

FINDING OPTIMAL MULTI-PERIOD PROJECT PORTFOLIO INVESTMENT STRATEGY UNDER UNCERTAINTY

*Konstantin N. Nechval*¹, *Nicholas A. Nechval*², *Gundars Berzins*², *Maris Purgailis*²,
*Juris Krasts*², *Uldis Rozevskis*², *Vladimir F. Strelchonok*³, *Natalie Zolova*²

¹ *Transport and Telecommunication Institute
Department of Mathematical Methods and Modelling
Lomonosova Str. 1, LV-1019, Riga, Latvia
E-mail: konstan@tsi.lv*

² *University of Latvia
Mathematical Statistics Department
Raina Blvd. 19, LV-1050, Riga, Latvia
E-mail: nechval@junik.lv*

³ *Baltic International Academy
Informatics Department
Lomonosova Str. 4, LV-1019, Riga, Latvia
E-mail: str@apollo.lv*

Project portfolio selection is a crucial decision in many organizations, which must make informed decisions on investment, where the appropriate distribution of investment is complex, due to varying levels of risk, resource requirements, and interaction among the proposed projects. In this paper, we present a mathematical model of investment for a set of projects under conditions of uncertainty. The model addresses the problem of an investor with access to a limited pool of capital, which makes decisions on investments. The problem is to decide how much to invest in each project so as to maximize the total expected return by the end of the horizon in relation to a given utility function. We discuss optimal investment decisions for the cases where the return from investment is a random variable. The single-period and multi-period cases of investment decisions are considered. The paper presents closed form solutions for commonly adopted utility functions.

Keywords: project, portfolio, multi-period investment, optimal control

1. Introduction

Project portfolio investment is the periodic activity involved in investing a portfolio of projects, that meets an organization's stated objectives without exceeding available resources or violating other constraints. Some of the issues that have to be addressed in this process are the organization's objectives and priorities, financial benefits, intangible benefits, availability of resources, and risk level of the project portfolio [1].

Difficulties associated with project portfolio investment result from several factors: (i) there are multiple and often-conflicting objectives, (ii) some of the objectives may be qualitative, (iii) uncertainty and risk can affect projects, (iv) the project portfolio may need to be balanced in terms of important factors, such as risk and time to completion, (v) some projects may be interdependent, and (vi) the number of feasible portfolios is often enormous.

In addition to these difficulties, due to resource limitations there are usually constraints such as finance, work force, and facilities or equipment, to be considered. As some researchers have noted [2], the major reason why some projects are selected but not completed is that resource limitations are not always formally included in the project selection process. In cases where resource limitations are at fault for a failed project, a selection model that incorporated resource limitations could have aided the decision maker in avoiding such mistakes [1]. Portfolio selection becomes more complex when resource availability and consumption are not uniform over time.

There are many different techniques that can be used to estimate, evaluate, and choose project portfolios [3-4]. Some of these techniques are not widely used because they address only some of the above issues, they are too complex and require too much input data, they may be too difficult for decision makers to understand and use, or they may not be used in the form of an organized process [5]. Among all of the techniques that are available, optimisation techniques are the most fundamental quantitative tool for project portfolio selection [6] and address most of the important issues. However, they have largely failed to gain user acceptance [7], and few modelling approaches, from a variety of optimisation approaches that have been developed, are being utilized as aids to decision making in this area [8]. According to Hess [9] "management science has failed altogether to implement project selection models; we have proposed more and more sophistication with less and less practical impact". One of the major reasons for the failure of traditional optimisation techniques is that they prescribe solutions to portfolio selection problems without allowing for the judgment, experience and insight of the decision-maker [7].

A literature review we conducted in this field [10] clearly showed that, although there are many different methods for project evaluation and portfolio selection that have their own advantages, no single technique addresses all of the issues that should be considered in project portfolio selection. Among published methodologies for project portfolio selection, there has been little progress towards achieving an integrated framework that: (a) simultaneously considers all the different criteria in determining the most suitable project portfolio, (b) takes advantage of the best characteristics of existing methods by decomposing the process into a flexible and logical series of activities and applying the most appropriate technique(s) at each stage.

