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Inventory control system analysis using different simulation modelling paradigms

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ANNOTATION

This work is aimed at summation and systematization of experience with different simulation software types on the basis of so-called simulation paradigms, which can be used to build flow-oriented models of inventory control systems. Identification and analysis of such paradigms is the fundamental scientific element of this work.

In the overview part of this work, there are basic types of material inventories that are created and used in systems of production and distribution of goods. General types of modern manufacturing and goods distribution control systems (for which inventory control is one of the most important functions) are listed in this work. Next, signs and factors that determine the diversity of real inventory control systems and their models are considered.

In this work it is stated that two classes of mathematical models – they are analytical and simulation models – are implemented for the analysis and optimization of inventory control systems. Examples of all presently known types of models that belong to these two classes are considered.

The body part is dedicated to the development of a method for selection and implementation of the three simulation paradigms, which are fundamentally different from each other in ways of scheduling and realization of events and flows within the models. In simulation software ExtendSim package, these paradigms are called "continuous", "discrete event" and "discrete rate". This work includes a detailed analysis of models built with the implementation of all three paradigms that display overall basic conceptual model of inventory control system. This analysis served as a basis for the recommendations posed regarding the selection and implementation of simulation paradigms.

The last part of this work provides examples of inventory control systems' simulation models designed by the author and being different in their levels of difficulty and function. The majority of the examples contain descriptions of the corresponding analytical models. These examples involve the results of quantitative experiments, targeted at the analysis and optimisation of the considered inventory control systems.

The promotional work consists of an introduction, 4 chapters, a conclusion, a list of references, and 2 appendices. The work comprises 167 pages and includes 66 figures, 23 tables and 30 formulas. The list of references includes 102 sources.

ANOTĀCIJA

Darbs ir virzīts uz dažādu imitācijas modelēšanas programmatūru, kuru pamatā ir tā saucamās modelēšanas paradigmas un kuras var izmantot krājumu vadības sistēmu plūsmas modeļu uzbūvei, pielietošanas pieredzes apkopošanu un sistematizāciju. Šādu paradigmu noteikšana un analīze ir šī darba galvenais zinātniskais komponents.

Darba pārskata daļā ir apskatīti materiālo krājumu pamatveidi, kuri tiek radīti un izmantoti preču ražošanas un sadales sistēmās. Uzskaitīti arī mūsdienu ražošanas un preču sadales vadības sistēmu pamata tipi, kurās krājumu vadība ir viena no būtiskākajām funkcijām. Tālāk ir apskatītas pazīmes un faktori, kas nosaka gan reālu krājumu vadības sistēmu, gan arī to modeļu daudzveidību.

Darbā parādīts, ka krājumu vadības sistēmu analīzei un optimizēšanai izmanto divas matemātisku modeļu klases: analītiskos un imitācijas modeļus. Apskatīti visu šodien zināmu modeļu tipu piemēri, kuri attiecas pie šīm divām klasēm.

Darba centrālā daļa ir veltīta trīs modelēšanas paradigmu izvēles un pielietošanas metodikas izstrādei, kuras principiāli atšķiras viena no otras ar notikumu plānošanas un realizācijas veidiem plūsmas modeļos. Imitācijas modelēšanas programmatūrā ExtendSim šīs paradigmas saucas „continuous“, „discrete event“ un „discrete rate“. Darbā tiek sīki analizēti modeļi, kas uzbūvēti, izmantojot visas trīs paradigmas, un atspoguļo vispārējo krājumu vadības sistēmas pamata konceptuālo modeli. Uz šīs analīzes pamata ir noformulēti priekšlikumi modelēšanas paradigmas izvēlei un pielietošanai.

Pēdējā darba daļā tiek parādīti autores izstrādāto krājumu vadības sistēmu imitācijas modeļu piemēri ar dažādu sarežģītību un iepriekš paredzējumu. Lielākā daļa no piemēriem satur atbilstošu analītisku modeļu aprakstu. Tiek parādīti skaitlisku eksperimentu rezultāti, kuru mērķis ir apskatāmo vadības sistēmu analīze un optimizācija.

Šīs promocijas darbs satur ievadu, 4 nodaļas, secinājumus, izmantotās literatūras un avotu sarakstu un 2 pielikumus. Šis darbs sastāv no 167 lapām, iekļaujot 66 attēlus, 23 tabulas un 30 formulas. Literatūras sarakstu sastāda 102 avoti

CONTENTS

ABBREVIATIONS

AIS	Analytical Information Systems
ERP	Enterprise Resources Planning
MRP	Material Requirements Planning
MRP II	Manufacturing Resource Planning
JIT	Just In Time
SCM	Supply Chain Management
TQM	Total Quality Management
ECR	Efficient Consumer Response
CD	Cross Docking
QR	Quick Response
VMI	Vendor Managed Inventory
EOQ	Economic Order Quantity
VBA	Visual Basic for Applications
RDC	Regional Distribution Centres
AB	Agent Based modelling
SD	System Dynamics

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INTRODUCTION

Analytical Information Systems (AIS) are created at enterprises with complex business structures, when effective decision-making process in the field of planning and management calls for the processing of large volumes of diversified data coming from multiple sources (Power, 2001). For the prompt analysis of data, such systems frequently apply OLAP (On-Line Analytical Processing) technology and a Data mining method for the intelligent information processing. AIS do not perform the function of basic scheduling and control in ERP-type systems (Monk, 2006), but they just supplement them, because structures of ERP-type systems ignore and disallow “extraordinary” data analysis experiments (ERP = Enterprise Resource Planning). Thereby, development environment and implementation of special models analysing and predicting the functioning of the enterprise is a merit of AIS-type systems. The further development stages are embodied in Business Intelligence (BI) systems, which allow solving a wide range of problems that are related to the operational activities of the enterprise, and to the development of its strategic plans (Watson, 2007; Rud, 2009).

Inventory control is one of the essential functions for both manufacturing and logistics enterprises, because decisions made in terms of this function determine the directions and volumes of flows of cargo and goods, so making a connection between the enterprise and its suppliers. The search of effective inventory control strategies for certain enterprises is performed with the help of special models that do not belong to the standard ERP-type software, but they are part of relevant AIS. Most often such models are developed by dint of simulation software that is intended at simulation of processes in manufacturing and logistics systems. This work is aimed at summation and systematization of experience with different simulation software types on the basis of so-called simulation paradigms, which can be used to build flow-oriented models of inventory control systems. Identification and analysis of such paradigms is the fundamental scientific element of this work.

Actuality and motivations of the research

Creation of inventory and its management is an integral part of many kinds of activities in production and logistics. It is widely known that creation of large inventory stocks results in increase of warehousing costs and blocking of capital. On the other hand, the decision to reduce inventory stocks leads to the necessity of more frequent commissions and orders for goods, which generally propagates the increase of total expenditures for its delivery, meanwhile, still preserving the risk of shortage of goods (situations like ‘out of stock’). Thus, some questions appear: what is the optimum size of the order, how frequently should orders be made and in which volume should goods (staple) be bought up with every order, so that to minimize total expenditures related to the receipt and storage of goods. To find principal answers for these questions, methods and models of inventory control theory are used. Based on this field's obtained results, practical algorithms and computer programmers for inventory management in systems of production and logistics are developed and implemented.

The number of possible variants of how to organize inventory control systems is practically endless, because each variant can be mixed with many constrains encountered in real life and it means the number of possible variants for inventory control systems management amounts to thousands.

As a matter of principle, for the quantitative research of inventory control systems can be implemented two classes of models, which are: analytical (purely mathematical) and simulation (computer) models. Both of them are associated with mathematical models, and each certain model is based on a specific theoretical (conceptual) model. In the class of analytic models available a large number of modifications and extensions for the classical Wilson's model, but inventory control task formulation can always appear and have no existing analytical model, or which, in principle, cannot be even created. In such a case, the only opportunity to study the effectiveness of the inventory control strategy is to implement one of the computer simulation types. The simulation model allows not only to reflect almost any features of the studied inventory control system, but also to formulate the system's optimality criteria based on the delivery process's financial model of any complexity.

Based on the aforesaid, it becomes clear why exactly the use of simulation modelling for the analysis on inventory control systems is a subject of this thesis. Only the simulation modelling and nothing else should now be considered as a universal method of analysis for the problem solving and inventory control systems optimization in real manufacturing and logistics systems. Currently characteristic is the fact that the vast majority of simulation models are based on the paradigm of "discrete event". Good known paradigm of "system dynamics" in the last 20 years of relatively rarely used, because it is not supported by the majority of software products, designed for simulation of production and logistics systems. The third paradigm of "discrete rate" relatively recently was formulated in the field of applied simulation and it has never been used to simulate the inventory management systems.

The relevance of the paper is based on the assumption that modern experts, who use simulation to analyze the inventory management systems, should be aware of the advantages and disadvantages of the three basic paradigms of simulation and be able to choose the most appropriate paradigm depending on the requirements for the accuracy of the display of real system in the model material, information and financial flows.

Goal and tasks of the research

The goal of this thesis is to develop and test the simulation methods implementation that support several paradigms aimed at building models of inventory control systems.

Based on this purpose, tasks of the study are listed below as follows:

- Overview of the studied problem
- Development of inventory control models taxonomy
- Comparative analysis of simulation modelling tools implemented for the analysis of inventory control systems
- Analysis of material flow system simulation paradigms that in principle may be applied for solving of the inventory control issues
- Development of recommendations for selection and implementation of simulation modelling paradigms when solving the inventory control issues
- Development of inventory control systems simulation models and their practical application

The object of study: Inventory control in manufacturing and marketing of products in logistic systems

The subject of study: Methods of simulation implementation as a solution for the inventory control tasks.

Methodology and the methods of research

The following theories and methods have been used in the work: the system approach, the methods of probability theory and mathematical statistics, analytical models, inventory management theory. The following tools were used: simulation software ExtendSim (versions 6 and 7) and AnyLogic (versions 6 and 7), the spreadsheet application software MS Excel 2010.

Scientific novelty of the work

In the course of the work, the following results, which are new to the transportation engineering science, have been obtained:

- Analysis of material flow systems simulation paradigms that may be applied for solving of the inventory control issues
- A new selection and implementation method of above-mentioned paradigms has been designed for the study of inventory control systems

- The development stages of conceptual and computer-performed models for inventory control systems have been studied and described in this thesis
- The analysis of simulation modelling packages carried out to solve the inventory control issues has been performed on order to identify the paradigms supported by these packages

Practical value and realization of the work

The developed selection and implementation method of simulation paradigms can be applied when working almost with any commercial simulation modelling package that would be chosen by an analytical department of a logistics entity for the solution of inventory control optimization issues and supply strategies

ExtendSim package libraries created in terms of this research can be practiced for the development of new models of inventory control systems for logistics entities and with a view to conduct a research with such models

Approbation of the research

The results received were presented at 8 international conferences in Latvia, Germany, Norway and Poland.

Structure of the thesis

The promotional work consists of an introduction, 4 chapters, a conclusion, a list of references, and 2 appendices. The work comprises 167 pages and includes 66 figures, 23 tables and 30 formulas. The list of references includes 102 sources. The structure of the work is as follows:

Introduction is dedicated to considering the relevancy of the subject of thesis, formulating the research goal and objectives; the object and subject of research and motivation.

The *first chapter* describes inventory control tasks usage in fields of production and logistics. Basic types of inventory stocks together with systems for production and logistics processes are examined in terms of various inventory management tasks that these systems deal with. There are also described production planning and methods of control and systems with the importance of production inventory stocks and methods and systems to adopt the interaction between the members in the trade supply chain and who focus on the inventories.

The *second chapter* is devoted to the review of mathematical modelling. This chapter provides the overview for both analytical and simulation models that are aimed at analysis and optimization of inventory control systems. These models explain the descriptive and optimization principles of inventory control systems and gives in certain circumstances easy-to-interpret results. There are explained various analytical models based on *Wilson's formula*. Also in this chapter, described main simulation concepts, they are: numerical methods based on Monte Carlo method in spreadsheet form, system dynamics and processes with discrete events. The last part of this chapter devoted to review of inventory control models based on mentioned concepts.

The *third chapter* is dedicated to simulation methodology in inventory control tasks. Deep analyses of simulation packages and justification for the use of the main simulation tool – ExtendSim in this research was done in the beginning of this chapter. Simulation paradigms implementation is described on example of ExtendSim *Continuous*, *Discrete Event* & *Discrete Rate* library sets and on the *basic conceptual inventory control model*. This chapter shows main advantages and disadvantages of using different simulation paradigms for solving inventory control problems.

The *fourth chapter* is devoted to practical implementation of inventory control models with different policies. Beginning of this chapter describes development of custom inventory control libraries based on stochastic *reorder point* and *fixed time between orders* models. The next step is the elaboration of basic inventory control models with mixed control policies, stochastic events combination and the expanded types of inventories. In last part of this chapter is given detailed example of *supply chain* and *divisible production* models with different optimization scenarios.

The work results are presented in this work's *conclusions*. Simulation software survey is in *Appendix No 1* and description of simulation tool selection on the basis of AHP method is in *Appendix No 2*.

Theses which are submitted for defense

1. The research of real inventory control systems in production and sales often have to be conducted at the condition that the demand is stochastic and unsteady, and the delivery time is prone to fluctuations because of the random delays of transport means on their way from the supplier to the warehouse. In such cases, capabilities of analytical models could not display the study object appropriately and that is the reason for implementation of **different types of computer simulation** as a research tool, which are overviewed in this thesis.
2. This thesis determines **three main paradigms of simulation modelling** that are based on fundamentally different ways of material flows and events representation in inventory control systems. These paradigms are called "continuous", "discrete event" and "discrete rate". Each simulation paradigm has its own characteristic level of detailed process display within the

model; while the integration of each paradigm implies use of special software, which is available in terms of different simulation modelling commercial packages.

3. **Selection of simulation paradigm** primarily relies on the indicators of system's performance, which are to be shown in model's running results. Every primary (natural), i.e. not yet in terms of money, indicator is related to particular physical processes within the system. In inventory control systems, such processes generally include transportation, reserves and service of material items that involve cargo (goods), means of transport and natural persons. Selection of simulation paradigm determines the level of detailed display of the corresponding physical processes within the model.
4. **The developed** simulation paradigm selection and implementation **method** can serve as a frame for the research of various inventory control systems which is proved by successful experience in development of multiple models. Some of them are described in this thesis.

Publications with authors' participations

1. Aivars Muravjovs, Eugene Kopytov (2013) Simulation Model of Current Stock of Divisible Products in ExtendSim Environment. *European Conference on Modelling and Simulation*, Alesund, Norway, pp. 664 – 669. **(SCOPUS)**

2. Eugene Kopytov, Aivars Muravjovs (2012) Multiple criteria analysis and choice of simulation tools for inventory control modelling using AHP method. *International Conference Reliability and Statistics in Transportation and Communication*, Riga, Latvia, pp. 211 – 220.

3. Aivars Muravjovs, Eugene Kopytov (2012) Comparative Analysis of Simulation Packages for Inventory Control System Modelling. *European Conference on Modelling and Simulation*, Koblenz, Germany, pp. 595 – 601. **(SCOPUS)**

4. Aivars Muravjovs (2012) Выбор системы моделирования для решения задач управления запасами. *Research and Technology – Step into Future*, Volume 7, Riga, Latvia, pp. 127 – 129.

5. Eugene Kopytov, Aivars Muravjovs (2011) Simulation of inventory control system for supply chain "producer–wholesaler–client" in ExtendSim environment. *European conference on modelling and simulation*, Krakow, Poland, pp. 580 – 586. **(SCOPUS)**

6. Eugene Kopytov, Aivars Muravjovs (2010) Supply Chain Simulation in ExtendSim Environment. *Proceedings of International Conference Reliability and Statistics in Transportation and Communication*, Riga, Latvia, pp. 447 – 456.

7. Eugene Kopytov, Aivars Muravjovs (2010) Supply chain simulation in ExtendSim environment. *International Conference Reliability and Statistics in Transportation and Communication*, Riga, Latvia, pp. 90 – 91.

8. Eugene Kopytov, Leonid Greenglaz, Aivars Muravjovs (2008) Modelling of the multiproduct inventory problem. *International Conference Reliability and Statistics in Transportation and Communication*, Riga, Latvia, pp. 139 – 145.

9. Айвар Муравьев (2008) Моделирование систем управления запасами в среде ExtendSIM. *Research and technology: step into the future*, Vol. 3, #2, Riga, Latvia, pp. 21 –22.

10. Eugene Kopytov, Leonid Greenglaz, Aivars Muravjovs, Leonid Burakov (2007) Investigation of two strategies in inventory control system with random parameters. *European conference on modelling and simulation*. Prague, Czech Republic, pp. 566 – 571. **(SCOPUS)**

11. Eugene Kopytov, Leonid Greenglaz, Aivars Muravjovs (2007) Multiproduct inventory control models with fixed time interval between moments of ordering. *Reliability and statistics in transportation and communication*, Riga, Latvia, p. 149.

12. Eugene Kopytov, Leonid Greenglaz, Aivars Muravjovs, Edvin Puzinkevich (2007) Modelling of two strategies in inventory control system with random lead time and demand. *Computer modelling and new technologies*, Vol. 11, #1, Riga, Latvia, pp. 21 – 30.

1. INVENTORY CONTROL SYSTEMS' THEORY AND PRACTICE

Creation of inventory and its management is an integral part of many kinds of activities in production and logistics. It is widely known that creation of large inventory stocks results in increase of warehousing costs and blocking of capital. On the other hand, the decision to reduce inventory stocks leads to the necessity of more frequent commissions and orders for goods, which generally propagates the increase of total expenditures for its delivery, meanwhile, still preserving the risk of shortage of goods (situations like 'out of stock'). Thus, some questions appear: what is the optimum size of the order, how frequently should orders be made and in which volume should goods (staple) be bought up with every order, so that to minimize total expenditures related to the receipt and storage of goods. To find principal answers for these questions, methods and models of inventory control theory are used (Muckstadt, 2010). Based on this field's obtained results, practical algorithms and computer programmers for inventory management in systems of production and logistics are developed and implemented.

Creation of inventory is always associated with expenditures. First, inventory stocks is actually blocked financial means; second, maintenance costs for specially equipped premises (warehouses) and payment to the personnel; third, the risk of damage and theft of stocks, which results in extra expenditures as well. At the same time, the absence of invention stocks can also result in expenditures, which appear in the form of different losses: because of production downtime in case of goods delayed delivery; losses from buying up at high rates and transportation of small consignments; losses because the goods are out of stock when they are in demand.

In the first part of this chapter, basic types of inventory stocks together with systems for production and logistics processes are examined in terms of various inventory management tasks that these systems deal with. The second chapter is dedicated to the pattern of theoretical models designed for optimum solutions to the inventory management tasks. Formulation of the research problem and justification of its topicality are stated in the end of the chapter.

1.1. Types of inventories

Inventory (inventory stock) – material values in the form of staple and semi-finished products at different stages of the production process and not applied in production at the moment, as well as finished products awaiting their enter into the production process or personal consumption (Nerush, 2001).

Inventory as a notion goes through all the stages of the production process and realization of products. Fig. 1.1 depicts classification of inventory stocks according to the following signs:

- inventory stocks assignment in relation to the production process (production and inventories);
- inventory stocks assignment in relation to the “smoothing” function of demand fluctuations (current stocks, insurance, seasonal stocks);
- the degree of material values “being stocked” (staple, semi-finished products, pre-cooked production of industrial and consumer function);
- in relation to the current time period (carryover storage, stock in transit);
- in relation to the possibility of usage in production or feasibility (preparatory inventory, excess inventory).

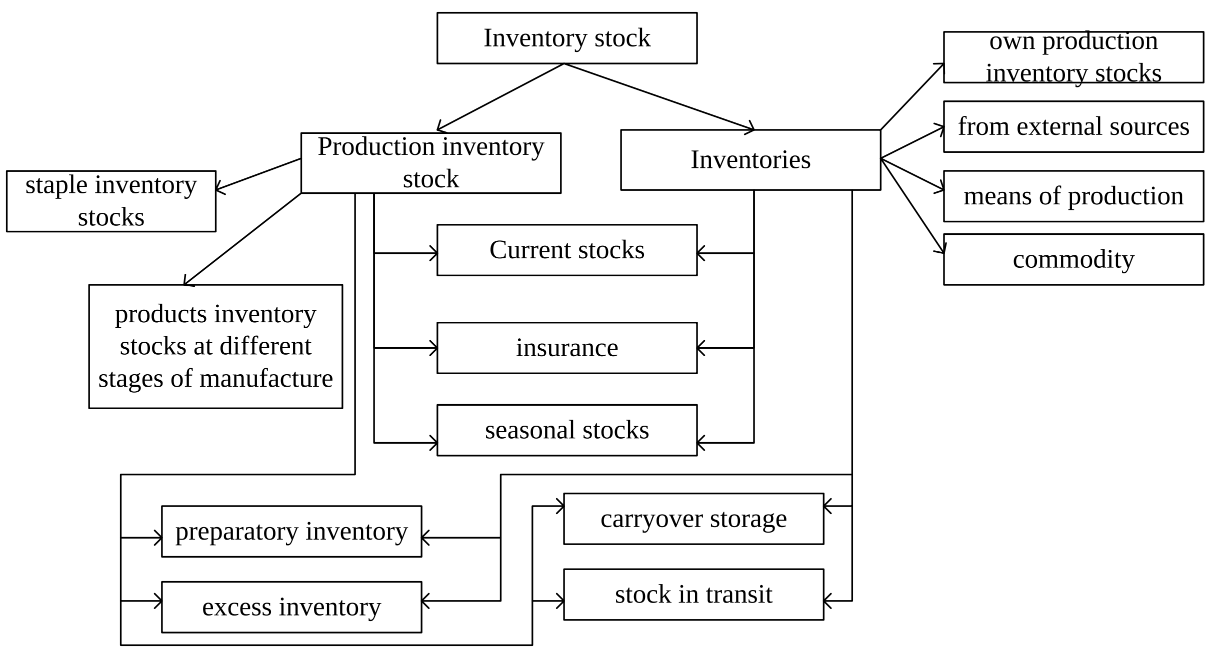


Fig. 1.1 Inventory stock (Hajinski, 2011)

Sustainable control of inventory stocks proposes the creation of such their level, which would ensure regular operation of the production process with minimum maintenance costs. In other words, replenishment of inventories should be implemented as long as the reduction of the risk of production and/or commerce process interruption because of the absence of inventory stocks exceeds the maintenance costs for one additional unit of inventory (Anikin, 1999).

Inventory stocks are included in the circulating assets of the enterprise and general principles of circulating assets' control are applicable to them. In the financial management theory, there are three principal approaches to the creation of inventory stocks:

- conservative approach – creation of large inventory stocks, but the efficiency of inventory usage in rather low;
- moderate approach – creation of normal inventory stocks (in compliance with calculated value of the standard) in case of most typical failures;
- aggressive approach – minimization of inventory stocks; in this case high risks are accompanied with high effectiveness of inventory usage.

There are many reasons for creation of material stocks and inventories in companies; however, what unites us is striving of production activity subjects towards the economic security. With this it should be noted that the cost of inventory stocks creation and ambiguity of terms of marketing conditions do not stimulate the increasing importance of expensive backup network "security" in the eyes of company's management, because they [marketing and inventory stocks] objectively contradict to the increase of production effectiveness (Baskin, 2005).

One of the main incentives for the creation of inventory stocks is the cost of their negative level (deficit). In case of inventory deficit, there are three types of possible expenditures, which are listed below according to their increasing negative influence:

- expenditures due to non-performance of an order (delay with sending the ordered goods) – additional expenditures for promotion and dispatch of goods of the order that cannot be performed on the means of the existing inventory and material stocks.
- expenditures due to marketing losses in cases when the regular customer applies for certain purchase in some other company (such expenditures are measured by the index, showing the loss of revenue because of unrealized bargain);
- expenditures due to loss of customer in cases when absence of inventory stocks results not only in waste of bargain of some sort, but also in the fact that customer starts looking for an alternative sources of supply (such expenditures are measured in terms of total revenue, which could have been gained from realization of all potential bargains between the customer and company).

First two types of expenditures are related to the number of so-called "temporary expenditures of the company as a result of adoption of an alternative course". It is hard to estimate the third type of expenditures numerically, as customers can hypothetically be substantially different as well as corresponding to them expenditures. However, it is essential for the company that the estimation of this specific type of expenditures be as close to the amount of costs (which could have been in reality) as possible. To keep in mind that the cost of inventory stocks deficit is bigger than just the price of missed bargains or unrealized orders. It includes the loss of time for the production and loss of working time, possibly with time losses due to expensive interruptions of production when transiting between complicated technical processes (Hajinski, 2010).

The role that inventory and material stocks from different fields of economy play in the production release is defined by the specific role that they have, since the differences of approaches to the investment policy in the field are explained, also prioritizing the tasks that are considered in the production process. The main objective of companies in certain fields of natural economy is the control of staple, meanwhile in other fields it is finished products; yet the enterprises in the fields are producing the investment goods, and the major part of the organizational efforts is focused on the control of WIP inventory.

From the point of view of practice, it is essential that inventory stocks are divided into production and stock inventory. **Production inventory stock** is a kind of stock that is located on all enterprises of the field of material production and intended for the industrial consumption. The purpose of production inventory stock creation is to ensure the regularity of the production process. **Inventories of goods** are the stocks of ready products of the manufacturers, stocks in transit, as well as stocks on the route from the supplier to the consumer, i.e. on the wholesale, small-scale wholesale and retail trade enterprises, and in the procurement organizations. The purpose of creation of such inventory stocks is to ensure the constant satisfaction of product demand from consumers (buyers).

Production planning and methods of control and systems will be considered below, with the importance of production inventory stocks and methods and systems to adopt the interaction between the members in the trade supply chain and who focus on the inventories.

1.2. Inventory stocks control in systems of production planning and control

A significant savings level can be achieved by applying corresponding production control systems, since the support of optimum inventory stock for each type of material resource requires additional current assets. Main types of such systems are considered below.

