

Session 1.

# **Business Systems Modelling**



# MARKOVIAN MODEL OF ASSET PRICES

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To price and hedge derivative securities, it is crucial to have a good model of the probability distribution of the underlying product. The most famous continuous time model is the celebrated Black–Scholes’s model, which uses the normal distribution to fit the log returns of the underlying asset. One of the main problems with this model is that the data suggest that the log returns of stocks are not normally distributed. So other more flexible distributions are needed. In the article a numerical model how to model dynamics of asset prices by Markov’s process in continuous time and with countable set of states based on phase type distribution is proposed.

**Keywords:** *phase type distribution, asset price dynamics, Markov’s process, numerical model*

## 1. Introduction

One of the main questions facing quantitative finance is how to model best the dynamics of prices in financial markets, such as individual stock prices, stock indices, interest rates, and exchange rates. Armed with the model of price dynamics, an investor can do the following:

- Calculate theoretical prices for derivative securities
- Measure the amount of risk associated with holding risky securities.

Applications in financial mathematics have relied heavily on diffusion process for each underlying asset price – that is, a stochastic integral or stochastic differential equation where the uncertainty is driven by Brownian motion, with a constant volatility. This implies a normal distribution for the continuously - compound stock return, or lognormal distribution for the stock prices. Unfortunately, it appears that asset prices in the real world are not driven by diffusion processes. The log returns of most financial assets do not follow a normal law. They are skewed and have an actual kurtosis higher than that of the normal distribution. There are more important reasons why the above mentioned approach is not adequate. Examples include fat tails, volatility clustering, large discrete jumps, parameter instability, and asymmetric correlations [1, 2]. The underlying normal distribution was replaced by a more sophisticated one. Examples of such, which can take into account skewness, excess kurtosis and other features, are the Variance Gamma [3], the Normal Inverse Gaussian [4], the CGMY (named after Carr, Geman, Madan and Yor) [5], the Hyperbolic Model [6] and the Meixner’s [7] distribution. Including such features makes analytic modelling less tractable, and potentially makes numerical modelling a more attractive alternative. In the following sections the algorithm of modelling asset prices by Markov’s chains is proposed.

## 2. Markov’s Property of Stock Prices

The dynamics of asset prices are reflected by uncertain movements of their values over time. Some authors [8, 9] state that Efficient Market Hypothesis (EMH) is one possible reason for the random behaviour of the asset price. The EMH basically states that past history is fully reflected in present prices and markets respond immediately to new information about the asset.

A Markov’s process is a particular type of stochastic process where only the present value of a variable is relevant for predicting the future. The past history of the variables and the way as the present has emerged from the past are irrelevant.

Stock prices are usually assumed to follow a Markov’s process. These processes are important models of security prices, because they are often realistic representation of true prices and yet the Markov’s property leads to simplified computations. If the stock price follows a Markov’s process, our predictions of the future should be unaffected by the price one week ago, one month ago, or one year ago. The only relevant piece of information is the price now. Predictions are uncertain and must be expressed in terms of probability distributions. The Markov’s property implies that the probability distribution of the price at any particular future time is not dependent on the particular path followed by the price in the past.

If stock price process  $S = \{S_t, 0 \leq t \leq T\}$  is Markovian and if denotes by  $F = \{F_t, 0 \leq t \leq T\}$  the natural filtration of  $S$  (intuitively,  $F_t$  contains all market information up to time  $t$ ), then we can write for a well-behaved function  $f$ :  $E[f(S_T) \mid F_t] = E[f(S_T) \mid S_t]$ . The stock price process takes values in some countable set  $E$ , called

the state space. If  $S_t = j \in E$ , we shall say "the process is in state  $j$  at time  $t$ ". The most common situation is for the state to be a scalar, but frequently it is more convenient for the state to be a vector.

### 3. Constructing the Probability Distribution of Stock Price Movement

Our aim is to construct the stock price dynamics as Markov's process with continuous time and countable space of states. To find the space of states and transition rates between them we have to construct price movement distributions up and down for a given stock. To get Markov's process the distribution of time length of stock price rising or decreasing must be exponential with parameter  $\mu$ . Unfortunately, usually it is insufficient, then a convenient representation for more general distributions is the Coxian's formulation [10]. This formulation, by means of fictitious phases, allows the duration of generating stock price rate of transition up or down to be described by a linear combination of stochastic variables. Thus, generation of price movement is a continuous succession of  $k$  phases, each having exponential service time distribution of rate  $\mu_j, j = 1, 2, \dots, k$ .

After phase  $j$ , a stock price leaves the phases with probability  $(1 - p_j)$ . The stock price can occupy only one phase at a time. Therefore, there can be at most one stock price within the set of phases at any time.

Let us consider a general probability distribution function  $G(t)$  of stock price movement. Useful approximation of this function can be obtained by the mixture and convolutions of exponential (phase-type) distributions. Then a Markov's chain with a countable space of states and continuous time can represent the evolution of stock price dynamics. Suppose we let  $m_k, k = \overline{1,3}$  denotes the  $k$ th non-central moment, i.e.  $E[X^k]$ , where  $X$  is a random variable of price movement time. Define a random variable  $X$  in this way:

$$X = \begin{cases} X_1 & \text{with prob. } p_2; \\ X_1 + X_2 & \text{with prob. } p_1, \end{cases} \quad (1)$$

where  $X_1$  and  $X_2$  are independent random variables having exponential distribution with parameters  $\mu_1$  and  $p_2\mu_2$  respectively;  $p_1 + p_2 = 1$ . It is easy to verify that the density function of  $X$  is given by

$$f(x) = p_2\mu_1 \left( \frac{\mu_2 - \mu_1}{\mu_2 p_2 - \mu_1} e^{-\mu_1 x} - \frac{\mu_2 p_1}{\mu_2 p_2 - \mu_1} e^{-p_2 \mu_2 x} \right). \quad (2)$$

**Note.** Duration of service time as a random variable given by (1) allows us to apply the method for automatic construction of numerical models for systems described by Markov's process.

Moment matching is a common method for approximating distributions [11, 12]. Though two-moment approximations are common, they may lead to serious error when the coefficient of variation  $v$ , (the standard deviation divided by the mean) is high. The first three moments of any non degenerate distribution with support on  $[0, \infty)$  can be matched by the distribution (2).

To obtain the values of the parameters  $\mu_1, \mu_2, p_1$  and  $p_2$  of approximation, a complex system of non-linear equations needs to be solved:

$$\begin{cases} \frac{1! p_2 \mu_1}{\mu_2 p_2 - \mu_1} \left( \frac{\mu_2 - \mu_1}{\mu_1^2} - \frac{\mu_2 p_1}{\mu_2^2 p_2^2} \right) = m_1; \\ \frac{2! p_2 \mu_1}{\mu_2 p_2 - \mu_1} \left( \frac{\mu_2 - \mu_1}{\mu_1^3} - \frac{\mu_2 p_1}{\mu_2^3 p_2^3} \right) = m_2; \\ \frac{3! p_2 \mu_1}{\mu_2 p_2 - \mu_1} \left( \frac{\mu_2 - \mu_1}{\mu_1^4} - \frac{\mu_2 p_1}{\mu_2^4 p_2^4} \right) = m_3; \\ p_1 + p_2 = 1. \end{cases} \quad (3)$$

The solution of the system is the following:

$$\mu_2 = \frac{g_2 - g_1^2}{g_1^3 - 2g_1 g_2 + g_3}, g_k = \frac{m_k}{k!}, k = \overline{1,3};$$

$$\mu_1 = \frac{1 + \mu_2 g_1 \pm \sqrt{(1 - \mu_2 g_1)^2 + 4\mu_2^2 (g_2 - g_1^2)}}{2g_1 - 2\mu_2 (g_2 - g_1^2)}; \quad (4)$$

$$p_1 = \frac{\mu_2 (\mu_1 g_1 - 1)}{\mu_2 (\mu_1 g_1 - 1) + \mu_1}; \quad p_2 = \frac{\mu_1}{\mu_2 (\mu_1 g_1 - 1) + \mu_1}.$$

The exponential stages are shown graphically on Fig.1.

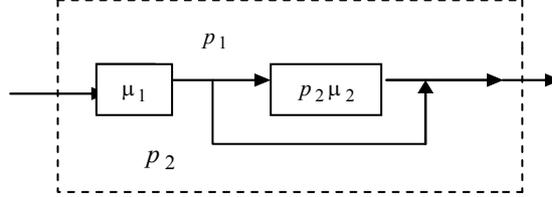


Fig. 1. The diagram of two exponential phases

#### 4. State Space Representation and Transitions

In this section we will use the so-called the event language to generate the space of stock prices and transition matrix between them. We approximate the distributions of price movement time up and down from the historical data of selected stock prices by the formulas respectively

$$f_u(y) = p_1^u \mu_1^u \left( \frac{\mu_2^u - \mu_1^u}{\mu_2^u p_2^u - \mu_1^u} e^{-\mu_1^u y} - \frac{\mu_2^u p_2^u}{\mu_2^u p_2^u - \mu_1^u} e^{-\mu_2^u p_2^u y} \right),$$

$$f_d(y) = p_1^d \mu_1^d \left( \frac{\mu_2^d - \mu_1^d}{\mu_2^d p_2^d - \mu_1^d} e^{-\mu_1^d y} - \frac{\mu_2^d p_2^d}{\mu_2^d p_2^d - \mu_1^d} e^{-\mu_2^d p_2^d y} \right).$$

A Markov's chain with the countable space of states and continuous time can describe the dynamics of stock price movement. To construct a numerical model of the system, the approach proposed in [13] will be applied.

The set of events in the system:

$$E = \{e_1^u, e_2^u, e_3^u, e_4^u, e_1^d, e_2^d, e_3^d, e_4^d\},$$

where

$e_1^{u(d)}$  – beginning of price movement up (down);

$e_2^{u(d)}$  – completed the stage of price movement up (down) with probability  $p_1^{u(d)}$  in the first phase;

$e_3^{u(d)}$  – completed the stage of price movement up (down) with probability  $p_2^{u(d)}$  in the first phase;

$e_4^{u(d)}$  – completed the stage of price movement up (down) in the second phase.

The set of transition rates:

$$Intens = \{\mu_1^u, p_2^u \mu_2^u, \mu_1^d, p_2^d \mu_2^d\},$$

where

$\mu_1^{u(d)}$  – completion rate of price movement up (down) in the first phase;

$\mu_1^{u(d)} p_2^{u(d)}$  – completion rate of price movement up (down) in the second phase.

Let us consider an asset observed on a discrete time scale  $\{0, 1, \dots, t, \dots, T\}$ ,  $T < \infty$  having  $S(t)$  as market stock value at time  $t$ . To model the basic stochastic process  $(S(t), t = 0, 1, \dots, T)$  we suppose that the asset has known minimal and maximal values so that the set of all possible values is the closed interval  $[S_{\min}, S_{\max}]$  [14]. For example, if  $S_0$  is the value of the asset at time 0, we can put

$$S_0 = (S_{\max} + S_{\min}) / 2$$

$$S_k = S_0 + k\Delta, k = 1, \dots, n$$

$$S_{-k} = S_0 - k\Delta, k = 1, \dots, n$$

$$\Delta = (S_{\max} - S_{\min}) / 2n$$

$n$  being chosen arbitrarily. This implies the total number of states is  $2n + 1$ . In what follows, we order these states in the naturally increasing order and use the following notation for the state space:

$$I = \{-n, -(n-1), \dots, 0, 1, \dots, n\}.$$

We can also introduce different step lengths following movements up and down and thus consider respectively  $\Delta, \Delta'$ . It is also possible to let  $S_{\max} \rightarrow \infty$  and  $T \rightarrow \infty$  particularly to get good approximation results.

To model the dynamics of stock prices we need know the number of the phase in which is the stock price. The state space of the system is completely specified by the set of triples

$$B = \{(b_1, b_2, b_3)\}, b_1 \in I,$$

where

$$b_2 = \begin{cases} 0, & \text{if the stock price is not changing up;} \\ 1, & \text{if the stock price is moving up in the first phase;} \\ 2, & \text{if the stock price is moving up in the second phase.} \end{cases}$$

$$b_3 = \begin{cases} 0, & \text{if the stock price is not changing down;} \\ 3, & \text{if the stock price is moving down in the first phase;} \\ 4, & \text{if the stock price is moving down in the second phase.} \end{cases}$$

The dynamics of stock price movement can be described in the event language. As an example, the description of the fourth event is represented bellow.

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e4u :      if  b2 = 2  and  b1 < n
              then b1 ← b1 + 1; b2 ← 0
              Intense ← μ1u p1
              end then
            end if
end e2u .
    
