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STRATEGIC CAPACITY PLANNING USING STOCK CONTROL MODEL

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The use of stock management model allows identifying the type of optimal strategy of production capacities. Namely, the optimal strategy of development for the infinite planning period is determined by two values: s and S. The development of capacities takes places only when the level of stocks decreases to the critical level s and the stocks are replenished up to the level S. In the case of a finite planning period, the optimal strategy of replenishing of stocks depends on the initial level of stocks and is determined by the set of pairs (s_τ, S_τ), i.e., the planning period of each year has its own critical level of stocks s_τ and the level S_τ, up to which the stocks are replenished.

Keywords: *capacity planning, stock control, stochastic demand*

Introduction

The aim of strategy capacity planning is to provide methods of determining a general capacity level of capital-intensive resources, namely, industrial premises, equipment and the aggregate volume of workforce, which would best support the long-term competitive strategy of a company. The selected level of production capacity exerts an enormous influence on the ability of a company to respond to actions of competitors, the structure of its expenses, the policy of stock management and the necessity of organisation of efficient work of the managerial personnel and the entire staff. If the capacity is insufficient, a company could lose customers due to their slow servicing or allow competitors to enter its sales market. If the production (or servicing) capacity is excessive, it is highly likely that, in order to remain in operation and stimulate the demand for its products, a company would be forced either to lower the price of its products (or services), or to make only a partial use of labour resources, or to create excessive stocks of goods and materials, or to manufacture additional and less profitable products [1,2].

The strategic managerial decisions in business are made on the basis of the information which is either insufficient or uncertain or both. In respect of the degree of certainty, the information used when solving managerial tasks is usually subdivided into three main groups [3-8]:

- Deterministic information or conditions of full certainty, when the information required for decision-making is known in its entirety and is highly specific and certain;
- Conditions of risk, when possible values of factors and probabilities of these values are known;
- Conditions of uncertainty, when no certain information is available about external factors.

The methods and models of mathematical programming (linear, non-linear, integer and dynamic programming) are applied to facilitate decision-making under deterministic conditions. When making decisions under risk conditions, which is typical of finance management tasks, as there is a possibility of long-term observation of statistical characteristics of fluctuations on a stock exchange being the consequence and also the cause of fluctuations of other financial quantities, possible values of uncontrolled factors and probabilities of these values are known. In such cases, that is, under risk conditions, the methods of decision matrix, decision tree, Bayesian analysis, stock management models, simulation modelling and various stochastic programming models are widely applied for the making of managerial decisions.

Heuristic rules, simulation modelling, methods of fuzzy sets, assessment of subjective probability, etc. are used for decision-making under conditions of uncertainty. A substantial uncertainty of information in respect of external factors is typical of strategic decisions. When no information is available about external factors, it is expedient to make certain assumptions about possible values and their probabilities and then to apply the methods which are analogous to the methods applied under risk conditions. Incomplete certainty of information as typical of managerial tasks may be taken into consideration when modelling a real situation in a variety of ways: introduction into the formulation of a task of probabilities of complying with individual resource constraints (considering that individual constraints are independent, because resources are not interchangeable), setting the probability of complying with the entire set of constraints (if resources may be interchangeable to some extent), formulation of the objective function in the form of mathematical expectation of a certain function or introduction into the objective function of various penalty coefficients for default on constraints, etc., formulation of different scenarios of stochastic and parametric programming tasks [4-7, 9, 10].

The system of capacity and production planning is of special importance when seeking to improve the function of a company's production planning, which aims at planning and monitoring of production process, which is planning and monitoring of resources, workforce, and equipment. A successful system of a company's activity planning enables managers to efficiently control the flow of raw materials, to effectively utilise human resources and machinery, to co-ordinate activities inside the company with the actions of suppliers, and to maintain contact with consumers with respect to market requirements. External environment forces, such as new technologies, products, processes, and global competition, constantly dictate competition conditions on the market under which a company should operate. The main factors that determine the competitiveness of the enterprise are quality and cost. A successful capacity and production planning reduces stocks and increases labour productivity, reduces production time, and improves equipment and employees' utilisation, which, responding to the fluctuations of market requirements, cuts down the cost and improves the quality and elasticity of production.

