OPTIMAL ADAPTIVE CONTROL OF PREVENTIVE MAINTENANCE PROCESS OF AIRCRAFT STRUCTURES

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Fatigue is one of the most important problems of aircraft arising from their nature as multiple-component structures, subjected to random dynamic loads. Fatigue damage is considered to initiate in a structural element when cracks develop, whether or not they are detected. The fatigue process then continues by crack propagation, resulting in strength degradation. Periodic inspections of aircraft are common practice in order to maintain their reliability above a desired minimum level. Thus, a catastrophic accident during flight can be avoided. In this paper, a control theory is used for planning inspections in service of fatigue-sensitive aircraft structure components under crack propagation. One of the most important features of control theory is its great generality, enabling one to analyse diverse systems within one unified framework. A key idea, which has emerged from this study, is the necessity of viewing the process of planning in-service inspections as an adaptive control process. Adaptation means the ability of self-modification and self-adjustment in accordance with varying conditions of environment. The adaptive control of preventive maintenance process of aircraft structures differs from ordinary stochastic control of preventive maintenance process in that it attempts to re-evaluate itself by force of uncertainties in the preventive maintenance process as they unfold and change.

Keywords: *aircraft, fatigue crack, preventive maintenance process, adaptive control*

1. Introduction

Based on engineering and macroscopic viewpoints, the mechanical properties of metallic materials are often considered homogeneous. However, a considerable amount of scatter has been observed in fatigue data even under the same loading condition. It may be attributed to the inhomogeneous material properties. As a result, probabilistic approaches for the fatigue crack growth have received great attention in recent years. Along with the development of fracture mechanics for the past three decades and the need of reliability or risk assessment for some important structures or components such as nuclear power plant and aircraft structures, the so-called 'probabilistic fracture mechanics' has thus arisen [1]. One of the important issues in the probabilistic fracture mechanics analysis lies in the probabilistic modeling of fatigue crack growth phenomenon. Many probabilistic models have been proposed to capture the scatter of the fatigue crack growth data. Some of the models are purely based on direct curve fitting of the random crack growth data, including their mean value and standard deviation [2]. These models, however, have been criticized by other researchers that, less crack growth mechanisms have been included in them. To overcome this difficulty, many probabilistic models adopted the crack growth equations proposed by fatigue experimentalists, and randomised the equations by including random factors into them [3–4]. The random factor may be a random variable, a random process of time, or a random process of space. It then creates a random differential equation. The solution of the differential equation reveals the probabilistic nature as well as the scatter phenomenon of the fatigue crack growth. To justify the applicability of the probabilistic models mentioned above, fatigue crack growth data are needed. However, it is rather time-consuming to carry out experiments to obtain a set of statistical meaningful fatigue crack growth data. To the writers' knowledge, there are only a few data sets available so far for researchers to verify their probabilistic models. Among them, the most famous data set perhaps is the one produced by Virkler et al. more than twenty years ago [5]. Two more frequently used data sets include one reported by Ghonem and Dore [6], and the other released by the Flight Dynamics Laboratory of the US Air Force [7]. Itagaki and his associates have also produced some statistically meaningful fatigue crack growth data, but have not been mentioned very often [8]. In fact, many probabilistic fatigue crack growth models are either lack of experimental verification or just verified by only one of the above data sets. It is suspected that a model may explain a data set well but fail to explain another data set. The universal applicability of many probabilistic models still needs to be checked carefully by other available data sets.

This paper describes a stochastic fatigue life methodology, SLAP (stochastic life approach), for aircraft structure components as an alternative to the current deterministic and partially stochastic approaches. Before presenting SLAP, some background information is given about the current life philosophies and general issues relating to deterministic and stochastic analysis.

1.1. Current Design Philosophies

Airworthiness regulations require proof that aircraft can be operated safely. This implies that critical components must be replaced or repaired before safety is compromised. The fatigue philosophies underlying the approaches for guaranteeing safety are called Safe-Life and Damage Tolerance.