```

The software for automatic construction of numerical models is created. The software consists of the following:

- The language of a model specification.
- A program for automatic generation of all possible states (the set of stock prices) and transition rates among them.
- A program for calculation of steady state probabilities of Markov's process.
- A program for calculation performance measures.

The states of Markov's process are generated from an initial state. All possible transitions from this state are considered. When this step is completed, the current state is marked, and one of the newly obtained states becomes the current state. The generation process terminates when all the states in the list have been marked and no new state is obtained. The stationary probabilities of the states are calculated solving the system of Chapman-Kolmogorov's differential equations when the derivatives equal zero. The probabilities of stock price states are calculated by the following formula:

$$P(k) = \sum_{b_2, b_3} \pi(k, b_2, b_3), k = -n, \dots, n,$$

where  $\pi(k, b_2, b_3)$  is the probability of the price at the fixed phase.

## 5. Conclusions

This paper gives a continuous time Markov's chain model for asset dynamics. It allows solving of a large size problem. The model can be applied for pricing of option financial products working in continuous time and with countable number of possible values for the imbedded asset, which is always the case from the numerical point of view. The main interest of this model is that it works even when there are possibilities of arbitrage, i.e. the most frequent cases. Further research will be devoted for pricing derivative securities based on the continuous time Markov's chain model.

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# MODELLING AND FORECASTING OF LABOUR MARKET IN LATVIA

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Problems of Latvian labour market and its possible development are analysed. An econometric model for labour force demand and supply forecasting is elaborated; it comprises 120 professions, 37 aggregated groups of professions and covers time period from 2007 till 2030. The results of the quantitative and qualitative Employers' Survey are analysed and taken into account.

**Keywords:** *labour market, labour force supply, labour force demand, production function, forecast*

## 1. Introduction

Development processes are underway in the economy of Latvia and comply, to a large extent, with guidelines of strategic documents on the development of national economy and employment in the country, reflecting goals and tasks defined by EU programmatic documents for the areas of competition and employment. The main goal in ensuring social development, prescribed by the long-term economic strategy of Latvia and the Single National Economy Strategy, is to raise the employment level to 70% during the coming 20-30 years and to reduce the unemployment rate to the natural unemployment level [1], [6].

During the development process of the national economy of Latvia the labour market structure undergoes not only quantitative changes but also qualitative changes creating disproportions between the labour force supply and the labour force demand in sectors of national economy, which generates a range of problems in implementing national employment and social development programmes [2], [4].

In these conditions enterprises have to invest not only in capacity development but mainly in efficiency improvement to reduce the negative impact of the growth of labour force costs on competitiveness and to address the problem of labour force deficit. The question of a more efficient restructuring of sectors of national economy, that would promote a more rapid development of sectors with a higher value-added as well as the question of developing a more flexible and balanced labour market, taking into account the actual demographic situation of the society and its development forecasts become all the more topical. In order to develop successfully in future, the national economy of Latvia needs significant structural changes and reorientation to the production of products and services with a higher value-added, essentially increasing labour productivity and competitiveness in the European and global markets. A more detailed analysis of national employment and education policies highlights certain changes required in these areas [5], [7].

## 2. Econometric Model of Forecasting

The labour force demand forecast for Latvia undertaken during the research is based on production functions developed that were determined for 15 sectors of national economy in line with the convergence and slow convergence scenarios for the development of national economy developed by the Ministry of Economy. During the demand forecasting the number of the economically active population by groups of profession is established by applying economic development forecasts and demographic forecasts. Assuming that preferences of choice remain unchanged, labour force demand forecasts are formulated for groups of professions [12].

Production functions are formulated by sector of national economy sectors for forecasting purposes in the course of conducting the analysis on the disparity between the labour force supply and demand by 120 professions and the 37 aggregated groups of professions, taking as the basis the existing studies on the labour market in Latvia (the Survey of Professions of the Central Statistical Bureau and the survey of 20 025 employees within the frame of the study "Compliance of the Professional and Higher Educational Programmes to the Demands of the Labour Market" of the project of the Ministry of Welfare of the Republic of Latvia "Labour Market Studies" of the National Programme "Labour Market Studies" of the European Union Structural Funds), as well as using methodical guidelines of analogous foreign studies (the experience of Ireland, Sweden, the Czech Republic and other countries) [8], [9], [10].

The Cobb-Douglas production function (hereinafter C-D production function) is very widely used in economic studies and forecasting, analysing the return of the production factor (capital and the labour force), as

well as in estimating the level of technological process and the trend in the country. It is general practice to use linear design methods for assessing the C-D production function. In order to assess the production function the standard C-D production function with constant returns of scale is frequently applied, which has the following presentation:

$$Y_t = A_t K_t^\alpha L_t^{1-\alpha}, \quad (1)$$

where  $Y_t$  is GDP in comparable prices,  $A_t$  is the total productivity of factors or the technological progress,  $K_t$  is the actual volume of the accumulated capital,  $L_t$  is the labour force in the economy,  $\alpha$  is the return on capital.

Testing of Latvian data showed that the C-D production function with constant returns of scale could not be applied for each individual sector of economy. The best results are given by the C-D production function with a variable return of scale:

$$Y_t = A_t K_t^\alpha L_t^\beta, \quad (2)$$

where the variables remain the same as for the K-D production function (1), in which  $\alpha$  is flexibility or the return on capital and  $\beta$  -flexibility or return on the labour force.

In designing the linear type C-D production function, it is assumed that the aggregate development of the productivity of factors in the course of time or the technological progress is determined as follows:

$$A_t = A_0 e^{t^{\frac{1}{\lambda} + \varepsilon_t}}, \quad (3)$$

where  $A_0$  is the primary level of technology,  $e$  is the exponent,  $t$  is the trend of productivity, which is dependant on time,  $\lambda$  determines the increase in the productivity trend,  $\varepsilon_t$  is the assessment error – a stochastic process.

By combining the equations (2) and (3) and applying a logarithm, we obtain an equation, which is used in the linear designing, in determining returns of the production factor and technological process:

$$\log(Y_t) = a + \lambda \frac{1}{t} + \alpha \log(K_t) + \beta \log(L_t) + \varepsilon_t. \quad (4)$$

The inverse production function against employment in theory will be represented as follows:

$$\log(L_t) = \frac{1}{\beta} \log(Y_t) - \frac{a}{\beta} - \frac{\lambda}{\beta} \frac{1}{t} - \frac{\alpha}{\beta} \log(K_t) - \frac{1}{\beta} \varepsilon_t. \quad (5)$$

Changes in the capital depend on the extent of returns on investments in the respective sector. A sector with a relatively higher level of return attracts investments and demonstrates a relatively higher capital growth, which at the given growth of the sector and the technological progress rate means a relatively lower increase of the employment level. Similarly a sector with a relatively higher rate of technological progress means a relatively lower increase in employment. It is necessary to note that the value-added in the sector of services is estimated on the basis of the amount of production factors involved in the production, because the technological progress in this sector is of relatively less importance. Thus the value-added and employment in the sector of services has a tendency to increase with an identical rate.

The labour force demand is forecasted in the break-down by profession on the basis of labour force demand forecasts by sectors. To implement this, an employment matrix of sectors-professions is developed, containing ratios of representatives of each profession in each sector.

The above-mentioned employment matrix of sectors-professions consists of the following elements  $L_{ij,t}$ :

$$\left| \begin{array}{cccccc|c} L_{1,1} & L_{1,2} & \cdot & \cdot & \cdot & L_{1,15} & L_1^O \\ L_{2,1} & L_{2,2} & \cdot & \cdot & \cdot & L_{2,15} & L_1^O \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ L_{120,1} & L_{120,2} & \cdot & \cdot & \cdot & L_{120,15} & L_{120}^O \\ \hline L_1^I & L_2^I & \cdot & \cdot & \cdot & L_{15}^I & L^A \end{array} \right|,$$

where  $L_{ij,t}$  is the number of the employed in the profession  $i$  ( $i=1,..,120$ ) and sector  $j$  ( $j=1,..,15$ ) during the time period  $t$ . Respectively

$$L_{j,t}^I = \sum_{i=1}^{120} L_{ij,t} \quad (6)$$

is the number of the employed in the sector  $j$  during the period  $t$ ,

$$L_{i,t}^O = \sum_{j=1}^{15} L_{ij,t} \quad (7)$$

is the number of the employed in the profession  $i$  during the period  $t$ , and

$$L_t^A = \sum_{i=1}^{120} L_{i,t}^O = \sum_{j=1}^{15} L_{j,t}^I \quad (8)$$

is the total number of the employed during the period  $t$ .

For 37 aggregated profession groups all calculations are carried out in a similar manner as for 120 professions, by replacing 120 professions with 37 aggregate groups.

The calculation of the matrix and evaluation of the effect of the ratio of the sector and professions is undertaken according to the following algorithm: based on the data of the number of the employed in the breakdown by sector and profession for the time period from 1997 till 2005, the trend time-series of the number of the employed for each sector  $j$  ( $j = 1.., 15$ )  $L_{j,t}^I$  and every profession  $I$  ( $I = 1..120$ )  $L_{i,t}^O$  are developed.

The sector-profession employment matrix is obtained from the temporary sector-profession employment matrix, using the sequence and column totals obtained in the 1st step (respectively  $L_{j,t}^I$  and  $L_{i,t}^O$ )  $L_{j,t}^I$ , using RAS (*restricted additive Schwarz*) method.

Time-series with ratios of 120 professions in sectors are obtained as follows:

$$S_{ij} = L_{ij,t} / \sum_{i=1}^{120} L_{ij,t} \quad (9)$$

It allows estimating the mean annual variations of the specific ratio trends of professions  $\Delta s_{ij}$ , which will be considered constant within the forecast period.

We will assume that the year 2007 is the beginning of the forecast period ( $t=1$ ) and the year 2020 is the end of the period ( $t = 13$ ). Growth rates of the sector labour force demand  $\gamma_{j,t=1}^I$  for the time period of 2007 to 2020 were obtained, using forecasts produced by the Ministry of Economics and with the help of the rates appraised by the production function. The number of the employed was calculated as  $L_{j,t=1}^I = [1 + \gamma_{j,t=1}^I] L_{j,t=0}^I$ , ratios of professions as  $S_{ij,t=1} = S_{ij,t=0} + \Delta S_{ij}$  and elements of sector-profession employment matrix as:  $L_{ij,t=1} = S_{ij,t=1} L_{j,t=1}^I$ . The sum totals of the sector-profession employment matrix elements by sequences form the time-series for the breakdown of the number of the employed by profession, whereas the sum totals of sector-profession employment matrix elements by columns constitute the time-series  $L_{j,t=1}^A$  of the number of the employed by sector.

The previous step is being repeated for each subsequent year, up to and including 2020.

The growth rates of the number of the employed are calculated as follows:

$$\gamma_T^A = [L_{20}^A - L_0^A] / L_0^A \quad (10)$$

$$\gamma_{j,T}^I = [L_{j,20}^I - L_{j,0}^I] / L_{j,0}^I \quad (\text{in the breakdown by sector}), \text{ and} \quad (11)$$

$$\gamma_{i,T}^O = [L_{i,20}^O - L_{i,0}^O] / L_{i,0}^O \quad (\text{in the breakdown by profession}). \quad (12)$$

The total effect of the sector ratio for a profession is calculated as follows:

$$e_{i,T}^I = \left[ \sum_{j=1}^{15} L_{ij,t=0} (1 + \gamma_{j,T}^I) - L_{i,t=0}^O \right] / L_{i,t=0}^O - \gamma_T^A \quad (13)$$

where the total  $\gamma_T^A + e_{i,T}^I$  is the growth rate of the number of the employed in the profession  $i$  on condition, that the distribution of professions in sectors in the forecast period remains unchanged.

The total profession ratio effect for the profession  $i$  is calculated as follows:

$$e_{i,T}^O = \gamma_{i,T}^O - \gamma_{i,T}^A - e_{i,T}^I, \tag{14}$$

where  $e_{i,T}^O$  is the part of the growth rate of the number of the employed in the profession  $i$ , which is explained by the changes in the distribution of professions in sectors during the forecast period.

When developing the forecast for the supply, a very simple method is selected, based on demographic forecasts. In many countries the supply is forecasted on the basis of forecasts of university graduates in terms of specialities however it does not allow to determine the supply by profession, because in practice there are cases, when upon graduating a certain discipline the person works in a totally different field.

Thus, in the present study according to the selected methodology the number of the inhabitants is projected first, then – the level of economic activity, which determines the forecasts for the labour force supply in the national economy in general. It is assumed in the forecasts that preferences of the inhabitants in the choice of professions will not change and are remaining the same as in 2005. Taking into account that a decrease in the number of the population is projected in future, the forecast is that the labour force supply will decrease in all professions.

The research study establishes the forecast demand and supply of the labour force in 120 selected professions and in 37 aggregate groups during the period from 2007 till 2013 as well as trends and visions for 2020 and 2030.

The production function has been identified for each sector of national economy in two variants in line with the approved national economy development scenarios of convergence and the slow convergence. As economy of scale may be observed in economy in general, but in the analysis by specific sector it may not be the case, the general Cobb-Douglas functions have been identified. Each sector of national economy has been identified its own function. The assessed production function of each sector was tested for the significance of coefficients assessed by these functions. The undertaken analysis of equations of production functions shows that the distribution of the balance not explained by equations is, to a large extent, with the normal distribution in all equations. A more profound analysis of errors in production function equations shows that, irrespective of the short time-series from the CSB (1997-2005), equations do not suffer from serial correlation, autocorrelation and heteroskedasticity. Tests prove the quality of equations. On the whole, it may be concluded that production functions have been structured in line with a specific task and may provide reliable forecasts according to the approved scenarios of economic development.

The acquired production functions are at the basis for the calculation of coefficients  $C, \alpha_0, \alpha_1, \alpha_2$  for the logarithmical equation of the labour force

$$\log L = C + \alpha_0 \frac{1}{t} + \alpha_1 \log K + \alpha_2 \log Y. \tag{15}$$

**Table 1.** Coefficients of the labour force logarithmical equation

Sector	C	$\alpha_0$	$\alpha_1$	$\alpha_2$
A	11.82	0.90	0.21	-0.67
B	6.49	-0.44	0.28	1.50
C	6.07	8.34	0.38	1.28
D	5.17	-30.29	0.06	1.44
E	9.75	-30.03	0.01	0.51
F	6.93	-11.53	0.17	1.00
G	-0.89	0.15	0.00	2.69
H	7.40	-35.39	0.03	1.02
I	1.08	-37.22	0.37	2.18
J	9.64	-25.95	-0.03	0.72
K	7.36	0.01	0.04	1.12
L	8.11	-10.14	0.02	0.84
M	7.91	-9.25	0.01	0.73
N	9.21	-8.95	0.05	0.33
O	10.01	-21.54	-0.04	0.41

*Note:* Calculations of the production functions and the labour force logarithmical equation are based on the data of the Central Statistical Bureau (CSB) of the Republic of Latvia (1997-2005).

The coefficient  $\alpha_1$  in Table 1 shows the change in the use of the labour force in percentage terms if the accumulated capital (K) increases by 1% but GDP in comparable prices remains unchanged. In a similar way the coefficient  $\alpha_2$  shows the change in the use of the labour force in percentage terms if GDP in comparable prices increases by 1% while the accumulated capital (K) remains unchanged. Table 1 shows that, for example, the use of the labour force in sector B increases by 0.28% if the accumulated capital (K) is increased by 1% but GDP in comparable prices remains unchanged; and in the same sector the use of the labour force increases by 1.50% if GDP in comparable prices increases by 1% while the accumulated capital (K) remains unchanged.

Coefficients of the acquired production function and the labour force logarithmical equation are used for the calculation of the labour force demand forecasts. Like production functions the labour force forecasts in sectors of national economy are also calculated in two variants according to the convergence scenario of the development of national economy and the slow convergence scenario of the development of national economy.

The forecasting of the labour force supply is based on the data of surveys on the number of employees, gender and the age structure, employment by profession and preferences in choosing an occupation (profession).

The comparison of results of the labour force demand forecasts with the acquired forecast results on the labour force supply in specific years of the forecast period provides information about the potential imbalance of the labour force demand and supply in the identified 120 professions or in the aggregate groups of professions in the forecast period.

The average values of changes in the ratios of groups of professions are calculated by using the historic CSB data for the period from 2002 till 2005. The mean values of changes in the ratio of groups of professions are retained for the whole forecasting period, calculating a new distribution of ratios for each forecast year.

The labour force supply is projected by using demographic forecasts. They allow establishing the number of the population and consequently to determine the number of the economically active population by employing economic development forecasts. On the assumption that priorities of the choices made by people remain unchanged, the labour force supply forecasts are formulated.

The description of the research methodology shows that there are a quite large number of employees in Latvia working in two and more jobs. Coefficients have been calculated in the course of the research that determines the ratio of jobs against the actually working employees. Taking into consideration this fact, it is possible to adjust the forecast required labour force amount. Forecast adjustments have been made for the 120 selected professions as well as the 37 aggregate groups of professions.

The analysis of the acquired forecasts shows that in 2007 the insufficiency of employees appears in 82 professions of the 120 and in 2013 it might be experienced in 99 professions. The highest deficit of employees – in excess of 50% – is forecast in 2007 for the following professions: senior specialists of natural sciences (a deficit of 1567 employees), foresters and employees of related professions (a deficit of 2452 employees), and agricultural, ground work, lifting and other loading/unloading equipment operators (a deficit of 13946 employees). In 2007 a significant surplus, provision exceeding 110% might develop in such professions as civil security and defence specialists (a surplus of 1376 employees), pharmacists (a surplus of 497 employees), stall and market salespersons (a surplus of 1 741 employees), manufacturers of furniture and other wooden products (a surplus of 779 employees) and employees of other types of services (a surplus of 248 employees).

Figure 1 shows the forecast on the labour force demand and supply for those 6 aggregated groups of professions where the highest insufficiency of employees might appear. The number of employees has been given on the vertical axes. These graphs show the forecast number of employees (assuming that employees work in several jobs on part-time basis).

The large discrepancies in the results of the medium-term forecasts (2007-2013) for the labour force demand and supply in separate professions can be explained by several factors:

- the low “prestige” of professions in the preferences of the population that is significantly influenced by the low remuneration level;
- employment of employees in several additional jobs that is not shown with sufficient precision by the initial data;
- the insufficient assessment of the development of the technological progress;
- the insufficient activity of entrepreneurs in capitalizing the production processes in practice may lead to a significant decrease in the labour force demand.