Well-known pragmatic difficulties that make project selection challenging include the following: (1) Success uncertainty (unknown rewards): Whether (or how well) a project will succeed, both technically and in the market, may be uncertain. (2) Changing opportunities (randomly arriving opportunities): The opportunity of funding a project may be uncertain. New projects are continually being proposed throughout the year, stimulated by changes in technology and market opportunities that might help the company to achieve its goal. (3) Need for quick funding decisions (on-line decision): The organizations proposing projects lack a comprehensive view of all projects activities company-wide. It is essential for top management to give the organization prompt accept/revise/reject decisions, since such feedback helps to coordinate their activities. (4) Combinatorial complexity: The costs and benefits from different projects may interact.

2. Problem Statement

The problem we will examine can be formulated as follows. Let w_0 denote the initial wealth (measured in monetary units) of the investor and assume that there are m projects (opportunities) for investments which belong to a set of m risk categories, with corresponding random rates of return r_1, r_2, \dots, r_m , among which the investor can allocate his wealth. The investor can also invest in a project of a riskless category offering a sure rate of return s . If we denote by u_1, \dots, u_m the corresponding amounts to invest in the projects belonging to the set of m risk categories and by $(w_0 - u_1 - \dots - u_m)$ the amount of investment in the project of the riskless category, the final wealth is given by

$$w_1 = s(w_0 - u_1 - \dots - u_m) + \sum_{j=1}^m r_j u_j, \quad (1)$$

or equivalently

$$w_1 = s w_0 + \sum_{j=1}^m (r_j - s) u_j. \quad (2)$$

The objective is to maximize over u_1, \dots, u_m ,

$$E\{g(w_1)\}, \quad (3)$$

where g is a known utility function for the investor, w_1 is a random variable. We assume that the given expected value is well defined and finite for all w_0, u_i , and that g is concave and twice continuously differentiable. We will not impose constraints on u_1, \dots, u_m . This is necessary in order to obtain the results in convenient form. A few additional assumptions will be made later.

If a given amount w_0 of the initial wealth is available for investment within N time periods, we are interested in determining decision rules on how much to invest in each project so as to maximize the total expected return by the end of the horizon in relation to a given utility function.

3. Investment Decisions for the Single-Period Case

Let us consider the preceding problem for every value of initial wealth and denote by $u_j^* \equiv u_j^*(w_0)$, $j=1, \dots, m$, the optimal amounts to invest in projects belonging to the set of m risk categories when the initial wealth is w_0 . We say that the investment portfolio $\{u_1^*(w_0), \dots, u_m^*(w_0)\}$ is partially separated if

$$u_j^*(w_0) = c_j h(w_0), \quad j=1, \dots, m, \quad (4)$$

where $c_j, j=1, \dots, m$, are fixed constants and $h(w_0)$ is a function of w_0 (which is the same for all j).

When partial separation holds, the ratios of amounts invested in projects belonging to the set of m risk categories are fixed and independent of the initial wealth; that is,

$$\frac{u_j^*(w_0)}{u_k^*(w_0)} = \frac{c_j}{c_k}, \quad \text{for } j, k \in \{1, \dots, m\}, \quad c_k \neq 0. \quad (5)$$

Actually, in the cases we will examine, when partial separation holds, the investment portfolio $\{u_1^*(w_0), \dots, u_m^*(w_0)\}$ will be shown to consist of affined (linear plus constant) functions of w_0 that have the form

$$u_j^*(w_0) = c_j[a + bsw_0], \quad j=1, \dots, m, \quad (6)$$

where a and b are constants characterizing the utility function g . In the special case where $a=0$ in (6), we say that the optimal investment portfolio is completely separated in the sense that the ratios of the amounts invested in projects belonging to both the set of m risk categories and the riskless category are fixed and independent of initial wealth. Here the following theorem holds.