The Safe-Life philosophy is based on the concept that significant damage, i.e. fatigue cracking, will not develop during the service life of a component. The life is initially determined from fatigue test data (S–N curves) and calculations using a cumulative damage ''law''. Then the design Safe-Life is obtained by applying a safety factor. When the service life equals the design Safe-Life the component must be replaced. However, there are two major drawbacks to this approach: (1) components are taken out of service even though they may have substantial remaining lives; (2) despite all precautions, cracks sometimes occur prematurely. This latter fact led the Air Forces to introduce the Damage Tolerance philosophy [9].

The Damage Tolerance philosophy recognizes that damage can occur and develop during the service life of a component. Also, it assumes that cracks or flaws can be present in new structures. Safety is obtained from this approach by the requirements that either (1) any damage be detected by routine inspection before it results in a dangerous reduction of the static strength (inspectable components), or (2) initial damage shall not grow to a dangerous size during the service life (non-inspectable components). For Damage Tolerance analysis to be successful it must be possible to:

- Define either a minimum crack length ad that will not go undetected during routine inspections, or else an initial crack length ai, nominally based on pre-service inspection capability.
- Predict crack growth during the time until the next inspection or until the design service life is reached.

An adjunct to Damage Tolerance is Durability analysis. This is an economic life assessment for components that are not safety-critical. The prediction of crack growth is similar to that for Damage Tolerance, except that a much smaller initial crack length is used.

1.2. Deterministic Analysis

Safe-Life and Damage Tolerance analyses are basically deterministic, i.e. they do not consider the variability of parameters used in the analyses. Instead, they apply scatter and safety factors to the obtained fatigue lifetime or crack growth life. In general, such analyses result in over-conservative life estimates and inspection intervals. Even so, the reliability (safety level) of the structure and its components is not properly known. Moreover, the safety factors applied are quite arbitrary, although historically successful probably because of the high degree of conservatism [10]. Other disadvantages of deterministic approaches are

- Lighter and more efficient structures can require new materials and design methodologies with different (unknown) safety factors.
- Deterministic analyses are often compared with test data. However, the comparisons are questionable, especially when the data are limited and have significant scatter. In such cases agreement between a deterministic analysis and test data can be fortuitous. It would be better, or essential, to do a stochastic analysis to compare with the test data.

1.3. Stochastic Analysis

The disadvantages of deterministic approaches provide a strong case for stochastic analysis. Good stochastic tools are now available, but they are not widely used. One reason is unfamiliarity. Another is the lack of data for generating proper distribution functions. However, this problem can often be overcome by combining available data with engineering judgement, and allocating the distribution type according to the type of phenomenon and goodness-of-fit tests. Even so, there should be experimental programmes to collect sufficient statistical data. This is still not common practice in fatigue engineering. The following steps can be distinguished in any stochastic analysis, Grooteman et al. [11]:

- *Choice of random variables and their distribution functions*. As stated above, it is often possible to do this. In cases where it is unclear whether an analysis parameter, or combination of parameters, should be treated as a stochastic variable, a sensitivity analysis can be done. In other words, does the variation in these parameters cause any significant scatter in the result? If not, these parameters can be treated deterministically.

In-service inspections provide non-failure and failure data, both of which are used to improve the estimated failure distribution, as discussed below. The non-failure data consist only of usage times at the moment of inspection, while failure data will consist of the inspection times and detected crack lengths. These latter data are not actual failures, but extrapolations using a validated fatigue crack growth model.

The failure distribution function for a component can be determined from experimental data provided there are sufficient failure points. In-service data can be used to update and improve the estimate of the failure distribution.

- *Choice of failure distribution functions*. There are many goodness-of-fit tests, for example Kolmogorov-Smirnov and Anderson-Darling. The Anderson-Darling test is more sensitive to deviations in the tails of a distribution than the older Kolmogorov–Smirnov test. These tests do not tell you that you do have a certain type of distribution function; they only tell you when the data make it (un)likely that you do.