The comparison of forecasts for 120 professions and 37 aggregate groups of professions shows that, undoubtedly, the forecasting for the aggregate groups of professions provides more precise results. It is confirmed by the first forecasts that have been acquired, i.e., the forecasts on the ratios of professions. Significant deviation errors in excess of the permissible precision interval appear for the forecasting period that exceeds 3 years, for 120 professions. Therefore forecast calculations for specific professions are permissible for a very short period – two to three years. When calculating forecasts for the 37 aggregate groups of professions, already the forecasts on the ratios of groups of professions show that deviation errors do not appear. Thus,

undeniably, forecasting for the 37 aggregate groups of professions provides more precise forecast results for short-term period and in particular for the medium-term and long-term period.

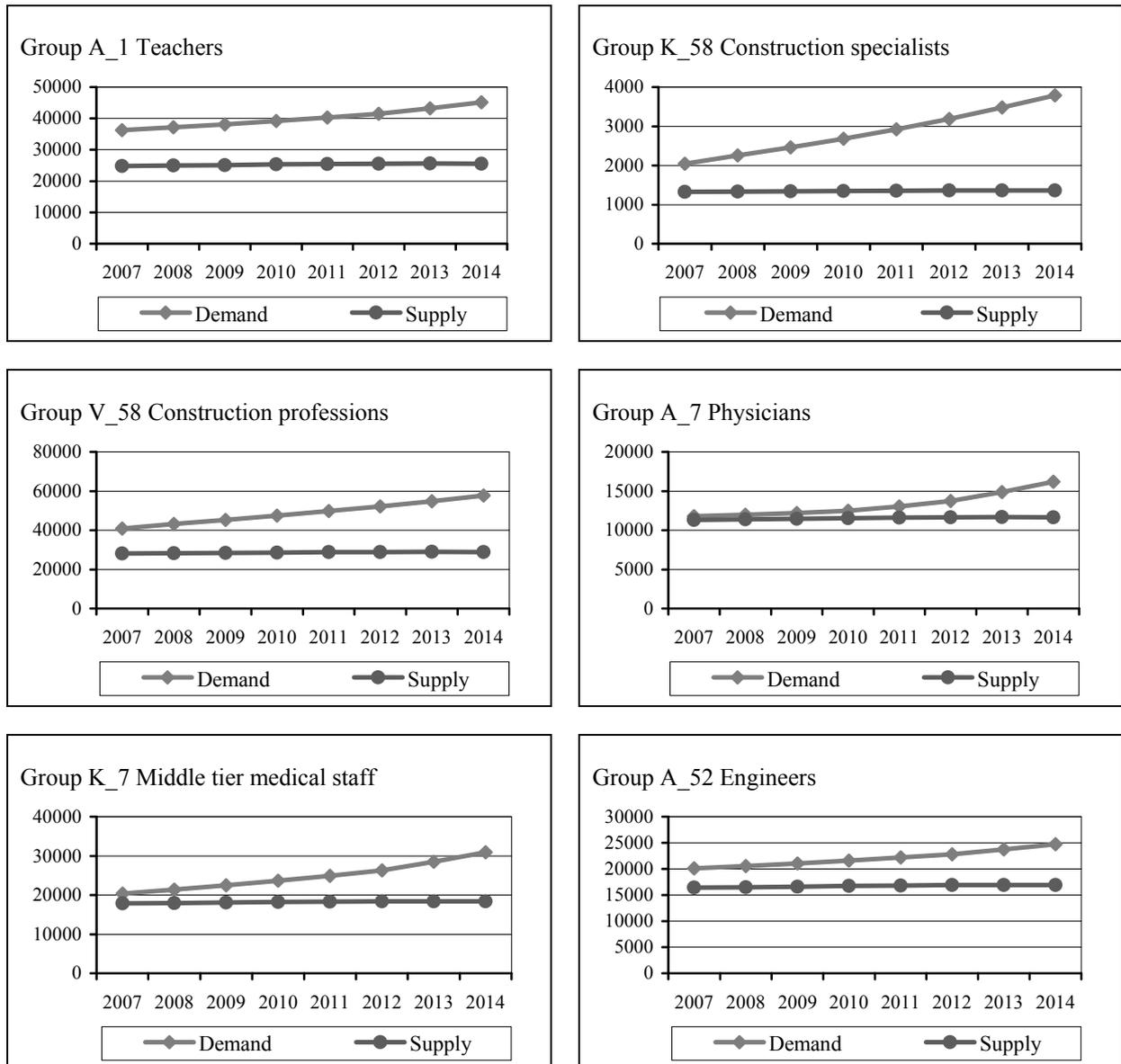


Fig. 1. Forecasts of the labour force demand and supply by group of professions

Source: Calculations done by the researchers.

The medium-term forecasts (2010-2013) show that the highest growth of the labour force demand is expected in the sectors of construction, manufacturing and mining and quarrying. In order to satisfy the labour force demand in these sectors, the educational system must create the necessary capacity for the training of specialists in the required professions. The additional demand for the labour force might be reduced by increasing the capital in agriculture, fishing, mining and quarrying, manufacturing, wholesale and retail sale, transport and communications as well as financial intermediation. The increase of capital will have the most significant impact on sectors of industry. The improvement of labour productivity is the most realistic option for reducing the number of employees in construction, wholesale and retail sale, in the sector of hotels and restaurants.

The analysis of the acquired short-term forecasts has allowed detecting that in 2007 the insufficiency of employees appears in 106 professions of the 120. For example, the highest insufficiency of the labour force is found in the following professions – building construction workers, construction electricians, senior project management specialists, labour safety inspectors, physicians, and electro-mechanic specialists. In the medium-term 2010-2013 the highest increase in the insufficiency of labour force is projected in the following professions:

construction engineers, electric engineers, dentists, other specialists of physics and engineering, computer equipment operators. Coefficients of the labour force logarithmical equation allow assessing the impact of various factors on changes in the labour force demand. An increase in the capital in agriculture, fishing, mining and quarrying, manufacturing, wholesale and retail sale, transport and communications as well as financial intermediation will allow reducing the number of the labour force employees in these sectors. In this respect the increase in capital will have the greatest impact on both sectors of industry.

The results of employers' qualitative and quantitative surveys are used in assessing the results acquired as a result of forecasting in particular in the long-term (2014-2020) in cases of the possible imbalance in the labour force demand and supply, that also confirmed difficulties encountered by practitioners in assessing trends in the development of the labour market and employment by professions or groups of professions in the long-term. The interpretation of the acquired forecasting results may be significantly influenced by the situation that has developed. Employers' surveys confirm that almost 52 % of employees do not work in professions of their acquired education. Labour market studies also reveal a situation that employees occupy simultaneously several jobs (in 2005 it was on the average 14.6% of the total number of employees in national economy) and adjustment coefficients are applied to specific professions during the forecasting process.

### **3. Conclusions**

Data bases of the official statistics on the labour market of Latvia provide information, which until now has not been analysed in the country. Project implementers used data provided by the CSB to develop for the first time in the country time-series (1997-2005) about the number of occupied jobs, the number of employees working in principal jobs and other indicators characterising the labour market, and analysed these indicators by sector of national economy, by profession and by group of professions.

Upon the assessment of results provided by forecast, the achievement degree of tasks identified by the forecasting of the labour force demand and supply by time horizons and the reliability of forecasts, researchers established that forecasts by 120 professions are more appropriate for short-term and medium-term forecasting, forecasts by 37 aggregate groups of professions – for medium-term and long-term forecasting as well as for the formulation of the labour market development visions.

The considerable disparities in forecast results on the labour demand and supply by specific professions can be explained by the following factors: the "low" prestige of specific professions in the preferences of the population that is, to a considerable degree, influenced by the low remuneration and the assessment of the significance of these professions in the community; employees working in several that is not reflected by statistical data with sufficient precision; the insufficient assessment of the development of the technological process; the insufficient activity of employers in capitalising the production process that can maintain an unjustifiably high labour force demand in specific professions in future.

The analysis of results of the Employers' Survey focus attention on insufficiently resolved issues in the labour market policy and generates reflections about respondents' understanding of relevant aspects promoting the development of economy, for instance, the impact of the technical progress on labour productivity. Results of the Survey confirm the orientation of employers towards an extensive increase in the labour force in future, which contradicts conclusions of demographic forecasts about the expected decline in the number of the population and the number of inhabitants capable of work. It is nationally important to achieve the understanding of employers about the actual demographic processes expected in the near future and to stimulate reorientation from an extensive approach (an increase in the number of employees) to an intensive approach (the improvement of labour productivity) in using labour resources.

Data of the qualitative survey of employers undertaken within the frame of the research provide an assessment of the current situation of the Latvian national economy. Positive factors that are mentioned are as follows:

- the growth of investments,
- the improvement of the quality level of goods and services,
- increased cooperation with foreign enterprises and
- the growth of exports.

In their turn, negative trends that are mentioned include the arbitrariness of officials in Latvia in interpreting European Union requirements, symptoms of the "overheating" of the economy (the high and lasting inflation, unjustifiably easy access to loans, in particular in the real estate market, the high ratio of speculative business in entrepreneur activities), difficulties encountered by employers in paying the wages required by the employee as well as the under-motivated labour force.

When describing problems of the Latvian labour market policy, respondents-employers emphasize the excessive protection of employees' rights provided by legislation, as a result it is difficult for employers to get

rid of unprofessional and negligent employees. The vulnerability of employers is emphasized as concerns their investment in the professional growth of employees that stimulates the use of the illegal labour force and the system of “salaries in envelopes”, reduces motivation to send employees to upgrading courses and reduces the motivation of employers to work effectively within the frame of an official employment contract.

Data of the Employers' Survey show that the existing disproportions in the labour market development, violations of labour legislation and the existence of the “shadow economy” distort, to a certain extent, the labour market (12.3% of employees working in the public sector have had violations of labour law), creating high-risk professions and even sectors (the main risk sectors are security, construction, agriculture and forestry). The labour force in general as well as in separate professions is influenced by internal and external migration factors creating a “relative” labour force deficit in the main sectors of national economy. Respondents' answers concerning ways of addressing the labour force shortage problem that in almost 20% of cases the hiring of new employees is simultaneous with the reduction of requirements concerning applicants' education and experience, is a disturbing symptom that in future may lead to a significant decline in the quality and competitiveness of the labour force.

The considerable disparities in forecast results on the labour demand and supply by specific professions can be explained by the following factors: the “low” prestige of specific professions in the preferences of the population, i.e. to a considerable degree, influenced by the low remuneration and the assessment of the significance of these professions in the community; employees working in several that is not reflected by statistical data with sufficient precision; the insufficient assessment of the development of the technological process; the insufficient activity of employers in capitalising the production process that can in future maintain an unjustifiably high labour force demand in specific professions.

Labour market studies confirm the fact that in Latvia a large number of employees simultaneously occupy several jobs. In 2005 the number of people working in extra jobs constituted 14.6% of the total number of the working population in Latvia. During the forecasting process adjustment coefficients that determine the actual number of employees, have been calculated and used for specific professions and sectors.

According to production functions an absolutely identical development in both scenarios of convergence and the slow convergence is projected in such sectors as public administration and defence, education, health and social care, community, social and individual services.

The analysis of the labour force demand by region and by gender for the selected 120 professions, confirmed in numerical terms the statement about the existence of the so-called “male” and “female” professions as well as numerically small professions and professions with marked employment fluctuations. Professions of the last two groups make forecasting more difficult as in numerically small professions ratios are practically 0 while professions where employment fluctuates, have very changing ratios that may also cause imprecision in forecasting. The fluctuation of professions was discovered through the analysis of the labour force demand by factors of impact at the regional level.

It is necessary to disperse the stereotype prevailing among labour market research commissioners and often also among project implementers that a new research study means also a new survey. From the point of national economy it is more efficient to channel funds consistently into the improvement of the official statistics for regular labour market forecast needs. It is appropriate establish samples of labour market – related companies, institutions and organisations on the basis of the Enterprise Register of the CSB as the population source. The Employers' Survey and the Job-seekers' Register of the Employment State Service, conjuncture surveys of the Latvian Institute of Statistics may be effective information sources also in future for the improvement of the labour market in the country.

It is recommendable to use the method for calculating the coefficient of the number of employees in principal work against the number of occupied jobs in sectors (regions) as well as in professions in the subsequent labour market studies. They have been used in forecasting the labour market supply, which will give an opportunity to considerable increase the reliability of forecast results.

The researchers of the given work recommend organising the Register of Employees (the State Revenue Service being the potential holder of the register) as the most perspective source of data for labour market analysis and forecasts.

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# SIMULATION OF SUPPLY CHAIN RELIABILITY

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This paper investigates problems related to supply chain risk identification and simulation-based risk assessment. Accordingly, the paper is divided into two logical parts. The first part represents earlier researches about risk recognition within the sphere of supply chains. The distinction between terms “uncertainty” and “risk” is discussed. Based on the predefined supply chain functions, risk is recognized as possible disruptions that can affect supply chain’s ability to function normally. Thus, it is advised to use a system reliability analysis as an approach to supply chain risk assessment. Within the second part of the paper, the use of simulation is advised as an effective approach for supply chains reliability evaluation. A simplified supply chain system is defined and corresponding supply chain reliability simulation model is created. Simulation results are validated using a mathematical description that proves the adequacy and effectiveness of simulation application. Then, it is stated that the mathematical reliability models application is limited due to potential complicity and risk quantification features.

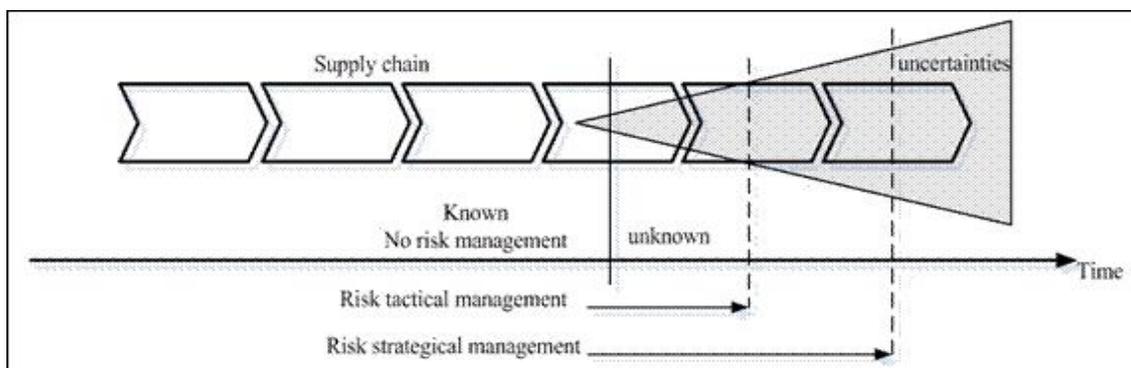
**Keywords:** *uncertainty, risk, reliability, supply chains and simulation*

## 1. Introduction

Nowadays information technologies help in managing large and complicated supply systems. Supply chain management provides wide profit increase opportunities for all potential members of its structure. But actually, as the range of available information becomes larger, the amount of uncertain factors increases too, and disruptions, which can be produced by those factors, become more destructive. Currently, risk handling is becoming a very important part of the supply chain management. Still, the term supply chain risk is not strongly defined yet; it is a topic for the latest researches. Here, it is suggested that the definitions and goals for supply chain managements under uncertainty and risk can be prescribed only from the predefined terms about supply chain, supply chain risk and uncertainty factors. These terms are not established too. So, at the beginning of this paper, a description for these terms is provided with the goal to estimate the term – supply chain risk.

### 1.1. Uncertainty and risk

Though risk and uncertainty has been an object of very extensive researches in various scientific directions for many years, the clear unique formulations of its meaning have not been stated yet. The attempts to define the terms uncertainty and risk have spawned a large variation of approaches, evidence the broad themed emanation from various academic disciplines [2]. There are at least two schools of thought on the issue, if risk and uncertainty have the same meaning; thus the clear definitions of the mentioned terms have to be provided.



**Fig. 1.** Risk management horizon

One way to distinguish between the two terms relies on the ability to make probability assessments. Then, risk corresponds to events that can be associated with given probabilities; and uncertainty corresponds to events for which probability assessments are not possible. This suggests that risky events are easier to evaluate, while uncertain events are more difficult to assess. According to another school of thought, there is no sharp distinction

between risk and uncertainty because of the problem that there is no clear consensus about probability term [3]. The current paper investigates a modified idea about risk and uncertainty collaboration. It is suggested that in general, the risk can be described by a few parameters like risk origin reasons, risk circumstances and risk origin probability [7]. Then, uncertainty can be declared as the risk causing factor, which forms a changing environment, in which risky events may occur. Any risky event is defined as an event that is not known for sure ahead of time, but the risk itself is defined as the potential harm that may arise in future due to some present processes or some future events[3, 5]. The time is defined as risk general characteristic, which declares things that have already occurred and are not the object of risk management anymore. At the same time uncertainty may vary over time; accordingly those changes may affect the ability to make probability assessments for risky events. This gives a reason to distinguish tactical and strategic risk managements, but not the risk and uncertainty management terms (see Figure 1). It is suggested that in the studied system any risk appears due to a changing environment formed by different uncertainties. Each uncertainty can cause a risk event in the future. Yet, not all uncertainties produce risk for certain system functionality.

## 1.2. Supply chain

Similar uncertainties and risk are considered in supply chain systems. Business environment can be characterized as turbulent that means an environment with high uncertainty, rapid change, novel markets, high margins and very demanding customers. Thus, it consists of many uncertain causing risks factors (see Figure 3). The possible destructiveness might be supply chain profit decline or unsatisfied end customer demand.

As for many complex systems, there is no single definition established for a supply chain, different researchers provide various definitions, which more or less reflect the goals of supply chain management. When different perceptions of supply chain nature are possible, the mean of supply chain risk can't be established. Accordingly, there are many various opinions about supply chain risk in different references.

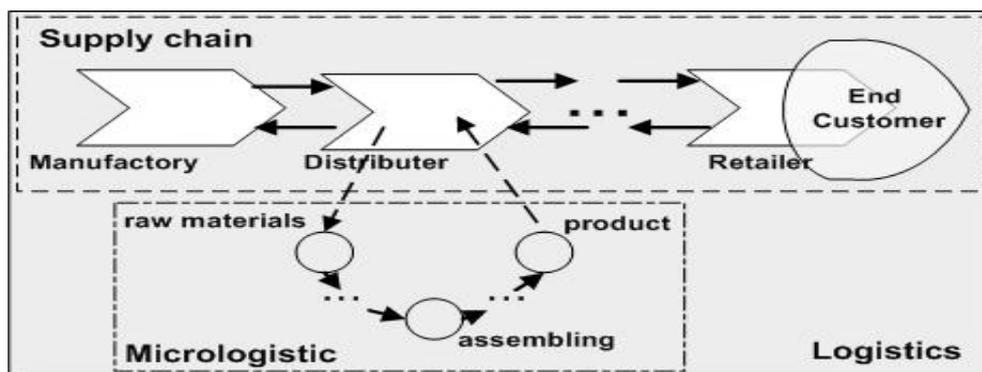


Fig. 2. Supply chain as a part of total logistics system

Within the current research, the supply chain is defined as a network of organizations involved into managing material, information and financial flows, that is necessary for realization of basic logistics operations (manufacturing, storing, transportation etc.), aimed to furnish the end customer with needed products or service produced by those organizations. It is very important to distinguish between supply chain management and detailed logistics (micro-logistic) inside separate organizations. Here, it is accepted that the function of supply chain management is to make optimisations aiming to minimize total logistics costs raised by finished production (finished within certain supply chain member) storing, transportation, etc. At the same time, supply chain management is part of total logistic management and ditto it is very similar to logistics systems inside separate organizations (see Figure 2).

## 1.3. Supply chain risk recognition

A precise understanding of risk essence is the most important point in risk management. Speaking about supply chain risks, most researchers observe risks connected with separate disruptions within discrete supply chain members. Then, supply chain management influence on particular risks values is considered. For instance, such risks can be prescribed as deficiencies in inventory, forecast failures, manufacturing quality failures, etc. Those risks are well-known to managers in the sphere of logistics. A more detailed discussion of particular risks is presented in our previous work [6]. The obvious disadvantage of such supply chain risk management is difficulties in recognition, how particular risks affect other supply chain members. So, these cases don't make difference between logistic individual risks within separate organization and a risk of disruption for a whole supply system. Accordingly, in best cases such perception allows evaluation of failure rates for single supply chain nodes, but not for the whole system.

On the other hand, risk management should be directed to increase reliability of the whole system (see Figure 3). For example, within the management process, logistic specialists might be interested to know: what a real reliability for the whole supply system is and how many critical failures occur during the chosen period of time; which suppliers are vital and should be provided by particular detailed management; how the chosen supply chain can be reconstructed in order to reduce critical failure risks, etc.

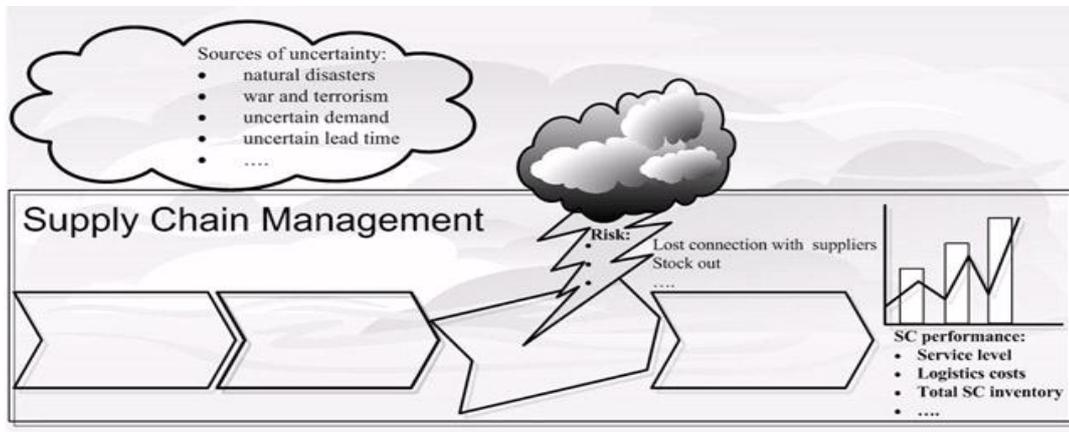


Fig. 3. Supply chain under uncertainty and risk

Let us advise three conditional states, which characterize supply chain system's ability to work properly. Normal condition defines supply chain normal work, which means material, financial and information stream run within the defined rules. At the same time some supplier failures are possible. For such cases, reserve suppliers or substitution materials should be used (see Figure 4a). Then, an emergency condition will be established, within which a final profit decreases, but the supply chain still continues to function. Cases, in which reserve or substitution are not possible, are called crises in the sense that it affects the ability to satisfy final customer. Here, it is accepted that risks are associated with the ability of supply system to change its conditional state in worse direction. So, supply chain risk is possible disruptions that can affect supply chain ability to function normally.

## 2. Simulation-Based Supply Chain Risk Evaluation

Risk evaluation is the next sophisticated step after risk recognition and identification. Commonly, risk assessment and evaluation are based on calculating a risky event occurrence probability with corresponding possible disruption values. There are three possible modes of probability evaluation: empirical, theoretical and subjective evaluation [4]. For previously defined concept of supply chains, risk evaluation might be based on varied mathematical methods. For instance, to evaluate characteristics of system transition between different conditional states, Markov process mathematics can be adopted; system stability for certain conditional states can be described by reliability theory. However, necessary mathematics becomes too labour intensive in large and volatile systems such as supply chains. At the same time, some researchers state that risk quantification builds on knowledge elicitation rather than on data collection. Data must only be considered as part of the available knowledge, the main source being human expertise [4]. Thus, the use of simulation is advised as an effective tool for development of a supply chain model, which not only simplify risk evaluation task within complicated systems, but also comply with expert opinion for input data realization.