In MRP-type systems implement (Material Requirements Planning) the methodology of material requirements planning, which lies in resource final demand according to the volume and calendar production schedule (Ptak, 2011). Materials come into production based on the planned requirements but without the expressed demand. Inventory stock control is put into practice on the production line. With this the materials are supplied in such a way, so that there are always some inventory stocks on the place of production. The main drawback of the MRP method is that enterprise keeps too large inventory stocks. That is why it would be expedient to use this method only in cases of large quantities of low-value staple and materials being processed relatively equally during the production and only for one type of insignificantly changed product.

The strategy of MRP II production scheduling (Manufacturing Resource Planning) ensures extended scope of enterprise resources rather than MRP strategy (Waldner, 1992). Unlike the MRP, MRP II systems do the planning of both material and financial aspects. The basis for planning the material security is the operational and calendar scheduling of production, stock on hand, rates of use of materials and components for the product. As a result, there are sorted in time sequence orders for all of the components. Through the development of the schedule from the end to the beginning of production taking into account the known production lead time for each product it is possible to calculate the needed date of placing the order. The main principle of MRP II is that “production pulls”, i.e. materials come to the production line only on its demand.

The main advantage of MRP II system is that in places of product manufacturing there are as much material as needed for the production process. The minimization of the amount of inventory stocks greatly facilitates their warehousing and storage.

The reasons that reduce the efficiency of MRP II system are the following: frequent changes in the production scheduling forces the suppliers to change their supply schedules, and consequently the materials supply time reduces; often it is difficult to timely identify the absence of certain material types in the current production, which can interrupt the production process and cause the material inventory stocks creation that are currently not needed.

The ERP (Enterprise Resources Planning) strategy is considered to be a developed version of MRP II (Monk, 2006). MRP II has new characteristics added, that is financial resource management and marketing. ERP is the first concept that focuses on business management instead of production. ERP implies the integrated scheme fulfilling the functions provided with all older concepts. An important difference from the MRP II methodology is the opportunity to dynamically analyze and dynamically alter the plan throughout the whole supply planning chain. Certain capabilities of the ERP methodology substantially rely on the software implementation. The ERP concept is more ‘hazy’ than that of MRP II. If MRP II has a distinct orientation to production companies, then the ERP methodology is applicable to the trade, service, and financial sectors.

The inventory stocks control and buying activity are the functions of ERP. They allow organizing the contracts maintenance, implementing the scheme of centralized buying, ensuring the record taking and optimization of warehouse inventory, etc.

The need to reduce stocks on hand, relatively high cost of capital and industrial areas in Japan resulted in the development of JIT (just-in-time) production and supply management system. The system is based on the following principles: “coordination of products manufacturing and staple flow” and “zero production backlogs” (Hirano, 2006). Products manufacturing and supply of staple and materials are initially planned as a whole thing. If during the production any kind of violation or alteration occurs, then the logistics maintenance system immediately responds to them, consequently changing the supply of staple and materials respectively. If there are violations or alterations in the supply of materials, then the start-release of other products in the production is carried out, or delivers the improvement of the previous technological process.

This kind of system in Japan is called Kanban (Waldner, 1992). This system refers to “pulling” variety of JIT system, with which the size of consignment, rate and terms of supply are determined not by the supplier, but business-to-consumer enterprise. Thereby, the whole process is arranged uninterruptedly with the constant rate, what can provide negligible inventory stocks. Kanban is an enterprise management system based on the principle of “zero production backlogs”. Enterprises receive parts and components daily or even several times a day. If a typical American enterprise restocks its production inventories 10-20 times a year, then enterprises using this system do it 50-100 times a year.

Widely spread today is SCM concept (Chopra et al, 2001). The term SCM (Supply Chain Management) was proposed by two companies – i2 Technologies and Arthur Andersen LLP in early 1980s. SCM is complex concept, which examines the logistics of industrial enterprises not from the point of view of simple supply of ready materials, component parts and assembly units, but as an active search on a competitive basis for the optimum partners to place the respective orders with. As a result, there is created not static, but constantly updated and modernized network of partner enterprises supplies. Without the effectively functioned and managed supply chains, basic modern industrial business control technologies cannot exist at all. JIT (just-in-time) and TQM (Total-Quality-Management).

1.3. Inventory management systems of distribution of goods

At the same time as SCM, which gained most of its advancement in the industrial sector, distribution and trade sectors pay more and more attention to the ECR concept. ECR (Efficient Consumer Response) is an concept that emerged in early 90s in two sectors: distribution and trade of customer goods in the US. In fact, ECR means a number of marketing and logistics strategies joined together in order to create efficient interface between the producers and distributers (suppliers and consumers) for minimization of total expenses and maximization of customer service (Hieber, 2001). The distinct difference between ECR and SCM concepts are not so much in the content as in the focus on the specific sphere of application. “Know-how” gained with regard to ECR or SCM can be successfully used in both concepts. An example of successful “know-how” transfer from the ECR concept to SCM are, such strategies as “Cross-Docking”, “Quick Response”, “Continuous Replenishment” and “Vendor Managed Inventory”.

“Cross-Docking” (CD) strategy subsequently was first implemented by Wal-Mart company (retail chain stores for the sale of consumer goods). The gist of this strategy is that large consignments of goods are delivered from regional distribution centers to so-called “Cross-Docking” terminals where they are sorted in accordance with a specific order of a particular store and without long-term storage in a warehouse are sent in corresponding stores (Chopra et al, 2001). This way, it creates the opportunity to avoid warehouse storage expenses, and allows to use transport more effectively, reduce associated with inventory capital and increase the customer service level by cutting delivery times. It essential that the CD strategy requires very high level of coordination and synchronization of vehicles that ensure physical implementation of input and output streams of terminals.

The “Quick Response” (QR) concept is a logistical coordination between the retailers and wholesalers aimed at improvement of promotion of finished products in their distribution networks in response to additional shift in demand (Suri, 1998). The realization of these concepts is put into practice through monitoring of sales in retail and transfer of information about sales volumes coded in specific nomenclature and assortment to wholesalers, who, in their turn – to the manufacturers of finished products. The implementation of QR concept allows to reduce expenses on finished products up to the desired level, but not less than the quantity that would permit to quickly satisfy consumer demand and significantly increase inventory stocks turnover at the same time.

“Continuous Replenishment” (CR) concept is a modification of QR concept and is intended to dismiss the necessity in orders for replenishment of finished products (Andraski, 1994). The aim of CR is to define the effective plan targeted at replenishment of inventory stocks of ready products from retailers. The necessary total amount and range of products are calculated. Then the agreement is reached between the suppliers, wholesalers, and retailers on replenishment of finished products through the signing of purchase commitments.

"Vendor Managed Inventory" (VMI) concept is one of the most advanced inventory control strategies used now and used by both trade and industrial companies (Franke, 2010). The essence of that lies in the responsibility of the supplier for the delivery and management of supplies and controlling of the inventory stocks level on the warehouse of the consumer. Based on the modern information technologies, the data on inventory stocks level, consumption level for a certain period of time and forecasted demand is given to the suppliers by the consumer. Thereby, the supplier has all the necessary information to manage its consumer's inventory stocks in the effective manner. The application of VMI strategy is profitable for both parties. The supplier gets an opportunity to reduce total expenses and use vehicles more efficiently, herewith improving the customer service, and planning their activities more effectively. Consumers, in their turn, disclaims the functions associated with delivery and cuts expenses on the storage of goods, and as a result receives a high level of customer service (Hieber, 2001).

1.4. Review and classification of inventory control strategies

The strategy of inventory control is a combination of rules according to which is made the decision to replenish the inventory stocks minding the time and the necessary volume of inventories to be ordered. Together with the description of conditions and restrictions of the inventory control process, the strategy transforms into an inventory control model for representation of which the following three forms can be used, while each of them includes all the elements of the previous starting from the second.

- **theoretical** (conceptual) model contains text, pictures and logic diagrams for explaining the terms and principles of inventory control system's functioning (Ziplin, 2000, Sprancmanis, 2011, Praude, 2013);
- **analytical** (purely mathematical) model contains formulas for calculations (reckoned to be optimum at some point) of time moments and the volumes of purchase orders (Gringlazs, 2005, Tersine, 1994);
- **simulation** (computer) model that also refers to the class of mathematical models and is intended for the research of inventory control system's functioning dynamics in actual terms,

or in conditions set in the framework of the simulation model by the researcher. (Law and Kelton, 2004)

Signs and factors that determining the variety of inventory control systems on the stage of their conceptual modelling will be considered below.

- Character of the processes. The model, which has at least one probability parameter (demand, delivery time, etc.) is statistically distributed, or otherwise – determined. Most of the interest to researchers lies in the statistically distributed model as maximally approximated to real cases.
- The number of nomenclatures. Models can be single product if they operate with only one type of inventory stocks, and they can be diversified if they operate with more than one type of inventory stocks.
- Safety stock. A number of models allows for the safety stock in case of inability of basic alternating stock to meet the demand.
- Deficit. Inventory control allows for deficit when there are not enough inventory stocks to meet the demand until the next delivery, but cases when deficit is not permissible are also possible.
- Dynamic pricing. Recently models with one optimization parameter acting as a price for single product unit have become popular, which allows to influence the volume of demand.
- Supply chains. In inventory control systems, there are some cases possible when inventory stock is supplied not directly from the manufacturer, but through intermediary, who has its own stocking strategy that differs from the manufacturer's plans for products sales.
- Limitations. Introduction of limitation on the model's factors significantly influences the inventory control strategy. The presence of such limitations is related to the particularities of the transported cargo and goods, delivery methods, manufacturing manner, and terms of supply agreements. Among the limitations are distinguished:
  - Limitations on storage space can appear when storing bulky goods or in case of deficit of warehouse space. In this case, mathematical model should focus on the formation of short-period orders with less volume.
  - Limitations on the storage appear in cases, when the supplier delivers perishable goods and characteristics of goods delay become primary. This problem can be

treated with the problem of supply route choice, when several routes exist with different duration of transportation.

- Products return for processing. This factor presents in the models related to the manufacturing in cases when backflow of used products exists, which can be processed and used for the manufacturing of new products.
- Repurchasing of excess stock. This particularity is characteristic of inventory control models that are connected with military technologies, when one of the requirements is the compulsory repurchase of unused stock by the supplier.

Table 1.1 shows some other evident signs and factors that need no additional explanations.

Table 1.1 Classification of inventory control models (Law and Kelton, 2004)

Signs	Model Types
Number of nomenclatures	Mononomenclative model
	Diversified model
Supply chains	Supply model without intermediaries
	Supply model with intermediaries
Demand	Determined
	Random
Delivery time	Determined
	Random
Deficit	Models that do not allow the deficit
	Models that allow the deficit
Safety stock	Model without the safety stock
	Model with the safety stock
Selling price	Models with fixed selling price
	Models with dynamic selling price
Discounts	Models that take discounts into account
	Models that do not take discounts into account
Limitations on storage space	Models without limitations on inventory storage space
	Models with limitations on inventory storage space
Limitation on storage time	Models without limitations on inventory storage time
	Models with limitations on inventory storage time
Limitation on the volume or weight of ordered replenishment	Models without limitations on the volume of ordered consignment
	Models with limitations on the volume of ordered consignment
The costs of stock	Models that do not take into account the costs of stock
	Models that take into account the costs of stock
Products return for processing	Models with products return for processing
	Models without products return for processing
Repurchasing of excess stock	Models without repurchasing of excess stock
	Models with repurchasing of excess stock

The time of ordering	Periodically, through certain time slot
	Provided that the necessary level has been reached by the inventory stocks
	Periodically, provided that the necessary level has been reached by the inventory stocks
Order volume	Fixed order
	Order up to a certain level

On Fig. 1.2 the relation between the levels of stock traditionally examined in the models.

Maximum desired stock determines the level of inventory stocks, and is economically expedient in the given inventory control system. In many inventory control systems, this quantity is used as a reference point when calculating the volume of the supply.

Reorder point (threshold level of stock) is a quantity that indicates the current inventory level, and it shows the need in forming and placing the order for replenishment.

Safety (warranty) **stock** is aimed at reducing the risks level related to the unforeseen fluctuations of demand for the finished products, non-performance of contractual obligations to deliver material resources, failures on the logistics cycles and other circumstances.

Current stock corresponds to the stock level at any inventory moment and is aimed at ensuring the continuous production process or marketing between the next two supplies.

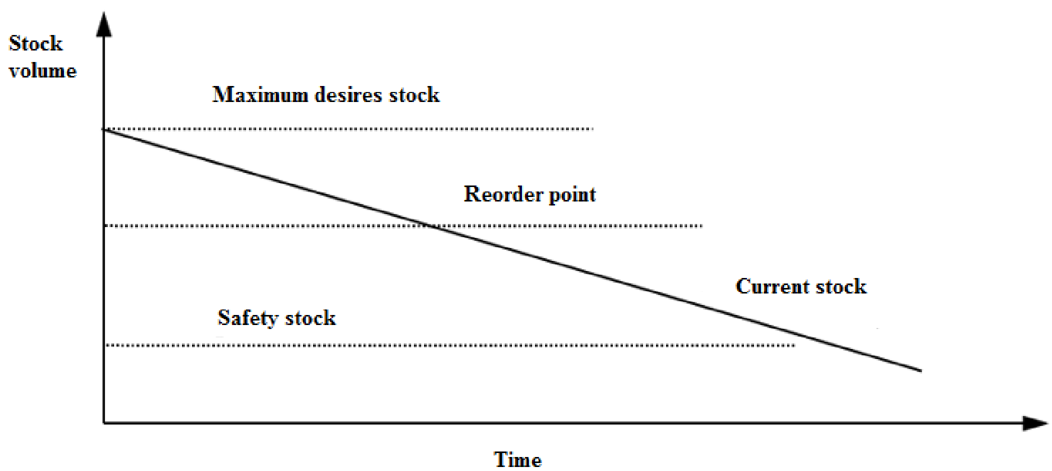


Fig. 1.2 Types of inventory stocks on time accounting (Gringlazs, 2005)

Each inventory control system is designed with the purpose of continuous delivery of certain type of material resources to the consumer. Implementation of such goal is reached through the following tasks:

- setting the level of maximum desired stock;
- determining the level of safety (warranty) stock;
- organizing the inventory of the current stock level;
- defining the time moment of orders formation;
- calculating the size of the orders.

The optimum inventory control means that this type of control corresponds to a certain criterion (rule) of optimality. The simplest model is the formula for the optimal size of the order published in Wilson's works in 1934 (Muckstadt, 2010), but only in certain cases it turns out to be useful for the decision-making in inventory control on the real enterprise. The inventory control optimization criterion on a certain enterprise should best match its highest economic goals and aspirations. The most complete optimization is possible on the delivery process's financial model, which considers the purchasing organizational expenditures, transportation expenditures, warehousing and storage, the costs of building and equipment rental, and personnel expenditures.

1.5. Chapter's conclusions and rationale for this research

Preliminary analysis of situation in the fields of theory and practice of inventory control leads to the following conclusions:

1. Systems of inventory control are widely used in many fields of production and logistics, and their effectiveness largely determines the effectiveness of the basic production and logistics processes. A large group of modern planning and production controlling systems is considered, as well as systems of planning and controlling the transportation of goods in which inventory control takes one of the central places.
2. The number of possible variants of how to organize inventory control systems is practically endless, because each variant can be featured by the signs provided in the Table 1.1, and thus can reach a number of 16. Provided that each sign can include one of the two values and one of signs can include one of the three values, the number is a combinatorial number of options that results in $2^{15} \times 3 = 98304$. Certainly not all formal combinations of listed in Table 1.1 signs can be implemented on practice, but even allowing for this fact, the number of possible variants for inventory control systems management amounts to thousands.

The following well-known facts are fundamentally important in terms of the rationale for this study:

1. As a matter of principle, for the quantitative research of inventory control systems can be implemented two classes of models, which are: analytical (purely mathematical) and simulation (computer) models. Both of them are associated with mathematical models, and each certain model is based on a specific theoretical (conceptual) model. The quantitative research itself can be conducted in order to estimate the inventory control system's functioning values, and also so that to determine the parameters values of such a system, which could allow to reach the optimum operating mode corresponding to the set (one or several) optimality criteria.
2. As an optimality criterion for inventory control system in Wilson's classical model, minimum of total expenses is exercised, including ordering and storing expenses. In this case, only one parameter's value is calculated – that is the optimum volume of the order. Wilson's classical model can be used only when executing a large number of limitations, which refer to the delivery period and pull of demand (both values should be constant), likewise to the other characteristics and parameters of the model.
3. In the class of analytic models available a large number of modifications and extensions for the classical Wilson's model, but inventory control task formulation can always appear and have no existing analytical model, or which, in principle, cannot be even created. In such a case, the only opportunity to study the effectiveness of the inventory control strategy is to implement one of the computer simulation types. The simulation model allows not only to reflect almost any features of the studied inventory control system, but also to formulate the system's optimality criteria based on the delivery process's financial model of any complexity.

Based on the aforesaid, it becomes clear why exactly the use of simulation modelling for the analysis on inventory control systems is a subject of this thesis. Only the simulation modelling and nothing else should now be considered as a universal method of analysis for the problem solving and inventory control systems optimization in real manufacturing and logistics systems.

The review of mathematical modelling in the next chapter, however, starts with analytical models due to implementation of the basic inventory control strategy building principles also in the simulation models, but first were designed with the help of the analytical models. While there were developed coherent systems of classification for analytical models and the methods of their implementation (calculation on given formulas) have no principal difficulties, such situation is not observed in the field of simulation models implementation to solve these inventory control problems. Although in the publication on this topic the are plenty of evidence proving the creation and implementation of various simulation models, there are no works dedicated to the methods of models creation to solve inventory control problems based on different and reputedly standard simulation modelling paradigms.

The particularity of the simulation modelling lies in the fact that the model accurately reflects one specific original system, but cannot be directly used to study any other system, hence, it requires the creation of its own new specialized model. That is why in the field of inventory control systems modelling, the value is concluded not in “strange” models, but in the method of creation of such models, and in case of new models study specialists involved with it will have to face the following problems:

- choosing the type (paradigm) of simulation modelling (Monte Carlo method, method of system dynamics or discrete event simulation);
- choosing the software package for model realization on the computer;
- realize all stages of development and implementation of the model through the work with the chosen software package.

The paper investigates the use of simulation methods for the analysis of inventory management systems, and it contains a vast amount of material that can be used as an effective support in solving all the problems stated above.

2. MATHEMATICAL MODELS OF INVENTORY CONTROL PROCESSES

Mathematical models of practically any processes in production, transportation and systems of logistics are usually divided into analytical and simulation models. The basis for analytical models are the final formulas or iterative procedures used for the calculation of numerical characteristics of the studied process, with set parameters values in a system where this process in observed. The use of a computer in this instance can make the calculation procedure easier, but it is not a principally vital operational condition when dealing with this class of models. Simulation models are models of mathematical processes, which from the moment of their conceptual definition are focused of the realization in the form of algorithms and their relevant computer programs. The results of the study process with the help of a simulation model can be gained only through its representation on a computer.

This chapter provides the overview for both analytical and simulation models that are aimed at analysis and optimization of inventory control systems.

As analytical models are not the subject of studies of this thesis, only the simplest determined models of this class are considered as examples. These models explain the descriptive and optimization principles of inventory control systems and give in certain circumstances easy-to-interpret results. The application of an analytical model for stochastic systems is accompanied with complex financial models of the supply process, and most often turns out to be inexpedient. In such cases, even "professional mathematicians" recommend designing and implementing corresponding simulation models (Sterligova, 2005a).

As simulation models in this case, there are considered relatively simple ones realized using spreadsheets, as well as arbitrary complex models based on the use of classical paradigms of processes simulation modelling, which implies models of system dynamics and models with discrete events processes.

2.1. Analytical models

The simplest model, an example of which shows a deduction of the formula for the optimum quantity of the order (Wilson's formula), is a single-product static model with immediate replenishment of the stock and lack of deficit. There are many modifications of this model, most known of which are the following (Sterligova, 2005b, Gringlazs, 2005):

1. Model taking into account losses from the deficit.

2. Model with gradual stock replenishment plan.

3. Model taking into account deficit with gradual stock replenishment plan.

4. Diversified order model.

5. Model taking into account wholesale discounts.

6. Model taking into account VAT tax.

The simplest model with two of its modifications from the list will be considered below.

2.1.1. Single-product static model

The simplest inventory control model in characterized by three properties: (Tersine, 1994)

- time-constant demand;
- immediate stock replenishment;
- lack of deficit.

In this case, the model with fixed order quantity and a model with fixed frequency behave absolutely in the same way, because the intensity of the demand and the duration of the preparatory period do not change. (Gringlazs, 2005, Sprancmanis, 2011):

The warehouse inventory traffic schedule of such a case is depicted on Fig. 2.1. On the Figure there are:

q – the quantity of the consignment;

$Z_{CP} = \frac{q}{2}$ – determining the average level of stock;

λ – tangent of the corresponding angle, pull of demand (the amount of products consumed per unit of time);

S – “Order point”;

Θ – the duration of the replenishment period;

I – the duration of the order cycle (planned period).

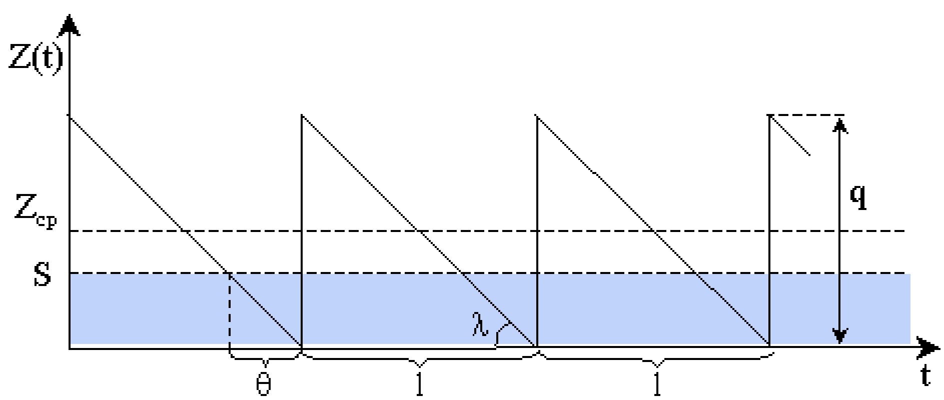


Fig. 2.1 Inventory movement in a single-product static model

For such a model, the quantity of the order at some point can be calculated by the formula:

$$Z(t) = Z(0) - \lambda t + W(t) \quad (2.1)$$

where $W(t)$– the total income of the product in a period [0,t].

The amount of total income is determined from the following equation:

$$W(t) = qn(t) \quad (2.2)$$

where $n(t)$ – the total number of deliveries in a period [0,t].

Herewith $1 = q/\lambda$, i.e. the inventory level reaches zero q/λ units of time later after the receipt of the order of q quantity.

The total number of deliveries:

$$n(t) = \left[\frac{t}{1}\right] = \left[\frac{\lambda t}{q}\right] \quad (2.3)$$

where [ ] – the integer part of a number.

From the (2.1), (2.2) и (2.3) we get:

$$Z(t) = Z(0) - \lambda t + q\left[\frac{\lambda t}{q}\right]. \quad (2.4)$$

Equation (2.4) fully describes the considered inventory storage system.

The optimization lies in the choice of the most economic quantity of the consignment q. The smaller q, the more frequently new orders should be placed. Herewith the average inventory level will decrease. On the other hand, the inventory level increases if q grows as well, but orders are placed more rarely.

As expenses depend on the frequency of orders and the volume of stocked inventory, the q value should be determined from the condition of a balance between the two types of expenses.

Hence, C_0, as above, – ordering expenses that appear whenever the order is placed; b – storage expenses for a unit of product per specific unit of time; C_1 – purchasing price for a unit of product; $d(t)_0$– the total volume of consumed products per period [0,t].

Let's express total expenses $V(t)$for the period [0,t] and aim to find the minimum of these expenses:

$$V(t) = C_0 n(t) + bZ_{CP}t + C_1 d(t) \to min.$$

Using the equations (2.3) and (2.4) and passing to the expenses per unit of time (for this let's divide the above formula by t), and we will get:

$$V = C_0\frac{\lambda}{q} + b\frac{q}{2} + C_1\lambda \to min.$$

It is worth noting that we had to neglect the integral part in formula (2.3) in order to get the differentiable function.

Then we will find the derivative of the function with the help of q, and equate it to zero:

$$\frac{dV}{dq} = -C_0 \frac{\lambda}{q^2} + \frac{b}{2} = 0$$

from where we will find q:

$$q^* = \sqrt{\frac{2C_0\lambda}{b}} \tag{2.5}$$

It is important that the second derivative at point q^* is strictly positive, what tells us that the minimum of the fucntion is now found.

Equation (2.5) is considered to be called as **Wilson EOQ Formula** (Tersine, 1994), which takes a central place in the whole inventory control theory. Thus, the optimum strategy of the model foresees ordering q^* of product units after every $I^* = q^*/\lambda$ unit of time.

Order placing strategy in the given model should determine "order point" as well. It can be shown that "order point" for this case is determined as follows:

$$S^* = \lambda\Theta. \tag{2.6}$$

When using formulas (2.5) and (2.6) it is necessary to control the pull of demand λ and storage costs b would be refered to the same period, for instance – the same year, month, day.

2.1.2. Model taking into account losses from the deficit

In the above considered simplest model, the deficit of products is not allowed. In the general case, when losses from the deficit are comparable with the expenses on the maintenance of inventory – the deficit is allowed. (Gringlazs, 2005, Sprancmanis, 2011)

Inventory traffic schedule for such a case is provided on Fig. 2.2, where q_Θ marks the amount of products consumed during the preparatory period.