The Anderson–Darling test statistic value is determined by (e.g. [12]):

$$A^2 = -\left[\sum_{i=1}^{n}(2i-1)(\ln(F(x_i)) + \ln(1-F(x_{n+1-i})))\right]n^{-1} - n, \tag{1}$$

where $F(.)$ is the cumulative distribution function and n is the number of observations. The result from (1) needs to be modified for small sampling values. For the Weibull's distribution the modification of A^2 is

$$A_m^2 = A^2\left(1 + 0.2/\sqrt{n}\right), \tag{2}$$

when the unknown parameters of scale and shape are estimated from the sample data. The determined A_m^2 value has to be less than the following critical values for acceptance of goodness-of-fit:

α	0.1	0.05	0.025	0.01
A_{crit}^2	0.637	0.757	0.877	1.038

- *Solution of the stochastic problem*. There are many stochastic methods, Bjerager [13]. The most well-known is the Monte Carlo method, but it is inefficient when dealing with the small probabilities of failure relevant to engineering structures. More efficient method, such as (Reliability Analysis Program (RAP), Grooteman [14]) has been developed. RAP includes many of the more modern methods. Also, it operates on top of any deterministic tool and needs no modifications to do this. The extra input compared to deterministic analysis consists of specification of the random variables and their distribution functions, specification of the failure function(s), and selection of the stochastic method to be applied.
- *Interpretation of the results*. An important issue is the allowed probability of failure (POF). Table 1 [15] gives some target probability values for lifetimes, based on the relative costs of safety measures and the consequences of failure. In general, the target lifetime POF will be about 10^{-3} for engineering problems. For military aircraft structures a typical lifetime is 10^4 flight hours, giving a POF ~ 10^{-7} per flight hour (FH).

Table 1. Lifetime Probability of Failure (POF) targets according to ISO 2394

	Consequences of failure			
Relative costs of safety measures	Small	Some	Moderate	Great
High	1	10^{-1}	10^{-2}	10^{-3}
Moderate	10^{-1}	10^{-2}	10^{-3}	10^{-4}
Low	10^{-2}	10^{-3}	10^{-4}	10^{-5}

2. Experimental Fatigue Crack Growth Data

For constant amplitude fatigue tests, experimental crack growth curves are shown in Figure 1.

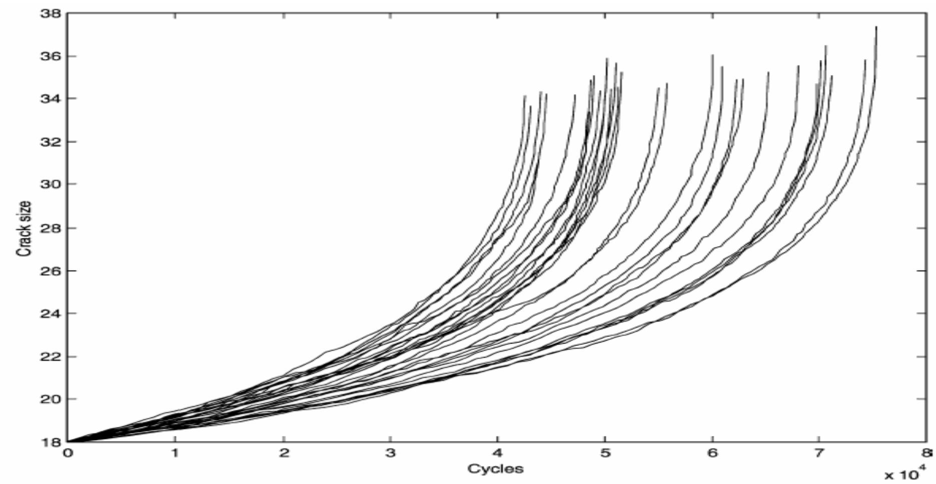

Figure 1. Crack propagation curves

A considerable degree of scatter can be seen from the figure. Some selected discrete data points of test including crack depth in wing spar of RAAF aircraft as a function of flights [16] are tabulated in Table 2.

Table 2. Depth of crack in wing spar of RAAF aircraft as a function of flights

Flight number	1233	1274	1278	1281	1333	1398	1500	1629	1716	1770	1888	1889	1904
Crack depth (mm)	3.169	3.600	4.246	4.400	5.692	6.031	7.200	12.00	14.98	17.72	18.58	20.62	78.07

3. Stochastic Modelling

To capture the statistical nature of fatigue crack growth, different stochastic models have been proposed in the literature. These models may have been verified by only one data set, and therefore not appreciated by other fellow researchers. Part of the reason is the difficulty and time consuming in obtaining the statistically meaningful fatigue crack growth data.