Theorem 1. If the utility function satisfies

$$-\frac{g'(w_1)}{g''(w_1)} = a + bw_1, \quad \text{for all } w_1, \quad (7)$$

where g' and g'' denote the first and the second derivatives of g , respectively, and a and b are some scalars, then the optimal investment portfolio is given by (6). Furthermore, if $G(w_0)$ is the optimal value of the problem, i.e.,

$$G(w_0) = \max_{u_1, \dots, u_m} E\{g(w_1)\}, \quad (8)$$

then we have

$$-\frac{G'(w_0)}{G''(w_0)} = \frac{a}{s} + bw_0, \quad \text{for all } w_0. \quad (9)$$

Proof. Let us assume that an optimal investment portfolio exists and is of the form

$$u_j^*(w_0) = c_j(w_0)[a + bsw_0], \quad j=1, \dots, m, \quad (10)$$

where $c_j(w_0)$, $j=1, \dots, m$, are some differentiable functions. We will prove that $dc_j(w_0)/dw_0 = 0$ for all w_0 and hence the functions c_j must be constant. We have for every w_0 , by the optimality of $u_j^*(w_0)$, for $j=1, \dots, m$,

$$\frac{dE\{g(w_1)\}}{du_j} = E\left\{g' \left[sw_0 + \sum_{k=1}^m (r_k - s)c_k(w_0)(a + bsw_0) \right] (r_j - s) \right\} = 0, \quad j=1(1)m. \quad (11)$$

Differentiating the m equations in (11) with respect to w_0 yields

$$\begin{aligned} & E \left\{ \begin{bmatrix} (r_1 - s)^2 \cdots (r_1 - s)(r_m - s) \\ \cdots \quad \cdots \quad \cdots \quad \cdots \\ (r_m - s)(r_1 - s) \cdots (r_m - s)^2 \end{bmatrix} g''(w_1)(a + bsw_0) \right\} \begin{bmatrix} dc_1(w_0)/dw_0 \\ \vdots \\ dc_m(w_0)/dw_0 \end{bmatrix} \\ &= - \begin{bmatrix} E \left\{ g''(w_1)(r_1 - s) s \left[1 + \sum_{k=1}^m (r_k - s)c_k(w_0)b \right] \right\} \\ \vdots \\ E \left\{ g''(w_1)(r_m - s) s \left[1 + \sum_{k=1}^m (r_k - s)c_k(w_0)b \right] \right\} \end{bmatrix} = - \begin{bmatrix} E \left\{ g''(w_1)(r_1 - s) s \left[1 + \sum_{j=1}^m (r_j - s)c_j(w_0)b \right] \right\} \\ \vdots \\ E \left\{ g''(w_1)(r_m - s) s \left[1 + \sum_{j=1}^m (r_j - s)c_j(w_0)b \right] \right\} \end{bmatrix}. \quad (12) \end{aligned}$$

Using relation (7), we have

$$g''(w_1) = - \frac{g'(w_1)}{a + b \left[s w_0 + \sum_{j=1}^m (r_j - s) c_j (w_0) (a + b s w_0) \right]} = - \frac{g'(w_1)}{(a + b s w_0) \left[1 + \sum_{j=1}^m (r_j - s) c_j (w_0) b \right]}. \quad (13)$$

Substituting in (12) and using (11), we have that the right side of (12) is the zero vector. The matrix on the left in (12), except for degenerate cases, can be shown to be nonsingular. Assuming that it is indeed nonsingular, we obtain

$$\frac{dc_j(w_0)}{dw_0} = 0, \quad j = 1, \dots, m, \quad (14)$$

and $c_j(w_0) = c_j$, where c_j are some constants, thus proving (6).

We now turn our attention to proving relation (9). We have

$$G(w_0) = E\{g(w_1)\} = E\left\{g\left(s\left[1 + \sum_{j=1}^m (r_j - s) c_j b\right]w_0 + \sum_{j=1}^m (r_j - s) c_j a\right)\right\} \quad (15)$$

and hence

$$G'(w_0) = E\left\{g'(w_1) s \left[1 + \sum_{j=1}^m (r_j - s) c_j b\right]\right\}, \quad (16)$$

$$G''(w_0) = E\left\{g''(w_1) s^2 \left[1 + \sum_{j=1}^m (r_j - s) c_j b\right]^2\right\}. \quad (17)$$

The last relation after some calculation and using (13) yields

$$G''(w_0) = - \frac{E\left\{g'(w_1) s \left[1 + \sum_{j=1}^m (r_j - s) c_j b\right]\right\} s}{a + b s w_0}. \quad (18)$$