The goal of the second part of the paper is supply chain reliability model development with different simplifications, which allows for necessary model validation with the help of mathematical evaluations. Further, the validated model can be used as a basis for more complicated and realistic system simulation.

### 2.1. System formulation

For further discussion, a simplified supply system should be described (see Figure 4a). It consists of six main and two substitution supply chain members. The first two suppliers provide the system with raw materials of two different types. Then, the rest relevant supply chain members assemble final product and deliver it to customers. The substitution supplier (Supplier 3) can deliver both types of material directly to the assembly if necessary. The usages of substitution supplier and substitution transportation intermediary (Delivery 3) increase supply chain total costs.

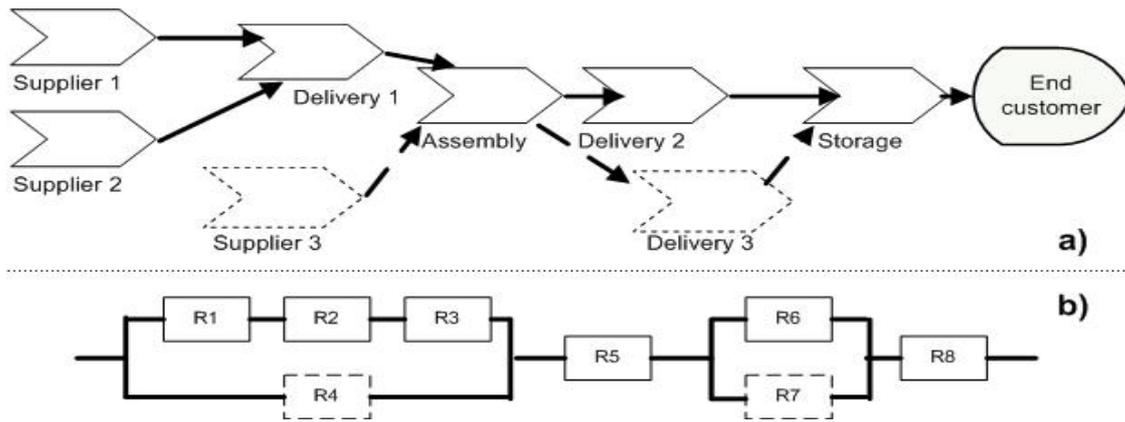


Fig. 4. Simple supply chain network

Table 1 provides individual characteristics about organizations involved in this network. It is assumed that the data about failure rates in particular supply chain nodes can be gained from organization managers (expert opinion). In a simple case, there are only two conditional states (failure and functioning); thus, characteristic parameters of the defined system reliability are an object of interest.

Table 1. Functioning characteristics of supply chain members

ID	Component mark	Supply chain member name	Failure rates, $\lambda$ days <sup>-1</sup>	
			Experiments 1-2	Experiment 3
1	R1	Supplier 1	$3,87 \cdot 10^{-4}$	$1,35 \cdot 10^{-4}$
2	R2	Supplier 2	$3,80 \cdot 10^{-4}$	$1,15 \cdot 10^{-4}$
3	R3	Delivery 1	$3,75 \cdot 10^{-4}$	$0,90 \cdot 10^{-4}$
4	R4	Supplier 3	$2,55 \cdot 10^{-4}$	$0,45 \cdot 10^{-4}$
5	R5	Assembly	$2,75 \cdot 10^{-4}$	$0,35 \cdot 10^{-4}$
6	R6	Delivery 2	$2,90 \cdot 10^{-4}$	$1,80 \cdot 10^{-4}$
7	R7	Delivery 3	$2,70 \cdot 10^{-4}$	$1,55 \cdot 10^{-4}$
8	R8	Storage	$1,85 \cdot 10^{-4}$	$0,25 \cdot 10^{-4}$

The system is defined as not repairable. Then, survival rate for a certain period of time is taken as a basic system reliability parameter that will be used for simulation model validation. The conditional state is defined in the following way:

$$X(t) = f(x) = \begin{cases} 1, & \text{if a system is functioning at time } t \\ 0, & \text{if a system is in a failed state at time } t \end{cases} \quad (1)$$

The other two assumptions are: a “burn-in” and a “wear-out” periods can be not taken into account, time failure functions can be describable by exponential distribution (as it is the most commonly used life distribution for technical system reliability analysis) [1, 8].

## 2.2. Reliability theory

As it was stated above, mathematics provides a wide range of methods for evaluating systems reliability characteristics. In this paper, reliability theory is applied to study the simple system. In practice, supply chain systems consist of more complicated networks. Moreover, in a real business, the assumption that a supply chain is a “non-repairable” system is not acceptable. Nevertheless, there are labour calculations even for a system with these assumptions, so it is possible to conceive about irrationality to perform more complex evaluations. At the same time, basic mathematical methods (which are provided in this paper) are necessary for further validation of the created simulation models.

So, within the studied supply chain, the reliability is defined as a probability that a system will not fail before predicted time moment T:

$$R(t) = \Pr(T \geq t) = \int_t^{\infty} f(u)du = e^{-\lambda t} \quad (2)$$

In the same way, from a known probability distribution (exponential) it is possible to define a mean time to failure:

$$MTTF = \int_0^{\infty} R(t)dt = \int_0^{\infty} e^{-\lambda t} dt = \frac{1}{\lambda}. \quad (3)$$

The above mathematics is usually used for describing a single component's (in most cases, a mechanical or electronic equipment) reliability. Still, supply chains consist of several separate nodes. The redrawn network of defined supply system is presented above (see Figure 4b). Then, a structural analysis is needed.

### 2.2.1. Combinations of simple series and parallel structures

When a network consists of only combinations of series and parallel structures, its analysis can be carried out in stages. Each stage simplifies the network by combining series and parallel sections. By using equations for series and parallel combinations, network sections can be obtained at each stage. The reduction process continues until one "super-component" remains, which links the start and end component [8]. The performance of the studied system is identical to the developed "super-component" (see Figure 5).

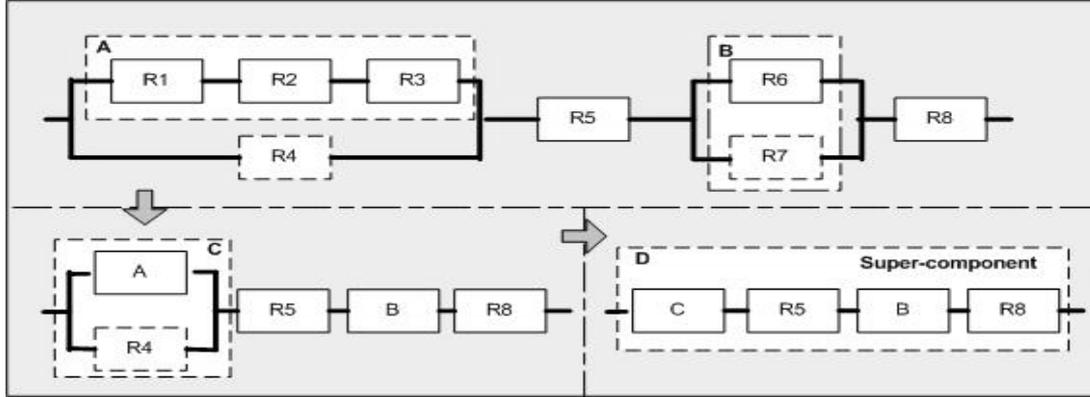


Fig. 5. System reliability network

Equations for calculating reliabilities of simple series and parallel networks of  $n$  components with reliabilities  $R_i$  are the following:

$$R_{series} = \prod_{i=1}^n R_i, \quad (4)$$

$$R_{parallel} = 1 - \prod_{i=1}^n (1 - R_i). \quad (5)$$

The accumulative formula for the whole supply system reliability calculation is shown below; the corresponding evaluated parameters are provided in Table 3.

$$R_{sys} = (1 - (1 - R_1 R_2 R_3)(1 - R_4)) R_5 (1 - (1 - R_6)(1 - R_7)) R_8, \quad (6)$$

$$R_{sys}(t) = (1 - (1 - e^{-\lambda_1 t} e^{-\lambda_2 t} e^{-\lambda_3 t})(1 - e^{-\lambda_4 t})) * e^{-\lambda_5 t} * (1 - (1 - e^{-\lambda_6 t})(1 - e^{-\lambda_7 t})) * e^{-\lambda_8 t}.$$

### 2.2.2. Minimal path sets

Unfortunately, the presented above method is hardly adoptable for complex systems. Thus, in practice, more universal methods are used. The "minimal path" method has a wider range of applications and it is adoptable for systems, which do not only consist of combinations of series and parallel structures. The disadvantage is that it only provides an approximation of reliability values; this method becomes useless in cases, when systems components have low reliability rates [1]. The "minimal path" set (a part of total component set) ensures that the system is functioning. In Table 2 the "minimal path" set for the predefined supply chain is provided.

Table 2. "Minimal path" set

Path name	Path structure
Total path set	{R1;R2;R3;R4;R5;R6;R7;R8}, {R1;R2;R3;R4;R5;R6;R8},...{R4;R5;R7;R8}.
"Minimal path" set	{R1;R2;R3;R5;R6;R8}, {R1;R2;R3;R5;R7;R8}, {R4;R5;R6;R8}, {R4;R5;R7;R8}

According to the "minimal path" method, the following equations should be used for the network structure function evaluation:

$$\phi(X) = \prod_{j=1}^p p_j(X) = 1 - \prod_{j=1}^p (1 - p_j(X)), \quad (7)$$

$$p_j(X) = \prod_{i \in p_j} R_i \text{ for } i = 1, 2, \dots, s, \tag{8}$$

where  $p_i$  is minimal path set and  $R_i$  is reliability (failure function) for a particular component.

Using the above equations, it is possible to define structural function (9), whose approximated value can be used for considering system reliability:

$$Q_{sys} \leq \phi(X) = 1 - (1 - X_1 X_2 X_3 X_5 X_6 X_8) * (1 - X_1 X_2 X_3 X_5 X_7 X_8) * (1 - X_4 X_5 X_6 X_8) * (1 - X_4 X_5 X_7 X_8),$$

$$Q_{sys}(t) \leq 1 - (1 - e^{-\lambda_1 t} e^{-\lambda_2 t} e^{-\lambda_3 t} e^{-\lambda_5 t} e^{-\lambda_6 t} e^{-\lambda_8 t}) * (1 - e^{-\lambda_1 t} e^{-\lambda_2 t} e^{-\lambda_3 t} e^{-\lambda_5 t} e^{-\lambda_7 t} e^{-\lambda_8 t}) * (1 - e^{-\lambda_4 t} e^{-\lambda_5 t} e^{-\lambda_6 t} e^{-\lambda_8 t}) * (1 - e^{-\lambda_4 t} e^{-\lambda_5 t} e^{-\lambda_7 t} e^{-\lambda_8 t}).$$

Evaluated reliability characteristics are given in Table 3.

### 2.3. Supply chain reliability simulation

For the problem under consideration, discrete-event simulation model was created using Promodel software (see Figure 6). The simulation process includes three experiments; each of them consists of five hundred replications. The basic simulation time period is one day; each replication continues until simulated system gets into a failed conditional state. The necessary reliability parameter values are evaluated using statistics gained from those five hundred replications.

The aim of the first experiment is to find a probability that the system will survive for the time period of 730 days. In the second experiment the necessary survival time period is decreased by 200 days. Then, failure rates for separate network components are increased in the third experiment with the goal to make 9<sup>th</sup> formula (9) more suitable (T=730 days) for system reliability evaluation. The calculated experimental data are shown in the table below:

Table 3. Simulation results

Exp. No.	Mathematical evaluations				Experimental data	
	$R_{sys}$	$Q_{sys}$	$MTTF_R$	$MTTF_Q$	$R_{Sim}$	$MTTF_{Sim}$
1	62,4 %	≤85,4%	1304 days	1693 days	61,2 %	1277 days
2	72,5 %	≤93,5%	1304 days	1693 days	73,4 %	1277 days
3	93,3 %	≤99,6%	5263 days	5841 days	94,0 %	5525 days

By observing simulation and mathematical evaluation results, some conclusions can be made. First of all, it is possible to conclude about the created simulation model adequacy following from the comparing simulation and “combinations of simple series and parallel structures” methods results. Then, in cases of more complicated supply chain systems, this mathematical method would be too labour intensive. Moreover, the method of simple structures combinations is limited and cannot be used for every network structure analysis. On the other hand, the method of “minimal path” set, which is more usable for network structural analysis, cannot be used for components with large failure rates; therefore it is useless in most supply chains reliability analysis tasks.

Then, the advantages of reliability simulation are obvious. The created simulation model can be restructured with the aim to analyse different supply chains with no limit on network complicity. In addition, simulation can be used for “repairable” or “multi-conditional states” systems.

### 3. Conclusions

The current paper investigates problems related to risk recognition and simulation-based risk evaluation in the sphere of supply chains. Though a lot of methods for supply chain risk management have been developed, there is no a universal solution for this problem yet. Thus, the definitions of risk, uncertainties and supply chains terms are discussed. Using these definitions, the supply chain risk is defined as possible disruptions that can affect supply chain ability to function normally. Risk evaluation can be considered as separate sophisticated process within risk management. In the current research, the use of simulation techniques is advised as an effective instrument for supply chain risk evaluation. It is very important that the simulation model can fully consider expert opinion through the input data about risks in separate supply chain members. By observing these data, the model provides risk evaluation values for the whole supply network. The output results of the model are statistical data about structure reliability of the simulated supply chain. In the same way, reliability parameters were evaluated using the analytical mathematics. For all that, the corresponding mathematical formulas application becomes too labour intensive in large and volatile systems. Still mathematical model application helps in validation of simulation models.

In the current paper advised models have many simplification and assumptions, what make them hardly adoptable in practise. Thus, the future research is directed on extending of simulation-based supply chain reliability models. The final goal of the research is to provide supply chain managers with powerful simulation-based software for supply networks reliability analysis.

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# INFORMATION CRITERION FOR VARIABLE SELECTION IN ECONOMETRIC MODELS AND ITS APPLICATIONS

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The analysis of regression data usually involves a comparison of several competing or candidate variables. Because the true variables are seldom known a priori, there is need for objective data-driven methods to aid in the selection of "good" variables. The problem of determining the "best" subset of variables has long been of interest to applied statisticians and, primary because of the current availability of high-speed computations, this problem has received considerable attention in the recent statistical literature. Often referred to as the problem of subset selection, it arises when one wants to model the relationship between a variable of interest and a subset of potential explanatory variables or predictors, but there is uncertainty about which subset to use. Several papers have dealt with various aspects of the problem but it appears that the typical regression user has not benefited appreciably. One reason for the lack of resolution of the problem is the fact that it is has not been well defined. Indeed, it is apparent that there is not a single problem, but rather several problems for which different answers might be appropriate. The intent of this paper is not to give specific answers but merely to present a new variable selection criterion. The variables, which optimise the criterion, are chosen to be the best variables. We find that the new criterion performs consistently well across a wide variety of variable selection problems. Practical utility of this criterion is demonstrated by numerical examples.

**Keywords:** regression model; variable selection; criterion

## 1. Introduction

Variable selection refers to the problem of selecting input variables that are most predictive of a given outcome. Variable selection problems are found in all machine learning tasks, supervised or unsupervised, classification, regression, time series prediction, two-class or multi-class, posing various levels of challenges.

In the recent years, variable selection has become the focus of a lot of research in several areas of application for which datasets with tens or hundreds of thousands of variables are available. These areas include text processing, particularly in application to Internet documents, and Genomics, particularly gene expression array data. The objective of variable selection is three-fold: improving the prediction performance of the predictors, providing faster and more cost-effective predictors, and providing a better understanding of the underlying process that generated the data.

Suppose that  $Y$ , a variable of interest, and  $X_1, \dots, X_v$ , a set of potential explanatory variables or predictors, are vectors of  $n$  observations. The problem of variable selection, or subset selection as it is often called, arises when one wants to model the relationship between  $Y$  and a subset of  $X_1, \dots, X_v$ , but there is uncertainty about which subset to use. Such a situation is particularly of interest when  $v$  is large and  $X_1, \dots, X_v$  is thought to contain many redundant or irrelevant variables.

The variable selection problem is most familiar in the linear regression context, where attention is restricted to normal linear models. Letting  $\mathcal{G}$  index the subsets of  $X_1, \dots, X_v$ , and letting  $p_{\mathcal{G}}$  be the number of the parameters of the model based on the  $\mathcal{G}$ th subset, the problem is to select and fit a model of the form

$$Y = X_{\mathcal{G}}\beta_{\mathcal{G}} + \epsilon, \quad (1)$$

where  $X_{\mathcal{G}}$  is an  $n \times p_{\mathcal{G}}$  matrix which columns correspond to the  $\mathcal{G}$ th subset,  $\beta_{\mathcal{G}}$  is a  $p_{\mathcal{G}} \times 1$  vector of regression coefficients, and  $\epsilon \sim N_n(0, \sigma^2 I)$ . More generally, the variable selection problem is a special case of the model selection problem where each model under consideration corresponds to a distinct subset of  $X_1, \dots, X_v$ . Typically, a single model class is simply applied to all possible subsets.

The fundamental developments in variable selection seem to have occurred directly in the context of the linear model (1). Historically, the focus began with the linear model in the 1960s, when the first wave of important developments occurred and computing was expensive. The focus on the linear model still continues, in part because its analytic tractability greatly facilitates insight, but also because many problems of interest can be posed as linear variable selection problems. For example, for the problem of non-parametric function estimation,  $Y$  represents the values of the unknown function, and  $X_1, \dots, X_v$  represent a linear basis, such as a wavelet basis or a spline basis.

One of the fascinating aspects of the variable selection problem has been the wide variety of methods that have been brought to bear on the problem. Because of space limitations, it is of course impossible to even

mention them all, and so we focus on only a few to illustrate the general thrust of developments. An excellent and comprehensive treatment of variable selection methods prior to 1990 was provided by Miller [25]. As we discuss, many promising new approaches have appeared over the last decade.

A distinguishing feature of variable selection problems is their enormous size. Even with moderate values of  $v$ , computing characteristics for all  $2^v$  models is prohibitively expensive, and some reduction of the model space is needed. Focusing on the linear model (1), early suggestions based such reductions on the residual sum of squares, which provided a partial ordering of the models. Taking advantage of the chain structure of subsets, branch and bound methods such as the algorithm of Furnival and Wilson [14] were proposed to logically eliminate large numbers of models from consideration. When feasible, attention was often restricted to the "best subsets" of each size. Otherwise, reduction was obtained with variants of stepwise methods that sequentially add or delete variables based on greedy considerations (e.g., Efroymson [12]). Even with advances in computing technology, these methods continue to be the standard workhorses for reduction.

Once attention was reduced to a manageable set of models, criteria were needed for selecting a subset model. The earliest developments of such selection criteria, again in the linear model context, were based on attempts to minimize the mean squared error of prediction. Different criteria corresponded to different assumptions about which predictor values to use and whether they were fixed or random (see Hocking [20]; Thompson [40] and the references therein). Perhaps the most familiar of those criteria is the Mallows

$$C_p = \frac{\text{RSS}_g}{\hat{\sigma}_{\text{full}}^2} + 2p_g - n, \quad (2)$$

where  $\text{RSS}_g$  is the residual sum of squares for the model based on the  $g$ th subset and  $\hat{\sigma}_{\text{full}}^2$  is the usual unbiased estimate of  $\sigma^2$  based on the full model. The standard texts, such as Draper and Smith [10], Montgomery and Peck [26] and Myers [28], recommend plotting  $C_p$  against  $p$  for all possible regressions and choosing an equation with low  $C_p$  or with  $C_p$  close to  $p$ . If  $\sigma^2$  is known, any model which provides unbiased estimates of the regression coefficients, i.e. which contains all important regressors, has  $E(C_p) = p$ .

Two of the other most popular criteria, motivated from very different viewpoints, are the Akaike's information criterion (AIC) and the Bayesian information criterion (BIC). Letting  $\hat{L}_g$  denote the maximum log-likelihood of the  $g$ th model, AIC selects the model that maximizes  $(\hat{L}_g - p_g)$ , whereas BIC selects the model that maximizes  $(\hat{L}_g - (\log n)p_g/2)$ . Akaike [1] motivated AIC from an information theoretic standpoint as the minimization of the Kullback-Leibler distance between the distributions of  $Y$  under the  $g$ th model and under the true model. To lend further support, an asymptotic equivalence of AIC and cross-validation was shown by Stone [38]. In contrast, Schwarz [34] motivated BIC from a Bayesian standpoint, by showing that it was asymptotically equivalent (as  $n \rightarrow \infty$ ) to selection based on Bayes's factors. BIC was further justified from a coding theory viewpoint by Rissanen [33].

Comparisons of the relative merits of AIC and BIC based on asymptotic consistency (as  $n \rightarrow \infty$ ) have flourished in the literature. As it turns out, BIC is consistent when the true model is fixed (Haughton [19]), whereas AIC is consistent if the dimensionality of the true model increases with  $n$  (at an appropriate rate) (Shibata [37]). Stone [39] provided an illuminating discussion of these two viewpoints.

For the linear model (1), many of the popular selection criteria are special cases of a penalized sum of squares criterion, providing a unified framework for comparisons. Assuming  $\sigma^2$  known to avoid complications, this general criterion selects the subset model that minimizes

$$\frac{\text{RSS}_g}{\sigma^2} + cp_g, \quad (3)$$

where  $c$  is a preset "parametric dimensionality penalty." Intuitively, (3) penalizes  $\text{RSS}_g/\sigma^2$  by  $c$  times  $p_g$ , the parametric dimension of the  $g$ th model. AIC and minimum  $C_p$  are essentially equivalent, corresponding to  $c = 2$ , and BIC is obtained by setting  $c = \log n$ . By imposing a smaller penalty, AIC and minimum  $C_p$  will select larger models than BIC (unless  $n$  is very small).

Further insight into the choice of  $c$  is obtained when all of the predictors are orthogonal, in which case (3) simply selects all of those predictors with  $t$ -statistics  $t$  for which  $t^2 > c$ . When  $\mathbf{X}_1, \dots, \mathbf{X}_v$  are in fact all unrelated to  $\mathbf{Y}$  (i.e., the full model regression coefficients are all 0), AIC and minimum  $C_p$  are clearly too liberal and tend to include a large proportion of irrelevant variables. A natural conservative choice for  $c$ , namely  $c = 2 \log v$ , is suggested by the fact that under this null model, the expected value of the largest squared  $t$ -statistic is approximately  $2 \log v$  when  $v$  is large. This choice is the risk inflation criterion (RIC) proposed by Foster and George [16] and the universal threshold for wavelets proposed by Donoho and Johnstone [8]. Both of these articles motivate  $c = 2 \log v$  as yielding the smallest possible maximum inflation in predictive risk due to selection