It will be noted that many probabilistic models of fatigue crack growth are based on the deterministic crack growth equations. The most well-known equation is

$$\frac{\mathrm{d}a(t)}{\mathrm{d}t} = Q(a(t))^b \tag{3}$$

in which Q and b are constants to be evaluated from the crack growth observations. The independent variable t can be interpreted as stress cycles, flight hours, or flights depending on the applications. It is noted that the power-law form of $Q(a(t))^b$ at the right hand side of (3) can be used to fit some fatigue crack growth data appropriately and is also compatible with the concept of Paris–Erdogan law.

The service time for a crack to grow from size $a(t_0)$ to $a(t)$ (where $t > t_0$) can be found by performing the necessary integration

$$\int_{t_0}^{t} dt = \int_{a(t_0)}^{a(t)} \frac{\mathrm{d}v}{Qv^b} \tag{4}$$

to obtain

$$t - t_0 = \frac{[a(t_0)]^{-(b-1)} - [a(t)]^{-(b-1)}}{Q(b-1)}. \tag{5}$$

For the particular case (when b=1), it can be shown, using Lopital's rule, that

$$t - t_0 = \frac{\ln[a(t)/a(t_0)]}{Q}. \tag{6}$$

Thus, we have obtained the Exponential model

$$a(t) = a(t_0)e^{Q(t-t_0)}. \tag{7}$$

The Exponential model is quite often used for calculation of growth of population/bacteria etc. The basic equation of it is

$$P_t = P_0 e^{rt}. \tag{8}$$

For prediction of crack growth the above equation is rewritten in [17] as

$$a = a_0 e^{mN}. \tag{9}$$

When the above model is used and tested, it is observed that the results obtained by the above model are in good agreement with the experimental results [17].

In this paper, we rewrite (6) as

$$\tau_{j+1} - \tau_j = \frac{\ln[a(\tau_{j+1})/a(\tau_j)]}{Q}, \quad j=0, 1, \dots . \tag{10}$$

where τ_j is the time of the jth in-service inspection of the aircraft structure component, $a(\tau_j)$ is the fatigue crack size detected in the component at the jth inspection.

4. Formulation of the Terminal-Control Problem

Rewrite (10) as

$$x_{j+1} = x_j + Qu_j, \quad j=0, 1, \dots , \tag{11}$$

where

$$x_j = \ln[a(\tau_j)], \tag{12}$$

$$u_j = \tau_{j+1} - \tau_j \tag{13}$$

represents the interval between the jth an (j+1)th inspections.

It is assumed that the parameter Q is a random variable, which can take values within a finite set $\{q^{(1)}, q^{(2)}, \dots, q^{(r)}\}$. However, in order to simplify the computation, only two values are chosen. Assume that, at the instant k, the random parameter Q takes on two values, $q^{(1)}$ and $q^{(2)}$, with probabilities p and 1-p, respectively, and that the value of the probability p is not known. It takes on two values p_1 and p_2 with a priori probability ξ and $1-\xi$, respectively.

Let us suppose that a fatigue-sensitive component has been found cracked on one aircraft within a warranty period. The detected fatigue crack size is $a(\tau_0)$. The maximum allowable crack size is $a^{\bullet}$. We plan to carry out N in-service inspections of this component and are in need to assign intervals, $u_0, u_1, \dots, u_{N-1}$, between sequential inspections so that the performance index

$$I = E\left\{(x^{\bullet} - x_N)^2\right\}, \tag{14}$$

where $x^{\bullet}=\ln(a^{\bullet})$, is minimized.

