MODELLING DISTURBANCES IN SYSTEM TRACK – RAIL VEHICLE

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All real systems work in disturbances conditions. "Disturbances" term belongs to special category of entry values which are not known before and do not make subject to control, it means they are uncontrolled entry signals. Disturbances are an important element of systems regulation, because usually lead to unexpected effects in functioning of steered systems.

An alternative approach has been proposed in this article in disturbing signals description, related on theory of wave disturbances, which allow to describe wide disturbances scale, occurred in suspensions of active vehicles.

Disturbances models based on new wave interpretation can describe wide class of real, indefinite disturbances which occur in suspensions of active vehicles.

Keywords: *suspensions, active vehicles suspensions, wavy interpretation of disturbances, Laplace transform, random disturbances*

1. Introduction

The basic task of suspensions is guarantee expected fluidity of vehicles move by isolation body from wheels vibration caused by road unevenness. In that way bodies decide about comfort of driving for driver and passengers and also reduce dynamic loads of vehicle sets increasing their durability.

In steering theory there is known a commonly fact that all real systems work in disturbances conditions. "Disturbances" term belongs to special category of entry values which are not known before and do not make subject to control, it means they are uncontrolled entry signals.

Disturbances are an important element of systems regulation, because usually lead to unexpected effects in functioning of steered systems. The typical disturbances examples which occur in vehicle driving are: gusts of wind and other aerodynamic forces which influence on vehicle, friction and neutrals in suspension system, unevenness of road pavement, moving centre of gravity and another undefined effects of displacements in mechanical sets of vehicles.

In science researches, run from the end of 70-ties in the last century, related to active bodies in vehicles deterministic or random interpretation of disturbing signals is accepted. If first of mentioned approach presents too simple meaning of disturbances nature then second complicate too much their description.

Statistics properties of disturbances such as average value, variation, spectral density and other supported on average are long term values. Meanwhile driver driving vehicle on chosen part of road by changing wind follow the current behave.

Effective steering in case of disturbances requires current information about their functioning. Statistic information supported on long-term observations doesn't meet current information demands and becomes in steering in real time totally useless.

An alternative approach proposed in disturbing signals description, related on theory of wave disturbances, which allow to describe wide disturbances scale, occurred in suspensions of active vehicles is presented in the lecture.

2. Wavy Interpretation of Disturbances in active vehicles suspensions

Disturbances which characterize wavy structure can be mathematically described by half-deterministic analytical dependences in the following way [1]:

$$w(t) = W\left\lfloor f_1(t), f_2(t), ..., f_M(t); .c_1 .., c_L \right\rfloor, \tag{1}$$

where $f_i\,(t), i = 1,2,..,M$ (M-finite value) – known time functions,

$c_k,\; k=1,...,L$ – unknown parameters, which can stepped change in a partly solid way their meanings. Mathematical models (1) type will be called as wave interpretation of disturbances $w(t)$.

Linear description (1) can be considered as interpretation $w(t)$ in functional area, where function set $\{f_i(t),...,f_M(t)\}$ is a base, and c_i – is partly solid weight factor. In other words disturbances $w(t)$ present linear weight combination of known basic functions $f_i(t$ and unknown weight factors c_i, which randomly in periodically-solid way change their meanings [3].

An example illustration of equation (1) is presented in Figure 1.

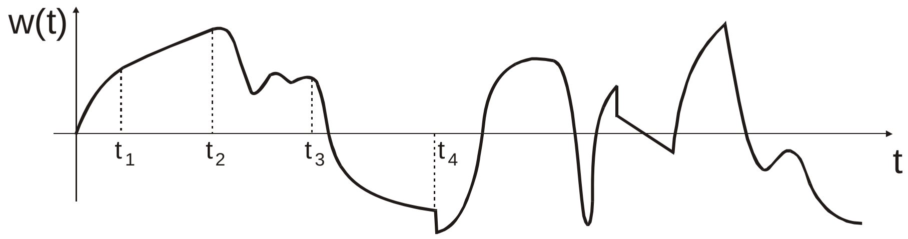

Figure 1. Wavy structure disturbances

Proposed interpretation of disturbances *w(t)* definitely differs from their traditional random treatment. Especially information range contented in equation (1) is qualitatively different from information contented in traditional statistic ideas, such as average value, variation, spectral density and others. Senses of factors ci in equation (1) are totally unknown (with the exception that they change in periodically-solid way). Wavy interpretation doesn't operate traditional statistical features and doesn't describe them.