(as  $v \rightarrow \infty$ ), a minimax decision theory standpoint. Motivated by similar considerations, Tibshirani and Knight [42] recently proposed the covariance inflation criterion (CIC), a nonparametric method of selection based on adjusting the bias of in-sample performance estimates. Yet another promising adjustment based on a generalized degrees of freedom concept was proposed by Ye [45].

Many other interesting criteria corresponding to different choices of  $c$  in (3) have been proposed in the literature (see, e.g., Hurvitz and Tsai [22-23]; Rao and Wu [32]; Shao [36]; Wei [44]; Zheng and Loh [48] and the references therein). One of the drawbacks of using a fixed choice of  $c$  is that models of a particular size are favoured; small  $c$  favours large models, and large  $c$  favours small models. Adaptive choices of  $c$  to mitigate this problem have been recommended by Benjamini and Hochberg [2], Clyde and George [5-6], Foster and George [13], Johnstone and Silverman [24], and Nechval et al. [30-31].

An alternative to explicit criteria of the form (3), is selection based on predictive error estimates obtained by intensive computing methods such as the bootstrap (e.g., Efron [11]; Gong [18]) and cross-validation (e.g., Shao [35]; Zhang [47]). An interesting variant of these is the little bootstrap (Brieman [3]), which estimates the predictive error of selected models by mimicking replicate data comparison. The little bootstrap compares favourably to selection based on minimum  $C_p$  or the conditional bootstrap, which performances are seriously denigrated by selection bias.

Another drawback of traditional subset selection methods, which is beginning to receive more attention, is their instability relative to small changes in the data. Two novel alternatives that mitigate some of this instability for linear models are the nonnegative garrotte (Brieman [4]) and the lasso (Tibshirani [41]). Both of these procedures replace the full model least squares criterion by constrained optimization criteria. As the constraint is tightened, estimates are zeroed out, and a subset model is identified and estimated.

The fully Bayesian approach to variable selection is as follows (George [16]). For the given set of models  $M(1), \dots, M(2^v)$ , where  $M(\mathcal{G})$  corresponds to the  $\mathcal{G}$ th subset of  $\mathbf{X}_1, \dots, \mathbf{X}_v$ , one puts priors  $\pi(\boldsymbol{\beta}(\mathcal{G}); M(\mathcal{G}))$  on the parameters of each  $M(\mathcal{G})$  and a prior on the set of models  $\pi(M(1)), \dots, \pi(M(2^v))$ . Selection is then based on the posterior model probabilities  $\pi(M(\mathcal{G}); \mathbf{Y})$ , which are obtained in principle by Bayes's theorem.

Although this Bayesian approach appears to provide a comprehensive solution to the variable selection problem, the difficulties of prior specification and posterior computation are formidable when the set of models is large. Even when  $v$  is small and subjective considerations are not out of the question (Garthwaite and Dickey [15]), prior specification requires considerable effort.

## 2. New Variable Selection Criterion

This criterion (denoted by  $N_{p_g}$ ) is given by

$$N_{p_g} = c_{p_g} p_g = \left( \frac{1}{R_{p_g}^2} - 1 \right) p_g, \quad (4)$$

where the coefficient of determination  $R_{p_g}^2$  ( $0 \leq R_{p_g}^2 \leq 1$ ) for the  $\mathcal{G}$ th subset model is computed as

$$R_{p_g}^2 = 1 - \frac{\text{RSS}_{p_g}}{\text{TSS}_{p_g}}, \quad (5)$$

$\text{RSS}_{p_g}$  is the residual sum of squares for the  $\mathcal{G}$ th subset model,  $\text{TSS}_{p_g}$  is the total sum of squares for the  $\mathcal{G}$ th subset model. The variables, which optimize this criterion, are chosen to be the best variables. Thus, the best subset of variables (denoted by  $\mathcal{G}^*$ ) is determined as

$$\mathcal{G}^* = \arg \inf_{\mathcal{G} \in \{\mathcal{G}\}} N_{p_g} = \arg \inf_{\mathcal{G} \in \{\mathcal{G}\}} \left( \frac{1}{R_{p_g}^2} - 1 \right) p_g. \quad (6)$$

Now let us assume that for each  $\mathcal{G}$ th subset we have available several models, each of which has the number of parameters equal to  $p_g$ , then the best model (denoted by  $p_{g^*}^*$ ) is determined as

$$p_{g^*}^* = \arg \inf_{p_g \in \{p_g: \mathcal{G} \in \{\mathcal{G}\}\}} N_{p_g} = \arg \inf_{p_g \in \{p_g: \mathcal{G} \in \{\mathcal{G}\}\}} \left( \frac{1}{R_{p_g}^2} - 1 \right) p_g. \quad (7)$$

### 3. Examples

#### 3.1. Example 1: Canadian Lynx Data

The data (Table 1) give the annual number of lynx trappings in the Mackenzie River District of North-West Canada for the period 1821 to 1934, yielding  $n=114$  observations.

**Table 1.** The Canadian Lynx Data

Year	Data	Year	Data	Year	Data	Year	Data	Year	Data	Year	Data	Year	Data
1821	269	1838	3409	1855	1638	1872	784	1889	39	1906	3794	1923	1132
1822	321	1839	1824	1856	2725	1873	1594	1890	49	1907	1836	1924	2432
1823	585	1840	409	1857	2871	1874	1676	1891	59	1908	345	1925	3574
1824	871	1841	151	1858	2119	1875	2251	1892	188	1909	382	1926	2935
1825	1475	1842	45	1859	684	1876	1426	1893	377	1910	808	1927	1537
1826	2821	1843	68	1860	299	1877	756	1894	1292	1911	1388	1928	529
1827	3928	1844	213	1861	236	1878	299	1895	4031	1912	2713	1929	485
1828	5943	1845	546	1862	245	1879	201	1896	3495	1913	3800	1930	662
1829	4950	1846	1033	1863	552	1880	229	1897	587	1914	3091	1931	1000
1830	2577	1847	2129	1864	1623	1881	469	1898	105	1915	2985	1932	1590
1831	523	1848	2536	1865	3311	1882	736	1899	153	1916	3790	1933	2657
1832	98	1849	957	1866	6721	1883	2042	1900	387	1917	674	1934	3396
1833	184	1850	361	1867	4245	1884	2811	1901	758	1918	81		
1834	279	1851	377	1868	687	1885	4431	1902	1307	1919	80		
1835	409	1852	225	1869	255	1886	2511	1903	3465	1920	108		
1836	2285	1853	360	1870	473	1887	389	1904	6991	1921	229		
1837	2685	1854	731	1871	358	1888	73	1905	6313	1922	399		

The natural logarithm of the series is commonly analysed. A comprehensive description of the data set as well as a review of the different analyses that have been published can be found in Tong [43] (ch. 7). For instance, evidence of non-linearity has been uncovered. We, however, assume here that the interest lies in fitting a linear auto-regression (hence possibly mis-specified) model of order  $k$ ,  $AR(k)$ , i.e., for  $i=k+1, \dots, n$ ,

$$y_i = a_0 + a_1 y_{i-1} + \dots + a_k y_{i-k} + \varepsilon_i, \tag{8}$$

where  $\{\varepsilon_i\}$  is an i.i.d. stochastic process with mean zero and variance  $\sigma^2 < \infty$ . The order  $k$  is unknown. The model selected by AIC is an  $AR(11)$ , while BIC and  $N_{p,q}$  choose  $AR(2)$ . Notice that historically, the  $AR(2)$  specification was first advocated by Moran [27].

#### 3.2. Example 2: Hudson Data

The data set  $(x_i, y_i)$ ,  $i=1(1)19$ , analysed here was simulated using the model:

$$y_i = 1 + x_i - 0.55x_i^2 + 0.001x_i^3 + \varepsilon_i, \tag{9}$$

where  $\varepsilon_i$ ,  $i=1(1)19$ , are independent and normal with mean zero and variance 1. The data taken from (Hudson [21]) are presented in Table 2.

**Table 2.** The Hudson Data

$i$	$x_i$	$y_i$									
1	2	2.84	6	12	7.39	11	22	7.35	16	32	9.99
2	4	5.50	7	14	6.67	12	24	6.11	17	34	10.31
3	6	5.96	8	16	5.72	13	26	6.67	18	36	12.03
4	8	4.50	9	18	7.95	14	28	9.67	19	38	13.51
5	10	6.45	10	20	5.93	15	30	7.35			

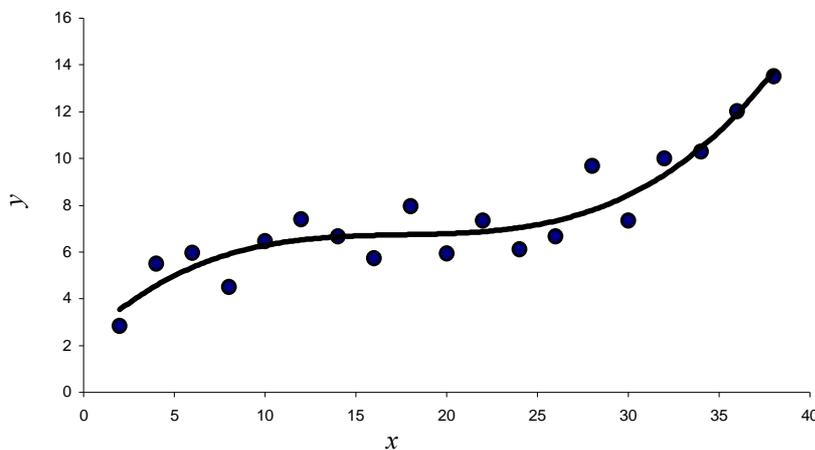
Assuming that

$$y = a_0 + a_1x + a_2x^2 + \dots + a_kx^k + \varepsilon, \quad k \geq 1, \tag{10}$$

the final choice of the best model, which minimizes (7), is

$$y = a_0 + a_1x + a_2x^2 + a_3x^3 + \varepsilon, \tag{11}$$

i.e.,  $p_g^* = 4$ , where  $\mathcal{G} = \{x\}$ . The Hudson data with the best regression curve are shown on Figure 1.



**Fig. 1.** The Hudson data with the best regression curve

### 3.3. Example 3: Hald Cement Data

Montgomery and Peck [26] (pp. 256-266) illustrated variable selection techniques on the Hald cement data and gave several references to other analyses. The data are shown in Table 3.

**Table 3.** The Hald Cement Data

$i$	$y_i$	$x_{i1}$	$x_{i2}$	$x_{i3}$	$x_{i4}$	$i$	$y_i$	$x_{i1}$	$x_{i2}$	$x_{i3}$	$x_{i4}$
1	78.5	7	26	6	60	8	72.5	1	31	22	44
2	74.3	1	29	15	52	9	93.1	2	54	18	22
3	104.3	11	56	8	20	10	115.9	21	47	4	26
4	87.6	11	31	8	47	11	83.8	1	40	23	34
5	95.9	7	52	6	33	12	113.3	11	66	9	12
6	109.2	11	55	9	22	13	109.4	10	68	8	12
7	102.7	3	71	17	6						

The response variable is the heat evolved  $y$  in a cement mix, and the four explanatory variables are ingredients in the mix. When a linear model

$$y = a_0 + a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 + \varepsilon \tag{12}$$

is fitted, the residuals show no evidence of any problems. But an important feature of these data is that the variables  $x_1$  and  $x_3$  are highly correlated ( $r_{13} = -0.824$ ), as are the variables  $x_2$  and  $x_4$  (with  $r_{24} = -0.973$ ). Thus we would expect any subset  $\mathcal{G}$  of  $\{x_1, x_2, x_3, x_4\}$  that includes one variable from a highly correlated pair would do as well as any subset that also includes the other member. It follows from (6) that the best model is

$$y = a_0 + a_1x_1 + a_2x_2 + \varepsilon, \tag{13}$$

i.e.,  $\mathcal{G}^* = \{x_1, x_2\}$  and  $p_{\mathcal{G}^*} = 3$ .

### 3.4. Example 4: Simulated Data

The data set  $(x_i, y_i)$ ,  $i = 1(1)100$  analysed here was simulated using the model:

$$y_i = 0.3 + 2x_i - 5x_i^2 + 3x_i^3 + \varepsilon_i, \tag{14}$$

where, for  $i=1(1)100$ ,  $x_i=i/100$  and  $\varepsilon_i$  are independent and normal with mean zero and variance  $0.15^2$ . The situation is such that the true model is known to belong to the class of models given by (10). The simulation data are given in Table 4 and are shown, with the true regression curve, on Figure 2.

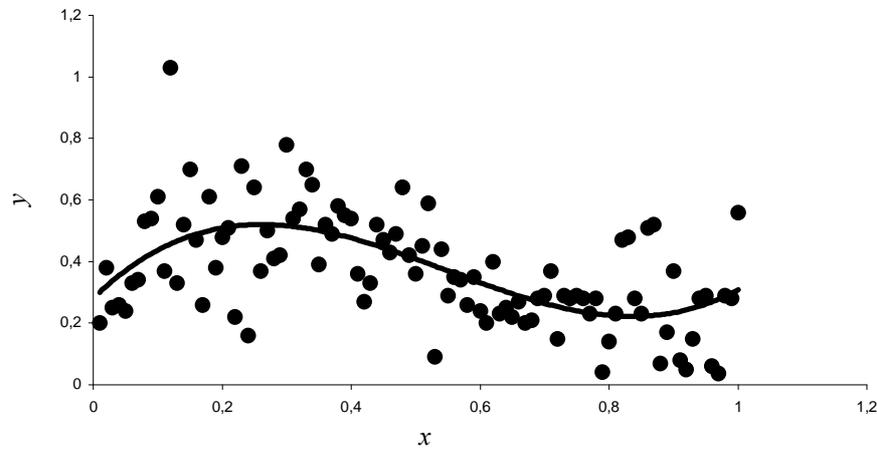


Fig. 2. Simulated data set with the best regression curve

Table 4. The Simulated Data

$x_i$	$y_i$												
0.01	0.20	0.16	0.47	0.31	0.54	0.46	0.43	0.61	0.20	0.76	0.28	0.91	0.08
0.02	0.38	0.17	0.26	0.32	0.57	0.47	0.49	0.62	0.4	0.77	0.23	0.92	0.05
0.03	0.25	0.18	0.61	0.33	0.70	0.48	0.64	0.63	0.23	0.78	0.28	0.93	0.15
0.04	0.26	0.19	0.38	0.34	0.65	0.49	0.42	0.64	0.25	0.79	0.04	0.94	0.28
0.05	0.24	0.20	0.48	0.35	0.39	0.50	0.36	0.65	0.22	0.80	0.14	0.95	0.29
0.06	0.33	0.21	0.51	0.36	0.52	0.51	0.45	0.66	0.27	0.81	0.23	0.96	0.06
0.07	0.34	0.22	0.22	0.37	0.49	0.52	0.59	0.67	0.20	0.82	0.47	0.97	0.04
0.08	0.53	0.23	0.71	0.38	0.58	0.53	0.09	0.68	0.21	0.83	0.48	0.98	0.29
0.09	0.54	0.24	0.16	0.39	0.55	0.54	0.44	0.69	0.28	0.84	0.28	0.99	0.28
0.10	0.61	0.25	0.64	0.40	0.54	0.55	0.29	0.70	0.29	0.85	0.23	1.00	0.56
0.11	0.37	0.26	0.37	0.41	0.36	0.56	0.35	0.71	0.37	0.86	0.51		
0.12	1.03	0.27	0.50	0.42	0.27	0.57	0.34	0.72	0.15	0.87	0.52		
0.13	0.33	0.28	0.41	0.43	0.33	0.58	0.26	0.73	0.29	0.88	0.07		

$x_i$	$y_i$												
0.14	0.52	0.29	0.42	0.44	0.52	0.59	0.35	0.74	0.28	0.89	0.17		
0.15	0.70	0.30	0.78	0.45	0.47	0.60	0.24	0.75	0.29	0.90	0.37		

BIC and  $N_{p_g}$  choose  $k=3$ , the true degree. AIC's final choice is  $k = 8$ , a clear overfitting.

### 3.5. Example 5: House Price Data

A set of 24 observations, given in Table 5, originally published by Narula and Wellington [29], is used to relate nine variables,  $x_1, \dots, x_9$ , to the sale price,  $y$ , of houses.

Table 5. The House Price Data†

$y$	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$x_6$	$x_7$	$x_8$	$x_9$
25.9	4.9176	1.0	3.4720	0.9980	1.0	7	4	42	0
29.5	5.0208	1.0	3.5310	1.5000	2.0	7	4	62	0
27.9	4.5429	1.0	2.2750	1.1750	1.0	6	3	40	0
25.9	4.5573	1.0	4.0500	1.2320	1.0	6	3	54	0
29.9	5.0597	1.0	4.4550	1.1210	1.0	6	3	42	0
29.9	3.8910	1.0	4.4550	0.9880	1.0	6	3	56	0
30.9	5.8980	1.0	5.8500	1.2400	1.0	7	3	51	1
28.9	5.6039	1.0	9.5200	1.5010	0.0	6	3	32	0
35.9	5.8282	1.0	6.4350	1.2250	2.0	6	3	32	0
31.5	5.3003	1.0	4.9883	1.5520	1.0	6	3	30	0
31.0	6.2712	1.0	5.5200	0.9750	1.0	5	2	30	0
30.9	5.9592	1.0	6.6660	1.1210	2.0	6	3	32	0
30.9	5.0500	1.0	5.0000	1.0200	0.0	5	2	46	1
36.9	8.2464	1.5	5.1500	1.6640	2.0	8	4	50	0
41.9	6.6969	1.5	6.9020	1.4880	1.5	7	3	22	1
40.5	7.7841	1.5	7.1020	1.3760	1.0	6	3	17	0
43.9	9.0384	1.0	7.8000	1.5000	1.5	7	3	23	0
37.5	5.9894	1.0	5.5200	1.2560	2.0	6	3	40	1
37.9	7.5422	1.5	5.0000	1.6900	1.0	6	3	22	0
44.5	8.7951	1.5	9.8900	1.8200	2.0	8	4	50	1
37.9	6.0831	1.5	6.7265	1.6520	1.0	6	3	44	0
38.9	8.3607	1.5	9.1500	1.7770	2.0	8	4	48	1
36.9	8.1400	1.0	8.0000	1.5040	2.0	7	3	3	0
45.8	9.1416	1.5	7.3262	1.8310	1.5	8	4	31	0

† $y$ , sale price (\$/1000);  $x_1$ , taxes (\$/1000);  $x_2$ , number of baths;  $x_3$ , lot size (ft<sup>2</sup>/1000);  $x_4$ , living space (ft<sup>2</sup>/1000);  $x_5$ , number of garage stalls;  $x_6$ , number of rooms;  $x_7$ , number of bedrooms;  $x_8$ , age (years);  $x_9$ , number of fireplaces.

It follows from (6) that the two-parameter model

$$y = a_0 + a_1x_1 + \varepsilon \quad (15)$$

would probably be chosen since it has the lowest  $N_{p,g}$ .

It will be noted that  $C_p$  (Gilmour [17]) chooses the three-parameter model

$$y = a_0 + a_1x_1 + a_2x_2 + \varepsilon. \quad (16)$$

#### 4. Conclusions

Today, variable selection procedures are an integral part of virtually all widely used statistics packages, and their use will only increase as the information revolution brings us larger datasets with more and more variables. The demand for variable selection will be strong, and it will continue to be a basic strategy for data analysis.

Although numerous variable selection methods have been proposed, plenty of work still remains to be done. To begin with, many of the recommended procedures have been given only a narrow theoretical motivation, and their operational properties need more systematic investigation before they can be used with confidence. For example, small-sample justification is needed in addition to asymptotic considerations, and frequentist justification is needed for Bayesian procedures. Although there has been clear progress on the problems of selection bias, clear solutions are still needed, especially for the problems of inference after selection (see Zhang [46]). Another intriguing avenue for research is variable selection using multiple model classes (see Donoho and Johnstone [9]). New problems will also appear as demand increases for data mining of massive datasets. For example, considerations of scalability and computational efficiency will become paramount in such a context. We suppose that all of this is good news, but there is also danger lurking ahead.

With the availability of so many variable selection procedures and so many different justifications, it has become increasingly easy to be misled and to mislead. Faced with too many choices and too little guidance, practitioners continue to turn to the old standards such as stepwise selection based on AIC or minimum  $C_p$ , followed by a report of the conventional estimates and inferences. The justification of asymptotic consistency will not help the naive user who should be more concerned with selection bias and procedure instability. Eventually, the responsibility for the poor performance of such procedures will fall on the statistical profession, and consumers will turn elsewhere for guidance (e.g., Dash and Liu [7]). Our enthusiasm for the development of promising new procedures must be carefully tempered with cautionary warnings of their potential pitfalls.

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# PROBLEMS PERTAINING TO MACROECONOMIC RISK ASSESSMENT IN THE PROCESS OF MODELLING OF INVESTMENT SOLUTIONS OF AIR CARRIERS

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The article deals with the problems of investment development of air companies. The author examines the aspects of modelling of investment solutions under conditions of developing aviation business, giving methods of modelling of demand, including those based on dynamics of macroeconomic indicators. The article contains the problems relating to assessment of macroeconomic risks, using models of assessment of discount rates. Variants of assessment of currency risks in the process of evaluation of efficiency of international investment solutions are analysed as well.

**Keywords:** *investment activity, risk, modelling, discounting, conversion, efficiency*

## 1. Introduction

Development of air companies is closely connected with the processes of transformation so typical for the world economy nowadays. Air companies promote development of the processes of globalisation of the world economy. Their progress and functioning create the conditions for more efficient internationalisation of economic life and international differentiation of labour. Air companies are to the known extent an image of countries of the universal community. Their development certifies of potential of the country, possibilities of its qualitative progress, stability of macroeconomic system. At the same time, development of air companies is influenced by the complex of risks, macroeconomic risks being the most serious ones.

The problems of revealing, systematisation and assessment of macroeconomic risks relating to functioning of air companies become the issue of the day. This article contains analysis of separate aspects of the mentioned problem.

## 2. Forecasts of Economic Development of Air Business

The most preferable form of development of companies nowadays is their qualitative dynamics. The latter is finally revealed as a growing ability of companies to offer services needed by the society, most rationally and efficiently using all kinds of resources, including finances.

Investment activity is a pre-condition for qualitative dynamics of companies. At the same time, investment activity should be first, expedient, second, efficient. Taking into account current problems of global aviation, air company's investments should facilitate implementation of the below aspects:

- to raise efficiency of use of air companies' assets;
- to reach higher level of effectiveness of used resources, first of all, aviation fuel;
- to settle the problems relating to ecologic aspects of aviation business;
- to lessen the problems relating to cyclical changes in activities of air companies;
- to solve the problems relating to systems "man-machine", "man-man" due to stricter requirements to the "human factor";
- to ensure compliance of all aspects of activities of air companies with the international standards and requirements relating thereto.

Results of activities of air companies are constantly growing. IATA expects increase in passenger traffic flow worldwide from 2,1 billion passengers in 2006 up to 2,6 billion passengers in 2010, noting that within 20 years park capacities will be doubled [1].

Specialists in the sphere of freight services point out that development of business activity in the world economy leads to higher demand for air carriages, reaching the new level. Indicators of intensity of freight traffic are in direct ratio to the indicators of gross domestic product (GDP), currently increasing by 6% per annum. According to Boeing and Airbus estimations, such indicator will grow in the longer term.

At the same time, it is stated that sea and road transport competitors become more and more severe, since they work up some market share, offering more attractive price policy [2].

The world market of technical maintenance and repair services grows intensively. According to expert's opinion, such market turnover will reach 51,1 billion USD in 2012 and 62,5 billion USD in 2017. Total profit of

companies functioning in the market of repair and maintenance services was not lesser than 2,5 billion USD in 2007. Company Team-SAI Inc. expects the tendency of increase in profit of companies in the aforesaid market within next 10 years.

Forecast of growth of the market of TM and R according to all types of aircrafts and dynamics of growth of the world park of aircrafts according to categories are given on Figure 1 [3].

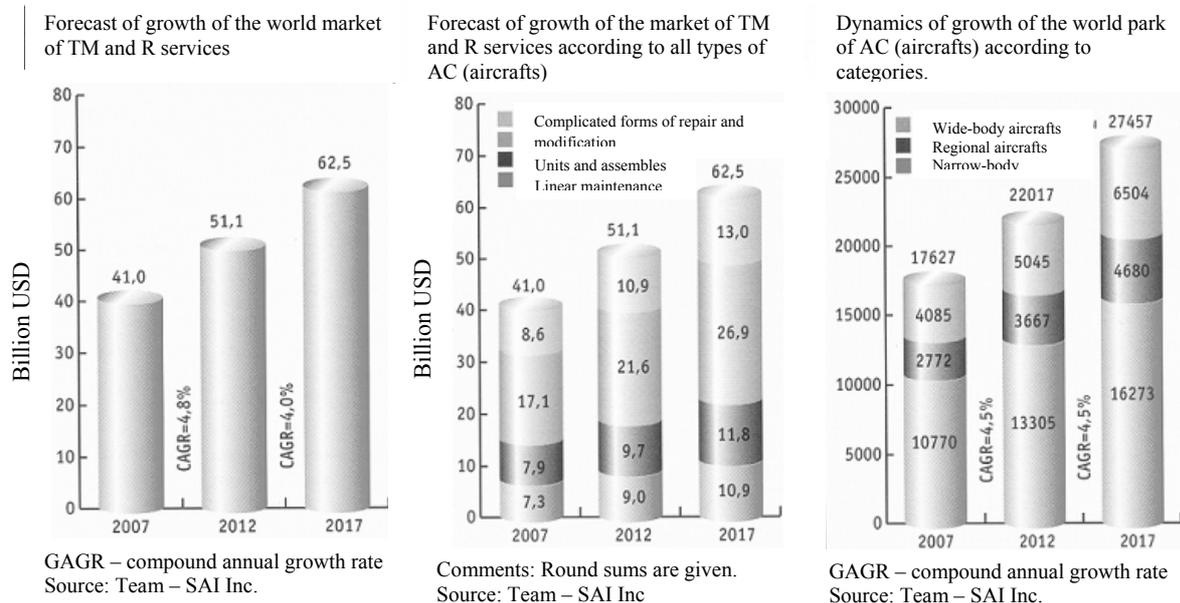


Fig. 1. Forecast of growth of the market of TM and RS

The increased tempo of business activity in any sphere is traditionally related to the growing investment activity in the same sphere. Air companies are not an exception. International aspect of activities of companies always requires special attention. Due to that, improvement of the process of assessment of efficiency of investment activities of companies is the matter of current importance.

### 3. Modelling of Demand

Efficiency of investment projects should be assessed on a qualitative information basis. The primary aspect for an air company is provision of services. In fact, the matter in question is modelling of demand in the circumstances of uncertainty of functioning of the market, with methods of modelling being of variable nature. One of them is formation of strategic investment programs on the basis of comparison of subjective "opinion" of customers and contract-basis orders.

Let's find below an example of modelling of the "product line" of civil aircrafts for 2008-2012 developed by the United Aircraft Corporation (Russia).

To define domestic demand for the new airliners, UAC conducted negotiations with 32 Russian air carriers. According to the UAC, results of questioning can be considered to be representative, since the companies participated in the negotiations are providers of 90% of services offered by the civil aviation of Russia [4].

The participants are offered to define their needs in the aircrafts produced or developed by the aircraft industry of Russia and ready for shipment within 2008-2012. The following payment provisions are offered: payment on instalment contract basis within 15 years with prepayment equal to 5% of the aircraft value; efficient monthly lease rate 0,85-0,90% of the aircraft value.

In addition to assessment of domestic demand, the United Aircraft Corporation gathers information on export orders, both contract-basis and planned for the same period, by leasing company *Ilyushin Finance, Co.* and directly by manufacturers and developers.

Orders are classified according to four groups on the basis of probability of signing of firm contracts. All orders are relevantly corrected through the reduction factors resulting in the forecast of demand for Russian aircrafts reduced from 755 to 452 machines [4].

**Table 1.** Stated demand for civil aircrafts

Type of AC	2008		2009		2010		2011		2012		Total	