By combining (16) and (18), we obtain the desired result:

$$- \frac{G'(w_0)}{G''(w_0)} = \frac{a}{s} + b w_0. \quad (19)$$

This ends the proof. \square

It can be shown that the following utility functions satisfy (19):
exponential:

$$-e^{-w/a}, \quad \text{for } b=0, \quad (20)$$

logarithmic:

$$\ln(w+a), \quad \text{for } b=1, \quad (21)$$

power:

$$[1/(b-1)](a+bw)^{1-(1/b)}, \quad \text{otherwise.} \quad (22)$$

Naturally in our problem only concave utility functions from this class are admissible. Furthermore, if a utility function that is not defined over the whole real line is used, the problem should be formulated in a way that ensures that all possible values of the resulting final wealth are within the domain of definition of the utility function.

4. Investment Decisions for the Multi-Period Case

It is now easy to extend the one-period result of the preceding analysis to the multi-period case. We will assume that the current wealth can be used to invest amounts in projects at the beginning of each of N consecutive time periods. If we denote:

w_ν the wealth of the investor at the beginning of the ν th period,
 $u_{j(\nu)}$ the amount to invest in project belonging to the j th risk category at the beginning of the ν th period,
 $r_{j(\nu)}$ the rate of return of project which belongs to the j th risk category at the beginning of the ν th period,
 s_ν the rate of return of project which belongs to the riskless category at the beginning of the ν th period, then we have (in accordance with the single-period model) the system equation

$$w_{\nu+1} = s_\nu w_\nu + \sum_{j=1}^m (r_{j(\nu)} - s_\nu) u_{j(\nu)}, \quad \nu=0, 1, \dots, N-1. \quad (23)$$

We assume that the vectors $\mathbf{r}_\nu = (r_{1(\nu)}, \dots, r_{m(\nu)})$, $\nu=0, \dots, N-1$, are independent with given probability distributions that result in finite expected values throughout the following analysis.

The objective is to maximize $E\{g(w_N)\}$, the expected utility of the terminal wealth w_N , where we assume that g satisfies for all w

$$-\frac{g'(w)}{g''(w)} = a + bw. \quad (24)$$

Applying the dynamic programming algorithm [11] to this problem, we have

$$G_N(w_N) = g(w_N), \quad (25)$$

$$G_\nu(w_\nu) = \max_{u_{1(\nu)}, \dots, u_{m(\nu)}} E \left\{ G_{\nu+1} \left[s_\nu w_\nu + \sum_{j=1}^m (r_{j(\nu)} - s_\nu) u_{j(\nu)} \right] \right\}, \quad \nu=0, 1, \dots, N-1. \quad (26)$$

From the solution of the one-period problem we have that the optimal policy at the beginning of period $N-1$ is of the form

$$\mathbf{u}_{N-1}^*(w_{N-1}) = \mathbf{c}_{N-1} [a + bs_{N-1}w_{N-1}], \quad (27)$$

where \mathbf{c}_{N-1} is an appropriate m -dimensional vector, $\mathbf{u}_{N-1}^* = [u_{1(N-1)}^*, \dots, u_{m(N-1)}^*]'$. Furthermore, we have

$$-\frac{G'_{N-1}(w)}{G''_{N-1}(w)} = \frac{a}{s_{N-1}} + bw. \quad (28)$$

Hence, applying the result of this section in (26) for the next to the last period, we obtain the optimal policy

$$\mathbf{u}_{N-2}^*(w_{N-2}) = \mathbf{c}_{N-2} \left(\frac{a}{s_{N-1}} + bs_{N-2}w_{N-2} \right), \quad (29)$$

where \mathbf{c}_{N-2} is again an appropriate m -dimensional vector.

Proceedings similarly, we have for the ν th period

$$\mathbf{u}_\nu^*(w_\nu) = \mathbf{c}_\nu \left(\frac{a}{s_{N-1} \cdots s_{\nu+1}} + bs_\nu w_\nu \right), \quad (30)$$

where \mathbf{c}_ν , $\nu=0, 1, \dots, N-1$, are m -dimensional vectors that depend on the probability distributions of the rates of return $r_{j(\nu)}$ of projects belonging to the risk categories and are determined by optimisation of the expected value of the optimal cost-to-go functions G_ν . These functions satisfy

$$-\frac{G'_\nu(w)}{G''_\nu(w)} = \frac{a}{s_{N-1} \cdots s_\nu} + bw, \quad \nu=0, 1, \dots, N-1. \quad (31)$$

Thus one can see that the investor, when faced with the opportunity to reuse sequentially his wealth, uses a policy similar to that of the single-period case.

