indicated factors. A moderate association was confirmed for complex factor ‘quality’. A small association was confirmed for the sub-factors ‘availability’, ‘price’, ‘reliability’, ‘complementary services’, ‘expertness of employees’ and ‘mail delivery time’. The assumption was not confirmed for the sub-factors ‘rapidity’ and ‘opening hours’. It was not possible to confirm or refute the assumption for the sub-factor ‘waiting time at the postal counter’, because the clause (3) was not fulfilled.
6.2. Internal Test of Independence inside Set 2 – Telecommunication Services

In the internal test of independence inside set 2, both qualitative statistic figure values related to telecommunication services. Several pairs of qualitative statistic figure values were identified. The following hypotheses were devised to assess their independence:

**H₀:** Average monthly expenditure on telecommunication services is independent of the telecommunication services provider

*(assumption: H₀ will be rejected)*

The assumption was not confirmed, so the average monthly expenditure on telecommunication services was independent of the telecommunication services provider.

**H₀:** The frequency of utilization of telecommunication service - 'data transmission' is independent of the telecommunication services provider

*(assumption: H₀ will be rejected)*

Respondents were asked to indicate in the questionnaire how often they used some selected telecommunication services. One of which was "data transmission". A small association was confirmed for this service. Therefore, the frequency of utilization of service 'data transmission' depends on the telecommunication services provider.

**H₀:** Satisfaction with telecommunication services is independent of average monthly expenditure on telecommunication services

*(assumption: H₀ will be rejected)*

It was not possible to confirm or refute this assumption. The pair of qualitative statistic figure values was not suitable for the application of a chi-square test of independence. The clause (3) was not fulfilled.

**H₀:** Satisfaction with telecommunication services is independent of factors which are considered strengths of telecommunication services

*(assumption: H₀ will be rejected)*

It was not possible to confirm or refute this assumption. The pair of qualitative statistic figure values was not suitable for the application of a chi-square test of independence. The clause (3) was not fulfilled.

7. Cross Test of Independence

In the cross test of independence, the first qualitative statistic figure value relates to postal services and the second qualitative statistic figure value relates to telecommunication services. Several pairs of qualitative statistic figure values were identified. The following hypotheses were devised to assess their independence:

**H₀:** Selection of the postal services provider is independent of the selection of the telecommunication services provider

*(assumption: H₀ can not be rejected)*

The assumption was confirmed. Respondents' selections of providers of these services were independent.

**H₀:** The indication of factors which are considered strengths of the postal services is independent of the indication of factors which are considered strengths of the telecommunication services

*(assumption: H₀ can not be rejected)*

The chi-square test of independence was applied to these pairs of factors:

- waiting time at the postal counter – signal coverage,
- quality of postal services – quality of telecommunication services,
- expertness of employees in the postal sector – expertness of employees in the telecommunication sector,
- price of postal services – price of telecommunication services.

The assumption was confirmed only in the first case. This means that nominating 'waiting time at the postal counter' as a strength of the postal services and nominating 'signal coverage' as a strength of the telecommunication services are two independent facts. For the remaining three pairs of factors a moderate association was confirmed. It can be concluded that the respondents consider these factors as strengths in postal services and in telecommunication services.

**H₀:** Average monthly expenditure on postal services is independent of average monthly expenditure on telecommunication services

*(assumption: H₀ can not be rejected)*

The assumption was not confirmed. The moderate association between respondents' average monthly expenditure on postal services and average monthly expenditure on telecommunication services suggests that respondents spend money proportionately on these services. Thus their 'communication activities' in the fields of postal and telecommunication services are aligned.
8. Test of Independence in the Regional Context

As mentioned above (chapter 2), the respondents in the marketing survey were small and medium enterprises located in the five districts of the Zilina region. By the test of independence in the regional context, regional disparities were identified in some cases.

8.1. Set 1 – Postal Services in Regional Context

In the test of independence in the regional context in set 1, the first qualitative statistic figure value relates to postal services and the second qualitative statistic figure value relates to the selected district. Several pairs of qualitative statistic figure values were identified. The following hypotheses were devised to assess their independence:

\( H_0: \) The selection of postal services provider is independent of the district where the respondent’s firm is located  
\textit{(assumption: } H_0 \text{ will be rejected)}

The assumption was not confirmed. The selection of postal services provider was actually independent of the district where the respondent’s firm is located. It can be concluded that the territorial availability of contact points of postal services providers is sufficient in the monitored districts. Regional disparities were not confirmed.

\( H_0: \) The frequency of selected postal service utilization is independent of the district where the respondent’s firm is located  
\textit{(assumption: } H_0 \text{ will be rejected)}

A small association was confirmed for some postal services: 1st class letter, 1st class parcel, express mail service and business parcel. Regional disparities, by contrast, were not confirmed for other postal services: 2nd class letter, 2nd class parcel, direct mail advertising and postal money order.

\( H_0: \) Knowledge of selected postal services - Track & Trace and Hybrid Mail - is independent of the district where the respondent’s firm is located  
\textit{(assumption: } H_0 \text{ can not be rejected)}

The assumption was confirmed. Knowledge of selected postal services - Track & Trace and Hybrid Mail - was actually independent of the district where the respondent’s firm is located.

\( H_0: \) The method of postal fee payment is independent of the district where the respondent’s firm is located  
\textit{(assumption: } H_0 \text{ will be rejected)}

Regional disparities were not confirmed in this case, although the results of double classification in the contingency table showed some non-uniformity in the utilization of certain methods of postal fee payment (especially by franking machine impression and by postage credit).