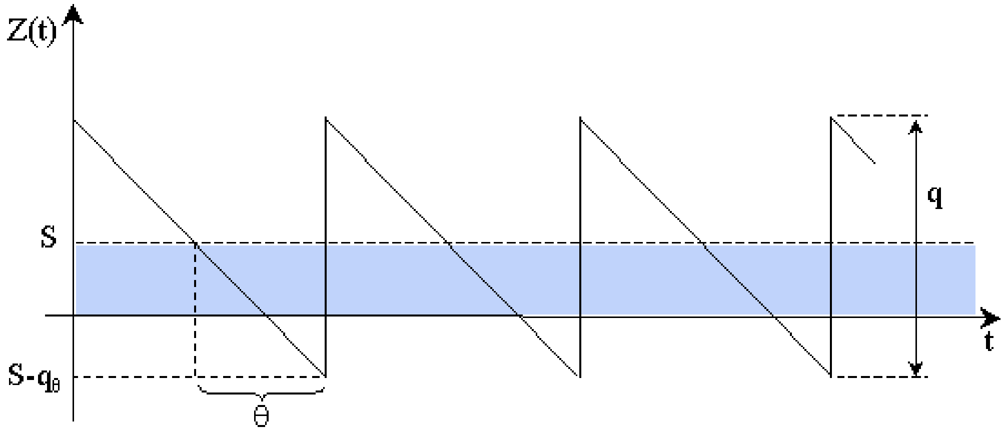


Fig. 2.2 Inventory movement in a single-product static model allowing for the deficit

Not making a detailed formula deduction, let's say the following:

In the case, when the minimized function considers expenses that appear due to the deficit, the optimum values of the parameters q^* and S^* have the following look:

$$q^* = \sqrt{\frac{2C_0\lambda}{b}} \times \sqrt{\frac{b+a}{a}} \qquad (2.7)$$

$$S^* = \lambda\Theta - \sqrt{\frac{2C_0\lambda}{(a+b)} \times \frac{b}{a}} \qquad (2.8)$$

where a – expenses due to lack of production units per the unit of time.

It is easy to notice that at high expenses of unmet demand, i.e. the deficit is not allowed ($a \longrightarrow \infty$), q^* and S^* in (2.7) and (2.8) strive to the certain values in (2.5) and (2.6).

2.1.3. Model with gradual stock replenishment

In some cases, for example, when an enterprise simultaneously produces and consumes the products, inventories are replenished gradually, not immediately. This means that in this case, one part of the production system functions as a supplier for the other part of system that plays a role of a consumer. (Gringlazs, 2005, Sprancmanis, 2011)

More often, the rate of production overcomes the rate of consumption. Inventory traffic schedule in such a system will have the following look according to the graph provided on Fig. 2.3. Let's refer to the necessary variables for further analysis:

q – the volume of the produced consignment, pcs.;

λ – pull of consumption, pcs. / time units;

ρ – rate of production, pcs. / time units; thereafter, $\rho - \lambda$ – the rate of inventory increase (pcs. / time units), on the graph – tangent of the respective angle;

Z_{max} – the maximum stock level;

b – expenses of the storage of one unit of products per unit of time, unit of cost;

C_0 – expenses on the commissioning works, unit of cost;

Θ – the duration of commissioning works, or order lead time, time units.

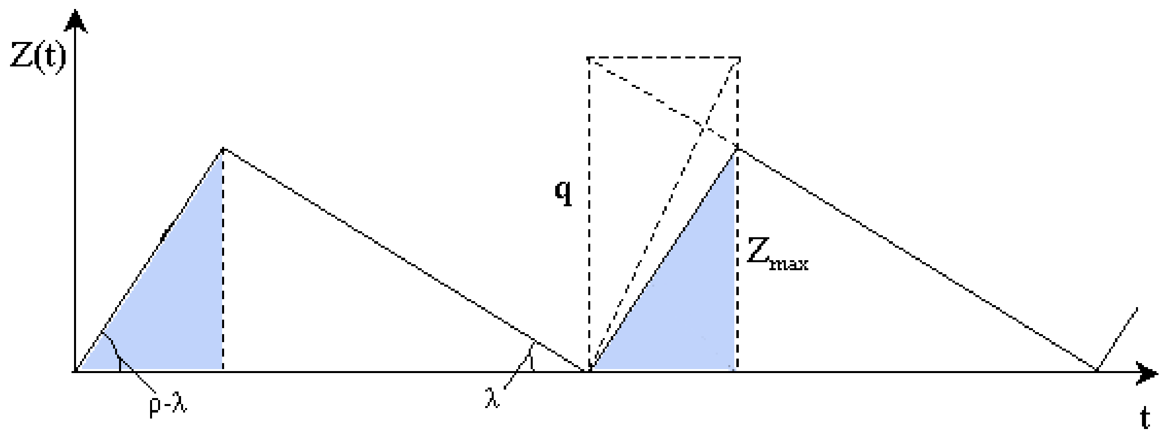


Fig. 2.3 Inventory movement in the model with gradual replenishment

When the company produces the products itself, then it has no order costs. However, for each production consignment there are preparation expenses – this is the preparation costs of the equipment for the given production process, and it includes the adjustment, tools replacement, etc. Another name for this kind of expenses is commissioning works. The cost of preparation in this case is the same as the cost of order, because it does not depend on the consignment quantity. Similarly, these values are used in calculations.

Let's pass to the definition of optimum parameters of the considered model. For this, we can use the same technique we have already applied in the section 2.1.1: form an expression that would show the dependence between the expenses V and model's parametres, and then find the derivative and equate it to zero.

This time we include only two types of expenses in the total expenditure: expenses on the execution of commissioning works and expenses on the storage of products. Expenditures proportional to the volume of the consignment (component that includes the quantity C_1) will not be included in the function. First, as we have seen above, this addend does not affect the total expression for the optimum parametres; secondly, in a situation when the enterprise is the producer and the consumer at the same time, such expenses actually are not related to the functioning of the inventory storage system.

Well, total expenses $V(t)$ for the period $[0,t]$:

$$V(t) = C_0 n(t) + bZ_{cp} t \longrightarrow min.$$

Using the equation (2.3) and passing to the expenses per unit of time (for this let's divide the above formula by *t*), and we will get:

$$V = C_0 \frac{\lambda}{q} + b \frac{Z_{max}}{2} \longrightarrow min$$

We express Z_{max} through q (the volume of production consignment). It is easy ti be done by the use of the inventory schedule provided on Fig. 2.3, namely considering some triangles and using the simplest trigonometric relations:

$$Z_{max} = \frac{q}{\rho}(\rho - \lambda),$$

from which:

$$V = C_0 \frac{\lambda}{q} + \frac{bq}{2\rho}(\rho - \lambda) \longrightarrow min.$$

Equation the derivative to zero:

$$\frac{dV}{dq} = -C_0 \frac{\lambda}{q^2} + \frac{b}{2\rho}(\rho - \lambda) = 0.$$

Expressing q:

$$q^* = \sqrt{\frac{2C_0\lambda}{b} \times \frac{\rho}{(\rho - \lambda)}}. \tag{2.9}$$

The expression (2.9) is used to determine the optimum consignment quantity form the model with gradual stock replenishment.

The optimum value of the "order point" S^* in this case as well as with single-product static model is found form the relation (2.6):

$$S^* = \lambda\Theta.$$

"Order point" in this case is the level of inventory at which it is recommended to start commissioning works.

2.2. Simulation models

As it was mentioned above, the common feature of all simulation model is the fact that the results of simulation can be acquired only through the processing on a computer with special simulation tools. The ways of creating these programs depend on the chosen philosophy (paradigm) of simulation modelling, and on a corresponding to this paradigm program package used to build a certain model.

All simulation models of inventory management processes are dynamic models. This means that in them events are acted out in the period of simulation time, also called the length of simulation run. This time can sometimes vary from several weeks up to several years. The results of simulation can be presented in the form of graphs of behaviour for any variables of the model in the length of simulation run, and in the form of averaged statistics gained through the registration of data about the events during the whole length of simulation run.

Most often simulation models of inventory management processes are stochastic models. This means that they use the generators of random numbers, which help to model numerical parameters that have certain and set by the model's developer rules of allocation. The typical case is the simulation of random demand for a certain type of product stocked in a warehouse.

With the help of **spreadsheets** (e.g., MS Excel) are generally created models of systems that are considered to be extremely simple from the point of view of modern logistics. This means that models show only input and output streams of a certain bin where the managed inventory is formed. Herewith inventory control logics can be rather complicated, as it is realized via the spreadsheet formulas or via specially written programs, for example, in VBA code for MS Excel. These models typically have a uniform countdown on the basis of Δt step with a unique line in the sheet corresponding to every new moment of time and reflecting the dynamics of simulated processes. In the models of this type, it is comparatively easy to allow for the influence of random factors on the inventory control process's flow; that is why Monte Carlo method is often used in order to get static results of simulation. This method implies the multiple repetition of model's simulation run with different flows of random numbers created by the generators.

In models of **system dynamics,** all the properties of models created with the help of spreadsheets are preserved: uniform countdown based on the Δt step, and consideration of random factors. One advantage of models of this class is an opportunity to reflect complex multi-stage supply chains, in which decisions to organize supplies are influenced by the direct and reverse information links. Models of system dynamics are most often created as single-product models, because multi-product models look bulky when appear in the environment of a specific package of simulation modelling. The most acknowledged packages for the creation of system dynamics models are AnyLogic, Dynamo, iThink/Stella, PowerSim and Vensim.

When building models based on the **processes with discrete events,** almost no limitations are imposed on the simulated supply chain's structure complexity and on the complexity of inventory control strategies currently under investigation. Also the stock-taking of almost any number of products is not principally difficult. Real problems when using models of this class appear due to large amount of preparatory works and when programming the models. Wherein, the main inconsistency of the simulation paradigm based on discrete events arises here: the more detailed and accurate is the reality created by the model, the more complex it becomes. Herewith, due to the "narrow focus", the model becomes less useful for the solution of similar problems in other inventory control systems. The developers of the models have to face the challenge of solving very complicated problem of how to choose the optimum level of complexity and detailed representation of the real inventory control system. Hence, if the complexity level is too low, then the potential effect from this class models implementation will not be met; but if the complexity level is too high, then the expected time-expenses for the creation of a model can become unacceptable. In order to create models based on discrete events, in most cases, such packages are used: AnyLogic, Arena, AutoMod, Delmia Quest, Enterprise Dynamics, ExtendSim, Flexsim, Plant Simulation, ProModel, Simul8 and Witness.

In the next part of the overview, there will be shown examples of simulation models development and implementation related to all abovementioned three classes.

2.2.1. Numerical models based on the Monte Carlo method in the form of a spreadsheet

"Spreadsheet simulation" refers to the use of a spreadsheet as a platform for representing simulation models and performing simulation experiments. In some cases, unknown parameters such as the interest rate at a future time or the demand for a product are actually random variables whose value cannot be predicted, i.e., the models are stochastic models. Many stochastic models in finance (including real estate and insurance), logistics and engineering can be conveniently setup in a spreadsheet for simulation. Spreadsheets are frequently used by actuaries, for example, to evaluate insurance rating methods. Consider, for example, an inventory model in which the demand for the product is stochastic. In order to evaluate a particular replenishment policy, this value must be sampled when the simulation experiment is run. An experiment would consist of sampling demand for the product and applying the inventory policy over a long period of time to compute observations of the periodic costs resulting from excess inventory and shortages associated with the policy. These observations would then be used to estimate the mean cost for the policy. The experiment would be repeated for several policies to find the inventory policy that produces the minimum mean cost. This is a typical stochastic model that can be analysed using simulation. Examples of spreadsheet-based models can be found in the papers (Esnaf, 2000; Mahamani, 2010; Liu, 2013). Other examples are discussed in detail below.

2.2.1.1. An inventory spreadsheet based model

Consider a single period inventory model where a quantity of a good will be purchased to satisfy a stochastic demand whose distribution is known. As an example, we could be placing an order for the number of hot dogs for a baseball game (Seila, 2006). Demand will be determined by many unpredictable factors, but data from past games shows that it has an Erlang distribution with parameters 4.0 and 2. This random variable has mean 8.0. We experience costs of ce = \$60 per case for an excess (if the amount ordered is greater than demand) and cs = \$160 per case for a shortage (if the amount on hand is less than the demand). Let D represent the demand - a random variable - and x represent the number of cases ordered - a decision variable. We can order cases in fractional amounts. Then, the realized cost, after we have attempted to satisfy demand, is

$$Y = c_e (x-D) \quad \text{if } x > D,$$

$$Y = c_s (D-x) \quad \text{if } x \le D.$$

We want to simulate this model in order to estimate the expected cost, given a specific order amount, x. If we do this for several values of x, we can select the order amount that provides the minimum cost.

For the single period inventory model which we are using as an example, our objective is to find the order quantity that minimizes the mean cost. Finding this value will require that we repeat the entire simulation experiment using various values for the order quantity, which is in cell D7. This is the place to use the Data-Table command. The result is shown on Fig. 2.4, in the range G11:I25. First, we created the sequence of order amount values in the range G13:G25. Then, we placed formulas =H5 and =H7 in cells H12 and I12, respectively, to copy the estimates of the mean and sampling error of the mean to these cells. Finally, we selected the range G12:H25 and invoked the Table command of the Data menu to produce the Table dialog on Fig. 2.4. In the field labelled “Column Input Cell”, we entered D7 to indicate that each value in the range G12:G25 should be placed into D7 before recalculating the spreadsheet. Upon clicking OK in the Table dialog, we get a table resembling that in Fig. 2.4.

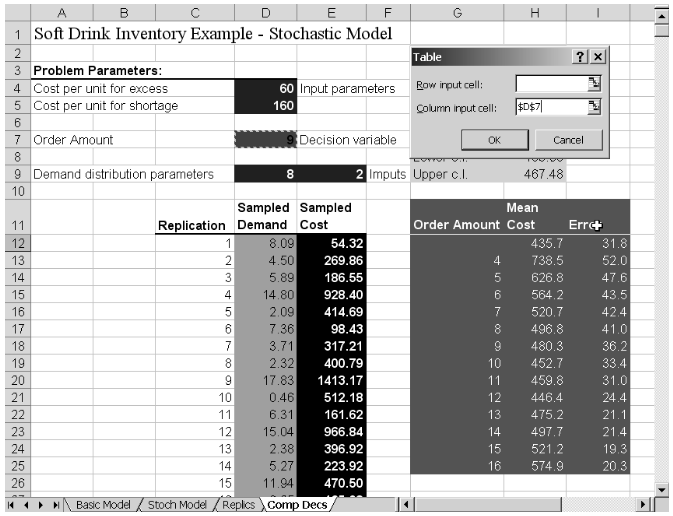


Fig. 2.4 Order Quantity Evaluation

Presentation generally includes some tables and graphs. Spreadsheets have extensive facilities that easily produce these types of presentations in high quality. The types of graphs or other displays will, of course, depend upon the data analysis and the objectives of the modelling effort. Fig. 2.5 Graph of Expected Cost Versus Order Quantity shows a graph of the results on Fig. 2.4. From this graph, you can not only identify the order quantity that minimizes cost, but also you can see that the cost is not very sensitive to the order quantity when it is close to the optimal value.

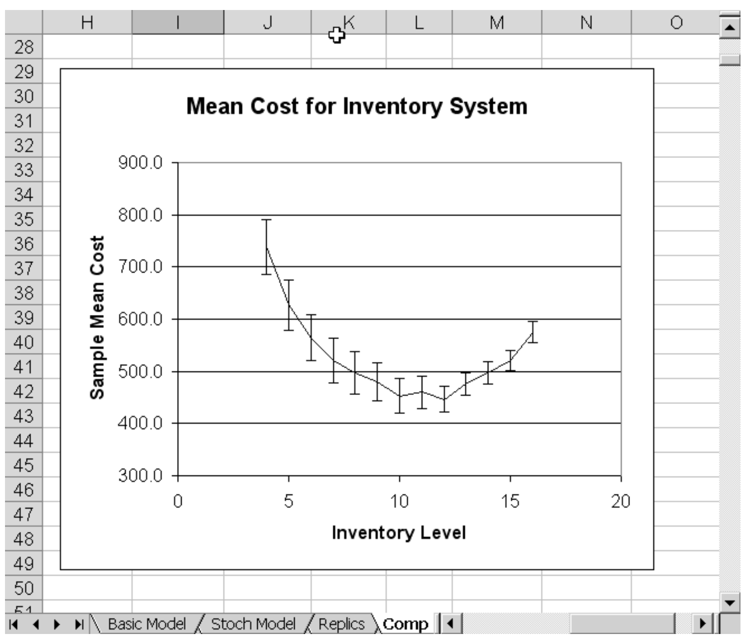


Fig. 2.5 Graph of Expected Cost Versus Order Quantity

It is important to note that in this example, is implemented a simulation model, run replications, collected output data, analyzed the output data to estimate the required parameters of the model and used the model to automate the evaluation of a series of decisions. With the exception of the formula for generating random variants, the entire process was done using built-in Excel features.

2.2.1.2. Implementation of Monte Carlo simulation in a spreadsheet

There are two decision variables: order quantity and reorder point and two probabilistic components: daily demand and reorder lead time (Zabawa, 2007). The main purpose of simulation runs is to try out various schemes of order quantities and reorder point and to find (to minimize) the smallest total inventory cost. That means that inventory cost includes ordering cost, holding cost and stock out costs.

The elements of pull system model are as follows:

- Daily demands for items, collected from observation of past days. That historical frequency will be converted into a probability distribution and cumulative distribution for the variable daily demand.
- Lead time (in days), collected from observations of past orders. That historical frequency will be converted into a probability distribution and cumulative distribution for the variable lead time.
- Beginning inventory (in units).
- Reorder point (in units).
- Order quantity (in units).
- Cost of placing order.
- Holding cost per unit.
- Cost of each lost sale (stock out).

Apart from the above mentioned, the additional assumptions were formulated:

- Orders (if necessary) are placed at the end of the day: the day's ending inventory is checked and if inventory is less or equal reorder point then order is placed.
- Lead time equal 1 day stands for that the order will arrive next morning after next day.
- After arriving at the morning, order quantity is added to inventory, before receiving of the new demand.

Stages of spreadsheet's simulation building were formulated for instance at (Seila, 2002). According to definition of Monte Carlo simulation (Lawrence, 2002), simulated events take place randomly and match the description of the theoretical probabilities derived from acquired experiences. The process of fundamental importance in Monte Carlo simulation is called random number mapping and consists in matching the random number with simulated events (when they occur and how long they last). We did similar task, but using more flexible formulas and data tables (see screenshots at Fig. 2.6, Fig. 2.7 and Fig. 2.8) for collecting simulation outcomes.

	A	B	C	D	E	F	G	H	I	J
30										
31		5	reorder point		30	highest random number (for demand)				
32		10	quantity order		80	highest random number (for lead time)				
33										
34		Demand	Frequency (periods)	Probabability	Cummulative Probability		Beginning of random-number interval	Ending of random-number interval		
35		0	10	0,033	0,033		1	1		
36		1	40	0,133	0,167		2	5		
37		2	60	0,2	0,367		6	11		
38		3	100	0,333	0,7		12	21		
39		4	60	0,2	0,9		22	27		
40		5	20	0,067	0,967		28	29		
41		6	10	0,033	1		30	30		
42		Total	300	1						
43										
44										
45										
46		Lead time (periods)	Frequency	Probabability	Cummulative Probability		Beginning of random-number interval	Ending of random-number interval		
47		1	10	0,125	0,125		1	10		
48		2	25	0,313	0,438		11	35		
49		3	15	0,188	0,625		36	50		
50		4	30	0,375	1		51	80		
51		Total	80	0,625						
52										

Fig. 2.6 Inventory management model as the MS Excel Spreadsheet

Fig. 2.6 presents the frequencies of demand and lead time, probabilities and cumulative probabilities and the way of assigning random-number intervals to get the proper outcomes in a computer simulation. Fig. 2.7 shows one of 20-day simulation runs. Average and total number of units received, simulated demand, ending inventory, stockouts and placed orders are calculated. Fig. 2.8 presents data table with one variable, used to collect 60 replications (runs) of simulated total stockouts during 20 days. A short procedure in Visual Basic is necessary to collect output values and to ensure correct calculation after sheet refreshing. Other (missing) values of problem description parameter, like reorder point, quantity order and beginning inventory are visible on Fig. 2.6 and Fig. 2.7

	A	B	C	D	E	F	G	H	I	J	K	L
1												
2	Period (day)	Number of units received	Beginning inventory	First random number (01-30)	Demand (in units)	Ending inventory units	Lost sales in current period	Order needed?	To place order really (no order is outstanding)?	Second random number (01-80)	Lead time	Planned arriving period
3						10						
4	1		10	19	3	7						
5	2		7	15	3	4		1	1	50	3	6
6	3		4	16	3	1		1				6
7	4		1	30	5	0	4	1				6
8	5		0	27	4	0	4	1				6
9	6	10	10	14	3	7						6
10	7		7	28	5	2		1	1	61	4	12
11	8		2	15	3	0	1	1				12
12	9		0	11	2	0	2	1				12
13	10		0	5	1	0	1	1				12
14	11		0	26	4	0	4	1				12
15	12	10	10	3	1	9						12
16	13		9	9	2	7						
17	14		7	15	3	4		1	1	21	2	17
18	15		4	12	3	1		1				17
19	16		1	21	3	0	2	1				17
20	17	10	10	18	3	7						17
21	18		7	6	2	5		1	1	62	4	23
22	19		5	3	1	4		1				23
23	20		4	2	1	3		1				23
24												
25	Average	1,5			2,75	3,05	0,9		0,2			
26	Total	30					18		4			

Fig. 2.7 Results of one 20-day simulation run

	N	O	P	Q	R	S
3		Total stockouts		Total stockouts		Total stockouts
4	Replication		Replication		Replication	
5	1	24	21	14	41	10
6	2	12	22	24	42	19
7	3	22	23	15	43	13
8	4	12	24	8	44	18
9	5	21	25	13	45	25
10	6	7	26	18	46	16
11	7	32	27	15	47	17
12	8	13	28	16	48	15
13	9	12	29	18	49	21
14	10	7	30	23	50	17
15	11	11	31	20	51	20
16	12	17	32	15	52	14
17	13	17	33	10	53	12
18	14	8	34	21	54	20
19	15	13	35	1	55	25
20	16	16	36	17	56	19
21	17	22	37	13	57	16
22	18	22	38	20	58	14
23	19	14	39	4	59	14
24	20	18	40	15	60	19
25					Average	16,07
26					Std. Dev.	5,49

Fig. 2.8 Data Table for 60 replications (runs) of simulated total stockout during 20 days

2.2.1.3. Excel spreadsheet as an inventory management simulation template

The baseline spreadsheet template (Oberstone, 2007) is shown on Fig. 2.9 and uses the following data to generate the best inventory management parameters, Q* (order quantity) and R* (reorder point):

- Product holding, ordering, and stockout costs (E9:G9)
- Four-tier supplier discount table (J9:K12) the wholesale product costs (L9:L12), and the product retail price, MSRP (M9)
- A specific combination of order quantity, Q (G11) and reorder point, R (G12)
- Data sets for the samples of random variables produce demand, D, and supplier lead time, L are organized into probability distributions that are used to randomly generate values of D and L during the 30-day simulation.

	Holding cost	Order cost	Stock-out cost
9	$5.00	$0.75	$50
11		Order Quantity, Q=	400
12		Reorder Point, R=	200

Discount Strata	From Order Quantity QBOT	To Order Quantity QTOP	Wholesale Cost	Retail price (MSRP)
Break #1	200	299	$85	$140
Break #2	300	399	$80	
Break #3	400	499	$75	
Break #4	500	600	$70	

	Day	Beg Inv, Q_1	D	End Inv, Q_2	Ave Inv, Qave	SO units	Place order?	Lead time, L	Days to receive order	Total inv costs,	Total Product Costs	Revenue	Total daily profit
15	1	400	23	377	388.5	0	NO	*	*	1,943	1,725	3,220	-448
16	2	377	59	318	347.5	0	NO	*	*	1,738	4,425	8,260	2,098
17	3	318	86	232	275	0	NO	*	*	1,375	6,450	12,040	4,215
18	4	232	77	155	193.5	0	YES	3	3	1,268	5,775	10,780	3,738
19	5	155	50	105	130	0	NO	*	2	650	3,750	7,000	2,600
20	6	105	77	28	66.5	0	NO	*	1	333	5,775	10,780	4,673
21	7	428	59	369	398.5	0	NO	*	0	1,993	4,425	8,260	1,843
22	8	369	41	328	348.5	0	NO	*	*	1,743	3,075	5,740	923
23	9	328	41	287	307.5	0	NO	*	*	1,538	3,075	5,740	1,128
24	10	287	86	201	244	0	NO	*	*	1,220	6,450	12,040	4,370
42	28	245	68	177	211	0	YES	1	1	1,355	5,100	9,520	3,065
43	29	577	59	518	547.5	0	NO	*	0	2,738	4,425	8,260	1,098
44	30	518	77	441	479.5	0	NO	*	*	2,398	5,775	10,780	2,608
45			1680			121						*Mean daily inventory profits=*	1,798
46												*Service Level=*	0.928
47												*% of "Loss" days =*	16.667

2. Raw_Data | 3. Org_Data | 4. Sim_Model | 5. VLOOKUP | 6. Reps

Fig. 2.9 Excel 30-Day Baseline Inventory Simulation Template (Q=400, R=200)

The simulation is initiated by using the calculate command F9 (for Windows). The 30-day business cycle is then automatically replicated 200 times for 25 specific Q-R combinations.

1. The five (5) Q-values selected are "edge" values of the discount table: the lower limit values for the four price breaks given in cells J9:J12 plus the largest value of the fourth break, K12.

2. In turn, each Q-value is paired with five (5) reorder point values, R, equal to 10, 30, 50, 70, and 90 percent of the Q value, e.g., the template values of Q=200, 300, 400, 500, and 600 are each examined with corresponding R values of .10Q, .30Q, .50Q, .70Q, and .90Q.