5. Multi-Period Project Portfolio Investment with Markowitz Mean-Variance Optimisation

To our knowledge, no analytical or efficient numerical method for finding the optimal multi-period portfolio policy for the constrained mean-variance formulation of Markowitz has been reported in the literature. This section presents an optimal solution to the constrained mean-variance formulation of the multi-period project portfolio investment problem. We consider a portfolio with $(m+1)$ risky projects, with random rates of returns. Let w_0 be an initial wealth of an investor at time 0. The investor can allocate his wealth among the $(m+1)$ projects. The wealth can be reallocated among the $(m+1)$ projects at the beginning of each of the following $(T-1)$ consecutive time periods. The rates of return of the risky projects at time period τ within the planning horizon are denoted by a vector $\mathbf{r}_\tau = [r_{\tau(0)}, r_{\tau(1)}, \dots, r_{\tau(m)}]'$, where $r_{\tau(j)}$ is the random return for project j at time period τ . It is assumed in this paper that vectors \mathbf{r}_τ , $\tau = 0, 1, \dots, T-1$, are statistically independent and return \mathbf{r}_τ has a mean $E\{\mathbf{r}_\tau\} = [E\{r_{\tau(0)}\}, E\{r_{\tau(1)}\}, \dots, E\{r_{\tau(m)}\}]'$ and a covariance

$$\text{Cov}\{\mathbf{r}_\tau\} = \begin{bmatrix} \sigma_{\tau(00)} & \cdots & \sigma_{\tau(0m)} \\ \vdots & \ddots & \vdots \\ \sigma_{\tau(0m)} & \cdots & \sigma_{\tau(mm)} \end{bmatrix}, \tag{32}$$

which can be found from the data of observations.

Let w_τ be the wealth of the investor at the beginning of the τ th period, and let $u_{\tau(j)}$, $j \in \{1, \dots, m\}$, be the amount invested in the j th risky project at the beginning of the τ th time period. The amount investigated in the 0th risky project at the beginning of the τ th time period is equal to

$$u_{\tau(0)} = w_\tau - \sum_{j=1}^m u_{\tau(j)}. \tag{33}$$

An investor is seeking a best multi-period investment strategy, $(u_{\tau(0)}, u_{\tau(1)}, \dots, u_{\tau(m)})$ for $\tau = 0, 1, 2, \dots, T-1$, such that either (i) the expected value of the terminal wealth w_T , $E\{w_T\}$, is maximized if the variance of the terminal wealth, $\text{Var}\{w_T\}$, is not greater than a preassigned risk level v^* , or (ii) the variance of the terminal wealth, $\text{Var}\{w_T\}$, is minimized if the expected terminal wealth, $E\{w_T\}$, is not smaller than a preassigned level e^* . Mathematically, a mean-variance formulation for multi-period project portfolio investment can be posed as one of the following two forms:

(i) Maximize

$$E\{w_T\} \tag{34}$$

subject to

$$\text{Var}\{w_T\} \leq v^*, \tag{35}$$

$$\sum_{j=1}^m u_{\tau(j)} \leq E\{w_\tau\}, \tag{36}$$

$$u_{\tau(j)} \geq 0, \quad j=1, \dots, m, \quad \tau=0, 1, 2, \dots, T-1, \tag{37}$$

with

$$w_{\tau+1} = \sum_{j=1}^m r_{\tau(j)} u_{\tau(j)} + \left(w_\tau - \sum_{j=1}^m u_{\tau(j)} \right) r_{\tau(0)} = r_{\tau(0)} w_\tau + \Delta'_\tau \mathbf{u}_\tau, \tag{38}$$

and

(ii) Minimize

$$\text{Var}\{w_T\} \tag{39}$$

subject to

$$E\{w_T\} \geq e^*, \tag{40}$$

$$\sum_{j=1}^m u_{\tau(j)} \leq E\{w_{\tau}\}, \quad (41)$$

$$u_{\tau(j)} \geq 0, \quad j=1, \dots, m, \quad \tau=0, 1, 2, \dots, T-1, \quad (42)$$