Production planning in a company usually covers several stages. At the initial stage, a highly aggregated production plan as a part of a company's activity plan is prepared. General production targets are provided for by the company's activity plan, which relates and co-ordinates the activities of various offices, such as design, marketing, production, finance. This strategic plan is developed into the strategic plans of functional fields and the budgets and capacities of the offices. A production plan is a constituent part of this plan and provides for production volumes necessary to reach the company's general goals. Production planning is most closely linked to marketing planning.

At the next stage, the main calendar production plan assessing the products demand, resources as well as aggregate capacities is prepared. The main plan is further developed into a more detailed plan of materials and capacity requirements, which reflects the amount of capacities and materials necessary for the implementation of the general plan. The last stage is the implementation of the mentioned plans in the company's shops and sales offices. At each stage of production planning, a demand of a different aggregation level is forecasted, with a whole complex of methods and techniques employed for this purpose.

The notion of production planning encompasses the planning of the equipment, raw materials, and human resources. By planning we seek to ensure the implementation of production targets. Production process depends on what method of production organisation is going to be selected. The selection of a method, in its turn, should take into consideration the kind of product that is going to be manufactured, the time needed for raw materials sourcing and production of components, and the type and number of operations to be carried out. Finally, the goals set for a particular production area should be considered.

Production planning can be of several kinds ranging from long-term production planning, which is necessary for strategic planning, to the preparation of a short-term working timetable or a calendar plan, which includes a week or even a day [11, 12, 13].

Capacity planning helps to prepare more accurate calendar plans of resources procurement and co-ordinate available capacities with requirements more efficiently. For this purpose it is necessary to determine if there are free capacities as regards the targets of the main production plan and successive actions that seek to relate the capacities with demand for them. Insufficient capacities present an important problem, which has to be quickly solved. However, a company also has to face certain difficulties when its capacities are too large.

The initial data for the solution of tasks of long- and medium-term planning of development of corporate production capacities usually include results of demand forecasting, which allows to present the demand for products in the form of probability distribution having regard to the growth of uncertainty in proportion to the increasing remoteness of the planned year.

Formulation of the Model

Process of strategic capacity planning could be viewed as the stochastic model of stock management. Let us suppose that the demand of product in the year τ (τ = 1, 2 …) is described as a stochastic process Q_τ with independent increments. Let us refer to the increment $Q_{\tau+1} - Q_\tau = \omega_\tau$ as the "demand" in the year τ of the planned period. Where the value of production capacities at the beginning of the year τ is equal to M_τ, the difference $M_\tau - Q_\tau$ will be referred to as the "level of stocks" in the year τ.

As the growth of capacities develops in the year τ is used to meet the need for products of the year $\tau+1$, the value of delay of supply in respect of the present model of stock management is equal to $\lambda = 1$ (the batch ordered in the segment τ is supplied in the segment $\tau + \lambda$).

Let us introduce the following symbols reflecting different stages of the process of replenishing of stocks:

i_τ – the level of available stocks at the beginning of the segment τ until supply of the previous year; $i_\tau = M_{\tau-1} - Q_\tau$ or, this may be considered as the level of stocks at the end of the first year;

q_τ – the quantity of stocks ordered in the segment τ, or the growth of production capacities in the year τ;

h_τ – the level of stocks at the beginning of the segment τ available to meet the demand in the segment of τ, that is, the available amount of stocks i_τ plus the supplied amount of stocks ordered at the beginning of the segment τ, or $h_\tau = i_\tau + q_{\tau-1}$; hence, h_τ is the level of available stocks after supply of the previous year; let us consider that such are stocks at the beginning of the year τ; if $\omega_0 = Q_1$, then the level of stocks at the beginning of the year τ is expressed by means of the demand $\omega_{\tau-1}$ in the following manner: $h_\tau = M_\tau - \omega_{\tau-1}$;