These baseline parameters are quickly "changed-out" or "flipped" with the result that the baseline inventory system is painlessly transformed into a completely fresh product simulation. It usually takes students less than two 90-minute class sessions to effectively manipulate the baseline model template. In two more sessions, they have gained a level of comfort with changing the baseline spreadsheet values to accommodate a different produce and to generate new inventory system parameters. They can see the baseline template mean daily profit of about $2,200 per day for a Q-R combination of 500 and 175 units (see Fig. 2.10).

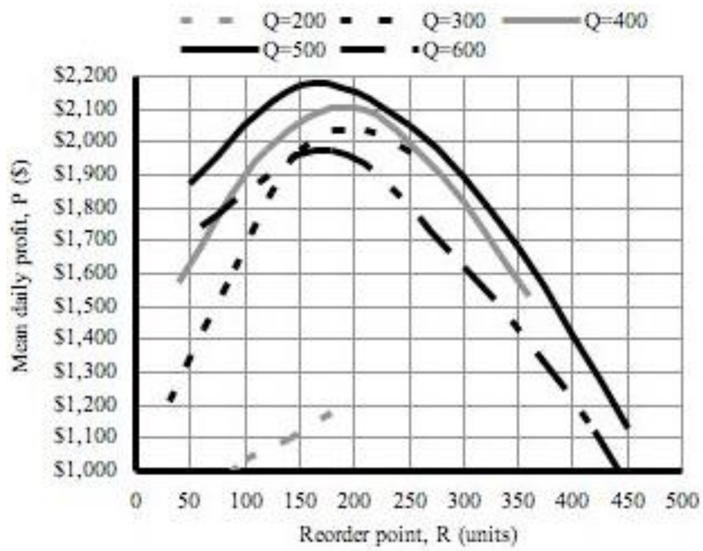


Fig. 2.10 Excel Inventory Management Simulation Mean Daily Profit (Baseline Template)

2.2.2. System Dynamics models

System Dynamics is a computer-aided approach for analysing and solving complex problems with a focus on policy analysis and design. Initially called Industrial Dynamics (Forrester 1961), the field developed from the work of Jay W. Forrester at the Massachusetts Institute of Technology. System Dynamics has its origins in control engineering and management; the approach uses a perspective based on information feedback and delays to understand the dynamic behaviour of complex physical, biological, and social systems.

Forrester (1961) defines Industrial Dynamics as "...the study of the information feedback characteristics of industrial activity to show how organizational structure, amplification (in policies), and time delays (in decision and actions) interact to influence the success of the enterprise. It treats the interactions between the flows of information, money, orders, materials, personnel, and capital equipment in a company, an industry, or a national economy..."

Even though the majority of scientific publications in the field of system dynamics have been issued in the period between 1965 and 1985, this method of simulation modelling is acknowledged as "classical" and implemented nowadays when solving many practical problems in the analysis of complex manufacturing, transportation and logistics systems. Examples of system dynamics method implemented in practice in order to solve inventory control problems can be found in the papers (Vlachos, 2007; Poles, 2009). Other examples are discussed in detail below.

2.2.2.1. The Sprague Electric Company Model

The Sprague Electric Company model (Zwicker, 1980) analyzes the production operations of a line of electronic components. The model (see Fig. 2.11) includes both customers and company behaviour. Main relationships are described firstly and then is examined each decision rule. Incoming customer orders are divided into two streams according to the "factory or inventory decision": one consisting of customer orders, which can be filled directly from inventory; the other, of orders which cannot be filled from inventory (i.e. must be specially produced). The latter forms the "backlog of special orders". The inventory, having been depleted by customer orders, is restocked by inventory management with the help of the "inventory reorder decision rule". These production orders compromise the "backlog of inventory orders". The rate of processing the two backlogs depends upon the production rate (PR) which in turn depends upon the number of men producing at factory (MENPF). MENPF is determined by the "employment change decision".

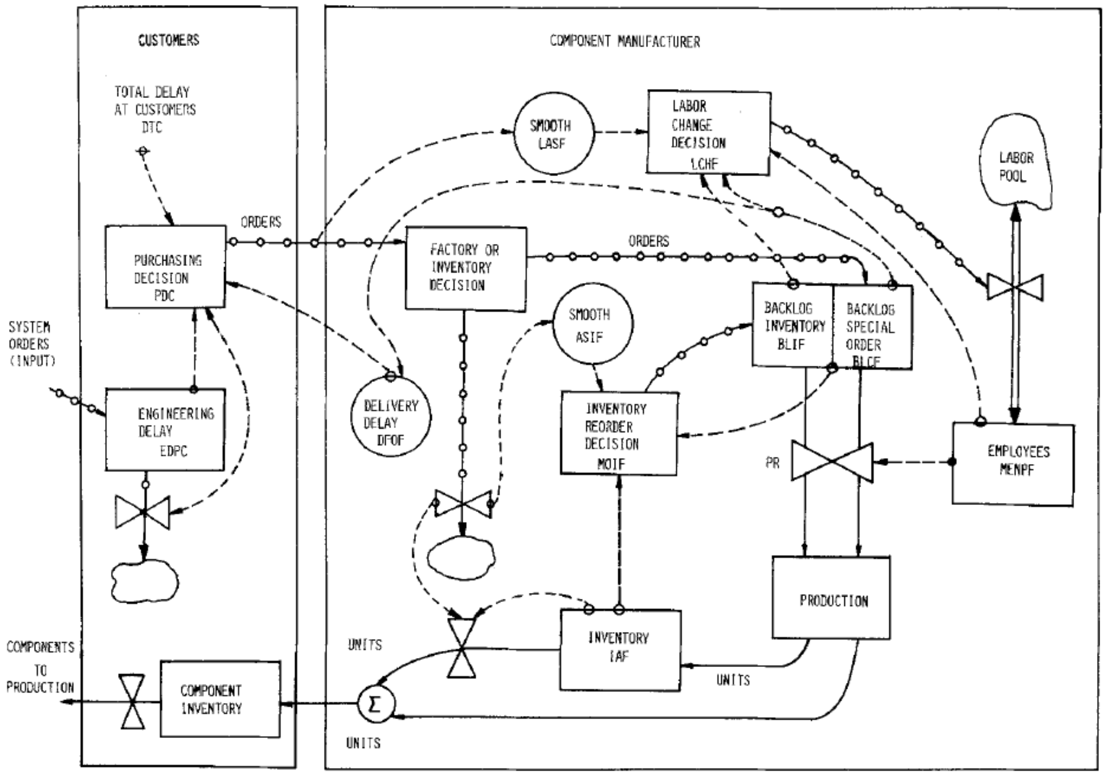


Fig. 2.11 Important structural relations of the Sprague Electric Company Model

The "purchasing decision of the customer" (PDC) is determined by formula:

$$PDC.KL = \frac{ECPC.K}{DEEDC.K} \tag{2.10}$$

- PDC - Purchasing decision (units/week)
- ECPC - Engineering design in process (units)
- DEEDC - Delay effective in engineering department (weeks)

DEEDC is – in somewhat simplified terms – determined by the difference between the target value of the "total delay at customer" (DTC), and the "delivery delay at factory" (DFOF). DEEDC decreases as DFOF increases, which creates an increase in the "purchasing decision rate" (PDC). Since an increase in the purchasing decision rate – ceteris paribus – causes an increase in the delivery delay - (DFOF), PDC shows self-induced growth.

Inventory management policy is directed by the inventory reorder decision. This is a linear is-ought decision rule:

$$MOIF.KL = ASIF.K + \frac{1}{TIAF} \times (IDF.K - IAF.K + OINF.K - OIAF.K) \tag{2.11}$$

- MOIF - Manufacturing orders for inventory (units/week)
- ASIF - Average shipments from inventory (units/week)
- TIAF - Time for inventory adjustment (weeks)
- IDF - Inventory desired (units)
- IAF - Inventory actual (units)
- OINF - Orders for inventory desired (units)
- OIAF - Orders for inventory actual (units)

OINF is determined by:

$$OINF.K = ASIF.K.\left(DPF + \frac{BLIF.K}{PIOF.K}\right) \tag{2.12}$$

- DPF - Delay in production (weeks)
- BLIF - Backlog for inventory (units)
- PIOF - Production of inventory orders (units/week)

in which the term $\frac{BLIF.K}{PIOF.K}$ is the (variable) delay in the backlog of inventory orders. As this, term increases, OINF also increases and – ceteris paribus – therefore MOIF. Since MOIF is the inflow rate of the inventory backlog (BLIF), MOIF, by means of this positive feedback relation, induces its own growth.

The model consists of 22 normal level equations, 24 rate equations, and 35 auxiliary equations. One simulation run affords 0,86s (DT=0.25 weeks, LENGTH=100 weeks) CPU-time on an IBM 370/158 with simulation system DYNAMO II.

2.2.2.2. Modelling of quick response order strategies in supply chains

Quick Response is a new supply chain management system designed to meet the changing requirements of an increasingly more competitive market in the apparel sector. (Hunter et.al., 1992 and Kincade et.al., 1993). The main objective of this study is to build a System Dynamics simulation model of the portion of the textile and apparel pipeline including the retailing and wholesaling processes to search for inventory decisions and policies that yield reduced costs/increased revenues in terms of the retailer. As seen on , the model not only includes the main components of supply chain, but also incorporates how product diversity may affect sales. There are two conflicting effects: first, as the product diversity of the store increases, the probability that customers' preferences will be matched increases toward 1.0 asymptotically. (See

Fig. 2.12(a)). This graph not only makes sense, but can also be obtained by probabilistic analysis, using Binomial probabilities (Barlas, 1999). The opposite effect of increasing diversity implies lower stocks of each product type ("TypeSupply"). Thus, as the ratio type supply/demand decreases, higher fractions of demand will be lost due to type stockouts (as shown on

Fig. 2.12(b)). Therefore, the conflicting effects of product diversity is potentially worth investigating dynamically.

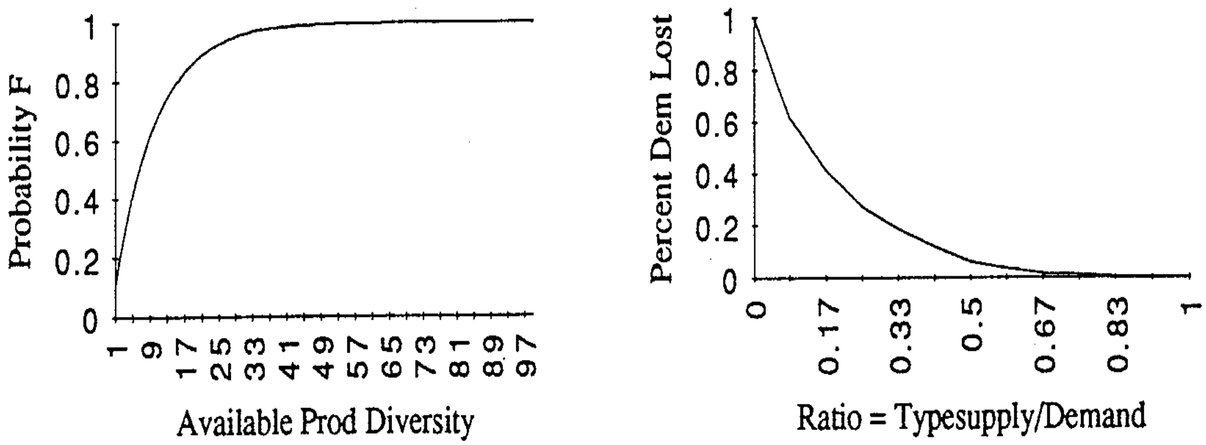


Fig. 2.12 (a) Probability of store's matching customer's preferences as a function of product diversity (b) Percentage of demand lost as a function of stock of each type/demand (Barlas, 1999)

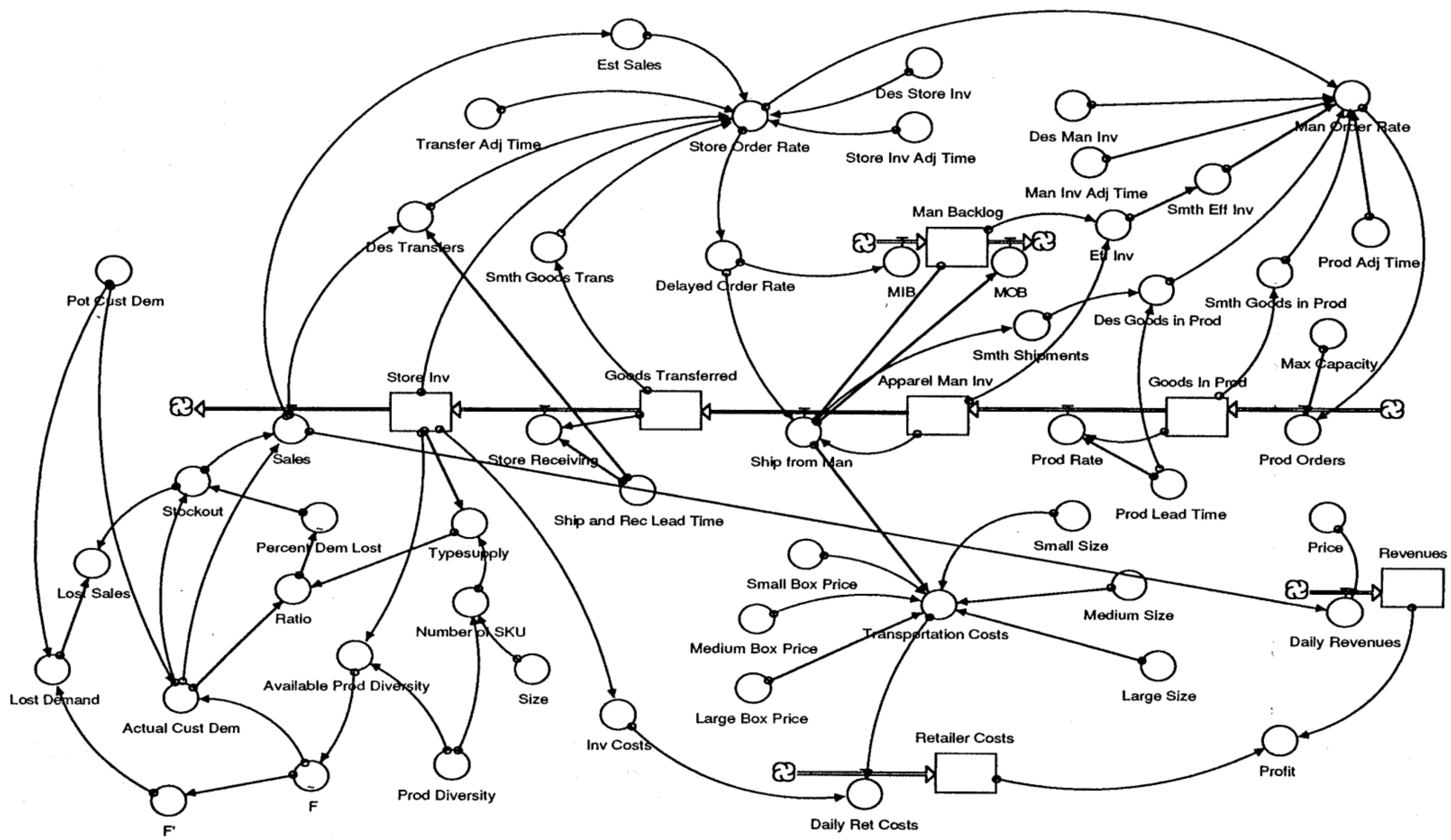


Fig. 2.13 Stock-flow diagram of the model (Barlas, 1999)

The second major originality of the model is that it involves two decisions made at discrete points in time (every seven days), intermixed with continuous flows of processing of goods in the supply pipeline. This "hybrid" system requires that the traditional order rate formulation -which yields steady-state errors in inventories- be modified. (See Fig. 2.14, especially the Apparel Manufacturing Inventory). The solution is to use the following modified order rate formulations:

Store_Order_Rate = IF (Time/7 - INT(Time/7))>0 THEN 0 ELSE

(Des_Store_Inv - Store_Inv)/Store_Inv_Adj_Time +

*(Des_Transfers Smth_Goods_Trans)/Transfer_Adj_Time + 7*Est_Sales*

and

Man_Order_Rate = IF (Time/7 - INT(Time/7))>0 THEN 0 ELSE

(Des_Man_Inv - Smth_Eff Inv)/Man_Inv_Adj_Time +

(Des_Goods_in_Prod - Smth_Goods in Prod)/Prod_Adj_Time +Store_Order_Rate

Observe that, in addition to multiplying the estimated daily sales by 7, three variables must be smoothed: Goods transferred, manufacturing inventory (effective) and goods in production. Only then, is it possible to have the inventories to reach their desired levels, as seen on Fig. 2.15

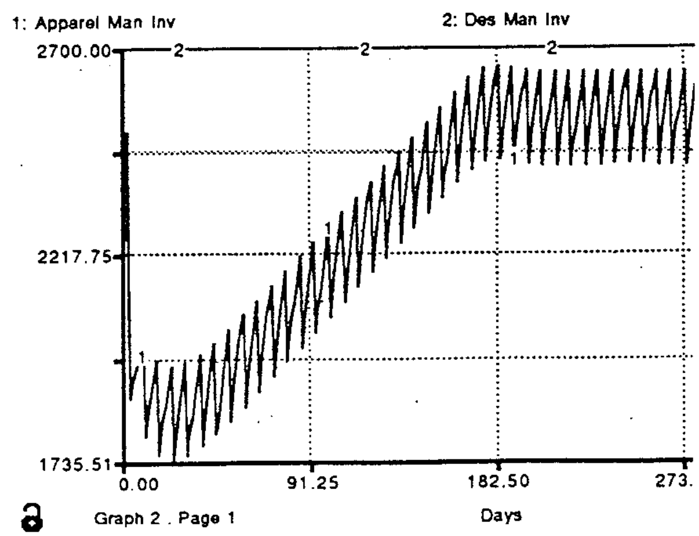


Fig. 2.14 The dynamic behaviour of inventories about their desired levels with standard ordering policies (Barlas, 1999)

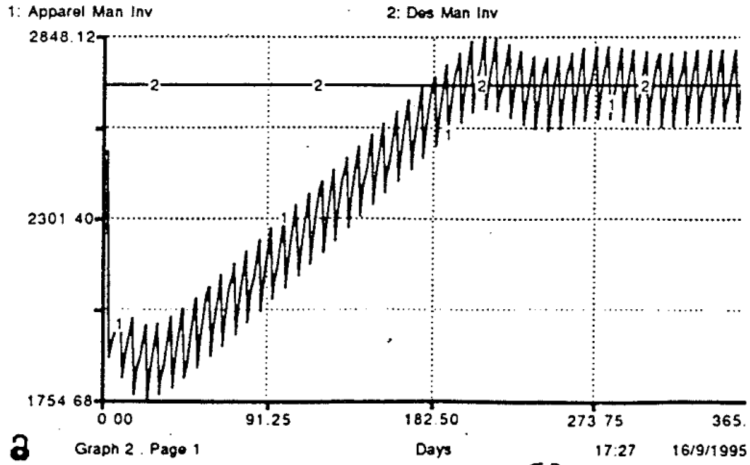


Fig. 2.15 The dynamic behaviour of inventories about their desired levels with improved ordering policies (Barlas, 1999)

2.2.2.3. Decision-Making in Stock Management

The use of System Dynamics Modelling in Supply Chain Management has only recently re-emerged after a lengthy slack period. Current research on System Dynamics Modelling in supply chain management focuses on inventory decision and policy development, time compression, demand amplification, supply chain design and integration, and international supply chain management. The paper (Angerhofer, 2000) first gives an overview of recent research work in these areas, followed by a discussion of research issues that have evolved, and presents a taxonomy of research and development in System Dynamics Modelling in supply chain management.

Sterman (1989) proposes that misperceptions of feedback account for poor performance in dynamic decision-making, as the decision processes are based on an anchoring and adjustment heuristic. Feedback is defined as not only outcome feedback, but also changes in the environment or condition of choice, which are caused by past action. Such multiple feedbacks are the norm in real problems of choice.

Sterman (1989) presents a generic model of a stock management system as shown on Fig. 2.16, which forms the basic structure in an environment for a decision-making experiment. This generic stock management structure is applicable to many different scenarios, including raw material ordering, production control, or at a macroeconomic level, the control of the stock of money. The model consists of two parts, the physical stock and flow structure of the system, and the decision rules used to control the system.

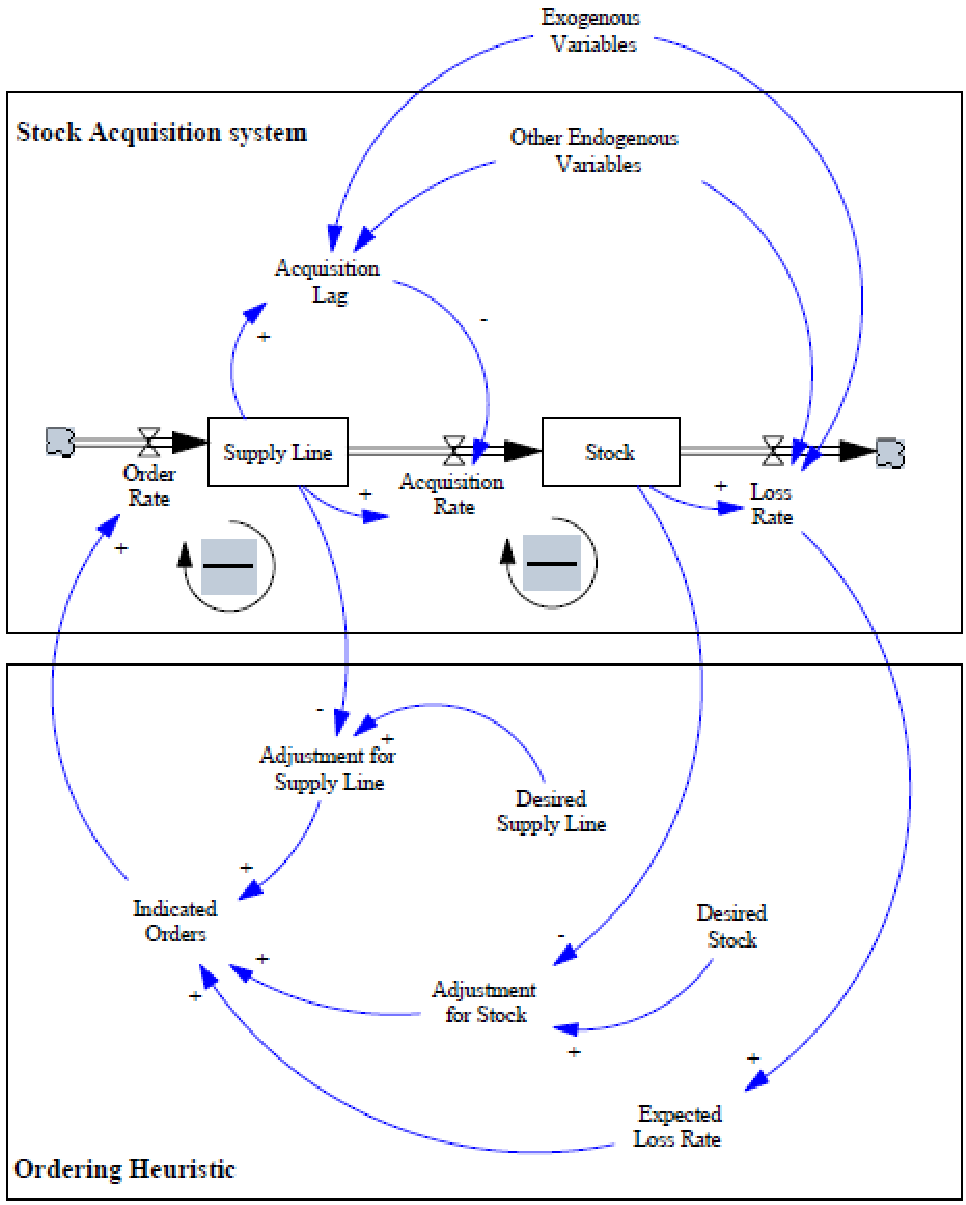


Fig. 2.16 Generic Stock Management System (Sterman, 1989)

Sterman (1989) states that "in most realistic stock management situations the complexity of the feedbacks among the variables precludes the determination of the optimal strategy", and proposes an order decision model based on a locally rational heuristics. An anchoring and adjustment policy is characterized by a mental simulation process, where an unknown quantity is estimated through recalling a known reference point (called the anchor), and then adjusting it according to other factors.

2.2.3. Models with Discrete Events

For the given model's class, there are four ways of planning and processing of events within the model (Schriber, 2002):

- The Activity-Oriented Paradigm
- The Process-Oriented Paradigm
- The Transaction Flow Paradigm
- The Event-Oriented Paradigm

Many modern simulation packages are based on discrete event processes. As it has already been mentioned above, such packages include AnyLogic, Arena, AutoMod, Delmia Quest, Enterprise Dynamics, ExtendSim, Flexsim, Plant Simulation, ProModel, Simul8 and Witness.

For example the basic concept of the event-oriented paradigm also known as event scheduling method is to advance time to the moment when something happens next (that is, when one event ends, time is advanced to the time of the next scheduled event). An event usually releases a resource. The event then reallocates available objects or entities by scheduling activities, in which they can now participate. Time is advanced to the next scheduled event (usually the end of an activity) and activities are examined to see whether any can now start as a consequence.

"Discrete event" process models comprise now a class of simulation models that are most frequently utilized for the analysis of complex manufacturing, transportation and logistics systems. Examples of discrete event process models implemented as a solution to the inventory control problems can be found in the papers (Chen, 2009; Sang, 2012; Cooper, 2013; He, 2013; Wu, 2013; Rossetti, 2013). Other examples are discussed in detail below.

2.2.3.1. Inventory Management Model with Cost Calculation and Optimization

The pull system model, described in section 2.2.1.2, is in (Zabawa, 2007), implemented with the help of Extend simulation package (see Fig. 2.17). The set of assumptions are as follows: the daily demand oscillates between 6 and 16 units, lead time oscillates between 1 and 5 day, initial inventory is 50 units, cost per day of unit holding is 10 cents, cost of lost sale (stockout) is $5, ordering cost is $20, reorder point is 35 and order quantity is 70 units. Note: in this situation, lead time equal to 1 day means that the order will arrive next morning.