	Domestic order	Export	Domestic order	Export	Domestic order	Export						
IL-96	1	3	0	4	3	5	3	4	2	2	9	18
TU-204	11	5	30	6	34	6	28	3	22	4	125	24
TU-334	21	0	21	0	25	8	24	14	23	5	114	27
AN-148	9	4	22	15	26	16	14	21	12	10	83	66
Superjet-100	6	0	30	0	76	14	34	59	0	70	146	143
<b>Total</b>	<b>48</b>	<b>12</b>	<b>103</b>	<b>25</b>	<b>164</b>	<b>49</b>	<b>103</b>	<b>101</b>	<b>59</b>	<b>91</b>	<b>477</b>	<b>278</b>

**Table 2.** Forecast of demand of Russian air companies and foreign customers in aircrafts produced in Russia within 2008-2012

Type of AC	Basic order according to groups of probability of signing of firm contract					Corrected forecast of demand according to years, taking into account probability of contracts					
	Group 1	Group 2	Group 3	Group 4	Total	2008	2009	2010	2011	2012	Total
IL-96	8	6	13	0	27	3	3	5	4	5	20
TU-204	25	58	49	21	153	12	23	25	23	18	101
TU-334	5	6	66	64	141	8	8	8	15	12	51
AN-148	22	12	63	52	149	8	16	24	17	10	75
Superjet-100	139	83	47	20	289	6	30	43	61	65	205
<b>Total</b>	<b>199</b>	<b>165</b>	<b>238</b>	<b>157</b>	<b>759</b>	<b>37</b>	<b>80</b>	<b>105</b>	<b>120</b>	<b>110</b>	<b>452</b>

In such case, we do not comment results of questioning. The method allowing referring desires of air companies to the real level of orders for aviation equipment is of interest.

#### 4. Macroeconomic Indicators as Model Variables

Mathematical models allowing assessing efficiency of investment solutions of air companies contain number of macroeconomic indicators as variables. For example, gross domestic product is a model variable used in world practice when planning volumes of air carriages.

Form of the basic model:

$$Y = a * X^b * Z^c, \tag{1}$$

where:

- Y – passenger-kilometres performed (PKP);
- X – gross domestic product in real terms (GDP);
- Z – income from passenger traffic per passenger-kilometre in real terms (PYIELD);
- a – constant factor got through the method of statistical appraisal;
- b, c – factors of price elasticity of demand with regard to relevant GDP and PYIELD.

Using logarithm instruments:

$$\ln Y = a + b * \ln X + c * \ln Z. \tag{2}$$

Air companies planning their results normally use the following model:

$$\ln PKP = 1.14 + 2.11 * \ln GDP - 0.62 * \ln PYIELD. \tag{3}$$

Company Boeing uses the following models:

$$\ln PKP = 3.21 + 1.88 * \ln GDP. \tag{4}$$

After volumes of air carriages are planned for the country in general, Air Company’s share in such carriages is defined on the basis of assessment of micro and meso-economic indicators [5].

Thus, risk of dynamics of business activity in macro system is taken into account when assessing efficiency of projects at the stage of forecasting of volume of work of Air Company.

To assess the efficiency of international investment projects, proper assessment of inflation and currency risks is required practically in all cases.

The time factor is traditionally included in estimations through the mechanism of discounting. In particular, within the framework of use of method of Net Present Value, being one of standard methods for assessment of efficiency of investments, we should do the following. Investment expenditures should be compared with the total amount of corrected future cash receipts generated by the same within the planned

period. With the desired discount rate (factor fixed independently by the analyst on the basis of desired or possible annual interest on the capital invested by him), it is possible to define present value of all cash outflows and inflows within economic life of the project as well as to compare the same. The result of such comparison will be a positive or a negative value (net present value) showing whether the assumed discount rate satisfies or not. [6].

Thus, within the framework of the given variant, discount rate is defined by the analyst independently on the basis of own preferences.

According to economic literature, discount rate is defined as maximum returns on alternative forms of capital investments. Our practice shows that method of selection of discount rate on the basis of the aforesaid provisions is most often preferred by entrepreneurs. In such case, they are attracted by the fact that there is no need to use indicators of meso-, macro- and mega- economic dynamics for assessments.

However, there are also other recommendations on how to assess risk factors and inflation for entrepreneurial activities [7]. In particular, we can do the same through assessment of corrected discount factor:

$$K_d = 1 + d_i = \frac{1 + \frac{r}{100}}{1 + \frac{i}{100}}, \quad (5)$$

where:

$d_i$  – discount rate;

$r$  – refinance rate set by the Central Bank of the country, %;

$i$  – inflation tempo for relevant year as stated by the Government of the country, %.

However, when using the said formula, we face some paradox. Discount factor will be more than 1 in case when  $r > i$ . In this context, the known thesis that today's monetary unit is more expensive than tomorrow's monetary unit proves to be true. However, if  $r < i$ , then  $K_d < 1$ , and in such case tomorrow's monetary unit becomes more expensive than today's monetary unit. With regard to discounted values, it turns out that the present value of generated cash flow is higher than its future value. This brings the question on appropriateness of use of the formula or incorrect interpretation thereof.

At the same time, we consider that it would be appropriate to assess discount rate on the basis of certain combination of inflation tempo and refinance rate. Taking into account such variables in the discount rate, an entrepreneur actually takes into account the state of currency circulation in the country where relevant project is implemented and returns on the government securities in such country.

It should be noted that some authors suggest that discount rate should be assessed taking into account activity risk level [7]. In such case, it is suggested that analyst should take into account a correction factor ( $p$ ), which level varies depending on goal of relevant project. In such case:

$$d = d_i + \frac{P}{100}, \quad (6)$$

where  $P$  – correction factor to take into account activity risk.

Thus, in fact we come to the idea of necessity to use cumulative method of assessment of discount rate, taking into account risk factor. Such method is widely used in developed economic systems, notably when implementing venture projects.

Specialists in such sphere consider that the problem of assessment of inflation risk recedes into the background in well-developed countries with low inflation tempo. The main problems in less developed countries are as follows:

- to define inflation risk premium;
- market risk premium;
- impact of quotations in the stock market on capital value;
- impact of relation between own capital and borrowed capital;
- possibility to rely on published factors;
- to correctly define risk-free rate;
- to calculate capital value in real and nominal terms [8].

Anyway, it should be taken into account that the farther we look into the future, the higher is the project risk. However, more complicated assessment methods do not necessarily raise efficiency of business appraisal.

Analysis of cash flow models is a task akin to scientific research. Dynamic models of such kind are a complicated object of analysis. They admit experimenting and "playing" various situations. They are a tool in analyst's hands which use requires systematic (scientific) approach, knowledge and skills [9].

## 5. Assessment of Currency Risk in the Process of Investment

The companies involved in the process of international investment use as a rule various currencies. This brings up a question when and how to take into account the process of currency exchange. According to the international practice, there are at least two methods of how to take into account currency exchange in the process of assessment of net present value of the project [10].

Let's examine such methods by the example of implementation of investment project of Latvian company in the territory of Russia, within which framework translation of currency: lat and rouble has taken place.

*First method:*

1<sup>st</sup> phase. We assess future value of generated cash flow in roubles.

2<sup>nd</sup> phase. We convert future value of generated cash flow into lats (according to expected exchange rate).

3<sup>rd</sup> phase. We calculate the discounted value (using lat discount rate).

*Second method:*

1<sup>st</sup> phase. We assess future value of generated cash flow in roubles.

2<sup>nd</sup> phase. We calculate the discounted value (using rouble discount rate).

3<sup>rd</sup> phase. We convert the discounted value into lats (using current exchange rate).

The theory again offers to an expert some variants – two methods consisting of similar number of phases and different in sequence thereof.

An expert faces a dilemma – how to act? What to do first: conversion or discounting? In fact, this question sounds this way: what should be assessed first: inflation or currency risk? Theory does not give unambiguous answer to such question. It is clear that an expert can assess net present value of the project using two methods and according to variation level can judge on the level of insurance risk, on the fact to which extent rate of currencies used in the project reflects real economic situation in relevant countries.

In most cases, an entrepreneur prefers the second method so as not to work with forecasted exchange rates.

Economic theory contains the research of various concepts on which basis it is possible to forecast currency rates. They comprise:

- Theory of Purchasing Power Parity – PPP;
- Theory of Interest Rate Parity – IPR;
- Fisher's Effect;
- International Fisher's Effect;
- Theory of Unbiased Forward Rate.

The aforesaid concepts are used to reveal interrelation between main indicators of the international currency market and on such base to find most optimal ways for capital investments.

Under present conditions, Theory of Unbiased Forward Rate, according to which nobody will give more precise forecast relating to exchange rate than the exchange market itself, is supported by more and more experts. Thus, forward rate converted by banks or future rate – by currency exchange is the best forecast on expected exchange rate [11].

Specialists in the sphere of practical currency management note that international investment solutions of companies should not depend on opinion of managers of such companies on how correctly currency value is defined by the market. Owners and managers of companies should not be governed by their own forecasts on exchange rate; they should substantiate their investment solutions by agreed forecasts of international currency markets [10]. It is practically always stressed that to implement international investment project, qualitative, efficient currency management is required.

Thus, both theory and practice admit the fact that forecasted currency rates often differ from those proven by real practice further on. There are various methods to forecast dynamics of exchange rates in modern theory: fundamental analysis, technical analysis, contingent valuation method, etc. Combination of such methods is often used. However, to define future exchange rate is the most complicated task requiring much efforts. Scientific researches of many scientists are devoted to such aspect [11].

## 6. Conclusions

The problem of development of investments solutions of air companies can be classified as the most topical nowadays. Relevant risks relating to business processes should be taken into account when developing investment solutions. The more we assess risks, the lesser level of profitability of the project we have at the stage of relevant assessments. However, finally businesses structures are ready to deal with risky situations in practice and do not lose their stability in extreme circumstances.

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# SIMULATION IN THE PROCESS OF PRICE FORMATION

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Problems of price formation theory and practice are considered in the article. Theoretical bases of the price as an economic category are given as containing preconditions of mathematical-statistical technique use in specific application problems of practical price formation. Practical price formation aspects are considered with the use of different methods of operation investigations. The application of non-linear programming methods is considered at determining manufacturing cost and prices of food assortment of homogeneous products. Balance methods application is investigated in conformity with the simulation of expenses and transfer prices under complex manufacturing structure conditions. Regression analysis is carried out at real data of local market of information technologies showing the interdependence of price with other economic figures.

**Keywords:** *simulation, price formation factors, regression equations, price differentiation*

## 1. Introduction

Simulation of price formation is an effective instrument of management and regulation of the most important economic processes – sale acts and purchasing goods and services, their productive and unproductive consumption, interaction of enterprises with workers, population and environment.

In determining the price as economic category two key notions are used. They are: “money expression of goods and services cost”, and “money expression of goods and services value (usefulness)”. Such duality is completely justified and it is the result of the price formation peculiarities under market conditions. The price of goods or services really fulfils both sides of transactions – demand and supply, consumer and producer, purchaser and seller.

Decision making of price and correspondingly behaviour of the mentioned parties participants are mediated by the presence, action and taking into account a great number of factors, important for each party. Besides two mentioned parties the process of price formation is under the influence of external (in reference to a concrete transaction) surroundings, where competitors play the most important part, correspondingly the following methods are used in price formation theory and practice:

- methods, competent on costs;
- methods, competent on purchaser;
- methods, competent on competitors.

The first group is often called expenditure (cost) methods, the second and third – market methods. But most effective is the integral approach to price formation in which the factors of supply, demand, competitors and external surroundings are taken into account. Because of wide application of simulation this group of methods is called econometric.

Generalizing [1, 2] it is possible to point out and determine the role of the following seven price formation factors:

- 1) costs (expenditures) – determine the lowest price level necessary for providing recoupment of the product within short-term and long-term periods;
- 2) demand – determines the possible upper level, accepted by purchasers and satisfying sellers;
- 3) competitors’ supply – price and volumes of offer and variety of goods;
- 4) enterprise goals – determine strategy and price policy of the enterprise;
- 5) price varieties, stipulated by the goods (services) peculiarities – price, tariff, rent, wage rates, fee, interest charges, etc.;
- 6) product (services) life cycle – price strategy change depending on life cycle phase;
- 7) regional peculiarities of a branch market – peculiarities in demand and supply behaviour in different regions.

Within the limits of this work on concrete examples simulation possibilities and quantitative estimation of the mentioned factors parameters influence are investigated.

## 2. Application of Nonlinear Programming Methods in Price Formation

One of the practical tasks often arising in price formation oriented on costs and demanding the application of mathematical-statistical methods is the determination of the manufacturing cost of different kinds of homogeneous products made at one enterprise. It is supposed that various kinds of products differ from each

other by a qualitative parameter subjected to severe quantitative evaluation and having a certain dependence upon manufacturing cost. Besides that, it is known that the greater part of the given kind of product in the total production volume, the lower is the manufacturing cost of each kind of product.

Lagrangian method as one of the methods of solving nonlinear problems can be used for solving this problem. The matter is that in economic models of investigation of dependence operations between constant and variable factors, linear ones can be considered only in the first-order approximation, at the deep investigation their nonlinearity is revealed. As a rule manufacturing cost is in nonlinear dependence upon the volumes of enterprises and resources expenses. Mathematical model of arising this nonlinear programming problem in a general view is formulated in the following way:

To find variables  $x_1, x_2, \dots, x_n$ , satisfying the system of equations (inequalities)

$$\varphi_i(x_1, x_2, x_3, \dots, x_n) \leq b_i, \quad i = 1, 2, \dots, m$$

and turning into maximum (or minimum) objective function, e.g.

$$Z = f(x_1, x_2, \dots, x_n) \rightarrow \max (\min).$$

Determination of conditional extremum begins with formation of auxiliary Lagrangian function  $L(X)$ , which in a tolerance range of solutions reaches maximum (minimum) for the same meanings of the variables  $x_1, x_2, \dots, x_n$ , as the objective function  $Z$ :

$$L(X) = f(x) + \sum_{i=1}^m \lambda_i \varphi_i(X),$$

where  $\lambda_i$  – multiplicative constant – Lagrangian multipliers.

Partial derived functions  $L$  over all variables under extremum conditions are equal to zero. Thus for practical determination of optimum it is necessary to calculate partial derivatives of the first order over all variables of function  $L$  and solve the system  $n + m$  equations:

$$\frac{dL}{d\lambda_i} = 0 \quad 1 \leq i \leq m$$

$$\frac{dL}{dx_k} = 0 \quad 1 \leq k \leq n$$

with  $n + m$  unknown  $x_1, x_2, \dots, x_n, \lambda_1, \lambda_2, \dots, \lambda_m$ .

As  $\frac{dL}{d\lambda_i} = q_i(x_1, x_2, \dots, x_n) - b_i$ , then the first  $m$  equations, which are the equations of connections, have

the form  $q_i(x_1, x_2, \dots, x_n) = b_i \quad 1 \leq i \leq m$ ,

and  $\frac{dL}{dx_k} = \frac{df}{dx_k} + \sum_{i=1}^m \lambda_i \frac{dq_i}{dx_k}$ , then the system of equations in a detailed form may be given:

$$q_i(x_1, x_2, \dots, x_n) = b_i \quad 1 \leq i \leq m,$$

$$\frac{df}{dx_k}(x_1, x_2, \dots, x_n) + \sum_{i=1}^m \lambda_i \frac{dq_i}{dx_k}(x_1, x_2, \dots, x_n) = 0 \quad 1 \leq k \leq n.$$

Conventionally the optimal solution consists of solving the given system.

For searching manufacturing cost of different kinds of production of the formulated above problem of price formation the equation of the form

$$\varphi(d_i) = x_0 + \frac{x_1}{d_i}, \quad 1 \leq i \leq n \quad \text{is used,}$$

where  $d_i$  – the meaning of  $i$ -kind product qualitative parameter;  $x_0, x_1$  – factors to be determined.

With regard to above formulated suppositions factors  $x_0, x_1$  are determined under the following conditions:

- total real manufacturing cost of all the enterprise production ( $c$ ) must coincide with the one calculated on the basis of theoretical formation

$$\sum_{i=1}^n \varphi(d_i) \cdot a_i = c,$$

where  $a_i$  – production volume of  $i$ -product variety.

- factors  $x_0, x_1$  must provide inverse proportional dependence between  $\varphi(d_i)$  and parts of  $i$ -kind product in total production volume.

Search for unknown factors  $x_0, x_1$  must provide as far as possible the least meaning difference  $\Delta i$

$$\Delta i = \varphi(d_i) - ka_i = \left( x_0 + \frac{x_1}{d_i} \right) - ka_i, \quad 1 \leq i \leq n,$$

where  $k$  – factor of proportionality.

Thus the mathematical model of the problem acquires the form:

$$\sum_{i=1}^n \left( x_0 + \frac{x_1}{d_i} \right) a_i = c$$

$$\sum \left( x_0 + \frac{x_1}{d_i} - ka_i \right)^2 \rightarrow \min.$$

Lagrangian function introduced for solving the problem acquires the form:

$$L(x_0, x_1, k, \lambda) = \sum_{i=1}^n \left( x_0 + \frac{x_1}{d_i} - ka_i \right)^2 + \lambda \left( \sum_{i=1}^n \left( x_0 + \frac{x_1}{d_i} \right) a_i - c \right).$$

Its partial derivative on  $x_0, x_1, k, \lambda$ :

$$\frac{dL}{dx_0} = 2 \sum_{i=1}^n \left( x_0 + \frac{x_1}{d_i} - ka_i \right) + \lambda \sum_{i=1}^n a_i,$$

$$\frac{dL}{dx_1} = 2 \sum_{i=1}^n \left( x_0 + \frac{x_1}{d_i} - ka_i \right) \frac{1}{d_i} + \lambda \sum_{i=1}^n \frac{a_i}{d_i},$$

$$\frac{dL}{dk} = -2 \sum_{i=1}^n \left( x_0 + \frac{x_1}{d_i} - ka_i \right) a_i,$$

$$\frac{dL}{d\lambda} = 2 \sum_{i=1}^n \left( x_0 + \frac{x_1}{d_i} \right) a_i - c.$$

Determination of the factors  $x_0, x_1$ , as well as  $k, \lambda$  meanings is provided by solving the system of equations:

$$Ax_0 + \sum \frac{a_i}{d_i} x_1 = c;$$

$$nx_0 + \sum_{i=1}^n \frac{1}{d_i} x_1 - \sum_{i=1}^n a_i k + \frac{A}{2} \lambda = 0;$$

$$\sum_{i=1}^n \frac{1}{d_i} x_0 + \sum_{i=1}^n \frac{1}{d_i^2} x_1 - \sum_{i=1}^n \frac{a_i}{d_i} k + \frac{1}{2} \sum_{i=1}^n \frac{a_i}{d_i} \lambda = 0;$$

$$\sum_{i=1}^n a_i x_0 + \sum_{i=1}^n \frac{a_i}{d_i} x_1 - \sum_{i=1}^n a_i^2 k = 0,$$

where  $A$  – a total volume of the product production  $A = \sum_{i=1}^n a_i$ .

### 3. Balance Methods in Transfer Prices Simulations

The other task of practical price formation oriented on expenses as well arises at the enterprises having complex production structure with the divisions to main and auxiliary departments. Products and services of auxiliary departments are consumed not only by the shops of the main enterprise but by other auxiliary

departments as well. As a result product and service cost of auxiliary departments includes in addition to their own expenses the costs connected with the use of products and services by other auxiliary departments. In order to determine the size of these expenses it is necessary to have the data of given products and services volumes and their prime cost. The determination of these product costs in its turn demands preliminary calculations of the product and service cost received by auxiliary departments from each other.

The solution of such problems is based on the balance model and matrix calculations. In the mathematical presentation of the problem the following designations are used:

- volume of products and services given by  $j$  department to  $i$  department -  $q_{ij}$ ;
- total expenses of consumer-departments -  $y_i$ ;