j_τ – the level of available stocks at the beginning of the segment τ plus the amount ordered until the beginning of τ and by the taking of the next decision on replenishing of the stocks; that is, $j_\tau = i_\tau + q_{\tau-1}$; we can see that the numerical values of h_τ and j_τ are equal, however j_τ stands for the value of stocks expected or planned by the taking of the decision, whereas h_τ stands for the actual value following the replenishment; the numerical values of the variables j_τ and h_τ are in this care identical, because the model does not identify as separate (there is no such necessity) the ordering in the year τ (taking of a decision regarding q_τ) and the delivery of the stocks ordered in the year τ, we can regard both as taking place simultaneously, from 31 December of the year $(\tau - 1)$ to 1 January of the year τ;

y_τ – the planned level of stocks at the beginning of the year τ plus the amount ordered in the segment τ, however without taking into account the demand of the current year, that is, $y_\tau = h_\tau + q_\tau$ or $y_\tau = M_\tau - Q_\tau + q_\tau$.

The amount of stocks available to meet the demand in the segment $\tau + 1$ will be equal to

$$h_{\tau+1} = y_\tau - \omega_\tau. \quad (1)$$

Therefore, in order to meet the demand of the year τ $\omega_\tau = Q_{\tau+1} - Q_\tau$ and by the taking of a decision regarding the next period $\tau+1$, we obtain the level of available stocks h_τ. Following a decision concerning the value of growth of capacities q_τ,, the level of stocks will be equal to y_τ, and to meet the demand of the next year we will have $h_{\tau+1} = y_\tau - \omega_\tau$ according to (1).

In the model of stock management, the objective function takes the form of the discounted aggregate amount of costs expected in each period. Let $c_\tau(q)$ be the price of the amount of stock q ordered in the segment τ. In the task of capacity development, $c_\tau(q)$ stands for the costs of development of new production capacities of the value q. Further, $H_\tau(h, \omega)$ is the costs of maintenance of capacities and losses in the segment τ in the case when the stocks h are available to satisfy the demand ω. Let us suppose that the costs $H_\tau(h, \omega)$ depend only on the difference between the values h and ω, that is

$$H_\tau(h, \omega) = \begin{cases} R(h-\omega), h-\omega \geq 0, \\ N(\omega - h), h - \omega < 0. \end{cases} \quad (2)$$

An important particular instance of (2) is the case when both functions in the right half are linear ones, that is, $R(h\text{-}\omega) = r\ (h\text{-}\omega)$, where r is the coefficient of the costs per unit of spare capacities, and $N(h\text{-}\omega) = k\ (h\text{-}\omega)$, where k is the coefficient of the losses due to the lack of the unit of production capacities.

Then the expected costs of stock maintenance and losses in the segment $\tau+1$, with $y_\tau = y$, equal to:

$$L_\tau(y) = \begin{cases} \sum_{\theta=0}^{y} r(y-\theta)p(\theta) + \sum_{\theta>y} k(\theta - y)p(\theta), y \geq 0, \\ \sum_{\theta=0}^{\infty} k(\theta - y)p(\theta), y < 0, \end{cases} \quad (3)$$

where $\theta = \varpi_\tau + \varpi_{\tau+1}$, a $p(\theta)$ is distribution of this amount.

The task of minimisation consists of finding the optimal volumes of the growth of capacities $q_\tau(j_\tau)$ for each segment τ and for each possible value of j_τ (or, which is the same, determination of the optimal values $y_\tau(j_\tau) \geq j_\tau$). Supposing that the dependence of y_τ on j_τ in implicit form is included below, we will get the following expression for the expected value of costs for the selected strategy y_τ, $\tau = 1, \ldots, T$ and the initial level of stocks j_1

$$\mathrm{E}\left\{\sum_{\tau=1}^{T} \beta^{\tau}[c_\tau(y - j_\tau) + L\tau(y_\tau)]\right\}. \tag{4}$$

Such an averaging is made with regard to all possible sequences of demand values, and the value β is the coefficient of discounting during the given period.