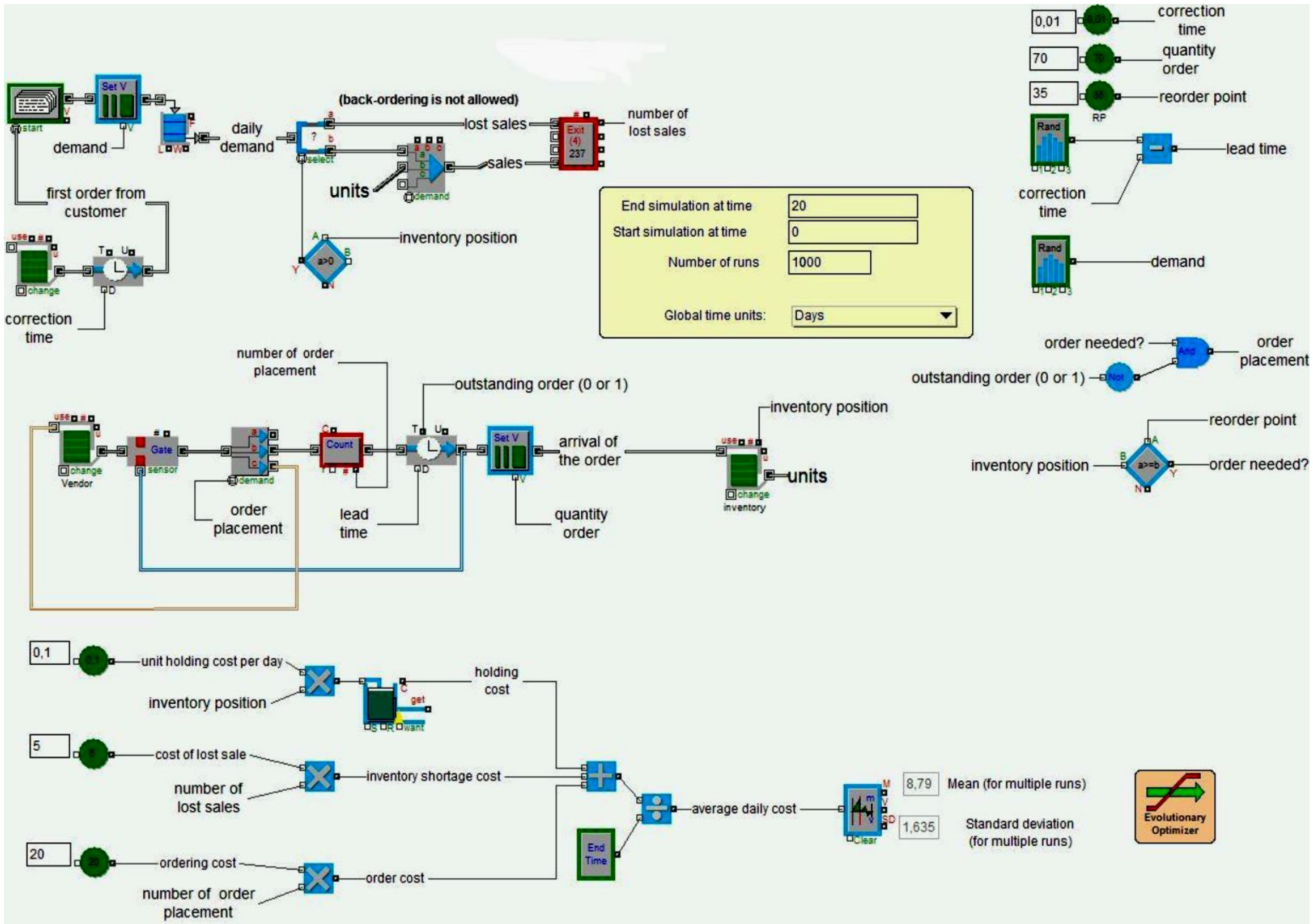


Fig. 2.17 Inventory Management Model with Cost Calculations, Ready for Optimization

The results of our experiment are presented on Fig. 2.17: after 1000 replications of 20-day length period the average total cost is \$8.79 with standard deviation equal to 1.635. The results of 1000 replications are: the average simulated total cost for the mentioned values of reorder point and order quantity was \$8.77.

The next experiment was more sophisticated and it concerned the optimisation of average total cost. We changed values of reorder point and order quantity. We took an opportunity to use Extend ability to evolutionary optimisation that is approachable in „Evolutionary optimizer" block.

We had to divide model parameters into two parts: independent variables (reorder point and order quantity) and dependent variable (in that case, goal function: average total cost of inventory) which will be minimized. We expanded length of simulation run to 1000 days because the beginning value of inventory could affect average cost and we would like to omit the bootstrap effect. Next step was to set minimum and maximum limit for variables. We assumed the reorder point (R) will be changed between 25 and 60 units and quantity ordered (Q) would be changed between 40 and 120 units. Our solution (maximum samples per case = 5, maximum cases = 1000, member population size = 10 and termination if best and worst within 0.95) was R=41 and Q=67 and estimated cost was \$8.30 (see Fig. 2.18).

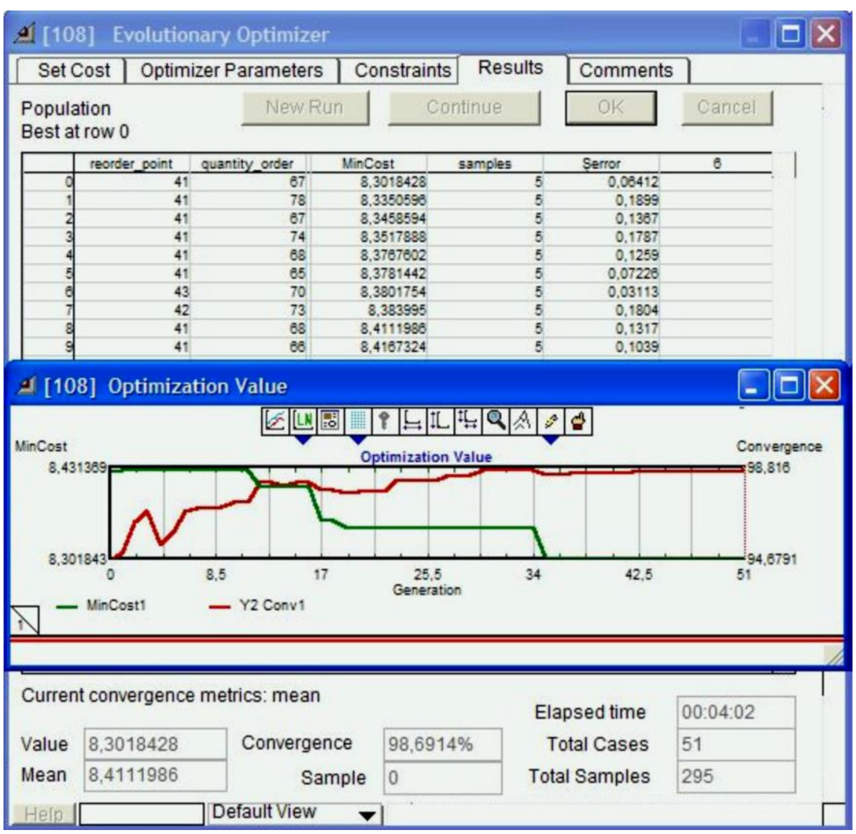


Fig. 2.18 Results of Optimization Process

2.2.3.2. Simulation of a Base Stock Inventory Management System

The first objective of this study (Lee et al., 2010) was to verify the key role of the interaction between inventory control and transportation strategy in different demand sizes for the multi-echelon logistics network model. This paper focuses on the interaction of demand sizes and trucking lot-sizes for the complex logistics network design. To manage the customer's service, there are three types of nodes in the logistics network considered in our model: factories, Regional Distribution Centres (RDC), and warehouses as shown on Fig. 2.19. It is assumed that factories are incapacitated, and RDCs and warehouses are capacitated.

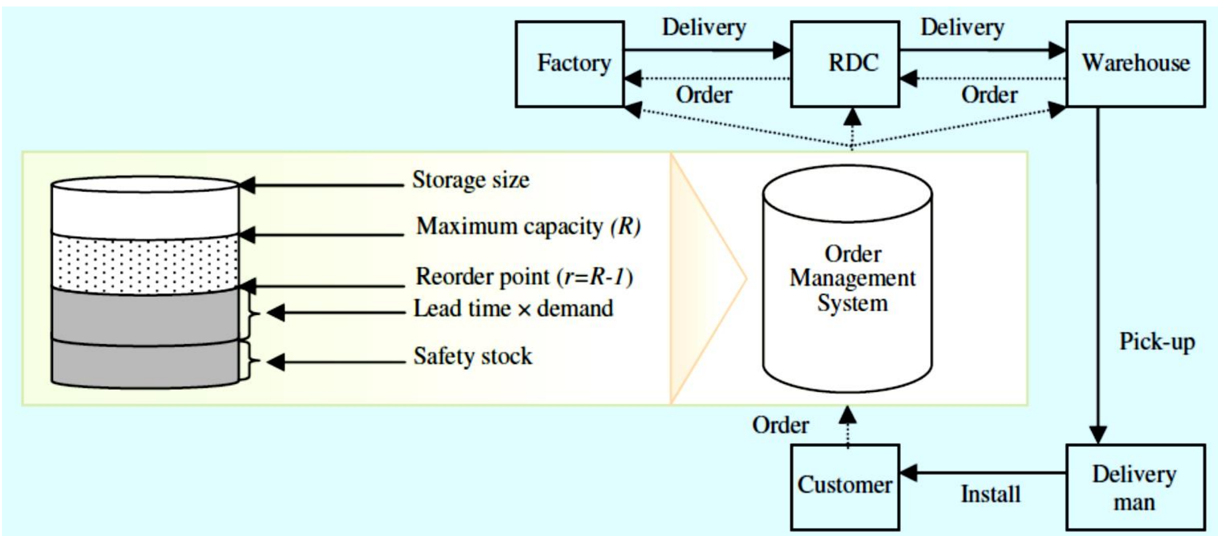


Fig. 2.19 Multi location model with inventory system

The objective of this simulation is to minimize the total cost of the overall systems in the supply chain from factories to warehouses. Total logistics cost is determined by adding the transportation cost, inventory cost, facility cost, and stockout cost.

The transportation and material handling at the factory and RDC operate 24 hours a day. The orders are closed at 6:00 p.m. every day by the information system. The model is denoted as (r, R) with $R = r+1$, where R is maximum capacity (upper bound) and r is the reorder point. Only one unit is ordered when the inventory level reaches the reorder point (r). For the simulation purposes, the R is set as the product of maximum utilization and the storage size of a location. Each warehouse is assumed to store 500 products without consideration for the item's physical dimensions. Maximum capacity (R) excludes the dock, the parking lot, dead space, office space, etc.

In the proposed simulation model, external demand occurs at the lowest echelon which is the warehouses in a normal distribution model. The total sale of products, however, is continuously increasing. The scenarios of demand sizes are assumed in units of 3, 5, and 7 million. The demand size is divided by the total business days (365) in a year for the daily arrival rate. Two years of outbound data are tested in the STAT:FIT utility of Promodel®. From the results, normal distribution is chosen as the distribution of choice to simulate the outbound data. The goodness of the fit test is conducted by Maximum Likelihood Estimates with an accuracy of the FIT of 0.0003 and a level of significance of 0.05. From the data, each day's demand of a product is set by:

$$A = N\left(\frac{DY}{NB}, \frac{DY}{NB} * RD\right), \tag{2.13}$$

where A = Daily arrival rate;

N = Normal distribution (mean, standard deviation);

DY = Expected demand per year;

NB = Business days per year;

RD = Rate of standard deviation of the mean.

Initial inventory is set up for a new business. It is set as a default value before the entities arrive in the simulation system. Inventory cost is classified into three categories: stockout cost, holding cost, and ordering cost. Stockout cost is assumed as 10% of the product margin. It is assumed holding cost consisted of storage managing cost as 7% of the average inventory, storage risk as 5% of the average inventory, and interest rate of 12% per year. Ordering cost is calculated by $1.00 per item.

Cost analysis is summarized on Fig. 2.20. Total cost is collected from the simulation, and unit cost is calculated to compare the transportation policies. The concern is how a company can keep the cost reasonable to maximize profit. Low cost does not mean that the company would have high profit per unit with that scenario. Trucking cost decreases as the shipping size increases and as the demand size decreases. The interaction effects between the shipping size and the demand size are not significant for the trucking cost. The material handling cost decreases as the shipping size increases and as demand decreases. The material handling cost is distorted when the shipping size is small and demand is up. The ratio of decreasing of the material cost is affected much more by demand size than the shipping size. The facility cost decreases by increasing shipping size and increasing demand size due to economies of scale. The holding shows the same pattern as the facility costs. In addition, the ratio of cost decrease is high at the demand size. The interaction between the large demand size and the large shipping size shows the lowest unit cost level.

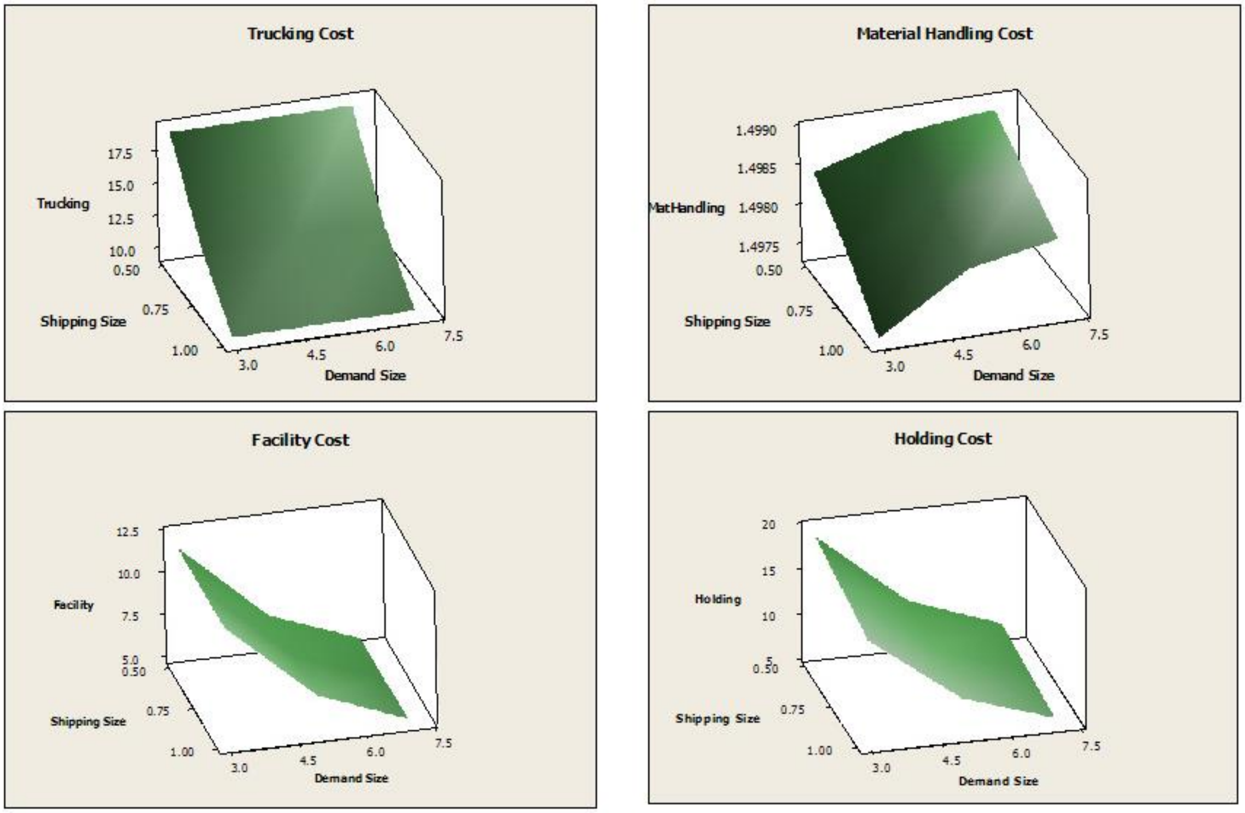


Fig. 2.20 Surface graph for the cost items of each scenario

2.2.3.3. Impact of various inventory policies on a supply chain with intermittent supply disruptions

This model considers a 4 level single-product supply chain that includes a retailer, a wholesaler, a distributor and a manufacturer. The system is depicted in the Fig. 2.21. The demand stream is shown in blue colour and supply stream in purple colour. The demand from the customer end is generated at retailer. The retailer demands from wholesaler, who in turn demands from distributor and at last distributor demands from the manufacturer. The manufacturer places the order to its shop floor and thus the supply starts downstream. The supply chain is prone to disruptions and when a player fails all kinds of flows in the chain are disrupted which means all incoming and outgoing flows through that player are stopped. The demands from the downstream and supply from upstream are not collected. They wait for the player to go back to normal and then all pending demands and supplies are delivered to it together. The period in which a player is available for transactions is termed as ON period and when it is disrupted is termed as OFF period. ON periods reflect supply disruption frequency, while OFF periods represent supply disruption duration. Frequency and duration are two indices for the severity of supply disruptions. The longer the ON periods, the less frequent the disruptions and the slighter the disruptions. Contrarily, the longer the OFF periods, the longer the disruptions last and the more severe the disruptions (Chen and Wang, 2010).

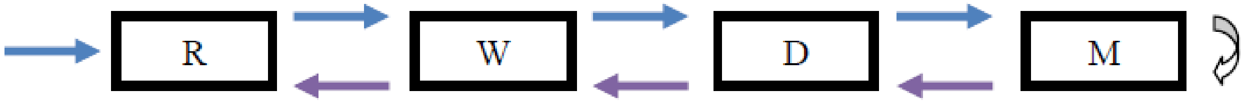


Fig. 2.21 A 4-level single product supply chain (Samvedi et al., 2011)

The periodic review inventory policy (r, S) is adopted at each tier of the chain, where *r* is an inter-review period and *S* is an order-up-to level. This policy means that, every a period of time *r*, the player reviews its inventory position and orders an appropriate quantity of products from the upstream supplier such that inventory position is increased to the order-up-to level *S*. Fig. 2.22(a) shows a standard (r, S) inventory policy that does not consider supply disruptions and Fig. 2.22(b) shows the one which considers them. Time points *t1*, *t2* and *t3* are inventory review points when inventory position is reviewed and an order of appropriate quantity is placed to the supplier. *t4*, *t5* and *t6* are time points when the ordered products are received by the retailer. Time periods *t4−t1, t5−t2* and *t6−t3* are three realizations of stochastic replenishment lead time L (Chen and Wang 2010). On Fig. 2.22(a), the solid line represents net inventory level, which is defined as on-hand inventory minus backorders (Hadley and Whitin 1963). When inventory level is positive, the retailer incurs holding cost that is proportional to holding duration and the quantity of held products. The dotted line in the figure represents inventory position, which by definition is equal to the corresponding net inventory level plus the quantity of products in the currently outstanding orders (Hadley and Whitin 1963). Outstanding orders stand for the orders that have been placed to the supplier but have not yet been received by the retailer.

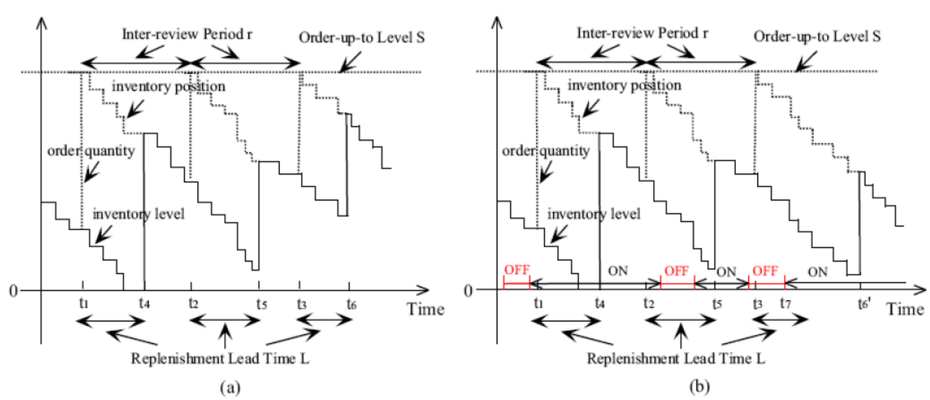


Fig. 2.22 (a) A standard (r,S) inventory policy (b) An (r,S) inventory policy with supply disruptions (Chen and Wang, 2010)

A simulation model has been developed to study the impact of failures at a level on other levels in a chain and the impact of various inventory policies during disruptions. This is a discrete event simulation with four events i.e. demand, supply, disruption and inventory review. The simulation starts from an initial pre-defined state with demands and supplies between levels scheduled. The simulation kicks off with first demand from the customer end. During the demand process, the player checks if there are any backlogged orders. These orders combined with the current demand are treated as the cumulative demand. The player then checks the availability in the inventory and supplies to the downstream level as much as possible i.e. if inventory is available to fulfil the cumulative demand then it is done. If less inventory is available then the entire available inventory is supplied and the balance is put in backlogged orders. If this was the retailer, then next demand from the customer end is scheduled and else the supply is scheduled to the downstream level. Supply event is simpler and requires just the updating of the inventory levels. The disruption event is called for a particular player. That player is flagged as disrupted and any further demand or supplies events will only update the demand in waiting and supply in waiting respectively. Only after the disruption flag is removed from the player, these demand and supplies in waiting will be considered for the updating of inventory levels. The next disruption is scheduled and the end of the current disruption is scheduled. At the end of the disruption, an inventory review is also scheduled. Inventory review event is called after a fixed review period for all players in the chain and during this event the inventory levels are checked if they are lower than the maximum inventory level defined. If the levels are lower they are updated to the level S.

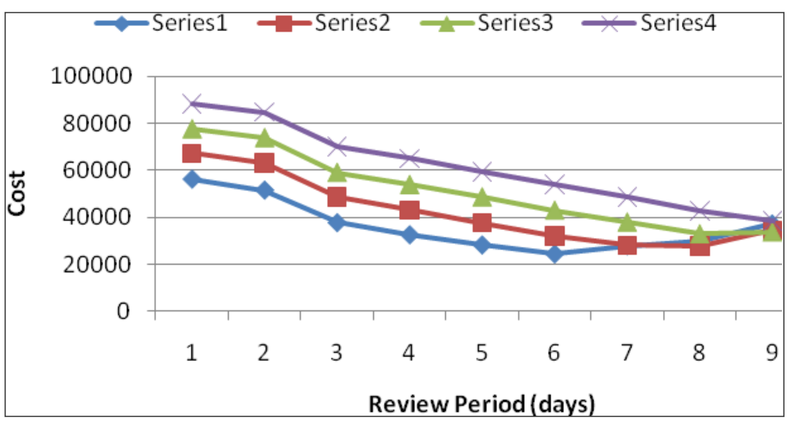


Fig. 2.23 Impact of changes in review period and maximum inventory level on cost

Simulation process example is presented on the diagram on Fig. 2.23. This chart illustrates the change in cost with the changes in review period and maximum inventory level. The four series in the figure represents different maximum inventory levels increasing from one to four. It can be noticed that the overall cost increases as the inventory level increases and it decreases as the review period increases. This can be deduced from the fact that having more inventory will definitely cost more and with increasing review period there will be less inventory replenishments and thus the inventory costs will come down. A resurgence in the cost is also seen as we go increasing the review period. This increase is more pronounced when maximum inventory levels is less. With less maximum inventory levels, the chances of backlogging will be more and this explains the late increase due to backlogging costs.

3. THE PRINCIPLES OF BUILDING THE MODELS OF INVENTORY CONTROL SYSTEMS BASED ON THE IMPLEMENTATION OF DIFFERENT PARADIGMS OF SIMULATION MODELLING

This chapter is fully devoted to the application of simulation modeling to analyze inventory management systems. It starts with the description of paradigms of process modeling in industrial and logistic systems, which are recognized by experts worldwide as standard. Then it is provided an overview of the properties of simulation modeling of software tool, which are most important from the point of modelers. Package ExtendSim has been selected as "Testing ground" for the experimental study of models of inventory management systems, since only this package provides opportunities to implement three fundamental modeling paradigms: ”Continuous”, ”Discrete Event” and “Discrete Rate”. It describes the basic conceptual model of inventory control system and on its basis the implementation and study of three computer models built with the use of all three paradigms of simulation is made. At the conclusion of this chapter it has been formulated conclusions concerning the properties of the models based on the use of different modeling paradigm, and it has been made recommendations for the choice of these paradigms.

3.1. Process simulation standard paradigms in manufacturing and logistics systems

There are three major methodologies used to build dynamic business simulation models: System Dynamics (SD), Process-centric (“Discrete Event”, DE) modelling, and Agent Based modelling (AB). The first two were developed in the 1950s and 1960s and both employ a system-level (top-down) view of things. The agent based approach, a more recent development, is a bottom-up approach where the modeler focuses on the behaviour of the individual objects (Borshchev, 2013).

“**System dynamics** is a perspective and set of conceptual tools that enable us to understand the structure and dynamics of complex systems. System dynamics is also a rigorous modelling method that enables us to build formal computer simulations of complex systems and use them to design more effective policies and organizations. Together, these tools allow us to create management flight simulators-micro worlds where space and time can be compressed and slowed so we can experience the long-term side effects of decisions, speed learning, develop our understanding of complex systems, and design structures and strategies for greater success.” (Sterman, 2000).

The System Dynamics (SD) methodology is typically used in long-term, strategic models and assumes a high level of aggregation of the objects being modelled. People, products, events, and other discrete items are represented in SD models by their quantities so they lose any individual properties, histories or dynamics. If this level of abstraction is appropriate for your problem, SD may be the right method to use.