5. Adaptive Optimization of the Terminal-Control Problem

The design is initiated with the determination of the *a posteriori* probabilities

$$\xi_{1:q^{(1)}} = \Pr\{p = p_1 | Q = q^{(1)}\} \tag{15}$$

and

$$\xi_{1:q^{(2)}} = \Pr\{p = p_1 | Q = q^{(2)}\}. \tag{16}$$

By the Bayes's theorem, it is found that

$$\xi_{1:q^{(1)}} = \frac{\Pr\{p = p_1\}\Pr\{Q = q^{(1)} \mid p = p_1\}}{\Pr\{p = p_1\}\Pr\{Q = q^{(1)} \mid p = p_1\} + \Pr\{p = p_2\}\Pr\{Q = q^{(1)} \mid p = p_2\}\}}$$

$$= \frac{\xi\, p_1}{\xi\, p_1 + (1-\xi) p_2} \tag{17}$$

and

$$\xi_{1:q^{(2)}} = \frac{\Pr\{p = p_1\}\Pr\{Q = q^{(2)} \mid p = p_1\}}{\Pr\{p = p_1\}\Pr\{Q = q^{(2)} \mid p = p_1\} + \Pr\{p = p_2\}\Pr\{Q = q^{(2)} \mid p = p_2\}\}}$$

$$= \frac{\xi(1-p_1)}{\xi(1-p_1) + (1-\xi)(1-p_2)}. \tag{18}$$

Let the minimum of I be denoted by $f_N(x_0,\xi)$, where x_0 is the initial crack size detected in the component. The minimum of I is a function of the initial crack size x_0 and the a priori probability ξ, and is given by

$$f_N(x_0, \xi) = \min_{(u_0, u_1, \dots, u_{N-1})} E\{(x^{\bullet} - x_N)^2\}. \tag{19}$$

At any sampling instant $k+1$, x_{k+1} takes on two values:

$$x_{k+1}^{(1)} = x_k + q^{(1)} u_k \tag{20}$$

and

$$x_{k+1}^{(2)} = x_k + q^{(2)} u_k . \tag{21}$$

For $k = 0$, x_1 takes on the value

$$x_1^{(1)} = x_0 + q^{(1)} u_0 \quad \text{with probability } p_0 \tag{22}$$

and the value

$$x_1^{(2)} = x_0 + q^{(2)} u_0 \quad \text{with probability } 1-p_0, \tag{23}$$

where p_0 is the expected value of p and is given by

$$p_0 = \xi\, p_1 + (1-\xi) p_2 . \tag{24}$$

Hence, for N=1,

$$f_1(x_0,\xi) = \min_{u_0} E\{(x^\bullet - x_1)^2\} = \min_{u_0} [p_0 (x^\bullet - x_1^{(1)})^2 + (1-p_0)(x^\bullet - x_1^{(2)})^2] . \tag{25}$$

For $N \geq 2$, invoking the principle of optimality yields

$$f_N(x_0,\xi) = \min_{u_0} [p_0 f_{N-1}(x_1^{(1)}, \xi_{1:q^{(1)}}) + (1-p_0) f_{N-1}(x_1^{(2)}, \xi_{1:q^{(2)}})] , \tag{26}$$

where $\xi_{1:q^{(1)}}$, $\xi_{1:q^{(2)}}$, $x_1^{(1)}$, and $x_1^{(2)}$ are defined in (17), (18), (22), and (23), respectively. As a result of the first decision, the process will be transformed to one of the two possible states $x_1^{(1)}$ or $x_1^{(2)}$ with probability p_0 or $1-p_0$. If the process moves to state $x^{(1)}$, the *a posteriori* probability $\xi_{1:q^{(1)}}$ is computed. If the process moves to state $x^{(2)}$, the *a posteriori* probability $\xi_{1:q^{(2)}}$ is determined.

In a one-stage process, the optimum decision is found by differentiating (25) with respect to u_0 and equating the partial derivative to zero. This leads to

$$p_0 q^{(1)} (x^\bullet - x_0 - q^{(1)} u_0) + (1-p_0) q^{(2)} (x^\bullet - x_0 - q^{(2)} u_0) = 0. \tag{27}$$