So interpretation (1) fills in "informative vacuum" in description of disturbances which occur in suspensions of active vehicles.

Especially equation (1) allows to describing wide range of possible wavy forms, which contain any unknown realization of disturbances *w(t)* in moment *t.* Besides, each separate realization *w(t)* in wavy interpretation can possess "its own" set of statistic features, thanks to which it can be used also to describe non-ergodic disturbing functions *w(t)* especially when each realization *w(t)* is a constant random value.

3. Models of wavy disturbances

Pointing out system of basic functions $\{f_i(t)\}$, is a first step in using wavy interpretation of disturbances as an instrument of regulation system. It can be done on the basis of visual and mathematical analysis of experimental recordings w(t) or by analysis of dynamic characteristics of physical process which generate w(t).

In the second step proper "state model" for equation (1) should be specified. The model is a differential equation met by function (1). In other words equation (1) should be considered as known "general solution" of looked for differential equation. Let's suppose that each chosen function $f_i(t)$ has Laplace transform $f_i(s)$, in the following way:

$$f_i(s) = \frac{P_{m_i}(s)}{Q_{n_i}(s)}, \tag{2}$$

where: $P_{m_i}(s), Q_{n_i}(s)$ - polynomials adequately *m* and *n* grade $m_i \le n_i$. If c_i temporarily suppose as permanent values then the transformation of Laplace's equation (2) is the following:

$$w(s) = c_1 f_1(s) + c_2 f_2(s) + \ldots + c_M f_M(s) = \sum_1^M c_i \frac{P_{m_i}(s)}{Q_{n_i}(s)}, \tag{3}$$

finally:

$$w(s) = \frac{P(s)}{Q(s)}, \tag{4}$$

where polynomial of numerator *P(s)* includes factors c_i, and polynomial of denominator *Q(s)* is the smallest general denominator in set of denominators polynomials $\{Q_{n_1}(s), Q_{n_2}(s), \ldots, Q_{n_M}(s)\}$ of equation (3). Such interpretation guarantee minimal size of final model of state *w(t),* which has the essentials of meaning from point of view of costs and apparatus complexity. So let's suppose that denominator polynomial *Q(s)* in equation (4) is following:

$$Q(s) = s^\rho + q_\rho s^{\rho-1} + q_{\rho-1} s^{\rho-2} + \ldots + q_2 s + q_1, \tag{5}$$

where $\rho \le \sum_1^M n_i$. From equation (4) comes that disturbances *w(t)* can be treated as „initial variable" of fictional linear dynamical system of operational transmission:

$$G(s) = \frac{1}{Q(s)}, \tag{6}$$

by initial conditions $\{w(0), \dot{w}(0), \ddot{w}(0), ...\}$. then disturbances (1) with taking dependences under consideration (3)-(5), meet the following homogeneous linear differential equation with constant parameters:

$$\frac{d^{\rho} w}{dt^{\rho}} + q_{\rho} \frac{d^{\rho-1} w}{dt^{\rho-1}} + q_{\rho-1} \frac{d^{\rho-2} w}{dt^{\rho-2}} + ... + q_2 \frac{dw}{dt} + q_1 w = 0 , \tag{7}$$

where factors q_i, $i=1,2,...,\rho$, are known, because don't depend on c_i and they are determined by system of basic functions $\{f_i(t)\}$, which are taken for given.

To take into consideration stepped factors changes ci in equation (7) we add to it external forced function ω(t), which is a progression of unknown, randomly appear pulse functions with randomly intensity (single, double, triple, etc. kind of Dirac's function).

So model of state $w(t)$ finally takes the form:

$$\frac{d^{\rho} w}{dt^{\rho}} + q_{\rho} \frac{d^{\rho-1} w}{dt^{\rho-1}} + q_{\rho-1} \frac{d^{\rho-2} w}{dt^{\rho-2}} + ... + q_2 \frac{dw}{dt} + q_1 w = \omega(t) . \tag{8}$$

It is important that pulse forced function ω(t), is unknown and introduced to model of state (8) just only in symbolic way with the purpose to mathematical description of steps ci in equation (1). Beside moments of appearance adjacent pulse functions are separated by minimal positive range $\mu > 0$.