with

$$w_{\tau+1} = \sum_{j=1}^m r_{\tau(j)} u_{\tau(j)} + \left(w_{\tau} - \sum_{j=1}^m u_{\tau(j)} \right) r_{\tau(0)} = r_{\tau(0)} w_{\tau} + \Delta'_{\tau} \mathbf{u}_{\tau}, \quad (43)$$

where

$$\Delta_{\tau} = [\Delta_{\tau(1)}, \Delta_{\tau(2)}, \dots, \Delta_{\tau(m)}]' = [r_{\tau(1)} - r_{\tau(0)}, r_{\tau(2)} - r_{\tau(0)}, \dots, r_{\tau(m)} - r_{\tau(0)}]'. \quad (44)$$

Formulation (i) or (ii) enables an investor to specify a risk level he can afford when he is seeking to maximize his expected terminal wealth or specify an expected terminal wealth he would like to achieve when he is seeking to minimize the corresponding risk.

A strategy of multi-period project portfolio investment is an investment sequence,

$$\mathbf{U}_T = [\mathbf{u}_0, \mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_{T-1}], \quad (45)$$

where

$$\mathbf{u}_{\tau} = [u_{\tau(1)}, u_{\tau(2)}, \dots, u_{\tau(m)}]', \quad \forall \tau=0(1)T-1. \quad (46)$$

More specifically, \mathbf{U}_T is a feedback strategy and \mathbf{u}_{τ} maps the wealth at the beginning of the τ th period, w_{τ} , into a project portfolio decision in the τ th period, i.e., $\mathbf{u}_{\tau} \equiv \mathbf{u}_{\tau}(w_{\tau})$. A multi-period project portfolio investment strategy, \mathbf{U}_T^* , is said to be efficient if there exists no other portfolio one, \mathbf{U}_T , such that $E\{w_T; \mathbf{U}_T\} \geq E\{w_T; \mathbf{U}_T^*\}$ and $\text{Var}\{w_T; \mathbf{U}_T\} \leq \text{Var}\{w_T; \mathbf{U}_T^*\}$ with at least one equality strictly. By varying the value of v^* in (i) or the value of e^* in (ii), the set of efficient multi-period project portfolio investment strategies can be generated.

An equivalent formulation to either (i) or (ii) in generating efficient multi-period portfolio strategies is
(iii) Maximize

$$E\{w_T\} - \mathcal{G} \text{Var}\{w_T\} \quad (47)$$

subject to

$$\sum_{j=1}^m u_{\tau(j)} \leq E\{w_{\tau}\}, \quad (48)$$

$$u_{\tau(j)} \geq 0, \quad j=1, \dots, m, \quad \tau=0, 1, 2, \dots, T-1, \quad (49)$$

with

$$w_{\tau+1} = \sum_{j=1}^m r_{\tau(j)} u_{\tau(j)} + \left(w_{\tau} - \sum_{j=1}^m u_{\tau(j)} \right) r_{\tau(0)} = r_{\tau(0)} w_{\tau} + \Delta'_{\tau} \mathbf{u}_{\tau}, \quad (50)$$

where $\mathcal{G} \in [0, \infty)$.

It will be noted that if \mathbf{U}_T^* solves (iii), then \mathbf{U}_T^* solves (i) with $v^* = \text{Var}\{w_T; \mathbf{U}_T^*\}$, and \mathbf{U}_T^* solves (ii) with $e^* = E\{w_T; \mathbf{U}_T^*\}$. Note that \mathcal{G} is equal to $\partial E\{w_T\} / \partial \text{Var}\{w_T\}$ at the optimal solution of (iii). Problem formulation (iii) is preferable to be adopted in investment situations where an investor is able to specify his desirable trade-off between the expected terminal wealth and the associated risk.

6. Numerical Example

Consider the case of a stationary multi-period process with $T = 2$. An investor has one unit of wealth at the very beginning of the planning horizon, i.e., $w_0 = 1$. The investor is trying to find the best allocation of his wealth among three risky projects, 0, 1, and 2 in order to maximize $E\{w_2\}$ while keeping his risk not exceeding 0.07; that is, $v^* = 0.07$. The observations of returns for risky projects, 0, 1, and 2 are given in Table 1.