- total volume of products (services) of supplier-department -  $Q_i$ ;
- personal cost of the department -  $C_i$ ;
- unit cost (service) -  $x_i$ .

Mutual provision of product (services) is given in Table 1.

Table 1. Mutual provision of services

Consumer departments	Personal costs	Supplier departments				Costs, total	Unit cost of service
		1	2	...	m		
1	$C_1$	$q_{11}$	$q_{12}$	...	$q_{1m}$	$Y_1$	$x_1$
2	$C_2$	$q_{21}$	$q_{22}$	...	$q_{2m}$	$Y_2$	$x_2$
...	...	...	...	...	...	...	...
m	$C_m$	$q_{m1}$	$q_{m2}$	...	$q_{mm}$	$Y_m$	$x_m$
Volume of services		$Q_1$	$Q_2$	...	$Q_m$		

Economical relations, embracing two groups of the unknowns: unit cost (unit cost of service) and total expenses of every department of an enterprise are given in the system of equations:

$$x_i = \frac{Y_i}{Q_i} (i = 1, 2, \dots, m),$$

$$Y_i = C_i + \sum q_{ij} \cdot x_j (j = 1, 2, \dots, m).$$

For solving the system can be brought to a standard form:

$$x_1 = \frac{C_1}{Q_1} + \frac{q_{11}}{Q_1} \cdot x_1 + \frac{q_{12}}{Q_1} \cdot x_2 + \dots + \frac{q_{1j}}{Q_1} \cdot x_j + \dots + \frac{q_{1m}}{Q_1} \cdot x_m;$$

$$x_2 = \frac{C_2}{Q_2} + \frac{q_{21}}{Q_2} \cdot x_1 + \frac{q_{22}}{Q_2} \cdot x_2 + \dots + \frac{q_{2j}}{Q_2} \cdot x_j + \dots + \frac{q_{2m}}{Q_2} \cdot x_m;$$

.....

$$x_m = \frac{C_m}{Q_m} + \frac{q_{m1}}{Q_m} \cdot x_1 + \frac{q_{m2}}{Q_m} \cdot x_2 + \dots + \frac{q_{mj}}{Q_m} \cdot x_j + \dots + \frac{q_{mm}}{Q_m} \cdot x_m.$$

After necessary transformations the system of equations can be given in a matrix form, for this purpose the following matrix forms are introduced:

$$q = \begin{pmatrix} q_{11} & q_{12} & \dots & q_{1m} \\ q_{12} & q_{22} & \dots & q_{2m} \\ \dots & \dots & \dots & \dots \\ q_{m1} & q_{m2} & \dots & q_{mm} \end{pmatrix} \quad Q = \begin{pmatrix} Q_1 & 0 & \dots & 0 \\ 0 & Q_2 & \dots & 0 \\ \dots & \dots & \dots_j & \dots \\ 0 & 0 & \dots & Q_m \end{pmatrix} \quad C = \begin{pmatrix} C_1 \\ C_2 \\ \dots \\ C_m \end{pmatrix} \quad x = \begin{pmatrix} x_1 \\ x_2 \\ \dots \\ x_m \end{pmatrix}$$

$$\text{Then, } (Q - q^T) x = C, \text{ a } x = (Q - q^T)^{-1} C.$$

#### 4. Example of Regression Analysis in the Research of Price Formation Factors

Realization of the integral approach to price formation seems to be advisable on the basis of the cost estimation simulation and econometric price formation methods integration. Econometric methods are realized by formation of regression equations including the statistical data accounted and estimated parameters.

The use of regression models in price formation is of special importance in reference to short product life cycles. The matter is that short product life cycle makes market demand parameters rather dynamic and should be taken into consideration and actively used in forming and realization of price policy. Just this feature is inherent in the goods on the information technologies market. Figures 1 and 2 show the change of price and volumes of processor P42.4/533 sales at Latgal market of IT (in Dauvgavpils). For two years the price of these goods was reduced twice from 141.45 lats in January 2003, to 77.08 lats in December 2004. Similar dynamics is observed in reference to other goods on the market of short product life cycle.

The change in volumes of sales in different periods is conditioned mainly by non-price determinants of demand. We can see a considerable growth of sales on Easter and Christmas holidays, as well as in September, at the beginning of the school year, i.e. in the periods, when there were no considerable price changes. The most favourable market conditions take place at the ending stage of life cycle when making a reduction in prices influences the growth of sales volumes. Knowledge of life cycle stages allows to forecast the demand parameters and come to a conclusion at purchasing and realization of IT products.

Investigation of IT market of Latgal region showed that the departments selling computers have different problems in realization of price policy, but common for the most of them is the application of the multiple-factor regression method in planning the demand and price policy formation.

As an example such research was carried out and shown regarding to the mentioned product realized in one of the companies on Latgal IT market. Initial data used in calculations are given in Table 2. They include the information on interconnected two pairs of indices subject to research:

- price of product – total compiled receipts;
- volume of product sales – total compiled receipts.

Interconnection price level – volume of sales is of secondary importance.

The given problem is brought to determination of the following regression equation parameters

$$x_0 = a + b_1x_1 + b_2x_2,$$

where:  $x_0$  – volume of receipts from the total volume of company realization;  $x_1$  – product price;  $x_2$  – volume of product sales.

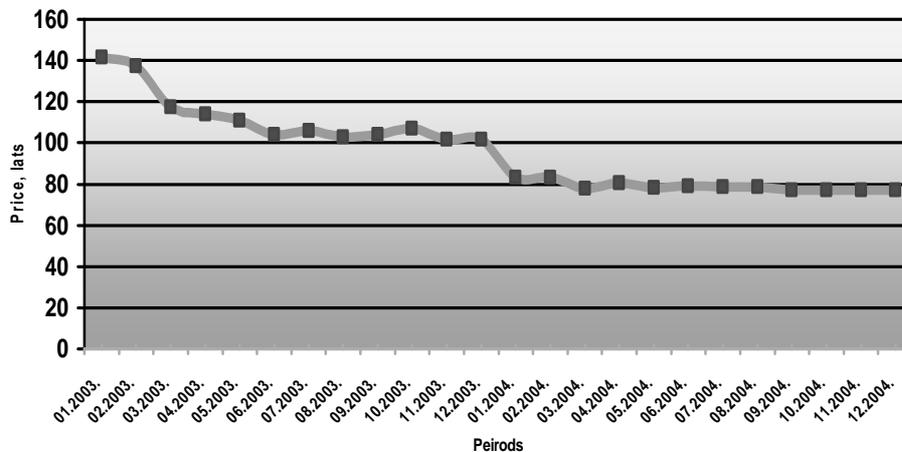


Fig. 1. Change of processor P42.4/533 price

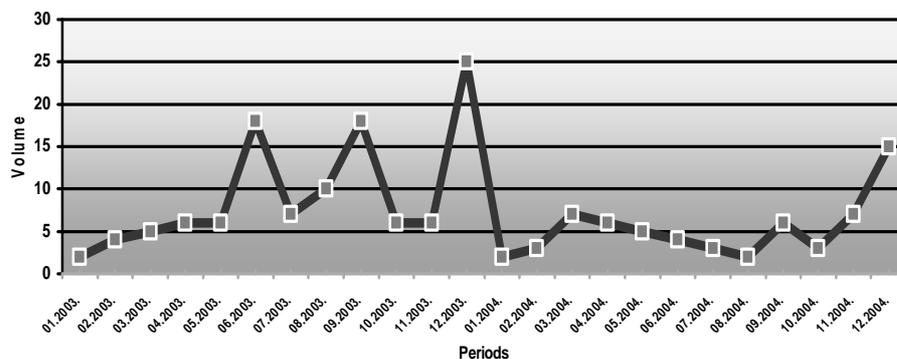


Fig. 2. Change in volumes of processor P4 2.4/533 sales

**Table 2.** Initial data for research of price and volume of processor P4 2.4/533 sales influence on the total volume of receipts

Period	Marks of period $n$	Product price, lats/product $x_1$	Volume of product sales, unit $x_2$	Total compiled receipts of the company, thousands of lats $x_0$
01.2003.	1	141,45	2	36,9
02.2003.	2	137,47	4	71,72
03.2003.	3	117,30	5	76,5
04.2003.	4	113,93	6	89,16
05.2003.	5	110,97	6	86,82
06.2003.	6	103,95	18	170,1
07.2003.	7	105,84	7	79,38
08.2003.	8	102,92	10	76,3
09.2003.	9	103,79	18	122,22
10.2003.	10	106,96	6	68,76
11.2003.	11	101,54	6	45,12
12.2003.	12	101,54	25	188
01.2004.	13	83,13	2	21,68
02.2004.	14	83,13	3	32,52
03.2004.	15	77,70	7	53,9
04.2004.	16	80,47	6	62,94
05.2004.	17	78,30	5	42
06.2004.	18	79,12	4	41,28
07.2004.	19	78,77	3	30,81
08.2004.	20	78,54	2	20,48
09.2004.	21	77,08	6	60,3
10.2004.	22	77,08	3	30,15
11.2004.	23	77,08	7	70,35
12.2004.	24	77,08	15	150,75

The product price is considered as the first factor, the volume of sales – as the second one. The data given in Table 2 are brought to a systematic view that make easier their further processing, the algorithm of which and the calculations are given in Table 3.

**Table 3.** Calculation of equation parameters

	$\sum x_0$	$\sum x_1$	$\sum x_2$	$\sum x_1^2$	$\sum x_2^2$	$\sum x_0^2$	$\sum x_1x_2$	$\sum x_0x_1$	$\sum x_0x_2$
1	36,9	141,45	2	20008,1	4	340,4025	282,9	5219,505	73,8
2	71,72	137,47	4	18898	16	321,4849	549,88	9859,348	286,88
3	76,5	117,30	5	13759,29	25	234,09	586,5	8973,45	382,5
4	89,16	113,93	6	12980,04	36	220,8196	683,58	10158	534,96
5	86,82	110,97	6	12314,34	36	209,3809	665,82	9634,415	520,92
6	170,1	103,95	18	10805,6	324	89,3025	1871,1	17681,9	3061,8
7	79,38	105,84	7	11202,11	49	128,5956	740,88	8401,579	555,66
8	76,3	102,92	10	10592,53	100	58,2169	1029,2	7852,796	763
9	122,22	103,79	18	10772,36	324	46,1041	1868,22	12685,21	2199,96
10	68,76	106,96	6	11440,44	36	131,3316	641,76	7354,57	412,56
11	45,12	101,54	6	10310,37	36	56,5504	609,24	4581,485	270,72
12	188	101,54	25	10310,37	625	56,5504	2538,5	19089,52	4700
13	21,68	83,13	2	6910,597	4	117,5056	166,26	1802,258	43,36
14	32,52	83,13	3	6910,597	9	117,5056	249,39	2703,388	97,56
15	53,9	77,70	7	6037,29	49	59,29	543,9	4188,03	377,3
16	62,94	80,47	6	6475,421	36	110,0401	482,82	5064,782	377,64
17	42	78,30	5	6130,89	25	70,56	391,5	3288,6	210
18	41,28	79,12	4	6259,974	16	106,5024	316,48	3266,074	165,12
19	30,81	78,77	3	6204,713	9	105,4729	236,31	2426,904	92,43
20	20,48	78,54	2	6168,532	4	104,8576	157,08	1608,499	40,96
21	60,3	77,08	6	5941,326	36	101,0025	462,48	4647,924	361,8
22	30,15	77,08	3	5941,326	9	101,0025	231,24	2323,962	90,45
23	70,35	77,08	7	5941,326	49	101,0025	539,56	5422,578	492,45
24	150,75	77,08	15	5941,326	225	101,0025	1156,2	11619,81	2261,25
	<b>1728.14</b>	<b>2295.14</b>	<b>176</b>	<b>2282.56</b>	<b>2082</b>	<b>3088.57</b>	<b>17000.8</b>	<b>169854.6</b>	<b>18373.08</b>

As a result of the calculations shown in table 3 the following parameters meanings are obtained

$$n = 24$$

$$\sum x_0 = 1728.14; \quad \sum x_1^2 = 2282.56; \quad \sum x_1x_2 = 17000.80;$$

$$\begin{aligned} \sum x_1 &= 2295.14; & \sum x_2^2 &= 2082; & \sum x_0 x_1 &= 169854.60; \\ \sum x_2 &= 176; & \sum x_0^2 &= 3088.57; & \sum x_0 x_2 &= 18373.08. \end{aligned}$$

Substitution of parameters allows to obtain the necessary system of equations :

$$\begin{aligned} 24a + 2295.14b_1 + 176b_2 &= 1728.14; \\ 2295.14a + 2282.56b_1 + 17000.80b_2 &= 169854.60; \\ 176a + 17000.80b_1 + 2082b_2 &= 18373.08. \end{aligned}$$

## 5. Conclusion

Total sequence of models structure and price calculation in such approach must include the following stages: setting goals and price formation tasks; estimation of demand parameters; evaluation of the enterprise costs (expenses); analysis of price and competitors' goods quality; choice of price formation methods; calculation of initial price; taking into account additional factors; determining the final price.

Introduction of simulation into the price formation process gives the possibility of multivariant calculations at which different principles of price differentiation can be realized: by consumers' groups, by time, by life cycle phase, by regions, in dependence on the size of a purchase, in dependence on the purpose of the goods use. The increase of the buying power of different market segments and the efficiency of the enterprise economic management will be the final result.

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# **THE COORDINATION OF AIMS AND RESULTS OF ACTIVITIES OF A TRANSPORT ENTERPRISE ON THE BASE OF THE MANAGEMENT TECHNOLOGY OF BUSINESS PROCESSES**

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The combination of coupled managerial approaches directed to designing a full-functional system of improving the business functioning is examined in this research. This improvement could be achieved on the basis of the business processes uninterrupted monitoring.

Such system is oriented to the establishment of the integral analytical business platform for providing the coordinated objectives setting, timely revealing and neutralization of the unforeseen events, decision-making at different levels of an organization.

**Keywords:** *coordination, business-processes, goal-setting, decision-making, requirements to executors*

## **1. Introduction**

In contemporary conditions against a background of the tense competition, a management of the company needs such management tools, which allow the efficient assessment of changes taking place in business and a constant monitoring of various activities of the company. Today the application of the old technologies by managers is often inefficient and does not give the company an opportunity to see timely the changes in its activity and the surrounding conditions of business.

Each year companies start using wider the process approach to management, which is based on the company's business-processes management [1]. Any business-process represents a totality of the interconnected types of activity transforming inputs into outputs interesting for the internal and external consumers. The advantages of the company's orientation to business-processes consist in the fact that:

- a value of the final product is formed in the very course of business-processes;
- each process has a consumer, and, a concentration on the process promotes a better satisfaction of consumers' demands;
- the process management allows better controlling time of the work execution and resources.

Therefore, a task of the analysis and optimisation of business-processes today is foremost for the arrangement of the company's efficient management. For today there is a wide spectrum of methods and technologies of management both in the management science and in the adjacent areas. For example, nowadays various systems of business-processes modelling and their optimisation on the reproduction basis on a model of different versions of the company's development strategy are very popular.

Nonetheless, to have such decision to be well grounded, the constructed model of business-process must be adequate and approximated to reality at maximum. That is why in the presence of a large number of modern software modelling tools a proper process of the computer model construction goes to the second ground relatively the development of a conceptual model on the basis of the integrated system analysis of business-processes.

Contemporary managers should have mechanisms allowing presenting business-processes precisely, visually, and in proper time. Due to a clear vision one can reach the effective interaction and mutual perfection both of the internal and the external industrial processes.

It is necessary to mention that in practice during the description of business-processes companies face the considerable difficulties. Often the IT department specialists mastering the art of the business-processes software description but having no relation to the real-time management perform the description of the enterprise's business-processes. The results of such description misrepresent a real situation of business due to the simplification of business-processes and description of the internal processes of the enterprise without taking into account clients' interests and suppliers.

On the other hand, when the description of business-processes is performed by the top-management of the company striving to optimise the organizational structure or to choose a new strategic way of development, the accuracy of the business-processes also is unsatisfying. It happens because the representatives of top-management do not have enough time for the opportune every-day supervising the changes occurred in the organizational business-processes. That is why only 10% of organizations implement their strategy in practice according to Kaplan, R. & Norton, D. [2].

The new management technology developed on the basis of the offered methodology of business-processes management (MBP) is suggested in this paper. Implementing the "management of business-processes" term is stipulated by designing the system of a description and support of business-processes by own forces on the basis of the non-traditional approaches. For the analysis of the opportunity of this technology implementation the pattern of the business processes management by a number of the Latvian transport enterprise is being investigated.

## 2. The Investigation of MBP Problems in Latvian Transport Enterprise

The analysis of situation with MBP has been performed in 12 successful Latvian transport enterprises. The choice of exactly this branch of industry is conditioned by the dynamics of the modern processes reflecting in the quickly changing environment of the transport sphere, and, as well by the fact that the authors' research activity is connected just with this area.

Two methods were used during the research: testing and interviewing of the management.

These methods were used for the estimation of the existing support structure of the business-processes with the aim to ground their efficiency, and as well for the investigation of opportunities of their improvement and visualization providing.

The professionals of three basic groups participated as the experts. The leaders of enterprises formed the first group, the managers of subdivisions formed the second group, and the interviewing business-consultants represented the third group.

The main results of testing and interviews are represented in Table 1.

**Table 1.** The estimation of the entire representation of the enterprise's business-processes

Major mechanisms influencing the business-processes monitoring	Leaders of enterprise	Managers of subdivisions	Business-consultants
1. The presence of the efficient mission of the enterprise	71%	54%	37%
2. The observance of the developed mission	6%	3%	1%
3. The presence of the efficient strategy.	65%	42%	24%
4. The implementation of the strategy	16%	11%	8%
5. Implementing the balanced system of characteristics (BSC)	15%	14%	13%
6. Implementing the process-oriented technology (ABC/M)	37%	32%	29%
7. Implementing the client relationship management technologies (CRM)	39%	34%	33%
8. The presence of the certificate of quality	29%	27%	29%
9. Application of the business-processes re-engineering tools	19%	17%	21%
10. Application of the "Lean Manufacturing" tools technology	2%	1%	3%
11. Implementing the cost estimation mechanisms	18%	12%	14%
12. Own software development	95%	92%	95%
13. The percentage of failures due to the business-processes non-coordination	50%	72%	77%
14. The need in the business-processes entire support technology	77%	81%	85%

As evident the top management of the enterprises estimates the arrangement and performance of the management processes slightly higher than the heads of subdivisions and business-consultants. In general, one can see the tendency of the enterprises to implement the efficient managerial tools and technologies. A high percentage of the opinions is connected with the necessity of the mission development and the choice of the strategic way of the development. But the practice of their implementing shows very low results now. It is impossible to follow the determined mission, because of non-concordance of the set aims and obtained results.

The unforeseen circumstances existing during implementing the strategy, the imperfective methodology of decision-making, non-coordination between the descriptive and real business-processes could be the main reason for it.

The management of the examined enterprises is striving to eliminate the given disadvantages by the purchase of the modern software products and information technologies. However, a high percentage of failures

arising during the elimination of malfunctions in case of non-concordance of business-processes characterize the inefficiency of their use. The thing is that the implementation of the new information technology often leads to new additional problems caused by the presence of the non-standard and incompatible software as well as by the absence of the computer platforms succession. Own software products do not help in such cases. Their development should be performed only after the execution of one interconnected description of really functioning business-processes. The given conclusion is confirmed by a high percentage of positive experts' opinions related to demand of the business-processes entire support technology application.

This is the aim of the suggested business-processes management technology, because with its help it will be possible not only to perform the efficient management of the business-processes, but also to use it as a conceptual base for business modelling and own software products planning.

### **3. The Business-Processes Management Methodology**

The main peculiarity of the MBP methodology consists in the uninterrupted effective supervising and perfecting the business-processes. In order to do this one should provide the visualization and transparency of business-processes by means of concordance of aims and final results of all operations implemented by managers of enterprises within the organization as well as beyond it.

The estimation of the efficiency of the obtained results is performed on the basis of cost measurement allowing to define not only the correctness of the made decisions, but to reveal the concealed levers of the organization development and to find out components of the specific losses.

The assignment of aims is being performed during the detailed investigation of possible defects, which can lead to the business-processes failures in future.

At that the interests of the parties involved in a certain business are taken into account. By that at the initial stage of business the requirements capable to provide the visualization and transparency of the future business-processes are developed. The development and monitoring of the business-processes are given to a special team of managers responsible for bringing together all operations in one taking into account the shift of priority towards the inter-team relationships, but not towards the existing operational cycle. It is achieved on the basis of the values creation maps where beforehand the elements capable to lead to the non-concordance of business conduct are taken into account. In other words the case in point is the arrangement of a constant monitoring with the help of the entire vision of all business-processes. In the given context a certain logical consequence of the distributed in time and space activities aimed at the obtaining a certain service and/or products is taken into account. So, for example, for the transport enterprise when describing the processes it is necessary to take into account not the traditional activities connected with the reflection of the transport systems and the involved personnel, but to be guided by all participants' mutual perception of objects and events images with their further representation in certain activities. First of all, all this accelerates the exchange of business information and reduces risks of untimely and erroneous activities leading to wrong and inefficient decisions.

The offered methodology was developed on the basis of the scientific methodology of prevision, adaptation to the mutual goals of the enterprise development and variable conditions of the external environment. The given methodology includes the elements of investigating the structure, logical organization, methods and means of activities.

In a wide sense the MBP methodology forms a system of notions about the maximal approximation of the described business-processes to real ones, which by its essence become a subject of study and rationalization. In the given context any methodological knowledge must appear, on the one hand, as a totality of prescriptions and norms where a content and sequence of certain stages of the business-processes construction are fixed, on the other hand, as a complex of descriptions of the factual execution of this processes. According to such an approach, the function of a concrete task consists of an internal arrangement and regulating the process of cognition of the practical creation or transformation of an investigated object.

Thus, the methodology of the science (MBP) should develop the characteristics of components of the scientific research. To such components the object of investigation, the object of analysis, the set tasks, the used means and procedures should be referred. All of them should be joined in such a way that it would be possible to understand the process of formation of the entire vision of a researcher during the concrete task solving by him/her.