The recursion equation of dynamic programming, which helps to obtain an optimal strategy minimising the expression (4), has the form

$$f_\tau(j) = \min\,[c_\tau(y-j) + L_\tau(y) + \beta \sum_{\omega=0}^{\infty} f_{\tau+1}(y-\omega)P_\tau(\omega)] \tag{5}$$

with $\tau = 1, 2, \ldots, T$, $f_{\tau+1}(j) = 0$.

The functions $f_\tau(j)$ are calculated in an ordinary way for each possible value of j, starting with $\tau = T, T-1, \ldots, 1$. In practice, the range of variation of ω is limited by such a finite value at which the function of demand distribution is close to 1. Similarly, the interval of variation of the value of j is imposed a restriction determined by the nature of a real-life situation.

The above-mentioned formulation of the task of capacity development in the form of a stock management task allows making use of results of the theory and draws certain general conclusions regarding the optimal strategy of replenishing of stocks in the cases of the finite and infinite planning periods [7].

Case of the Finite Planning Period

Let us introduce the following assumptions in respect of the function of costs making up the objective function of the task:

$$C_\tau(q) = \begin{cases} 0, q = 0 \\ k_\tau + c_\tau q, q > 0, k_\tau \geq \beta k_{\tau+1}, \end{cases} \tag{6}$$

$L_\tau(y)$ is convex and

$$(c_\tau - \beta c_{\tau+1})y + L_\tau(y) \to \infty \text{, if } |y| \to \infty, \tag{7}$$

where $k_\tau \geq 0$, $c_\tau \geq 0$, $c_{\tau+1} \equiv 0$.

Where $L_\tau(y)$ is determined by the expression (3), it can be easily noticed to be convex and $L_\tau(y) \to \infty$ if $|y| \to \infty$. As unit costs of development of new capacities for various years of the planning period differ only by the coefficient of discounting, i.e., $c_\tau = c$, then $(c_\tau - \beta c_{\tau+1}) = (1-\beta)c$ and the condition (7) is fully met.

In these assumptions, a model described by the recursion equation (5) has an optimal (s_τ, S_τ) - strategy, or a strategy such as for any period

$$\begin{aligned} &y_\tau(j) = j \text{ and} \quad q_\tau(j) = 0 \quad \text{if } j \geq s_\tau, \\ &y_\tau(j) = S_\tau \text{ and} \quad q_\tau(j) = S_\tau - j \quad \text{if } j < s_\tau. \end{aligned} \tag{8}$$

In such a way, if the level of stocks exceeds some critical level S_τ, replenishing of stocks will not take place. However, if the level of stocks decreases down to the 'critical' level s_τ, a decision is made on the replenishing of stocks, and the stocks will be replenished up to the level S_τ. Consequently, the optimal strategy for any segment τ is completely described by two parameters (s_τ, S_τ) and does not depend on the level of stocks in the initial segment of the planning period.

In the case when the optimal strategy of replenishing stocks has the (s_τ, S_τ) -structure, the optimisation process as described by the recursion equation (5) is considerably simplified. The algorithm of determination of an optimal strategy in the segment τ has the following form:

Step 1. Calculate

$$g_\tau(y) \equiv c_\tau y + L_\tau(y) + \beta \sum_{\omega=0}^{\infty} f_{\tau+1}(y - \omega) p_\tau(\omega) \tag{9}$$

for all values of y which are of interest and suppose that $S_\tau \geq 0$ meets the condition $g_\tau(y) = g_\tau(S_\tau)$.

Step 2. Take as S_τ the smallest of the numbers meeting the condition

$$g_\tau(S_\tau) \leq g_\tau(S_\tau) + K_\tau$$

(S_τ may have negative values).