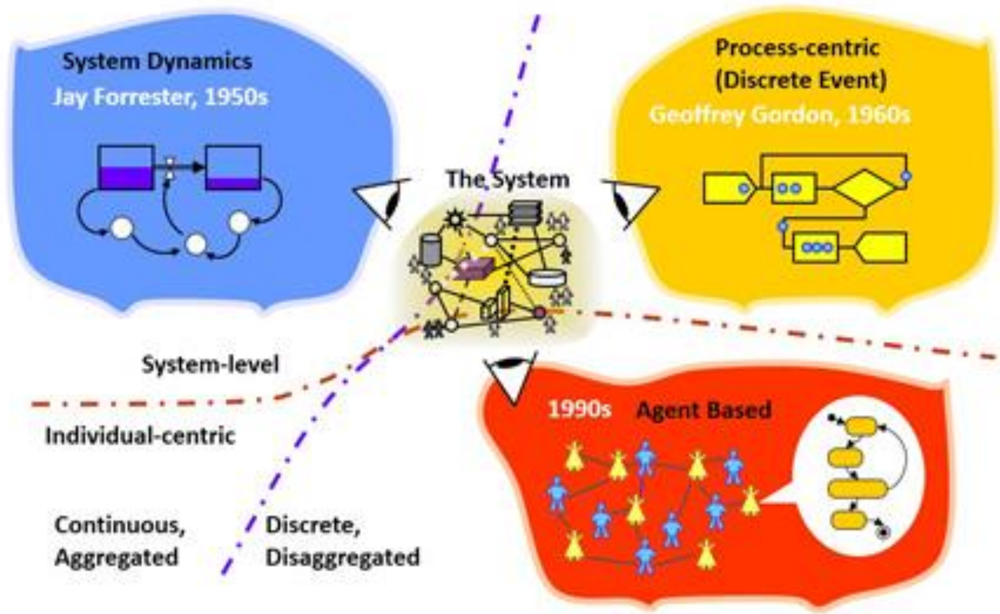


Fig. 3.1 Multimethod Simulation Approach (Borshchev, 2013)

The great majority of processes we observe in the world consist of continuous changes. However, when we try to analyze these processes it often makes sense to divide a continuous process into discrete parts to simplify the analysis. **Discrete Event Modelling** techniques approximate continuous real-world processes with non-continuous events that you define (Banks et al., 2000).

Here are some examples of events:

- a customer arrives at a shop,
- a truck finishes unloading,
- a conveyor stops,
- a new product is launched,
- inventory levels reaches a certain threshold, etc.

In discrete event modelling the movement of a train from point A to point B would be modelled with two events, namely a departure event and an arrival event. The actual movement of the train would be modelled as a time delay (interval) between the departure and arrival events. This doesn't mean however that you can't model the train as moving. In fact, with simulation software you can produce visually continuous animations for logically discrete events.

The term Discrete Event is however mainly used in the narrower sense to denote "Process-Centric" modelling that suggests representing the system being analysed as a sequence of operations being performed on entities (transactions) of certain types such as customers, documents, parts, data packets, vehicles, or phone calls. The entities are passive, but can have attributes that affect the way they are handled or may change as the entity flows through the process. Process-centric modelling is a medium-low abstraction level modelling approach. Although each object is modelled individually as an entity, typically the modeller ignores many “physical level” details, such as exact geometry, accelerations, and decelerations. Process-centric modelling is used widely in the manufacturing, logistics, and healthcare fields.

Discrete Event modelling techniques should be used only when the system under analysis can naturally be described as a sequence of operations. It is not always clear for any given modelling project which of the three modelling paradigms is best. For example, if it is easier to describe the behaviour of each individual entity than trying to put together a global workflow, agent based modelling may be the way to go. Similarly, if you are interested in aggregates and not in individual unit interaction, system dynamics may be applied.

Although you can find a number of various definitions of **Agent Based Modelling** (ABM) in the literature, from the viewpoint of practical applications agent based modelling can be defined as an essentially decentralized**,** individual-centric (as opposed to system level) approach to model design (Railsback and Grimm, 2011). When designing an agent based model the modeller identifies the active entities, the agents (which can be people, companies, projects, assets, vehicles, cities, animals, ships, products, etc.), defines their behaviour (main drivers, reactions, memory, states, ...), puts them in a certain environment, establishes connections, and runs the simulation. The global behaviour then emerges as a result of interactions of much individual behaviour.

Traditional modelling approaches treat a company’s employees, projects, products, customers, and partners as either aggregated averaged quantities or as passive entities or resources in a process. For example, system dynamics models are full of assumptions like “we have 120 employees in R&D, they can design about 20 new products a year”, or “we have a fleet of 1200 trucks that can move so much cargo in a month, and 5% of them need to be replaced each year”. In the process-centric (also known as discrete event) approach you would view your organization as a number of processes, such as: “a customer calls a call centre, the call is first handled by operator of type A, which takes an average of 2 minutes, then 20% of the calls need to be forwarded to…". These approaches are indeed more powerful than "spreadsheet-based modelling". They can capture organizational dynamics and non-linearity, but they ignore the fact that all those people, products, projects, pieces of equipment, assets, etc. are all different and have their own histories, intentions, desires, individual properties, and complex relationships. For example, people may have different expectations regarding their income and career, or may have significantly different productivity in different teams. R&D projects interact and compete and may depend upon one another and aircraft have individual and rigid maintenance schedules whose combination may lead to fleet availability problems. A customer may consult his family members before making a purchase decision. The Agent based approach is free of such limitations as it suggests that the modeller directly focus on individual objects in and around the organization, their individual behaviours, and their interactions. The agent based model is actually a set of interacting active objects that reflect objects and relationships in the real world and thus is a natural step forward in understanding and managing the complexity of today's business and social systems.

There is also the term **Hybrid Simulation**, by which represent various combinations of all of the above analytical and simulation models, as well as modeling systems, some of which are physical objects (machines, robots, etc.).

3.2. The analysis of simulation packages

According to overviews (OR/MS Today 2013) published on the Internet where the information is provided by the companies-producers of simulation software, nowadays, simulation technologies have around 150 units of analytical-type software featured on the market, and they are focused on the simulation. The range and variety of such software continues to grow, and they reflect the tendency of strong demand on this software.

The overview of (OR/MS Today, 2013) is demonstrated as 10 charts with features and characteristics of each product:

- Software Name
- Vendors
- Typical Applications of the software
- Primary Markets for which the software is applied
- Operating Systems
- Can the software use a multiprocessor CPU if available?

- Can the software utilize other software to perform specialized functions?
- If so, name which software:
- Can the software be controlled or run by an external program?
- If so, name which software:
- Can the software be customized by user using model primitives or programming languages?
- Does the software provide the ability to monitor how the CPU cycles are used during execution of a model?
- Model Building: Graphical model construction (icon or drag-and-drop), Model building using programming/ access to programmed modules, Run time debug
- Model Building (continued): Input Distribution Fitting (Specify), Output Analysis Support (Specify), Batch run or experimental design (Specify)
- Model Building (continued): Optimization (Specify), Code reuse (e.g., objects, templates), Model Packaging (e.g., can completed model be shared with others who might lack the software to develop their own model?), Tools to support packaging (Specify), Does this feature cost extra?
- Model Building (continued): Cost Allocation/Costing, Mixed Discrete/Continuous Modelling (Levels, Flows, etc.)
- Animation: Animation, Real-time viewing, Export animation (e.g., MPEG version that can run independent of simulation for presentation), Compatible animation software, 3D Animation, Import CAD drawings
- Support/Training: User Support/Hotline, User group or discussion area, Training Courses, On-site Training, Consulting Available
- Price: Standard, Student Version

The overview Table (OR/MS Today, 2013) that contains first four positions from the abovementioned list is provided in the Appendix 1. In this overview there are no packages that would be targeted at work with System Dynamics and Agent Based Modelling paradigms. For Discrete Event Simulation paradigm there are only Arena, Enterprise Dynamics, ExtendSim, FlexSim, Simio and SIMUL8 packages shown. It is known, however, that the list of the most distinguished packages form this group looks like this: AnyLogic, Arena, AutoMod, Delmia Quest, Enterprise Dynamics, ExtendSim, Flexsim, Plant Simulation, ProModel, Simul8 and Witness. Package AutoMod most often is used in the US when developing the standard models with manufacturing and logistics systems, and the Plant Simulation package is used for these purposes by all German carmakers. For the creation of System Dynamics paradigm, most often used are AnyLogic, Dynamo, iThink/Stella, PowerSim and Vensim packages, and the most popular package for the actualization of Agent Based Modelling concept is AnyLogic software package.

Other earlier overviews of OR/MS Today, e.g. for 2009, contain some other lists of packages. What is important from the point of view of this thesis is not the completeness of the ones, but discussed in the overviews features and characteristics of packages listed above.

Technological opportunities of modern simulation software determine the following properties:

- versatility and flexibility of basic and alternative to the basic concept structuration and formalization of the simulated dynamic processes that are inherent in the simulation system. Today process-oriented structuration concepts and – in continuous simulation systems – models and methods of system’s dynamics that are popular among simulation systems of discrete type; they are based on the network paradigms, automated approach and some other ones;
- by the presence of problem-oriented means when the simulation system contains a set of concepts, abstract elements, language constructions from the subject field of the corresponding research;
- by application of volume-oriented special programming language that support copyright (author’s) simulation and procedures of simulation process control;
- by the presence of the convenient and easy-to-interpret graphic interface, when the flow sheets of discrete models and system flow diagrams are uninterruptedly implemented on the ideographic level, and models’ parameters are determined through the submenu options;
- by the use of developed 2D and 3D animation in real time;
- the possibility to implement several levels of model’s representation, the means for the creation of stratified descriptions. Modern simulation systems apply structurally-functional approach, multilevel hierarchical nested structures, and other methods of models representation on different description levels;
- the presence of lines and tools to analyse the results of scenario and alternative calculations within the simulation model;

- mathematical and information support of incoming data analysis procedures; analysis of sensitivity and calculation procedures of a wide class related to the planning, organization and conduction of a directed computational experiment on the simulation model.
- experimental research on the simulation model is informative, that is why it is necessary to implement Simulation Data Base approach based on the access to the simulation databases. Technologically, this is solved with the help of simulation system's own specialized analytical blocks, or by integrating with other software environments;
- the process execution module can operate outside the model development media;
- by implementation of multi-user operational mode, interactive distributed simulation, designs in the field of simulation interacting with the World Wide Web, etc.

One of the most essential properties that every simulation software should have is the **flexibility during the simulation,** or the possibility to simulate systems with different levels of technological operations complexity. Packages of simulation with a fixed number of structural components and without an opportunity to program, will inevitably be inappropriate for some systems encountered in practice. Further, there are some possibilities giving the flexibility to the simulation package (Law and Kelton, 2004):

- Possibility to determine and alter attributes of objects and global variables, and rely on them when making a logical decision (e.g., by dint of if…then…else constructions).
- The possibility of mathematical expressions and functions application (logarithm, raisin to the power etc.).
- The possibility to create new library components and alter the existing ones; application of new and already altered components in given and future models.

The next important characteristic of simulation is the **ease of use** (ease of study, too), that is why many modern simulation packages are fitted with graphical user interface. Such a program should have library components (blocks) – not too "primitive" and not too "refined". The first case requires many library components to simulate even a simple situation; in the second one, the dialog window of each component will contain an excessive number of parameters required to ensure the relevant flexibility of the program.

Hierarchical modelling can be very useful for complex systems. The hierarchy allows grouping several components of the initial model into new structural components of the higher level. These new structural components then can be joined into structural components of even higher level, etc. The last structural components are placed in the library of available structural components, and they can be repeatedly applied in the current or future models. Repeated application of model's elements with the expansion of logical possibilities increases the effectiveness of simulation. The hierarchy is a crucial concept of many simulation packages.

Software should be equipped with powerful **debugging tools**, such as interactive debugger that allows:

- monitoring separate mobile units across the model beneficial to make sure that they are processed in a proper manner;
- checking the status of model at every single event to occur (for example, the time of delivery to the warehouse);
- setting up the values of certain attributes of variables, for instance, to make the object move to the end on logical path, which is unlikely to be encountered.

The speed of model's operation is highly important when simulating some systems. This applies to the military systems models and models with a number amount of units to be processed (e.g., high-speed communication network models) (Law and Kelton, 2004). Also in the case of model's optimization when thousands of runs are necessary, high-speed becomes the main requirement to achieve an appropriate result.

If the simulation model will be used by somebody except for the developer, it is desirable that exist the possibility to create a convenient **user interface** with the help of which specialist will easily enter the simulation parameters, such as product demand or simulation time.

If the simulation model is not provided with reliable tool of statistical analysis, then it would be impossible to get reliable data about the work of the system under simulation. Above all things, the program package should contain a powerful **generator of random numbers,** i.e. mechanism for generating independent values uniform-type distributed in the interval [0, 1]. Each random factor of the system shall be demonstrated in the simulation model by the allocating of probabilities, not only by average value. If it is possible to select a standard **theoretical distribution** allowing to display certain random factors in an optimum way, then such allocation should be applied in system's model. Software should support the following constant allocations: exponential, Weibull distribution, lognormal, normal, uniform, and triangular distribution, and the distribution of gamma and beta. Besides the continuous distributions, discrete distribution should be available as well: binominal, geometrical and negative binominal, also Poisson distribution and discrete uniform distribution (Law and Kelton, 2004). If no possibility observed to find the theoretical distribution, which successfully distributes some random factors, it is worth applying **empirical distribution** that is built on the given statistical data and is usually set by the developer or user of the model.

For the sake of getting a feedback from users on their experience with the model, program should be equipped with a function to create **standard reports.** The ability to create reports in formats defined by the user of the model should be foreseen as well.

Moreover, the package should also actualize the diverse **statistical graphics.** Above all, it is necessary to make use of possibility to create histograms for some data observed here. In the event of continuous data, histogram functions as an estimation tool for its integral probability distribution density function, and for discrete data – as a tool for the distribution raw probability estimation.

The same way **time-dependent plots** are of great importance. The representation of one of several output variables of the model on such a plot (for example, number of requirements in a certain queue) are displayed throughout the simulation time, enabling the image of dynamic behaviour of the system under simulation.

In addition, the opportunity to **import the initial simulation data** from the spreadsheets and databases should be provided, as well as **export of simulation results** to the spreadsheets, databases, statistical and graphical packages with a purpose to perform further analysis of these results.

3.3. Method of ExtendSim package application to build models of inventory control systems

As a simulation tool for the research, has been chosen a universal package of ExtendSim simulation supplied by Image That, Inc. (Lawrence, 2002; Krahl, 2007). The first version of the package (named Extend) had been released in 1988, and it had all innovative concepts and technologies of simulation of that time embodied in it. Extend simulation system had been the first to incarnate such features as:

- Specially designed graphical interface
- Development environment for building model elements (blocks)
- Hierarchical structure of the model
- Discrete-message-based event structure
- Graphical display of connections for the flow of units and data

ExendSim environment models (versions 7, 8 and 9) built with the help of graphical blocks stored in the libraries. Each block describes calculations or operations with mobile units in the process of simulation. Dialog windows of blocks are a basic mechanism of data enter and receipt of simulation results. Blocks are stored in libraries, and the ones, in their turn, are sets of blocks with similar characteristics. The developer has an opportunity to create his/her own libraries with standard or individually developed blocks. Blocks are moved from the library to the area of models creation by dint of “drag and drop” mechanism. A set of ExendSim regular library is shown in the Table 3.1

Table 3.1 ExtendSim 7 regular libraries

Library	Description
Item	Item processing blocks
Value	Value processing blocks
Plotter	Plots and charts
Animation 2D 3D	Animation for 2D and 3D environments
Rate H	High-speed, high-volume, or rate based processes
Utilities	Blocks that perform utility functions
Electronics	Electronic components

ExtendSim package allows simulating various configurations of systems, because they contain the ModL as an internal programming language for the setup of existing and creation of its own blocks, which could be used to build new models. New blocks are installed in new model, and then implemented in both these and other models.

Presently ExtendSim package enables two-dimensional and three-dimensional animation, exercising Proof Animation package as an auxiliary mean.

Every simulation model of ExtendSim package is tied to Notebook medium, in which it is possible to save elements of dialog windows and results of simulation. Thereby, Notebook can be used as an interface for data input and as a mean to display important results on screen of simulation during the run of a model.

ExtendSim package random numbers are not limited in their number. Furthermore, it is possible to access 26 standard theoretical distributions of probability and empirical distributions.

ExtendSim package has a simple way of performing independent repeated runs of the model, and also building of point estimates and confidential intervals for the indicators proving system is operating. The package allows building statistical plots including histograms and time-dependence plots.

ExtendSim package has been chosen as a mean to develop inventory control simulation models as it is the only package supporting three basic paradigms: continuous (system dynamics), discrete event and discrete rate. The first two paradigms have already been discussed above. Discrete rate paradigm is relatively new one (Damiron, 2008; Krahl, 2009), and so-called mesoscopic models of processes have been defined based on its basis in the prospect of building in manufacturing, transport and logistics systems (Schenk et al., 2009), (Schenk et al., 2010).

The use of the mesoscopic modelling concept in the queuing systems simulation is discussed in (Savrasov, 2007). The applications of the discrete rate and mesoscopic approaches to traffic flow simulation are described in the papers (Savrasov, 2008; Tolujew, 2008; Savrasovs, 2010; Savrasovs, 2011). An idea to mesoscopic supply chain simulation is formulated in (Hennis, 2014) and (Terlunen, 2014).

The essence of continuous, discrete event and discrete rate paradigms of ExtendSim package will be considered below, alongside with the representation of method developed in terms of this thesis that implies the application of all three paradigms when building inventory control models.

3.3.1. ExtendSim package simulation paradigms

The three main modelling methodologies are continuous, discrete event, and discrete rate. Continuous modelling (sometimes known as System Dynamics) is used to describe a flow of values. Discrete event models track unique entities. Discrete rate models share some aspects of both continuous and discrete event modelling.

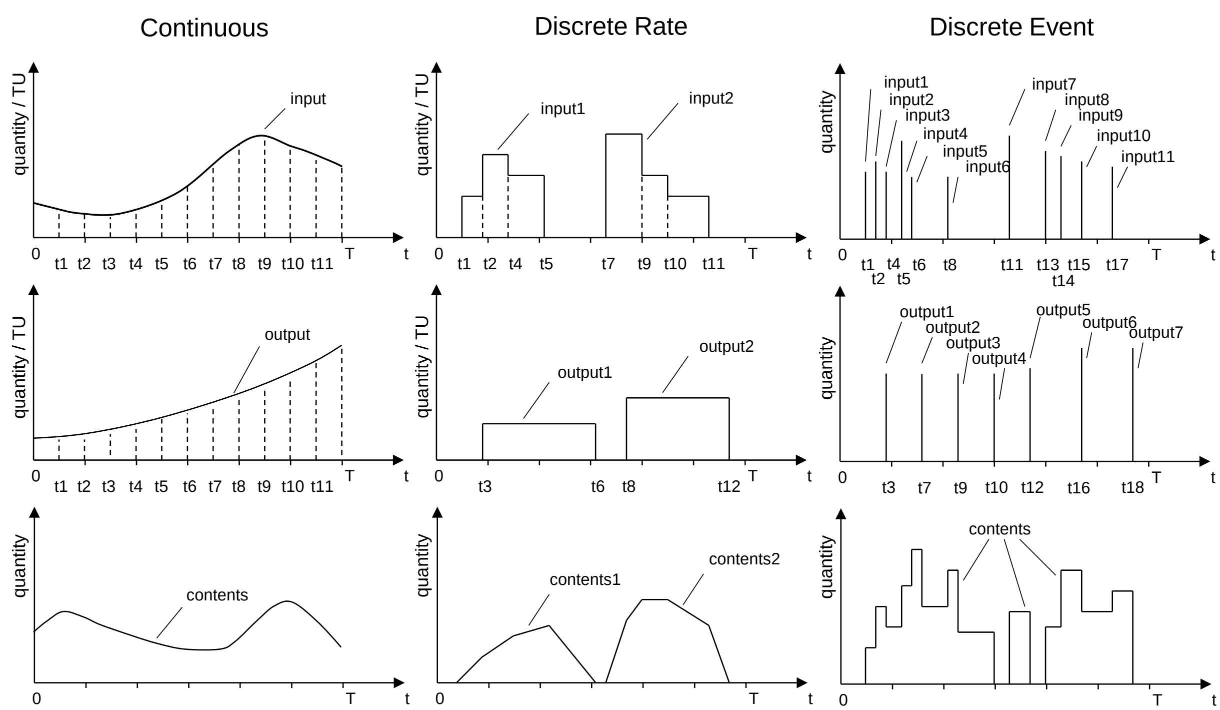


Fig. 3.2 Processes in simple storage structured stock for three simulation paradigms

In all three types of simulations, what is of concern is the granularity of what is being modelled and what causes the state of the model to change. Fig. 3.2 illustrates the flows at the input and output, and dynamics of the content (inventory) of a simple storage structured stock all utilizing three simulation paradigms.

In continuous models, the time step is fixed at the beginning of the simulation, time advances in equal increments, and values change based directly on changes in time.

In this type of model, values reflect the state of the modelled system at any particular time, and simulated time advances evenly from one time step to the next. For example, an airplane flying on autopilot represents a continuous system since its state (such as position or velocity) changes continuously with respect to time. Continuous simulations are analogous to a constant stream of fluid passing through a pipe. The volume may increase or decrease at each time step, but the flow is continuous. In discrete event models, the system changes state as events occur and only when those events occur; the mere passing of time has no direct effect on the model. Unlike a continuous model, simulated time advances from one event to the next and it is unlikely that the time between events will be equal. A factory that assembles parts is a good example of a discrete event system. The individual entities (parts) are assembled based on events (receipt or anticipation of orders). Using the pipe analogy for discrete event simulations, the pipe could be empty or have any number of separate buckets of water traveling through it. Rather than a continuous flow, buckets of water would come out of the pipe at random intervals.

Table 3.2 Comparison of main modelling methodologies

Modelling method	ExtendSim library	What is modelled	Examples
Continuous time	Value library Electronics library	Processes	Processes: chemical, biological, economic, electronics.
Discrete event	Item library	Individual items	Things: traffic, equipment, work product, people. Information: data, messages, and network protocols at the packet level.
Discrete rate	Rate library	Flows of stuff	Rate-based flows of stuff: homogeneous products, high speed production, data feeds and streams, mining.

Discrete rate simulations are a hybrid type, combining aspects of continuous and discrete event modelling. Like continuous models they simulate the flow of stuff rather than items; like discrete event models they recalculate rates and values whenever events occur. Using the pipe analogy for a discrete rate simulation, there is a constant stream of fluid passing through the pipe. But the rates of flow and the routing can change when an event occurs. The three main modelling methodologies are summarized in the Table 3.2.Although not definitive, the Table 3.3 will help to determine which style to use when modelling a system.

Table 3.3 Continuous, discrete event, and discrete rate differences

Factor	Continuous	Discrete Event	Discrete Rate
What is modelled	Values that flow through the model.	Distinct entities ("items" or "things")	Bulk flows of homogeneous stuff. Or flows of otherwise distinct entities where sorting or separating is not necessary
What causes a change in state	A time change	An event	An event
Time steps	Interval between time steps is constant. Model recalculations are sequential and time dependent	Interval between events is dependent on when events occur. Model only recalculates when events occur.	Interval between events is dependent on when events occur. Model only recalculates when events occur.
Characteristics of what is modelled	Track characteristics in a database or assume the flow is homogeneous.	Using attributes, items are assigned unique characteristics and can then be tracked throughout the model	Track characteristics in a database or assume the flow is homogeneous.
Ordering	FIFO	Items can move in FIFO, LIFO, Priority, time-delayed, or customized Order	FIFO
Routing	Values need to be explicitly routed by being turned off at one branch and turned on at the other (values can go to multiple places at the same time.)	By default, items are automatically routed to the first available branch (items can only be in one place at a time.)	Flow is routed based on constraint rates and rules that are defined in the model (flow can be divided into multiple branches.)
Statistical detail	General statistics about the system: amount, efficiency, etc.	In addition to general statistics, each item can be individually tracked: count, utilization, cycle time.	In addition to general statistics, effective rates, cumulative amount

Typical uses	Scientific (biology, chemistry, physics), engineering (electronics, control systems), finance and economics, System Dynamics	Manufacturing, service industries, business operations, networks, systems engineering	Manufacturing of powders, fluids, and high speed, high volume processes. Chemical processes, ATM transactions. Supply chains

3.3.2. Description of the basic conceptual inventory control model

To make it possible to compare the characteristics of models that are created with the help of different simulation paradigms that are offered in terms of ExtendSim package, in any case there will be displayed the same supply chain, which uses the same elementary strategy for inventory control. A set of properties of the system under simulation and which are retained in all particular models is usually represented in the form of so-called basic conceptual model. By the means of extension and refinement of the basic conceptual model, its own conceptual model will be created when designing every particular model.

Fig. 3.3 illustrates the logical structure of simulated system comprising four elements that are interconnected with material flows: supplier, transportation channel, warehouse, and customer. There are information flows shown in the structure of the system: information about the pull of demand for simulated daily demand, information about the inventory level in stock, and information which supply manager sends to the supplier as a replenishment order.