Thus, the main attention should be paid on the search for the right way, which is situated between the internal facilities of the enterprise and the possibilities, which are opened in the competitive sphere of its activities.

In other words, the rational approach of creating business-processes is used for determining the course of actions and the best extent of reaching goals.

The further implementation of the rational principles should be broadened on the basis of the systematic approach, which calls in question, the universality of the single model.

The goals of this model depend on social characteristics of the main designer of business-processes and the social model where he/she acts.

The realization of MBP starts with the preliminarily setting a task where the conditions of business-processes creating are formed taking into account their whole perception. In order to do that at first it is necessary to determine the clear borders of the research area with the obligatory instruction of the factual and desirable state of affairs. The factual state of affairs is determined by the real characteristics of the research area but the desirable situation is determined by a vision of the originator's of the certain problem. The major component of any arrangement is the exposure of the discrepancy that appears between the factual and desirable state. These states characterize two points of view formed during the perception of the mentioned state of affairs. It should be noticed that the presence of such discrepancy determines a fact that the problem exists. As a rule any problem might be viewed from the position of eliminating certain disadvantages that in totality represent the arisen discrepancy. The elimination of these disadvantages, as a rule, predetermines urgency of a problem and very often characterizes the novelty in task setting. In case of proving the actuality and novelty of a problem, it is necessary to offer the well-grounded decisions for elimination of disadvantages on the basis of various tools of the factor analysis originating the reasons and characteristics of a certain problem, as well as the methods of estimation and realization of the selected efficient means for forming and using the enterprise's resources. In such a way, one could reveal the composition of MBP on the stage of task setting. This composition is qualitatively determined by way of the essential signs, but quantitatively it is determined by way of the differential signs. In order to determine signs one can search for special, common and general features. This generalization due to the direction of the detailed elaboration could be determined in two ways: "top-down" relatively the higher categories or "bottom-up" regarding the lower categories. Thus, the selected signs allow finding the similarity of perception when reflecting the true reality in the subjective activity of the business-processes designer. Further this activity is reproduced at the quantitative level and estimated with the help of indices of the integral features of differences. As a result, the chosen indices should be used as a basis for the next decision.

The second stage of the MBP methodology represents the development of the subject of the research. Such a subject is constantly being changed according to the level of changes in the designer's vision of the real situation of business-processes. In spite of this, different changes of the internal business-processes relatively to the external environment are used as the subject of the MBP research. The investigation of the subject is held according to the determined problems, goals and tasks directed to solving problems in the best way. Thus, any change of the business-processes designer's vision caused by the happened changes of surroundings and internal environment should influence the processes of any decision-making. The subject of MBP could be represented in the way of the systematic approach of the strategic variables management (Fig. 1).

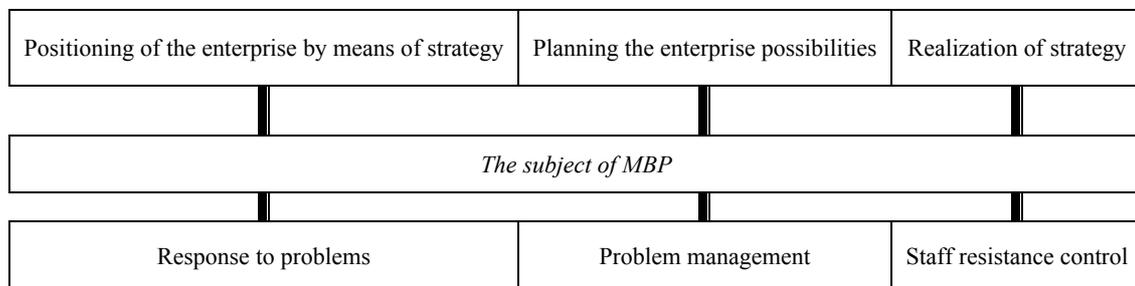


Fig. 1. The subject of MBP as the systematic approach to the strategic variables management

The subject of MBP is examined from the position of the systematic approach, in which within the frames of the strategy, forming a positioning of the enterprise in the community external environment is performed. For that, the strategic planning of the enterprise possibilities is implemented. Further in the process of the strategy implementation in real time mode the strategic response to problems, which must be eliminated during a continuous monitoring, is performed. Besides, the systematic control of the staff resistance relatively the implementation of the determined strategy is ensured.

The analysis of the object of the research is conducted on the third stage of the MBP methodology. The components of the business-processes are used as the object of the research. During their analysis the structure of the business-processes is determined and investigated at the level of the optimal modes of the separate elements and components functioning. At first, the characteristic elements of the certain business-process are determined from the position of MBP, and then its further comprehension is performed by means of the generalized components. It is necessary to notice that in this case only the composition of elements and components is grounded, while the existing links are determined only partly. The existence of the uncertainty factor and the informalized character of the business-processes structure, provoke it. Thus, it is necessary to reveal the essence of the existing problem. At that, the degree of the detailed elaboration of the problem is revealed at such extent that it is possible to determine the obvious consequences of further decision-making.

The final interfacing of the existing elements and components happens at the fourth stage of the MBP methodology. The formal aspects of the given methodology are connected with the formal structure. At that the methods of the components rearrangement in the old construction with the aim of obtaining a new one or joining new parts by new methods are used. At the qualitative level such actions represent the fixation of links between the elements, which were determined by the way of decomposition of signs. It allows focusing the subjective entire view on the existing business-processes. The final generalization is implemented during the application of the qualitative estimation procedures allowing establishing relations among the business-processes components. These relations are used for implementing reproduced properties and comprehending the components of the integral formation in the unforeseen circumstances.

Thus, the examined MBP methodology is represented as the following scheme (Fig. 2):

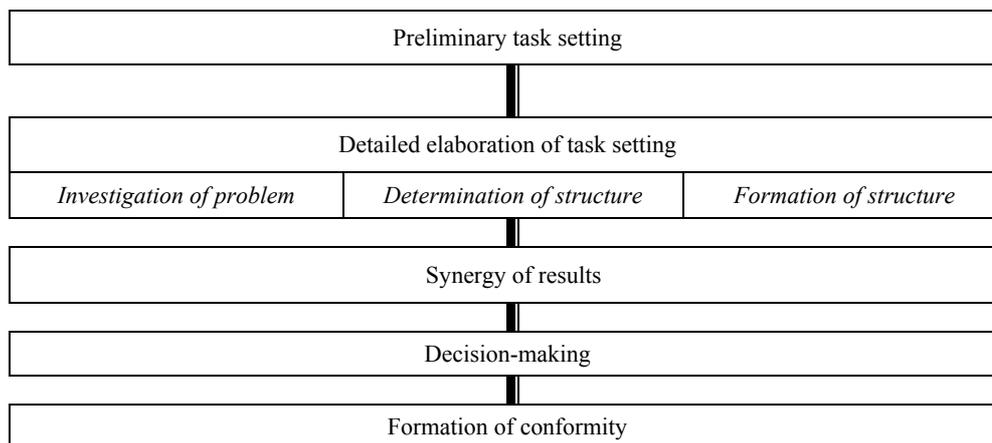


Fig. 2. The enlarged scheme of the MBP methodology

The appearance of the “Synergy of Results” stage is stipulated by the achievement of the positive cumulative effect  $(2+2=5)$ , which should significantly increase the combined results of different separate business-processes in comparison with the efficiency levels of each of them separately. Thus, the business-process contributes in the process of the extra cost making at the level of the enterprise that finally influences the results of synergy.

The possibility of making timely efficient decisions aimed to the achievement of the desired state of the business-processes in future is ensured at the “Decision Making” stage. The efficiency of the made decision depends on the professionalism of the specialist responsible for decision-making and his/her level of preparedness for decision-making. During the decision making it is necessary to observe the following rule. If the results of the managerial decisions analysis force us thinking about the possible development of the situation, and as well as the correctness of the set goals setting is doubtful, then it is necessary to revalue and change the enterprise strategy. The process of changing the strategy consists of two phases. In the first phase the new possibilities and dangers are taken into account, the resource base of the enterprise is redistributed, the system of values of the top management is defined more exactly and the society social responsibility of the enterprise is revalued. The second phase of the strategy changing includes the modification of the organizational structure, the more precise definition of the organizational processes configuration, and the management style change.

The “Formation of Conformity” stage represents the process of coordination of the determined strategy and MBP from the point of view of the arrangement of the efficient functioning of the enterprise. The capability of management to follow the elaborated strategy is investigated at this stage. The start for the successful implementation of strategy is formed just here. Therefore, the application of the MBP methodology represents the complicated process connected with the organizational factors, and as well as with the special managerial thinking forming. Such thinking should be developed not only by the employees designing and improving the business-processes but also by the entire staff.

#### 4. Broadening the Area of the System Character of the Business-Processes Management Methodology Owing to the Non-System Factors Investigation

The examined MBP methodology refers to the system methodologies class. This feature advantageously distinguishes it from other methodologies. However, at present the system methodology in spite of its entire value and the professional necessity is coming in for criticism increasingly from the position of its practical application [2].

The main reason for such criticism is the artificial bringing of this system to some balanced category, with the help of which one could get rid of the revealed contradictions capable of leading to the simple decisions

obtaining. Thus, it is possible to approach the estimation of system functioning in the conditions of uncertainty and constant unsteadiness provoked by the internal manifestations.

As a rule, such manifestations occur on the basis of redundancy or the lack of information that in its turn leads to the insufficient knowledge and as a result negatively influence the efficiency of the made decisions. The solution of this problem one can search beyond the frames of the traditional system methodology. At that it should be taken into account that the essence of the problem lies not in the limited nature of the scientific understanding but in the complexity of the practical implementation. Thus, the systematic approach implementing assumes the presence of the entire interconnection of a great number of components. These components are limited by the subjective perception of the true reality as well as by the imperfection and incompleteness of the applied tools. Besides, the existing links among the system elements are changing very often that finally results in its disordered integrity. In this case a special influence on the integrity destruction is caused not by the composition of elements (and components) forming the system or their structural constructions (links and relations), but the conceptual character of reasons and features of elements from the position of their impact directly on the system and its surroundings. The composition and structure are created on the basis of standard procedures and that is why the majority of them have the stable systematic nature (excluding the very complex systems).

The composition of the system is determined by the visions of its designers with their individual characteristics, personal values, emotions, spirits, and so on. All this leads to the mutually exclusive contradictions among separate individuals and different communities. Just these contradictions create the non-systematic component of controllability, which is the source of conflicts and disagreement of the “aims-and-results” compliance. Thus, the compositional aspects are the typical signs of the system disorganization. These signs should be studied before the system designing that will allow the qualitative foreseeing and perceiving the methods and ways that are impossible to systemize. In that way it is possible to determine not only the non-systematic problems in the process of intercepting the weak signs of their manifestations, but also in advance to determine the ways of restoration of the aims achievement coordination. In other words, the system character area is widening during the investigation of the non-system problems. Their further detailed elaboration should be directed to the estimation of the organizational defects owing to the reproduction of their properties at the qualitative level. The qualitative estimation of defects at the pre-planning stage will allow revealing the nature of dangers of the unknown origin and estimating the scale of their overcoming. As a result, the precision and formalization of requirements to the designing the future system are increased. It should be noticed that with such approach the process of the step-type detailed elaboration does not impede the system controllability because the external requirements are developed according to the activities of the certain executors.

Such activities taking into account the specificity of the quality factor sensitiveness and having the quantitative grounding of the consequences allow formalizing the future business-processes owing to the successive ensuring a transparency of the performing functions but not vice versa (when at the beginning the business-processes, for which the executors’ functions are formalized are described and then the general demands are made). It its turn, this leads to the improvement of the quality of the business-processes monitoring, decrease of a number of the fuzzy rules, reduction of the redundancy, improvement of the coordination of aims and the activity results.

In this connection the investigation of the non-system preconditions is included, which in conditions of the insufficient knowledge, growth of uncertainty and mutual dependency are directed to the non-standard methods allowing linking the future business-processes with the certain executors. Moreover, the immersion into the non-system environment at the initial stage of the task setting broadens the systematic character area where the certain priorities on the path to the system improving during the process of its designing are determined. With such approach before the certain activities implementing it is necessary to improve the methodological ways of the objective reality cognition and search for the efficient managerial decisions. Revealing and estimating the possible future defects stipulate their efficiency. Moreover, these possible manifestations were preliminary investigated because a set of the warning measures was developed for them. First of all, the tools, on the base of which one can solve problems in the unfavourable situations, are at the disposal of managers. In future, it improves the quality of the offered product promoting owing to the managerial thinking perfecting. In other words, the systems tools are supplemented with the off-system mechanisms allowing revealing the nature of the hidden events, which finally determine the area of the unstable state and development of the complex system. Just here the development of the system approach methodology takes place that ensures a stable organizational development in conditions of uncertainty and continuous changing the surrounding environment. Thus, the result-oriented designing where the problem situations and dangers are included increases the general controllability of the organization. In cases when the non-standard situation arise in practice that can disturb the equilibrium of the organization then the earlier grounded impacts for neutralization of the potential dangers are performed. In that way, the planned recovery of the organization takes place [3].

It is necessary to notice that the made decisions are investigated, information related to them is processed and accumulated. All this is a base for making the new performing demands, aims and results coordinating, the business-processes correcting that, in its turn leads to perfecting the practice of the MBP system methodology application.

### 5. The Interpretation of the MBP Goal-Setting Elements by Means of the Interdependent Balanced Indices Set

The efficiency of the results of the made decisions of the MBP methodology is determined on the base of the balanced interdependent indices set. One should choose such indices, which reflect their clear relationship with the specificity of the developed business-processes and meet the goals set in advance. The estimation of indices is held according to the set periodicity of registration and observations. Between the selected indices the analysis of the existing interrelations is held from the position of their influence on the initial, intermediate and final results of the enterprise activities, which, in their turn, should be coordinated with the determined goals. The indices calculation algorithms are designed taking into account all possible participants' demands made to business. Finally, these requirements are brought to a set of restrictions for the certain business-processes.

Figure 3 represents the scheme of the successive measurement of indices in the frame of MBP.

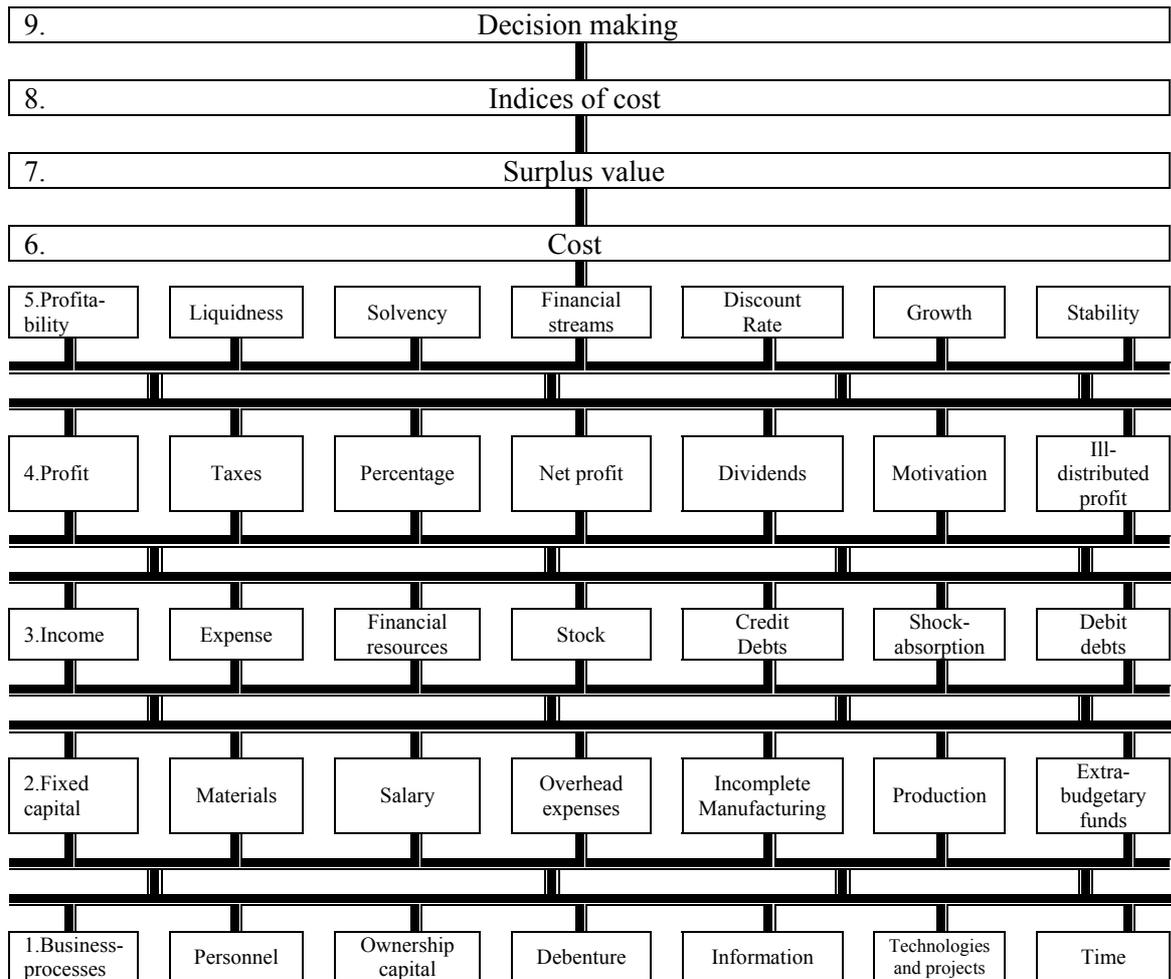


Fig. 3. Forming a balanced complex of MBP dynamic indices

The given scheme of the indices balanced complex formation to some degree meets the process of building a tree of aims. The major distinction is the technology of designing. The descending designing technology "top-down" is implemented during the process of opening multi-level system of aims.

The purport of creating and existence of the enterprise is investigated at the initial stage. This purport should open the desired ideal concept about the business-processes state. At that, the conditions of the balanced approach by a long-term growth and short-term results are determined, at which the set parameters must exceed the analogous indices of competitors. Then the mission of the enterprise reflecting the social destination of a working enterprise is formulated. The mission is the strategic instrument identifying the target market and the enterprise's business. The given tool should help the enterprise staff to perceive and interpret the business-processes in a similar way. Thus, the preconditions for the mutual cognition of values are created. Such cognition takes place on the basis of the standardized rules of behaviour determined by the sum of skills, personnel and a style of management. The standardized rules characterize the target directions successively revealing the strategic and structural aspects of the business-processes arrangement. First of all the most general

milestones are outlined in the planning period that are supposed to be achieved in full volume or in its general part. Thus, according to the set milestones obtained on the basis of the designed system of norms, the control of the desired final results of the enterprise's goals achievement is performed.

The considered order of goal setting is oriented to forming a hierarchical structure of indices that quantitatively reflects each level in the general system of aims. Moreover, the design of such structure is held taking into account the requirements of the performed goal setting. In the other words, demands made by the system of aims are the external specifications relatively the designed complex of indices.

The Figure 4 reflects the influence of goal setting mechanisms on the choice of quantitative indices.

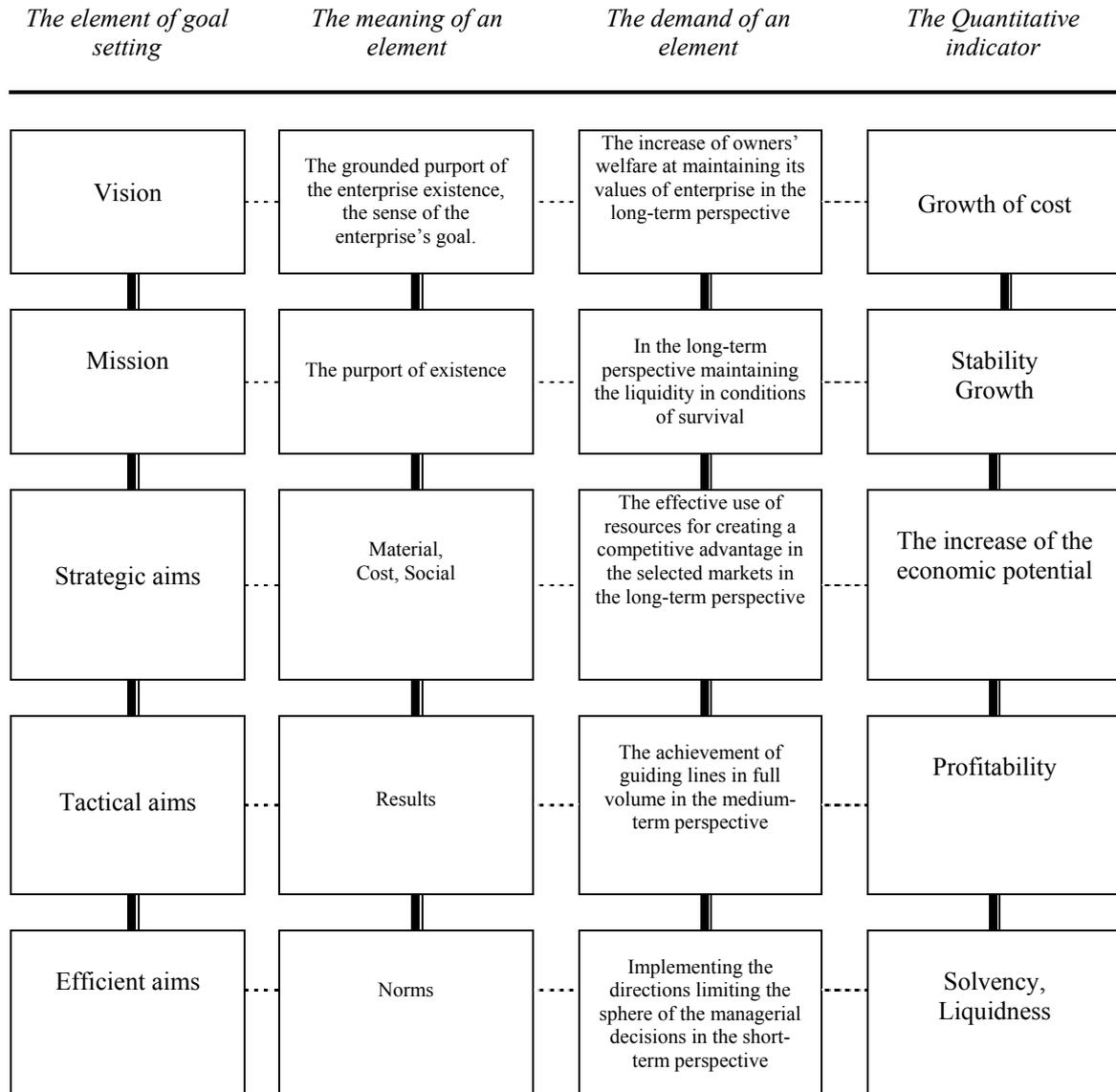


Fig. 4. The scheme of the representation of the elements of aims having a special order and quantitative indices in MBP

To illustrate the suggested scheme it is possible to consider the "Vision" term. The requirement to this element consists in promoting the welfare of owners and at the same time maintaining the value of the enterprise in the competitive market. The estimation of the owners' welfare is characterized by the increase of the enterprise cost. Measuring enterprise cost is implemented on the basis of the discounted cash flows, which are formed according to the algorithms determined in advance.

## 6. Conclusions

The suggested MBP represents the conception based on the combination of some popular managerial approaches: the process-oriented management, the balanced system of indices, the management of interaction with clients and suppliers etc. The major role of this concept consists in the complex dynamic examination of the

modern combined approaches directed to the full-service working system designing. It is necessary to mention that before using the fashionable approaches of business administrating it is necessary to develop the strategy and to form a set of measures for the feasible aims achievement. The application of the given technology will allow a company reproducing the images of the business-processes more precisely and coming closer to the true reality that, finally, will allow integrating the most necessary systems in a single whole.

First of all a company should create a common interrelated description of the real business-processes in force, and only then to select certain software products and technologies. Moreover, such selection should be done after the development of the general strategy and a concept of the balanced indices is formed. It will provide an opportunity to obtain and accumulate the necessary and valid business information. Besides, the transparency and visualization of data fundamentally influencing the management process are ensured. As a result all the participants interested in the certain enterprise are provided with an opportunity to see their mutual coordinated moving in one direction.

Since the preliminary stage has shown the interest of the Latvian transport enterprises to apply more efficient management systems, the next stage will be the introduction of the suggested technology in certain enterprises. It is assumed that the application of the MBP technology in practice will allow developing the company's state assessment and diagnostics integrated system allowing a considerable increase of a company's controllability level and quick response to changes of the state of the external and internal environment of a company.

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