Hence

$$u_0 = \frac{E\{Q\}}{E\{Q^2\}} (x^\bullet - x_0), \tag{28}$$

where

$$E\{Q\} = p_0 q^{(1)} + (1-p_0) q^{(2)} \tag{29}$$

and

$$E\{Q^2\} = p_0 [q^{(1)}]^2 + (1-p_0)[q^{(2)}]^2 \tag{30}$$

are functions of ξ. By defining

$$E_i\{Q\} = p_i q^{(1)} + (1-p_i) q^{(2)}, \quad i=1, 2, \tag{31}$$

it can readily be shown that $D\{Q\}$ can be written as

$$E\{Q\} = \xi\, E_1\{Q\} + (1-\xi) E_2\{Q\}. \tag{32}$$

Similarly, by defining

$$E_i\{Q^2\} = p_i [q^{(1)}]^2 + (1-p_i)[q^{(2)}]^2 , \quad i=1, 2, \tag{33}$$

$E\{Q^2\}$ can be expressed in terms of ξ as

$$E\{Q^2\} = \xi\, E_1\{Q^2\} + (1-\xi) E_2\{Q^2\}. \tag{34}$$

The minimum for the one-stage process is given by

$$f_1(x_0,\xi)=G_1(\xi)(x^\bullet-x_0)^2, \tag{35}$$

where

$$G_1(\xi)=1-\frac{E^2\{Q\}}{E\{Q^2\}}. \tag{36}$$

By defining

$$h_0(\xi)=\frac{E\{Q\}}{E\{Q^2\}} \tag{37}$$

the optimum decision u_0 may be written as

$$u_0=h_0(\xi)(x^\bullet-x_0). \tag{38}$$

It can be shown by mathematical induction that

$$f_k(x_0,\xi)=G_k(\xi)(x^\bullet-x_0)^2. \tag{39}$$

In view of (39),

$$f_k(x_1^{(1)},\xi_{1:q^{(1)}})=G_k(\xi_{1:q^{(1)}})(x^\bullet-x_0-q^{(1)}u_0)^2, \tag{40}$$

$$f_k(x_1^{(2)},\xi_{1:q^{(2)}})=G_k(\xi_{1:q^{(2)}})(x^\bullet-x_0-q^{(2)}u_0)^2. \tag{41}$$

The minimum for a (k+1)-stage process is

$$f_{k+1}(x_0,\xi)=\min_{u_0}[p_0G_k(\xi_{1:q^{(1)}})(x^\bullet-x_0-q^{(1)}u_0)^2+(1-p_0)G_k(\xi_{1:q^{(2)}})(x^\bullet-x_0-q^{(2)}u_0)^2],$$

k = 1, 2, …, N–1. (42)

From this recurrence relationship is found that the optimum decision is given by

$$u_0=h_k(\xi)(x^\bullet-x_0), \tag{43}$$

where

$$h_k(\xi)=\frac{E\{QG_k(\xi_{1:Q})\}}{E\{Q^2G_k(\xi_{1:Q})\}}, \tag{44}$$

$$E\{QG_k(\xi_{1:Q})\}=\xi\ E_1\{QG_k(\xi_{1:Q})\}+(1-\xi)E_2\{QG_k(\xi_{1:Q})\}, \tag{45}$$

$$E_i\{QG_k(\xi_{1:Q})\}=p_i\ q^{(1)}G_k(\xi_{1:q^{(1)}})\}+(1-p_i)q^{(2)}G_k(\xi_{1:q^{(2)}})\},\quad i=1,2, \tag{46}$$

$$E\{Q^2G_k(\xi_{1:Q})\}=\xi\ E_1\{Q^2G_k(\xi_{1:Q})\}+(1-\xi)E_2\{Q^2G_k(\xi_{1:Q})\}, \tag{47}$$

$$E_i\{Q^2G_k(\xi_{1:Q})\}=p_i\,[q^{(1)}]^2G_k(\xi_{1:q^{(1)}})\}+(1-p_i)[q^{(2)}]^2G_k(\xi_{1:q^{(2)}})\},\quad i=1,2. \tag{48}$$

From (42) and (43) it follows that

$$f_{k+1}(x_0,\xi)=G_{k+1}(\xi)(x^\bullet-x_0)^2, \tag{49}$$

where

$$G_{k+1}(\xi)=E\{G_k(\xi_{1:Q})\}-\frac{E^2\{QG_k(\xi_{1:Q})\}}{E\{Q^2G_k(\xi_{1:Q})\}}, \tag{50}$$

$$E\{G_k(\xi_{1:Q})\}=\xi\ E_1\{G_k(\xi_{1:Q})\}+(1-\xi)E_2\{G_k(\xi_{1:Q})\}, \tag{51}$$

$$E_i\{G_k(\xi_{1:Q})\} = p_i G_k(\xi_{1:q^{(1)}})\} + (1-p_i)G_k(\xi_{1:q^{(2)}})\}, \quad i=1, 2, \tag{52}$$

Equations (35), (36), (49), and (50) are recurrence relationships with which it is possible to evaluate the minimum for an N-stage process $f_N(x_0,\xi)$.