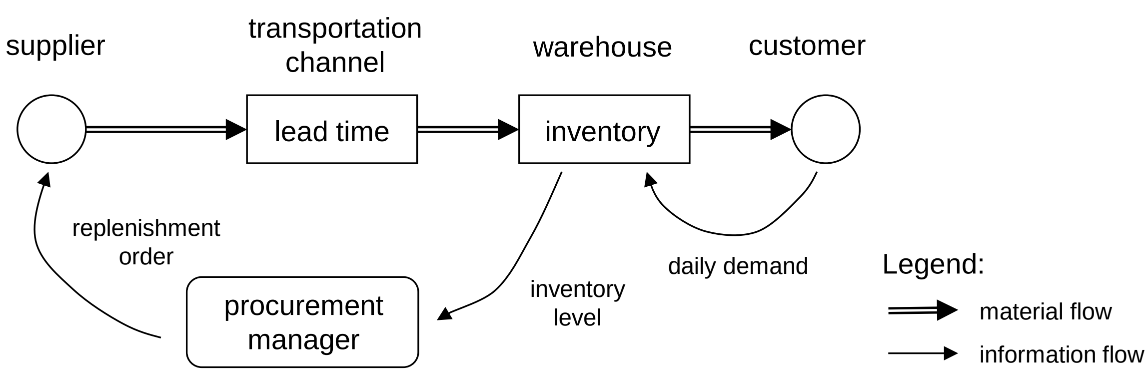


Fig. 3.3 Simulated system's structure

In all variants of the model there are some assumptions as follows:

- the model is single-product only;
- pull of demand is simulated as a random magnitude with even allocation with borders size [30, 80], and besides this, the same random sequence is applied for all variants of models; sequence is illustrated on Fig. 3.4;

- application of a strategy with a fixed size of the order, i.e. 300 equal units;
- reorder level, namely threshold level of inventory – allows to make a purchase order when this level is reached and when it equals to 150 units;
- control procedure of the current stock level is performed once a day when the process of customer service is over;
- if the supply occurs in a certain day, then it is supposed that the whole consignment of goods is delivered at the warehouse before the services delivery to customers, i.e. the whole consignment is available the same day as delivery;
- lead time is 3 days;
- daily demand that cannot be met the same day is reckoned to be lost (lost demand);
- the initial inventory level at the warehouse is 300 units;
- the process of simulation is 30 days long.

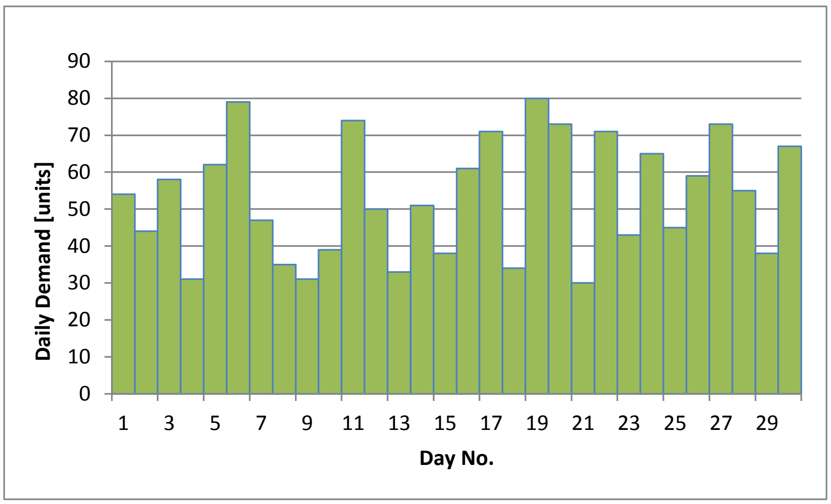


Fig. 3.4 Random daily demand for the 30 days of process

As a matter of principle, all models designed in this work can be used to conduct simulation experiments, because all aforementioned numerical parameters can be easily implemented in the simulated system. However, the result of simulation for all models will be displayed for a single above-mentioned set of parameters only. This is due to the aim of not to study a certain inventory control system, but to demonstrate the method focusing on the implementation of different paradigms for simulation modelling of processes supported by the means of ExtendSim package.

3.3.3. Implementation of "continuous" paradigm

Elaboration of conceptual model

Usually, rather abstract models of processes are created with the help of "continuous" paradigm, reflecting the work of real systems only at principal level. The most important decision is the choosing of simulation time Δt step as it determined the "resolution characteristic" of the model in terms of separated display of particular system events. In the models of "continuous" class, time takes only those values that are divisible by the magnitude of Δt step. In theory, even when simulating 30 days of process, 1 second can be taken as Δt step, but then within one run in the model there will be 2.592.000 steps (3600s×24h×30d), each of them having recalculated totally all variables of the model. Normally, maximum large value of Δt step is chosen to increase system's performance, at which no essential difference is observed between simulated systems and real-time processes.

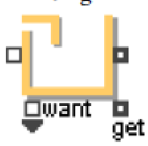
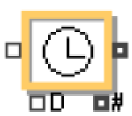
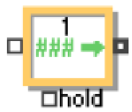
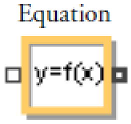
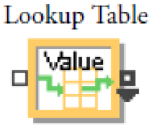
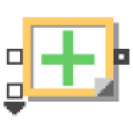
When considering the implementation of the given model, it was decided to choose 1 day of process as a Δt time step. This meant that the possibility to simulate certain events related to particular hours or minutes of the mentioned day will be dismissed. The results of accomplishment of such supposed events will be "calculated" only on the boundary of time between several days. In relation to the studied model, it is supposed that in order to properly identify the level of stock at the end of the day it would be enough to know three magnitudes: the level of stock at the beginning of the day, the amount of goods arriving at the warehouse during the day, and the amount of goods taken from the warehouse during the day. Such assumptions are typical of inventory control analysis strategy on the basis of continuous models that most often are designed as System Dynamics models (see section 2.2.2).

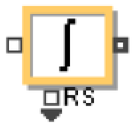
Choosing blocks from value.lix library

There are 31 block in the composition of "yellow" library with the name value.lix from the package ExtendSim 7. These blocks are designed to work with numerical values of variable models. All of them (except *Wait Time* block) can be used not only in models designed on "continuous" paradigm, but also in "discrete event" and "discrete rate" models. Value.lix library blocks in "discrete event" models fulfil auxiliary (mathematical and logical) functions, because moving items that make flows in such models cannot "enter" these blocks but interacting with them via information messages.

To create below described inventory control model, there were 7 blocks selected (see Table 3.4). Most important of them are *Holding Tank* and *Equation* blocks as they have all properties necessary to build a model of System Dynamics type. *Holding Tank* block has a connector *want*, which allows setting the desired value of the intensity of the output stream. This means that this connector serves as blocks of *valves*, which are in the structure of all flow-oriented models built according to the principle of System Dynamics. Connector *get* illustrates the input stream for *Holding Tank* block. On each step of simulation time, the following task is solved automatically: if the level of stock is not less than the *want* connector's set value, then exactly this portion is generated on the connector *get*. Otherwise, the whole inventory remnant in the block *Holding Tank* is left over. *Equation* blocks are intended for the calculation of intensity for the input and output streams of blocks *Holding Tank*. Thus, they operate as variables of *Auxiliary* type, also forming the basis of System Dynamics type models.

Table 3.4 Using value.lix library blocks in the model (ExtendSim 7, User Guide)

Block	Function
Holding Tank 	Accumulates the total of the input values, and allows you to request an amount to be removed if it is available. You can also choose to allow a request that would make the contents go negative (such as an overdraft). You can specify that the inputs are summed or integrated.
Wait Time 	Holds its inputs for a specified amount of simulation time (the delay) before passing them to the output. This block works like a conveyor with slots: values come into a slot, advance position based on an advance in simulation time, then exit when their slot reaches the end of the conveyor. Note: This block should not be used in a discrete event model.
Constant 	Generates a constant value at each step. You specify a constant value in the dialog (the default constant is 1.0). This block is typically used for setting the value for inputs to other blocks. For example, you can use it for a steady flow of fluid, cash, or a delay time value. If the ValueIn input on the left is connected, the input value is added to the constant in the dialog and the sum of those two numbers is output.
Equation 	Outputs the results of the equations entered in the dialog. You can use ExtendSim's built-in operators, functions, and some or all of the input values as part of the equation. The equations can have any number of inputs and any number of outputs.
Lookup Table 	Acts as a lookup table (x in and y out) that are used to calculate what the output value would be for the given input. Input values can come from an input connector (the default) or can be simulation time. The output can be discrete, interpolated or stepped.
Math 	Performs a selected mathematical operation on its inputs and outputs a result.

Integrate 	Integrates the input values over time using either Euler or Trapezoidal integration methods. If present, an initial value is added to the outputs.

Creation of the model

The ready model built on the basis of “continuous” paradigm is depicted on the Fig. 3.5. By way of constants, there are three input parameters of the model set: *Reorder Level*, *Order Size* and *Lead Time*. Daily demand in the form of variable *Demand* is given by the block which has *Lookup Table-like* entries of values, illustrated on the Fig. 3.4. The *Plotter 1* block is used for the creation of chart with the view to control the set sequence of values for the *Demand* variable. The rest of the *Plotter* blocks serve to demonstrate the dynamic behaviour of variable models which are included in the composition of model’s output data:

- *Delivery State* variable shows the current state of the channel by which the supply is realized; when goods are in transit, the value of this variable equals to the size of supply, otherwise – zero;
- variable *Sold Volume* shows cumulative amount of sold goods;
- variable *Inventory* shows the current value of stock level at the warehouse;
- variable *Order Backlog* shows cumulative amount of goods related to the lost demand.

General or also flow-oriented part of the model consists of five blocks which are given the corresponding names:

- *Procurement Manager* block implements the inventory control strategy by checking the time on every step of simulation, whether or not it is necessary to send a delivery order at that current moment; the order (variable *order_stock*) is formed if the two requirements written in the first line of the program and performed by *Procurement Manager* block are met. These requirements are:

```
if (( inventory <= reorder_level ) and ( delivery_state == 0 ))
  order_stock = order_size;
else
  order_stock = 0;
```

- *Lead Time* block ensures delay in supply for the time set in the initial data of the model;

- *Inventory Level* block simulates the dynamics of stock changes at the warehouse; on each step of simulation time, the amount of goods determined by the *Deliver Stock* variable comes to this block, and variable *Shipments* shows the amount that leaves it;
- *Sold Volume* block contains summarized values of *Shipments* variable, which are observed at all stages of process (in the standard case 30 steps simulated correspond to 30 days of the process);
- the assignment of the block and *Delivery State* variable have already been described above; the information about the supply channel's state is formed and is necessary for the operation of *Procurement Manager* block.

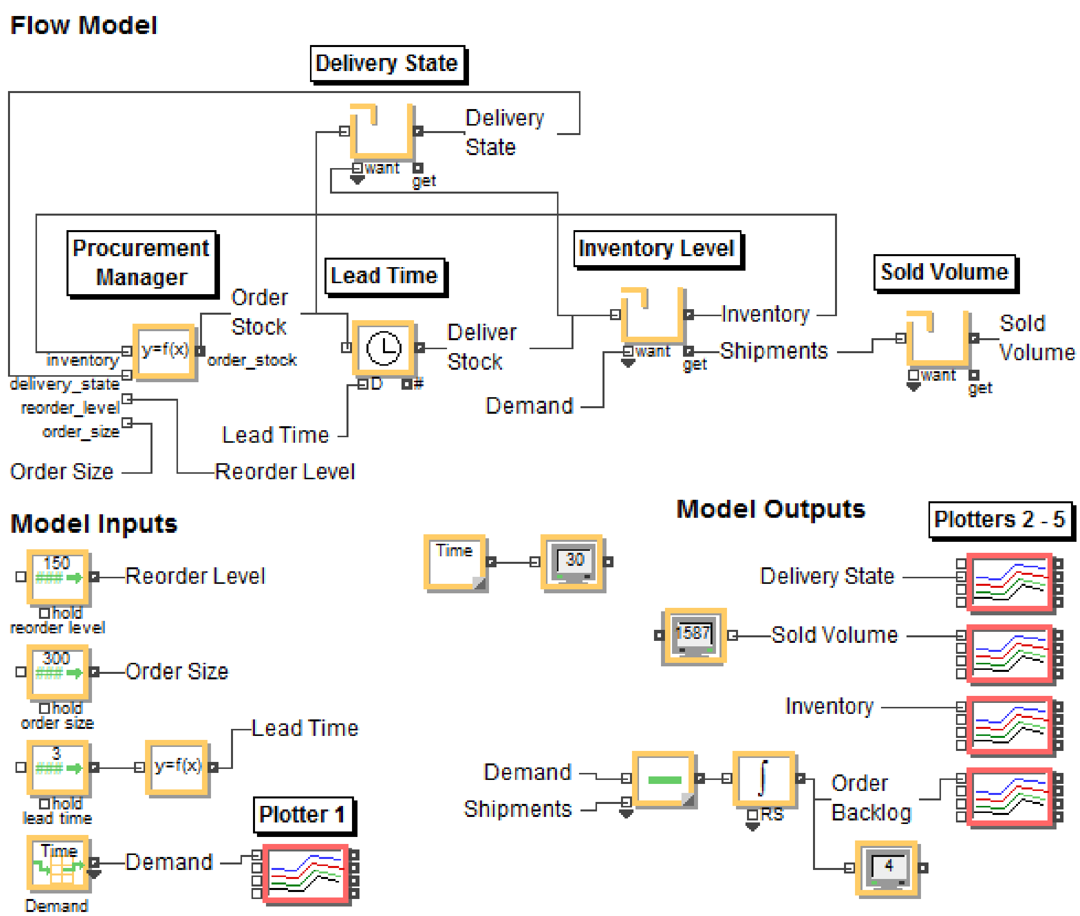


Fig. 3.5 Inventory control system's model on the basis of "continuous" paradigm

The analysis of simulation results

Main results of simulation are shown in the form of chart, in Table 3.5.

Demand diagram repeats the initial data, like on the Fig. 3.4.

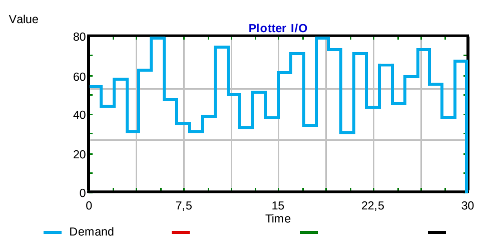
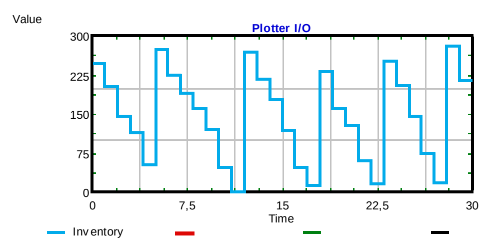
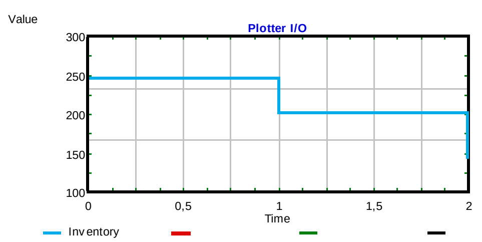
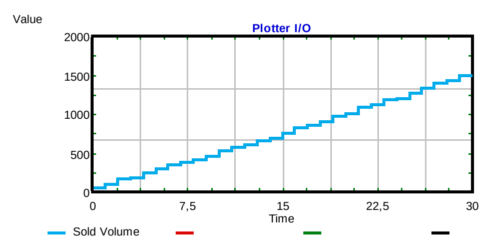
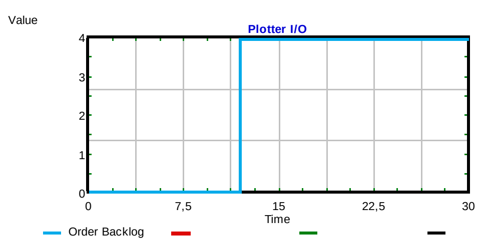
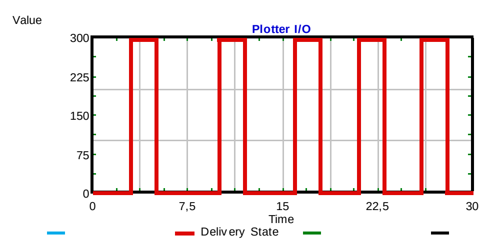
Inventory (30 Days) diagram shows the dynamics of changes in stock at the warehouse within 30 days. Each stage of the diagram corresponds to one process. It can be seen that delivery has been accomplished 5 times, and at the end of the day 11 the level of stock decreased to a value of 0.

Inventory (2 Days) diagram shows the “anatomy” of the customer service process during the first two days. When the first day is over (Time=1), the remnant amount at the warehouse equals to (300 – 54 = 246), as 300 units is the initial number of goods, and 54 is the daily demand of the first day illustrated on the Fig. 3.4.

Sold Volume diagram shows the cumulative amount of sold goods; *Display Value* block that is installed in the model and connected to the *Sold Volume* variable shows the total number of 1587 units of goods sold in the period of 30 days. By summarizing the data about the daily demand (see Fig. 3.4), it is possible to get the total amount of demand that equals to 1591 units.

Order Backlog on the diagram shows that 4 units of goods at a time at the end of day 11 could not have been sold, and this corresponds to the difference: (1591 – 1587 = 4).

Table 3.5 Results of control system’s simulation on the basis of “continuous” paradigm

Demand	Inventory (30 Days)
	
Inventory (2 Days)	Sold Volume
	
Order Backlog	Delivery State
	

Delivery State diagram shows the dynamics that corresponds to the auxiliary variable that reflects the state of the supply chain. It can be clearly seen that there were exactly 5 supplies. The width of the chart's stages corresponds to two days, i.e. the factual time of supply was 2 days with the set parameter *Lead Time* = 3. Such a "trick" exactly in the model of "continuous" type was necessary to actualize assumptions that the goods will be available for sale on the third day after the sale. Without changing the *Lead Time* value this way, the goods in "continuous" type model will be available for sale only the next day.

3.3.4. Implementation of "discrete event" paradigm

Elaboration of conceptual model

"Discrete event" paradigm most frequently finds its application in simulation modelling of processes in the production and logistics. It has a feature of the lowest level of abstraction, which characterizes the method exercising the display of real processes in the form of a computer program. Almost all simulated objects of the real system in such program have their own "virtual objects". This relates to the permanent objects in the system that are part of its resource and related to the mobile units (usually just temporarily present in the system), for the processing of which these resources are used. The essence of "discrete event" type models' functioning lies in execution of certain actions with particular mobile units. These objects are created, moved within the system, delayed, accumulated and finally deleted form the system in its certain points. All these actions are accompanied by the events that mean, for example, the start and end of operation on the mobile units. The same way models of "discrete event" type show the time without any restrictions, i.e. the notion "fixed Δt time step" is not applicable in such models. The event in the model can occur at any time, same way it happens in reality.

When actualizing the given model it was decided to give a detailed representation of the following processes related to two types of mobile objects characteristic of this model: units of goods and customers. Methods of appearance, storage and transportation of units of goods have already been in terms of basic conceptual model. Introduction of customer model in the composition of system marks the expansion of the conceptual model that is implemented on purpose to increase the adequacy when displaying the reality. The same way time in this model will be displayed as "real astronomical", i.e. each of 30 days of process will consist of 24 hours or 1440 minutes comprising these hours.

Minute will serve as a unit of timing, viz. all numbers displayed on the time axis of diagrams of processes will mean the number of minutes. Total duration of the simulated process is $1440 \times 30 = 43200$ minutes.

The process of inventory control system's operation within the model based on "discrete event" paradigm will be accompanied by the following additional details not mentioned in the basic conceptual model:

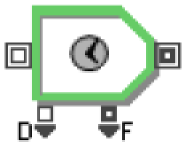
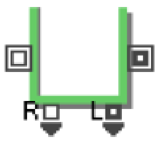
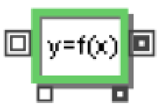
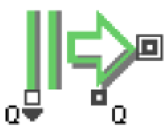
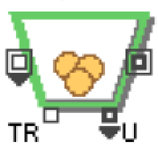
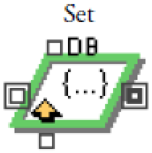
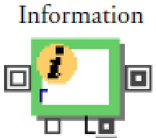
- customer service process will happen daily from 8 a.m. to 4 p.m., hereby it can end long before 4 p.m. in case the number of customers is law for the whole day;
- customers appear at the warehouse within the period between 15 and 45 minutes, and each of them request between 6 and 12 units of goods and it takes 10 minutes to serve this customer;
- number of clients and the number of requested units of goods is automatically adjusted in the model so that every 30 days the total amount of requested units correspond to the data of daily demand demonstrated on the Fig. 3.4;
- analysis of the current stock level at the warehouse and in case of necessity order picking for delivery is made at 4 p.m. same day;
- income of goods to the warehouse at the day of delivery occurs at 7 a.m. in the morning; this means that factual time of delivery should be set in hours, not in days, because its count starts at 4 p.m. and ends at 7 a.m. the next day; when the parameter *Lead Time* is set at *3 Days*, delivery time is defined as follows: $24 \times 3 - 9 = 63$ h.

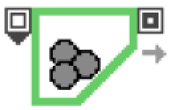
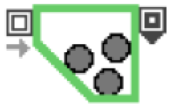
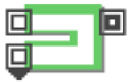
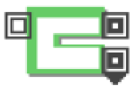
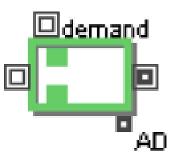
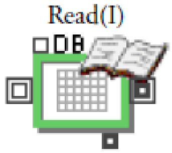
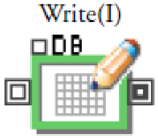
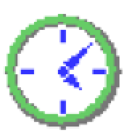
Choosing blocks from value.lix library

In the composition of item.lix "green" library, 31 blocks are in the ExtendSim 7 package. These blocks are designed to work with mobile values of the model known as items. Items appear from the sources (Create block) and leave the model in outflow (block Exit). All the other blocks belonging to the library item.lix, such as item-type objects, just "pass through" it.

To create the below-mentioned models of inventory control system there were 16 blocks chosen, they are displayed in Table 3.6. The most important ones are Activity, Queue and Equation(I), because they help to simulate the rest of the actions occurring together with the participation of units of goods and separate customers. Activity block allows delaying objects of item type for the time set as constant or allocation of a random magnitude. Queue type block is used in this model as a space for storage of inventory at the warehouse. Equation(I) block is activated by the transient item block at the moment when it is necessary to perform some mathematical operations or make management decisions, for example, send a purchase order.

Table 3.6 Using item.lix library blocks in the model (ExtendSim 7, User Guide)

Block	Function
Activity 	Holds one or more items and passes them out based on the process time and arrival time for each item.
Queue 	Queues items and releases them based on a user selected queuing algorithm, such as Resource pool queue, Attribute value, First in first out, Last in first out, and Priority. Options include reneging and setting wait time. If you need more advanced control over the queueing algorithm, consider using the Queue Equation block, below.
Equation(I) 	Calculates equations when an item passes through. The equations can use multiple inputs and properties of the item as variables, and the result(s) of the equations can be assigned to multiple outputs and properties of the item.
Create 	Provides items or values for a discrete event simulation at specified interarrival times. Choose either a distribution or a schedule for the arrival of items or values into the model.
Resource Item 	This block holds and provides items (cars, workers, orders, and so forth) to be used in a simulation. It can be used as part of an open or closed system.
Exit 	Passes items out of the simulation. The total number of items absorbed by this block is reported in its dialog and at the value output connectors.
Set 	Sets the properties of items passing through the block from input connectors, values in the dialog, or databases.
Information 	Reports statistics about the items that pass through it, such as cycle time and TBI (Time Between Items).

Batch 	Allows items from several sources to be joined as a single item. Useful for synchronizing resources and combining various parts of a job ("kitting").
Unbatch 	Produces multiple items from a single input item. This block can be used to disassemble a kit, break a message packet into component messages, route the same message to several places, or distribute copies of invoices.
Select Item In 	Selects items from one input to be output based on a decision.
Select Item Out 	Selects which output gets items from the input, based on a decision
Gate 	Limits the passing of items through a portion of the model, either by demand or by using a sensor connector to monitor how many items are in a section of the model.
Read(I) 	Reads data from a database when an item arrives. You can define an indefinite number of reads to be made by the block when an item passes through.
Write(I) 	Writes data to a database when an item arrives. You can define an indefinite number of writes to be made by the block when an item passes through.
Executive 	This block must be placed to the left of all other blocks in discrete event and discrete rate models. It does event scheduling and provides for simulation control, item allocation, attribute management, and other discrete event and discrete rate model settings.

Creation of the model

The ready model created on the basis of "discrete event" paradigm is displayed on Fig. 3.6. For the initial data input and output of the simulation results exactly the same constants and variables are used, likewise in the previous model on the basis of "continuous" paradigm.

General or also flow-oriented part of the model consists of three sections which are found in three horizontal layers. Wide lines of connections between the blocks show flows with moving separate *items* inside of them. Narrow lines are the connections with the help of which blocks exchange numerical data in between.

The bottom closed-loop of “green blocks” serves to simulate a special timer that would give signals at 8 a.m. and 4 p.m. daily corresponding to the description of the process provided above as an extension for conceptual model. The framework of timer consists of three *Activity* blocks, each of them delaying the circulating within the timer *item* exactly for 8 hours. From 8 a.m. to 4 p.m. the second *Activity* block supports the value of signal *Work Time* = 1, which is an authorization for simulation of input flow of customers in the upper part of the model. At 4 p.m., *item* starts *Equation(I)* block with the name *Procurement Manager,* in which the strategy for inventory control is implemented and made a decision on the necessity to send an order purchase. In the latter case, a special *item* is directed to the *Activity* block with a *Lead Time* designation, where later it is delayed for the set period of delivery time, e.g. 63 hours. After this time, *item* leaves the *Activity* block and with the help of *Unbatch* block creates all units of goods that relate to this supply, e.g. 300 units. All of these goods displayed as *items* are saved in the *Queue* block under the name of *Inventory.* They are extracted from there by dint of *Batch* block, where the information about customers’ goods comes from the upper part of the model. The *Sold Volume* block collects all the units of goods sold during the simulation.

The whole upper line of blocks serves for the simulation of incoming customers process in strict compliance with the description provided in the extended form of the conceptual model. In the *Equation(I)* block, pull of demand with the name *Customers Flow Model* is raffled by dint of random numbers generator for each customer, and the total amount of daily demand, illustrated on the Fig. 3.4, is also controlled.