So if base functions fi(t) in equation (1) has Laplace transform in kind (2) then with the purpose of finding state model for equation (1) it is necessary to define factors {q1, q2,...,qρ} from equations (1) and (5), and next use general state model (8).

Differential equation of the order of ρ (8) can be presented in the form of system of differential equations of the order of first. For example equation (8) can be written down equivalently in form of "totally observed" known canonic form:

$$\begin{aligned} & w = z_1 , \\ & \dot{z}_1 = z_2 + \sigma_1(t), \\ & \dot{z}_2 = z_3 + \sigma_2(t), \\ & \cdot \\ & \cdot \\ & \cdot \\ & \dot{z}_{\rho-1} = z_{\rho} + \sigma_{\rho-1}(t), \\ & z_{\rho} = -q_1 z_1 - q_2 z_2 - ... - q_{\rho} z_{\rho} + \sigma_{\rho}(t) \end{aligned} \tag{9}$$

where symbolic activity $\omega(t)$ in equation (8) is replaced in equation (9) by functions $\sigma_i(t)$, $i=1,2,...,\rho$, which are progressions of unknown, random Dirac's functions, and point means an operator d/dt.

In general case it should be expected that differential equation (8) or system of differential equations (9) will contain variable factors q_i and/or nonlinear elements below $w, dw/dt$ etc. So searched "state model" for disturbances $w(t)$, which have wave structure, can be presented in the form of one differential equation of the high order:

$$\frac{d^{\rho} w}{dt^{\rho}} + f(w, \frac{dw}{dt} ,..., \frac{d^{\rho-1} w}{dt^{\rho-1}} ,t) = \omega(t) , \tag{10}$$

or in form of system of equations of the first order:

$$\begin{aligned} & \omega = W(z,t), \\ & \dot{z} = Z(z,t) + \sigma(t); \end{aligned} \qquad z = (z_1,\ z_2,\ ...z_{\rho}). \tag{11}$$

Let's consider that model (11) has the advantage over model (10) because it uses methods of state variables. If $w(t)$ is a multi-dimension disturbance which contains p components $w = (w_1,w_2,...,w_p)$, then state model should be determined for each independent component $w_i(t)$.

4. Examples of State Models for Real Disturbances in Suspensions of Active Vehicles

State models expressed by equations (10) and (11) for real disturbances, occurred at least in short time intervals in suspensions of active vehicles, can be determined on base recordings of experimental oscilogram presented in Figure 2.

Example 1. In case shown in Figure 2a disturbances meet the following differential equation:

$$\frac{dw}{dt} = 0 \tag{12}$$

c in that case is a constant.

To take into consideration unknown random, stepped changes *c*, it should added to equation (12) the element σ(t), which composed of unknown progression of Dirac's function. State model of disturbances *w(t),* presented in Figure 2a has finally the following form:

$$\frac{dw}{dt} = \sigma(t). \tag{13}$$

Example 2. Disturbances presented in Figure 2b described by equation $w(t) = c_1 + c_2t$, which meet the following differential equation of second order:

$$\frac{d^2w}{dt^2} = \omega(t), \tag{14}$$

where $\omega(t)$ means unknown progression of random single and double impulse of randomly intensity. Equivalent model in system form (9) is the following:

$$w(t) = (1,0)\binom{z_1}{z_2} \tag{15}$$

$$\dot{z}_1 = z_2 + \sigma_1(t), \qquad \dot{z}_2 = 0 + \sigma_2(t), \tag{16}$$

Example 3. Disturbances *w(t),* presented in Figure 2c. of Laplace transform:

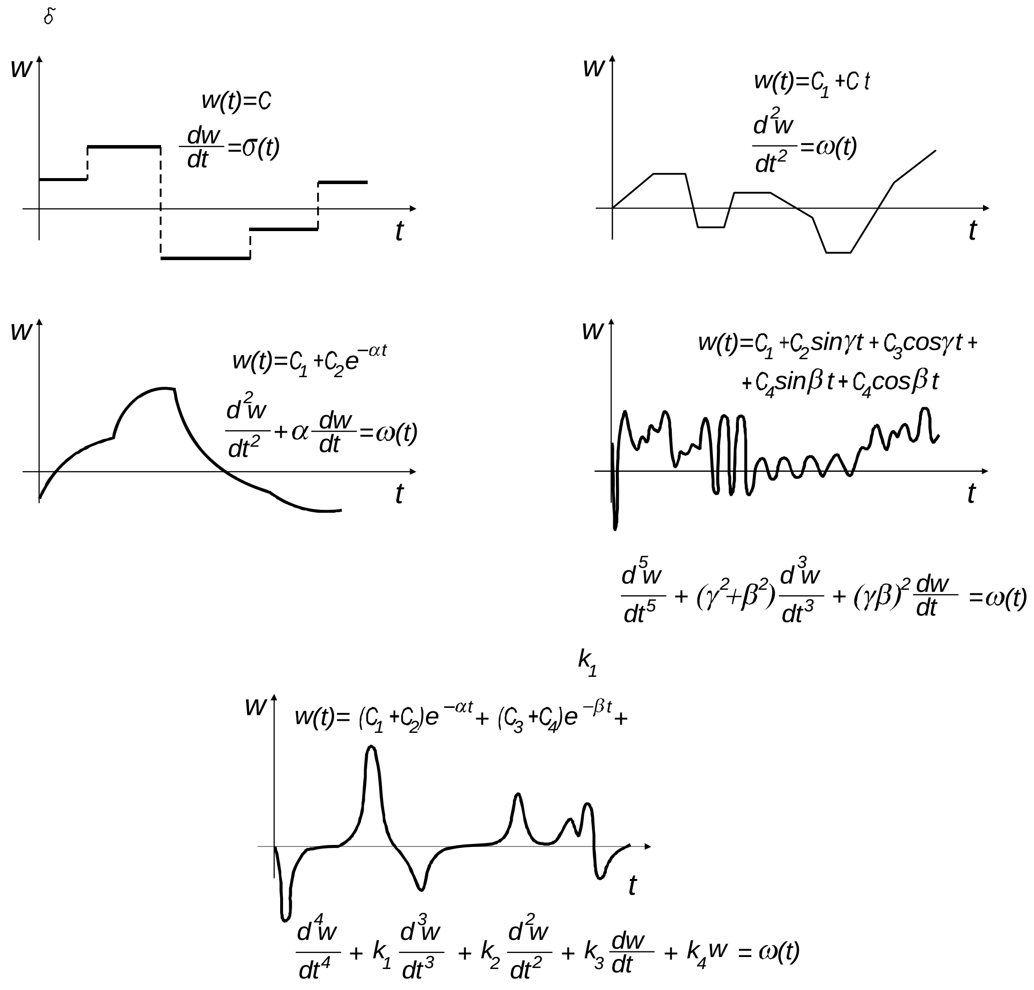

Figure 2. Disturbances of wavy structure in suspensions of active vehicles

$$w(s) = c_1\left(\frac{1}{s}\right) + c_2\left(\frac{1}{s+\alpha}\right). \qquad (17)$$

can be presented in equation form (4) as follows:

$$w(s) = \frac{\left[c_1(s+\alpha) + c_2 s\right]}{s(s+\alpha)}. \qquad (18)$$

In that case $Q(s) = s^2 + \alpha s$, and from equations (5) and (8) it follows that $w(t)$ meet the following differential equation of second order:

$$\frac{d^2 w}{dt^2} + \alpha\frac{dw}{dt} = \omega(t). \qquad (19)$$

Equivalent state model has the form:

$$w = (1,0)\begin{pmatrix} z_1 \\ z_2 \end{pmatrix} \qquad (20)$$

$$\dot{z}_1 = z_2 + \sigma_1(t), \qquad \dot{z}_2 = \alpha z_2 + \sigma_2(t), \qquad (21)$$

Example 4. Disturbances $w(t)$ in kind of small waves presented in Figure 2d has the following Laplace transform:

$$w(s) = \frac{P(s)}{s\left(s^2+\gamma^2\right)\left(s^2+\beta^2\right)}. \qquad (22)$$