With the initial state x_0 and initial information ξ, the first optimum decision is

$$u_0 = h_{N-1}(\xi)(x^\bullet - x_0), \tag{53}$$

where $h_{N-1}(\xi)$ is evaluated from (44) to (48) and (50) to (52), with k = N −1. The second optimum decision should be made after observation of the random variable Q in the first decision stage. If it is observed that $Q = q^{(1)}$, the *a posteriori* probability $\xi_{1:q^{(1)}}$ and the new state

$$x_1^{(1)} = x_0 + q^{(1)}u_0 \tag{54}$$

are used as the initial information for the remaining $N-1$ stages. The second optimum decision can be determined in similar manner and is given by

$$u_1 = h_{N-2}(\xi_{1:q^{(1)}})(x^\bullet - x_1^{(1)}) . \tag{55}$$

If the observed value of Q after the first decision is $q^{(2)}$, the *a posteriori* probability $\xi_{1:q^{(2)}}$ and the new state

$$x_1^{(2)} = x_0 + q^{(2)}u_0 \tag{56}$$

are used as the initial information and the initial state for the remaining $N-1$ stages. The second optimum decision is then given by

$$u_1 = h_{N-2}(\xi_{1:q^{(2)}})(x^\bullet - x_1^{(2)}) . \tag{57}$$

Thus, after the first inspection, the computer must calculate the *a posteriori* probability $\xi_{1:q^{(1)}}$ or $\xi_{2:q^{(2)}}$, the new state x_1 and the second optimum decision u_1.

If the observed value of Q after the second decision is $q^{(1)}$, the *a posteriori* probability $\xi_{2:q^{(1)}}$ and the new state

$$x_2^{(1)} = x_1 + q^{(1)}u_1 \tag{58}$$

are used as the initial information and the initial state for the remaining N — 2 stages, in particular, to determine the third optimum decision

$$u_2 = h_{N-3}(\xi_{2:q^{(1)}})(x^\bullet - x_2^{(1)}) . \tag{59}$$

In (58), if $x_1 = x_1^{(1)}$, then *a posteriori* probability is $\xi_{1:q^{(1)}}$ and u_1 is given by (55); and if $x_1 = x_1^{(2)}$, then *a posteriori* probability is $\xi_{1:q^{(2)}}$ and u_1 is given by (57).

If the observed value of Q after the second decision is $q^{(2)}$, the *a posteriori* probability $\xi_{2:q^{(2)}}$ and the new state

$$x_2^{(2)} = x_1 + q^{(2)}u_1 \tag{60}$$

are used to determine the third optimum decision, which is

$$u_2 = h_{N-3}(\xi_{2:q^{(2)}})(x^\bullet - x_2^{(2)}) . \tag{61}$$

By repeated observation and computation in the above manner, the optimum-inspection policy (u_0, …, u_{N-1}) for the fatigue-sensitive component, which has been found cracked on one aircraft within a warranty period, can be determined.

Each new optimum decision is made by using new information resulting from the observation of the random variable Q.

Conclusions

An analytical solution to the terminal-control problem is generally not easy to derive, and numerical procedures should be followed. In this paper, the design of adaptive and learning control processes is considered. The design of such processes is carried out on the basis of the Bayes's theorem and the functional-equation approach of dynamic programming. The adaptation and learning process consists in gathering information about the random variable and processing this information according to some decision rule. The decisions resulting from the information processing are used to improve the control, which is chosen initially according to an *a priori* distribution. Certain information patterns are derived by making a number of observations of the random variables. On the basis of the information pattern, the adaptive controller can determine the correct probability distribution through the process of learning. A preventive maintenance of cracked aircraft structure component is used to illustrate the design procedure.

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