Step 3. Calculate $f_\tau(j)$ according to the formula

$$g_\tau(S_\tau) + K_\tau - c_\tau j,\ j < S_\tau$$

$$L_\tau(y) + \beta \sum_{\omega=0}^{\infty} f_{\tau+1}(j - \omega) p_\tau(\omega),\ \ j \geq S_\tau. \tag{10}$$

In such a way, for each period τ the algorithm consists of a single operation of finding the minimum at Step 1, which replaces performance of an operation of minimisation for each j using the recursion equation (5), which describes a general model of stock management. The sequence of calculations according to the mentioned algorithm begins with τ=T, where $f_{T+1} \equiv 0$. It should be noted that in this case, there were no limitations imposed on distribution of demand $p_\tau(\omega)$.

Case of the Infinite Planning Period

In the process of planning of capacity development, each decision adopted is a part of the never-ending chain of actions. A choice made earlier influences a current one; the decisions made at the present moment influence the future, etc. Therefore, it is natural to attempt to consider the development strategies in which the duration of the planning period is infinite.

For a task with the infinite planning period to yield specific solutions, it is necessary to make some presumptions concerning a stationary state. In the simplest case, it is presumed that the general conditions of a system's functioning remain unchanged with time; in particular, the demand for products is described by a stationary random process. Although in real life the stationary state is rarely maintained during the periods of a long duration, when making decisions at the beginning of the planning period an approach based on the use of a stationary model with the infinite planning period is an efficient and relatively simple way of obtaining optimal planning solutions.

Let us make the same presumptions in respect of the function of costs as in the case of the finite planning period, but let us also consider that the economic parameters of the model and distribution of demand are stationary during the infinite planning period. In particular, let us suppose that $p_\tau(\omega) = p(\omega)$,

$$C(q) = \begin{cases} 0, q = 0 \\ K + cq, q > 0, K \geq 0, c \geq 0, \end{cases} \tag{11}$$

L(y) is convex function and

$$C(1-\beta)y + L(y) \to \infty \text{ if } |y| \to \infty. \tag{12}$$

These assumptions make it possible to use the theorem of optimal stationary (s, S)-strategy. Where a task contains a model which is characterised by the recursion equation (5) and in which economic parameters meet the conditions of (11), (12), there exists an optimal (s, S)-strategy for which s_τ=S and S_τ=S at any value of τ during the infinite planning period.

A generalisation of the recursion equation (5) in respect of the infinite planning period can be presented in the following way:

$$f(j) = \min[c(y-j) + L(y) + \beta \sum_{\omega=0}^{\infty} f(y-\omega)p(\omega)]. \qquad (13)$$

A complete algorithm of effective calculation of the optimal (s, S)-strategy for the infinite planning period is sufficiently complicated. However, the case of normal distribution of demand allows applying a similar method [7].

The use of stock management model allows identifying the type of optimal strategy of production capacities. Namely, the optimal strategy of development for the infinite planning period is determined by two values: *s* and *S*. The development of capacities takes places only when the level of stocks decreases to the critical level *s* and the stocks are replenished up to the level *S*. In the case of a finite planning period, the optimal strategy of replenishing of stocks depends on the initial level of stocks and is determined by the set of pairs (s_τ, S_τ), i.e., the planning period of each year has its own critical level of stocks s_τ and the level S_τ, up to which the stocks are replenished.

Conclusions

1. Formulation of a task of development of production capacities in the form of a task of optimal stock management allows making use of results of the theory of optimal stock management. The presented algorithms of solving a task in the cases of the finite and infinite period have been formulated in the terms of the task of capacity planning and could be used when making strategic plans of development of corporate production capacities.

2. The considered models of development of production capacities enable to analyse the prepared development plans with a view to determining the impact of some types of costs on the optimal solution and also comparing the results obtained when making use of various models to solve a task of production capacity development.

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