Table 1. The observations of returns for three risky projects

Year (<i>t</i>)	Project		
	0	1	2
1	1.3000	1.2250	1.1490
2	1.1030	1.2900	1.2600
3	1.2160	1.2160	1.4190
4	0.9540	0.7280	0.9220
5	0.9290	1.1440	1.1690
6	1.0560	1.1070	0.9650
7	1.0380	1.3210	1.1330
8	1.0890	1.3050	1.7320
9	1.0900	1.1950	1.0210
10	1.0830	1.3900	1.1310
11	1.0350	0.9280	1.0060
12	1.1760	1.7150	1.9080

At first consider the case of a stationary multi-period process with $T = 1$. The problem is to maximize

$$E\{w_1\} \tag{51}$$

subject to

$$\text{Var}\{w_1\} \leq v^* = 0.07, \tag{52}$$

$$\sum_{j=1}^2 u_{0(j)} \leq E\{w_0\} = w_0 = 1, \tag{53}$$

$$u_{0(j)} \geq 0, \quad j=1, 2, \tag{54}$$

with

$$w_1 = \sum_{j=1}^2 r_{0(j)} u_{0(j)} + \left(w_0 - \sum_{j=1}^2 u_{0(j)} \right) r_{0(0)} = r_{0(0)} w_0 + \Delta'_0 \mathbf{u}_0. \tag{55}$$

Using (55) and Table 1, it can be shown that

$$E\{w_1\} = E\{r_{0(0)}\} w_0 + E\{\Delta'_0\} \mathbf{u}_0, \tag{56}$$

where

$$E\{r_{0(0)}\} = 1.0891, \quad E\{\Delta'_0\} = [0.1246, 0.1455]; \tag{57}$$

$$\text{Var}\{w_1\} = [w_0, \mathbf{u}'_0] \text{Cov}\{[r_{0(0)}, \Delta'_0]'\} [w_0, \mathbf{u}'_0]', \tag{58}$$

where

$$\text{Cov}\{[r_{0(0)}, \Delta'_0]'\} = \begin{bmatrix} 0.0099 & 0.0015 & 0.0021 \\ 0.0015 & 0.0407 & 0.0374 \\ 0.0021 & 0.0374 & 0.0723 \end{bmatrix}. \tag{59}$$

Using Solver's software (MS Excel), we obtain from maximization of (51) a percent investment (on $w_0=1$) in three risky projects, 0, 1, and 2, at period $\tau=0$ as follows: $u_{0(0)}^* = 0\%$, $u_{0(1)}^* = 26.92\%$, and $u_{0(2)}^* = 73.08\%$. The corresponding expected terminal wealth and the risk level are given by $E\{w_1\} = 1.229$ and $\text{Var}\{w_1\}=0.07$, respectively.

Now consider the case of a stationary multi-period process with $T = 2$. The problem is to maximize

$$E\{w_2\} \tag{60}$$

subject to

$$\text{Var}\{w_2\} \leq v^* = 0.07, \tag{61}$$

$$\sum_{j=1}^2 u_{0(j)} \leq E\{w_0\} = w_0 = 1, \tag{62}$$

$$\sum_{j=1}^2 u_{1(j)} \leq E\{w_1\}, \tag{63}$$

$$u_{\tau(j)} \geq 0, \quad j=1, 2, \quad \tau=0, 1, \tag{64}$$

with

$$w_{\tau+1} = \sum_{j=1}^m r_{\tau(j)} u_{\tau(j)} + \left(w_{\tau} - \sum_{j=1}^m u_{\tau(j)} \right) r_{\tau(0)} = r_{\tau(0)} w_{\tau} + \Delta'_{\tau} \mathbf{u}_{\tau}. \tag{65}$$