\( H_0: \) The method of mail delivery is independent of the district where the respondent’s firm is located  
\textit{(assumption: } H_0 \text{ will be rejected)}

The assumption was confirmed. There are regional disparities in the form of small associations between the method of mail delivery and the district where the respondent’s firm is located. Detailed examination of the results of double classification in the contingency table shows that:

- mail delivery to a private letter-box is used proportionally in all districts,
- mail delivery to a Post Office Box is most used in the Namestovo district and least used in the Bytca district,
- mail delivery by collection at the counter shows only slight regional disparities.

\( H_0: \) Average monthly expenditure on postal services is independent of the district where the respondent’s firm is located  
\textit{(assumption: } H_0 \text{ will be rejected)}

As mentioned above, regional disparities in the frequency of some selected postal services utilization were confirmed. This was the basis for the assumption that average monthly expenditure on postal services will depend on the district where the respondent’s firm is located. This assumption, however, was not confirmed.

\( H_0: \) Satisfaction with postal services is independent of the district where the respondent’s firm is located  
\textit{(assumption: } H_0 \text{ will be rejected)}

It was not possible to confirm or refute this assumption. The pair of qualitative statistic figure values was not suitable for the application of a chi-square test of independence. The clause (3) was not fulfilled.
H₀: The nomination of factors which are considered strengths of the postal services is independent of the district where the respondent’s firm is located
(assumption: H₀ will be rejected)
The assumption was not confirmed for the factors of: 'quality', 'reliability', 'rapidity', 'expertness of employees', 'opening hours' and 'waiting time at the postal counter'. This means that respondents indicated these factors as strengths or weaknesses of the postal services independently of the district where their firms are located.

On the other side, a small association was confirmed for some other factors:
- 'availability' and 'mail delivery time', where dependency on the location of the respondent’s firm for obvious reasons
- 'price', which refers to the different purchasing power in different districts and
- 'complementary services'.

H₀: The assumed trends in postal services development is independent of the district where the respondent's firm is located
(assumption: H₀ can not be rejected)
It was not possible to confirm or refute this assumption. The pair of qualitative statistic figure values was not suitable for the application of a chi-square test of independence. The clause (3) was not fulfilled.

8.2. Set 2 – Telecommunication Services in the Regional Context

In the test of independence in the regional context in set 2, the first qualitative statistic figure value relates to telecommunication services and the second qualitative statistic figure value relates to the selected district. Several pairs of qualitative statistic figure values were identified. The following hypotheses were devised to assess their independence:

H₀: The selection of telecommunication services provider is independent of the district where the respondent’s firm is located
(assumption: H₀ will be rejected)
The assumption was not confirmed, just as with postal services (as mentioned above in chapter 8.2). Therefore, regional disparities were not confirmed in the selection of telecommunication services provider. It can be concluded that the territorial availability of contact points of telecommunication services providers is sufficient in the monitored districts.

H₀: The frequency of selected telecommunication service utilization is independent of the district where the respondent’s firm is located
(assumption: H₀ will be rejected)
The assumption of a statistically significant association was confirmed only in case of two telecommunication services – fax services and data transmission – where a small association was confirmed. Regional disparities were not confirmed in the case of the voicemail service. It was not possible to confirm or refute the null hypothesis regarding independence for the majority of telecommunication services (phone service, audio conferencing, video conferencing and Audiotex Service), because clause (3) was not fulfilled.

H₀: Average monthly expenditure on telecommunication services is independent of the district where the respondent's firm is located
(assumption: H₀ will be rejected)
As mentioned above, regional disparities in the frequency of some selected telecommunication services utilization were confirmed. This was the basis for the assumption that average monthly expenditure on telecommunication services is dependent on the district where the respondent's firm is located. This assumption, however, was not confirmed.

H₀: Satisfaction with telecommunication services is independent of the district where the respondent’s firm is located
(assumption: H₀ will be rejected)
It was not possible to confirm or refute this assumption. The pair of qualitative statistic figure values was not suitable for the application of a chi-square test of independence. The clause (3) was not fulfilled.

H₀: The nomination of factors which are considered strengths of the telecommunication services is independent of the district where the respondent’s firm is located
(assumption: H₀ will be rejected)
The assumption was not confirmed for any factor which could be considered strength of the telecommunication services. These factors were: 'quality', 'availability', 'price', 'reliability', 'rapidity',

170
'complementary services', 'expertness of employees', 'opening hours' and 'signal coverage'. This means that respondents indicated these factors as strengths or weaknesses of telecommunication services independently of the district where their firms are located.

H0: The assumed trends in telecommunication services development is independent of the district where the respondent’s firm is located
(assumption: H0 can not be rejected)

It was not possible to confirm or refute this assumption. The pair of qualitative statistic figure values was not suitable for the application of a chi-square test of independence. The clause (3) was not fulfilled.

9. Conclusion

The marketing survey of the utilization of postal and telecommunication services in a regional entrepreneurial environment, on which this paper is based, was conducted in order to obtain relevant information about existing postal and telecommunication services and about customer satisfaction with these services. The results of the marketing survey were double classified and then the chi-square test of independence was applied to the results. This methodology can also be used for other questionnaire surveys focused on other areas such as the utilization of postal and telecommunication services. The results of regular surveys can be used as groundwork for suggestions and recommendations concerning improving the quality of the postal and telecommunication services, to identify regional disparities in the utilization of postal and telecommunication services and also for market segmentation. Some of the assumed regional disparities were confirmed. Other regional disparities highlighted the need for further investigation of factors influencing customer behavior.

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