So according to equations (5)-(8), $w(t)$ meets the following differential equation of fifth order:

$$\frac{d^5 w}{dt^5} + \left(\gamma^2+\beta^2\right)\frac{d^3 w}{dt^3} + (\gamma\beta)^2\frac{dw}{dt} = \omega(t). \qquad (23)$$

Equivalent presenting the equation (23) has a form:

$$w(t) = (1,0,0,0,0)\begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \end{pmatrix} \qquad (24)$$

$$\begin{aligned} &\dot{z}_1 = z_2 + \sigma_1(t), \qquad \dot{z}_2 = z_3 + \sigma_2(t), \\ &\dot{z}_3 = z_4 + \sigma_3(t), \qquad \dot{z}_4 = z_5 + \sigma_4(t), \\ &\dot{z}_5 = -(\gamma\beta)^2 z_2 - \left(\gamma^2+\beta^2\right) z_4 + \sigma_5(t), \end{aligned} \qquad (25)$$

Example 5. Impulse type disturbances presented in Figure 2e of Laplace transform :

$$w(s) = \frac{P(s)}{\left(s^4 + k_1 s^3 + k_2 s^2 + k_3 s + k_4\right)} \qquad (26)$$

where k_i – are functions of two known parameters α i β, meet the following differential equation:

$$\frac{d^4 w}{dt^4} + k_1\frac{d^3 w}{dt^3} + k_2\frac{d^2 w}{dt^2} + k_3\frac{dw}{dt} + k_4 w = \omega(t) \qquad (27)$$

or

$$w(t) = (1,0,0,0)\begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{pmatrix} \tag{28}$$

$$\begin{aligned} \dot{z}_1 &= z_2 + \sigma_1(t), \\ \dot{z}_2 &= z_3 + \sigma_2(t), \\ \dot{z}_3 &= z_4 + \sigma_3(t), \\ \dot{z}_4 &= -k_4 z_1 - k_3 z_2 - k_2 z_3 - k_1 z_4 + \sigma_4(t) \end{aligned} \tag{29}$$

where k_i are known factors, depended on parameters α i β.

Conclusions

Achievements of microprocessor technology in the space of ten years gave another dimension of digital steering systems. Contemporary steering theory cannot omit problematic disturbances which occur in real complex multidimensional systems.

Counteracting to disturbances is old and important task in designing closed steering systems. Traditional methods of regulation base on deterministic interpretation of disturbances or presenting them by models of stochastic processes. First approach presents too simplified understanding of real phenomenon nature, while second reflects excessive pessimism and provides to complicated description of real disturbances.

This lecture presents alternative methods of indefinite disturbances description, more exact than deterministic approach and not so complicated as models of stochastic processes. Disturbances models based on new wave interpretation (fig.3) can describe wide class of real, indefinite disturbances which occur in suspensions of active vehicles.

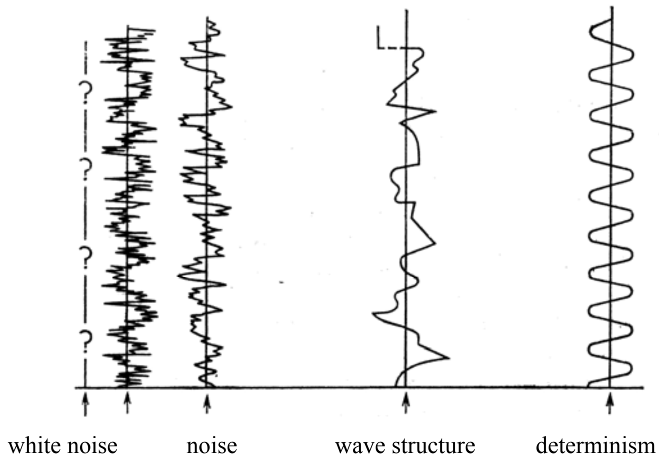

Figure 3. Place of wavy structure in spectrum of infinity

Using a wave method of disturbances modelling and contemporary methods of state variables can be built new effective class of closed steering systems called as regulators which adapt to disturbances; those disturbances can:

a) absorb external disturbances in automatic way,
b) minimize influence of external disturbances in real time,
c) use in optimal way external disturbances for controlling system of vehicle suspension.

Using active systems of vibro-insulation controlled by regulators which adapt to disturbances allow not only to monitoring vehicles dynamics in real time but also improve driving comfort.

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