Using (65) and Table I, it can be shown that

$$E\{w_2\} = E\{r_{1(0)} r_{0(0)}\} w_0 + E\{r_{1(0)} \Delta'_0\} \mathbf{u}_0 + E\{\Delta'_1\} \mathbf{u}_1, \tag{66}$$

where

$$E\{r_{1(0)} r_{0(0)}\} = 1.1960, \tag{67}$$

$$E\{r_{1(0)} \Delta'_0\} = [0.1371, 0.1605], \tag{68}$$

$$E\{\Delta'_1\} = [0.1246, 0.1455]; \tag{69}$$

$$\text{Var}\{w_2\} = [w_0, \mathbf{u}'_0, \mathbf{u}'_1] \text{Cov}\{[r_{1(0)} r_{0(0)}, r_{1(0)} \Delta'_0, \Delta'_1]'\} [w_0, \mathbf{u}'_0, \mathbf{u}'_1]', \tag{70}$$

where

$$\text{Cov}\{[r_{1(0)} r_{0(0)}, r_{1(0)} \Delta'_0, \Delta'_1]'\} = \begin{bmatrix} 0.0491 & 0.0028 & 0.0053 & 0.0022 & 0.0037 \\ 0.0028 & 0.0496 & 0.0490 & 0.0447 & 0.0427 \\ 0.0053 & 0.0490 & 0.0942 & 0.0426 & 0.0823 \\ 0.0022 & 0.0447 & 0.0426 & 0.0407 & 0.0374 \\ 0.0037 & 0.0427 & 0.0823 & 0.0374 & 0.0723 \end{bmatrix}. \tag{71}$$

Using Solver’s software (MS Excel), we obtain from maximization of (60) a percent investment (on $w_0=1$) in three risky projects, 0, 1, and 2, at period $\tau=0$ as follows: $u_{0(0)}^* = 100\%$, $u_{0(1)}^* = 0\%$, and $u_{0(2)}^* = 0\%$; and a percent investment (on $E\{w_1\}=1.0891$) in three risky projects, 0, 1, and 2, at period $\tau=1$ as follows: $u_{1(0)}^* = 39.80\%$, $u_{1(1)}^* = 47.24\%$, and $u_{1(2)}^* = 12.96\%$. The corresponding expected terminal wealth and the risk level are given by $E\{w_2\}=1.2807$ and $\text{Var}\{w_2\}=0.07$, respectively.

Conclusions

The problem considered in this paper is to decide how much of our available resources to invest in each project so as to maximize the total expected return by the end of the horizon in relation to a given utility function. We formulate the problem in terms of dynamic programming, which allows one to obtain optimal investment decisions for a set of projects under conditions of uncertainty in a simple form.

The derived optimal multi-period project portfolio investment strategy provides investors with the best strategy to follow in a dynamic investment environment.

References

1. Schniederjans, M., Santhanam, R. A multi-Objective Constrained Resource Information System Project Selection Method, *European Journal of Operational Research*, Vol. 70, 1993, pp. 244-253.
2. Lucas, H. C. *Computer-Based Information Systems in Organizations*. Chicago: Science Research Associates, 1973.
3. Cooper, R.G., Edgett, S.J., Kleinschmidt, E.J. Portfolio Management in New Products: Lessons from the Leaders-I, *Research Technology Management*, Vol. 40, 1997, pp. 16-28.
4. Dos Santos, B.L. Selecting Information System Projects: Problems, Solutions and Challenges. In: *Proceedings of the Hawaii Conference on System Sciences*, 1989, pp. 1131-1140.
5. Cooper, R.G. *Winning at New Products*. MA: Addison-Wesley, Reading, 1993.
6. Jackson, B. Decision Methods for Selecting a Portfolio of R&D projects, *Research Management*, Vol. 11, 1983, pp. 21-26.
7. Mathieu, R.G., Gibson, J.E. A Methodology for Large Scale R&D Planning Based on Cluster Analysis, *IEEE Transactions on Engineering Management*, Vol. 30, 1993, pp. 283-291.
8. Liberatore, M.L., Titus, G.J. The Practice of Management Science in R&D Project Selection, *Management Science*, Vol. 29, 1983, pp. 962-974.
9. Hess, S.W. Swinging on the Branch of a Tree: Project Selection Applications, *Interfaces*, Vol. 23, 1993, pp. 5-12.
10. Archer, N.P., Ghasemzadeh, F. *Portfolio Selection Techniques: A Review and a Suggested Integrated Approach*. Innovation Research Center, Working Paper 46. Hamilton, Ontario: School of Business, McMaster University, 1996.
11. Bellman, R., Kalaba, R. *Dynamic Programming and Modern Control Theory*. New York: Academic Press, 1964.