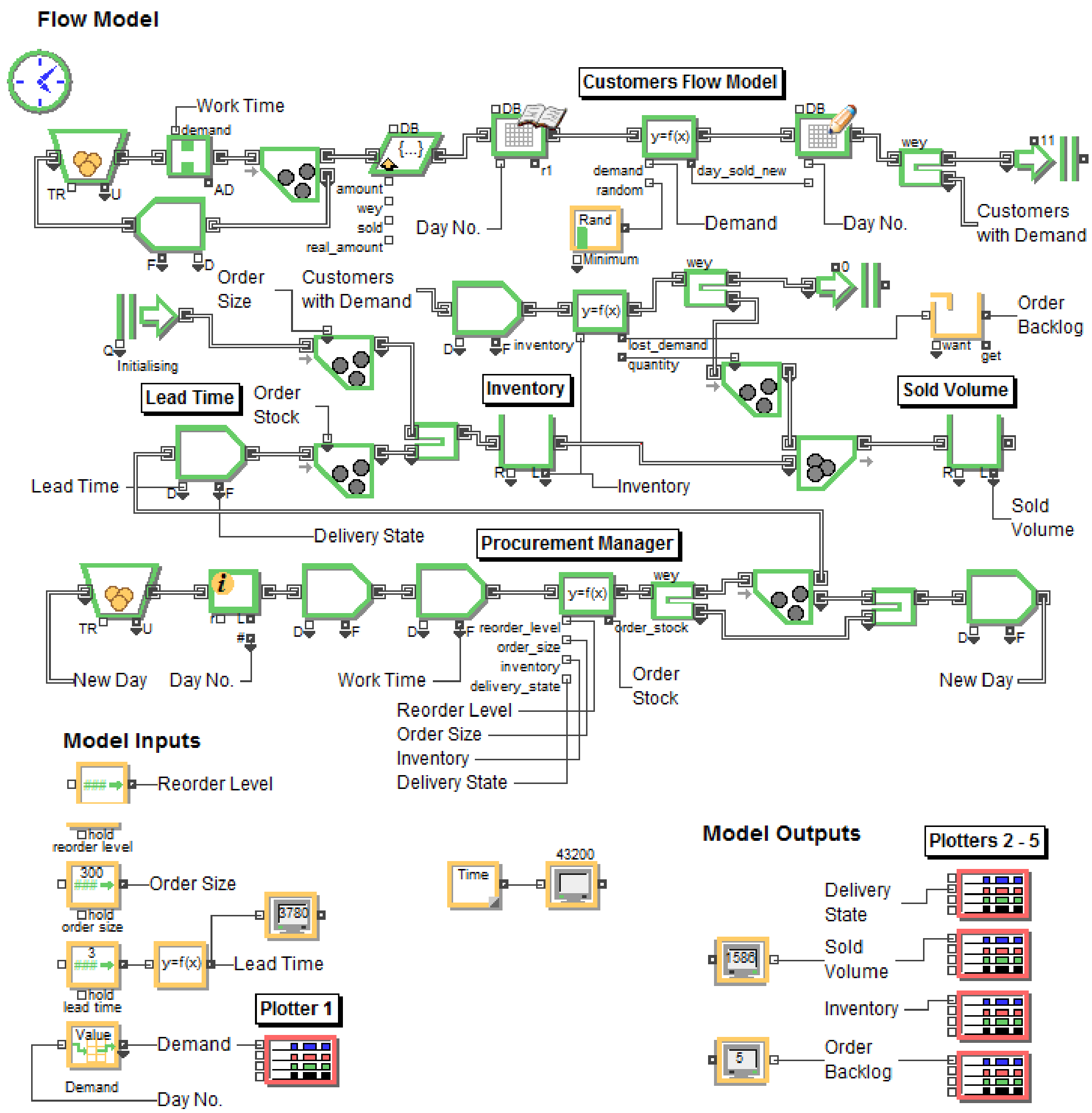


Fig. 3.6 Inventory control system's model on the basis of "discrete event" paradigm

The analysis of simulation results

Main results of simulation are shown in the form of chart, in Table 3.7. As it has already been mentioned above, contents of these diagrams totally agrees with what was described in the previous model based on the "continuous" paradigm. Also, numerical values of the totals, e.g. *Sold Volume,* practically totally coincide with what has been got with the previous model.

Diagrams provided on the Table 3.7 have some essential features characteristic of the model on the basis of "discrete event" paradigm.

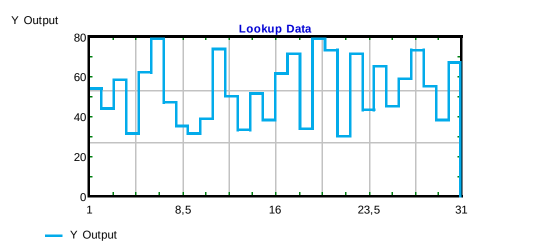
On the diagram *Inventory (30 Days)*, some separate stages showing the daily volume of sales are non-vertical, but sloping, because they display the number of units of goods sold to each customer.

More accurately, it is shown on the *Inventory (1 Day)* diagram, where it can be seen that during the first operation day of the model, daily demand comprising 54 units of goods had been allocated between six customers. In the table on the basis of which the diagram of *Inventory (1 Day)* has been built, there are some registrations of events related to customer service. The first customer left the warehouse at time 490 min (8h 10min), and the last customer – at time 618 min (10h 18min). Hence, the process of customer service had duration of 2 hours and 18 minutes.

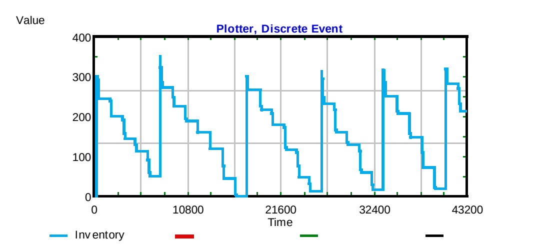
On the *Delivery State* diagram where the supply channel state is shown, it is observed that the width of diagram stages corresponds to the abovementioned 63 hours. Such time of delivery has been automatically calculated in the model on the basis of set delivery time, which is 3 days. The formula for the calculation of real time of delivery is provided in the last article of extended conceptual model's description.

Table 3.7 Inventory control system's results on the basis of "discrete event" paradigm

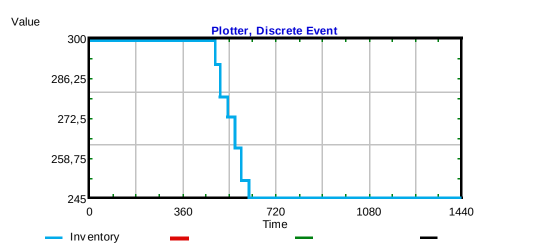
Demand



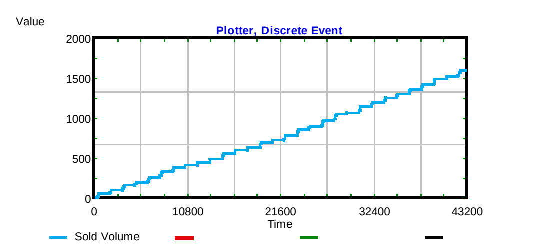
Inventory (30 Days)



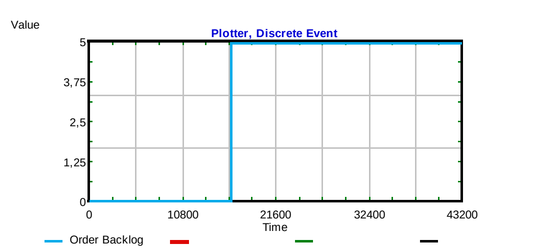
Inventory (1 Day)



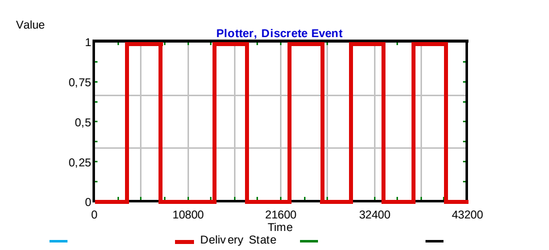
Sold Volume



Order Backlog



Delivery State



3.3.5. Implementation of "discrete rate" paradigm

Elaboration of conceptual model

"Discrete rate" paradigm has been defined as a part of ExtendSim package rather recently (Krahl, 2009). This package is the only simulation software that has special blocks for actualization of "discrete rate" type models (see the *choice of blocks from the rate.lix library* below). Models of such type are more like "continuous" type models rather than "discrete event" ones, because the flows in them are displayed with the help of intensity values (quantity/time unit). Separate objects like *items* are not used to form the flows. Main particularity of "discrete rate" models is a piecewise stationary character of flows (see Fig. 3.2). These models do not use the notion of "fixed Δt time step". This means that time changes its values at the moment when intensity changes in one of the flows. These moments can be at any point on the time axis of the model, i.e. the time in "discrete rate" models is displayed the same way it is in "discrete event" models. Changes in the intensity of flows can be displayed as events, and this is how the "discrete event" model works. For this reason, "discrete rate" models are also called hybrid models as they combine properties of "continuous" and "discrete event" models.

When developing "discrete rate" paradigm-oriented models, one fact should rather be taken into account: both processes of moving units of goods (from the supplier to the warehouse and then to the customer) is controlled through the *Valve* type block (see below). In order to let needed consignments of goods through such blocks, it is necessary that not only the volumes of these consignments be defined, but also the values of intensity of flows measured as quantity/time unit. As a result, there are two assumptions added to the basic conceptual model, they are:

- Shipment of goods on the side of supplier and unloading of goods on the side of warehouse happens with the speed 300 units per hour;
- Goods handling to the customers happen at an average speed of 10 units per hour.

Already actualized in "discrete event" model, elements of daily scenario of warehouse functioning are also added.

- customer service process will happen daily from 8 a.m. to 4 p.m., hereby it can end long before 4 p.m. in case the number of sold goods per day is relatively low;
- analysis of the current stock level at the warehouse and in case of order placement for delivery is made at 4 p.m. of the day;

- income of goods to the warehouse on the day of delivery occurs at 7 a.m. in the morning. This means that factual time of delivery should be set in hours, not in days, because its count starts at 4 p.m. and ends at 7 a.m. the next day. When the parameter *Lead Time* is set at 3 Days, delivery time is defined as follows: $24 \times 3 - 9 = 63$ h.

Separate customers in the “discrete rate” model will not be simulated as the amount of goods allocated to the customers will be presented in the form of flow with a daily cumulative volume being equal to the pull of demand, as it is shown on Fig. 3.4. Indeed, the volume of the output flow will be equal to the pull of demand, provided that the inventory stock volume at the warehouse is enough to meet the demand.

As a unit of timing will be 1 hour, i.e. all numbers displayed on time axis of diagrams of processes will mean the number of hours. Total duration of simulated process is $24 \times 30 = 720$ minutes.

Choosing blocks from value.lix library

There are 11 blocks in the composition of “blue” library rate.lix of ExtendSim 7 package. These blocks are designed to work with continuous flows, the intensity of which changes as quantity/time unit. The notion of “quantity” here may belong to the uninterrupted physical magnitude (e.g. the volume in m^3 or mass in kg), and to the variety of discrete units (e.g. the number of units of goods or cargo).

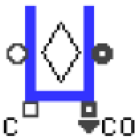
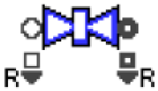
To create the inventory control model described below, three blocks were selected (see Table 3.8).

As it can be seen in this Table, *Tank* block can perform very different operations within the model. In the model provided below, it is used as a source and intermediate storage, or as a sink for accumulation of system’s output flows during the whole period of simulation.

Valve block is a controller of flows in “discrete rate” model. During the flow through the block, it should have a flow intensity set up. The duration of one portion of flow with a constant intensity is determined by the external management signals or by a specified volume of this portion.

Convey Flow block is most often used as a transportation element in “discrete rate” model. The delay time of the flow in the block is set up by the *Delay* parameter.

Table 3.8 Using rate.lix library blocks in the model (ExtendSim 7, User Guide)

Block	Function
Tank 	Acts as source, intermediate storage, or sink. As a residence type block the Tank has the capacity to hold defined amounts of flow as time advances. If a Tank has no outflow connection, by definition it is being used as a sink. Conversely, if a tank has no inflow connection, by definition it is being used as a source.
Valve 	Controls, monitors, and transfers flow. This block places an upper bound on the rate at which flow is allowed to pass through. Goals can be set up to control flow.
Convey Flow 	Add a delay to the flow available at the output of the block. The Convey Flow is FIFO and can be accumulating or non-accumulating.

Creation of the model

The ready model created on the basis of "discrete rate" paradigm is displayed on Fig. 3.7 Inventory control system's model on the basis of "discrete rate" paradigm for the initial data input and output of the simulation results, exactly the same constants and variables are used, likewise in the previous model based on "continuous" and "discrete event" paradigms.

The main flow part of the model with movement and storage of goods happening within it consists of six "blue blocks" referring to rate.lix library. The signals indicating the start of flows movement and data about their volume coming from the lower fragment of the model consisting of "green blocks" refering to item.lix library. Like in the previous "discrete event" model, these blocks serve to simulate a special timer that would give signals at 8 a.m. and 4 p.m. daily corresponding to the description of the process provided above as an extension for conceptual model.

At 8 a.m. in the morning the first *Equation(I)* block runs, where *Shipment* variables are calculated (the actual volume of sales for the current day) and *Lost Demand* (a magnitude of unmet demand). Also at the same moment the *Start Day* = 1 command is introduced, resulting in *Valve* block starting the flow of goods to the customers.

The second *Equation(I)* block starts at 4 p.m., and checks the terms of order placement. If the order is to be send to the supplier, the *Valve* block situated on the input of the model receives the *Quantity* variable value (order size) and *Start Order* = 1 signal, resulting in*Valve* block passing through the flows from the supplier to the transport element.

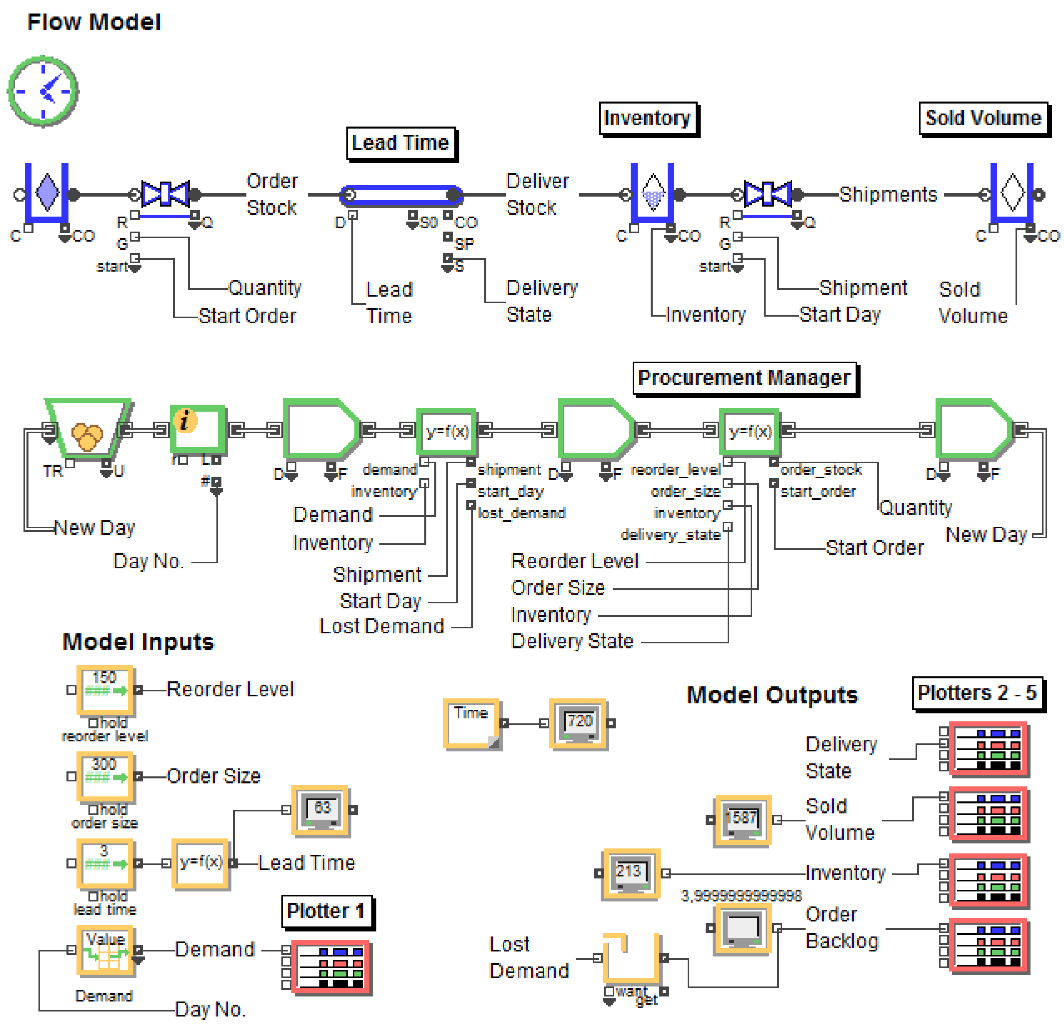


Fig. 3.7 Inventory control system's model on the basis of "discrete rate" paradigm

The analysis of simulation results

Main results of simulation are shown in the form of chart, in paradigm. As it has already been mentioned above, contents of these diagrams totally agrees with what was described in the previous model based on the "continuous" and "discrete event" paradigms. In addition, numerical values of the totals, e.g. *Sold Volume,* practically exactly coincide with what had been received with the previous model.

Diagrams provided on the Fig. 3.9 have some essential features characteristic of the model on the basis of "discrete event" paradigm.

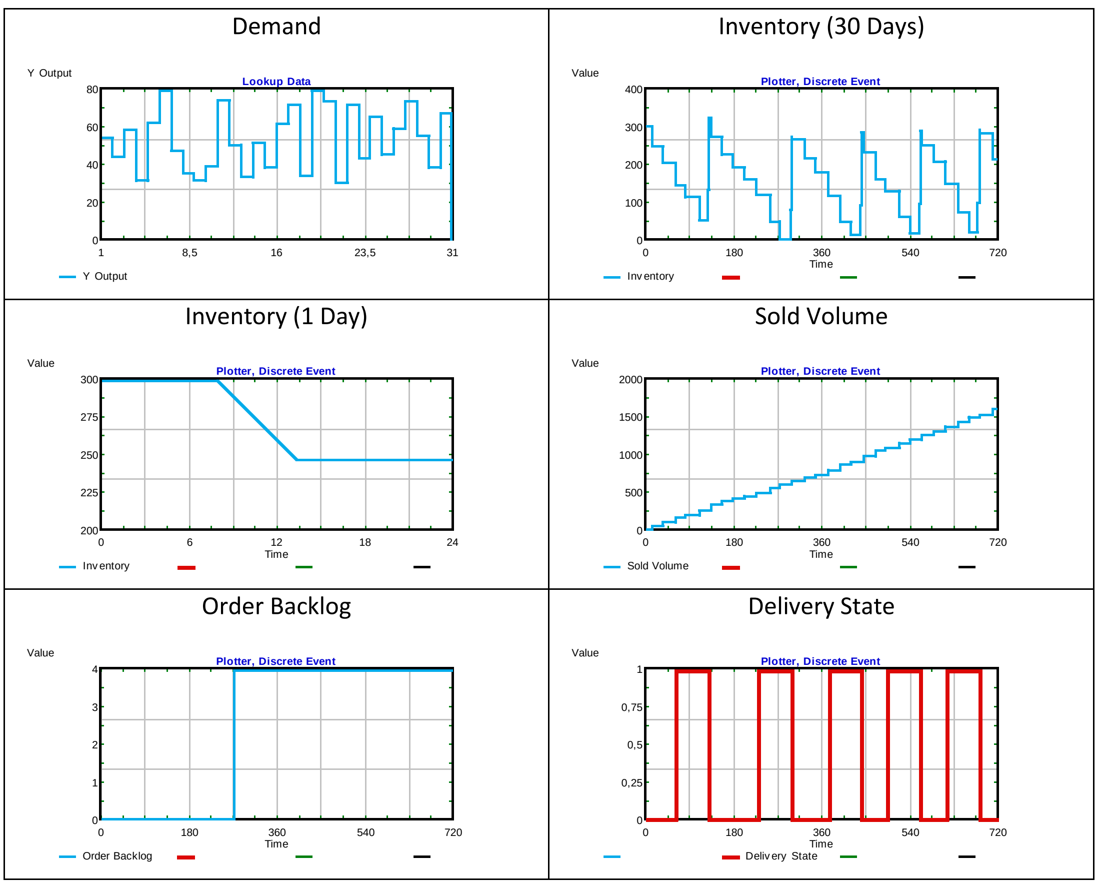
Inventory (30 Days) diagram stages showing goods income to the warehouse is not vertical, but sloping, because the unloading of goods on the side of the warehouse lasts for 1 hour if the supply is

300 units. The distance between the stages – which correspond to the moments of completion of goods handling to the customers at certain days – is not equal, as this daily process starts or ends earlier depending on the volume of sold goods.

It can be seen on the *Inventory (1 Day)* diagram that during the first operation day of the model, 54 units of goods had been handles to the customers as a continuous flow. In the Table on the basis of which the *Inventory (1 Day)* diagram is built, there are some registrations of events related to the changes in the intensity of system's output flow. The process of goods handling started at 8 a.m. in the morning (8.0h) and ended at 13.4h, this means that it was 5.4h long (5 hours and 24 minutes).

On the *Delivery State* diagram where the supply channel state is shown, it is observed that the width of diagram stages corresponds to 63 hours. Such time of delivery has been automatically calculated in the model on the basis of set delivery time, which is 3 days. The formula for the calculation of real time of delivery is provided in the last article of extended conceptual model's description.

Table 3.9 Inventory control system's results on the basis of "discrete rate" paradigm



3.4. The principles of building the models of inventory control systems based on Multimethod Simulation Approach

The method of building inventory control systems simulation models can be reckoned as successful at least because three above-described models have been created on its basis. The initial data and output variables remain the same for all three models, which makes the procedure of their comparison easier. Regarding the comparison of simulation numerical results, the models are almost identical. Further there will be discussed the qualitative discrepancies of the models that are caused by different simulation paradigms for their software implementation.

3.4.1. A comparison of the built models

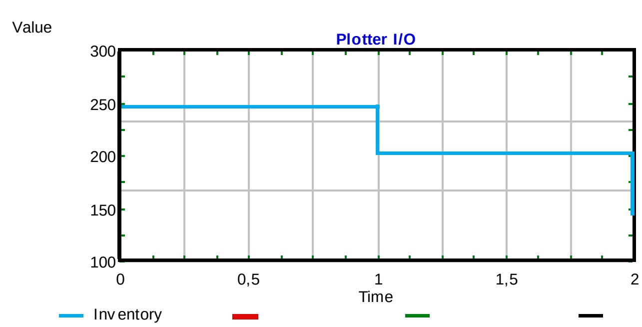
Model on the basis of "continuous" paradigm entirely correspond to the basic conceptual model of the studied inventory control system. Even the time unit in the model has been taken for 1 day, as no other units of time in the basic model are mentioned. In order to build the body part of the *Flow Model* where processes of movement and storage are being implemented, there were used only three types of blocks from the value.lix library. The total number of blocks is five, which allows associating the structure of the model with the lowest level of difficulty (see Fig. 3.5). The logics of the inventory control strategy itself is described as several lines of programming code, and it can be easily altered. The drawback of this model is that it is principally impossible to simulate the lead time in a form of a random magnitude at the condition that the existing model's structure is preserved. It also becomes clear that without the radical modification of the model it will not be possible to display the events of daily scenario, unlike in the other two models where these events are defined.

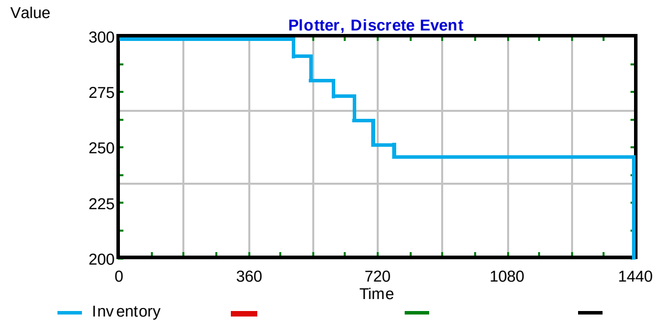
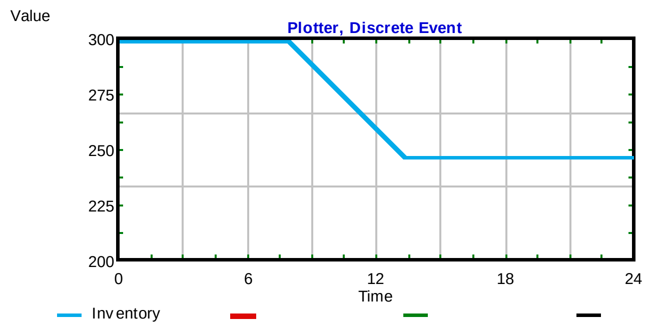
When building a model on the basis of "discrete event" paradigm, a goal was set to show typical of this model's class methods of how to display of real manufacturing and logistics systems, as well as flows of events tied to these objects. The model is built in such a way that every unit of goods and each customer are displayed as *items* in it. For this purpose, beforehand some substantial add-ons were added to the basic concept of the model, and only after that this new model has been implemented with ExtendSim package. The model has a comparatively complex structure. 16 blocks from the item.lix library have been chosen for its implementation, and the total number of blocks in model's body part is 35 (see Fig. 3.6). High complexity of the model is its only drawback. The main advantage of the one is that by changing its structure it is possible to empower almost any changes and add-ons performed within the scopes of the conceptual model.

Model based on “discrete rate” paradigm combines positive qualities of both above-considered models of “continuous” and “discrete event” types. The structure of the main flow part of the model has almost no difference from “continuous” model’s structure (see Fig. 3.7). It implements only three blocks from the rate.lix library, while the total number of blocks in model’s structure is six. To enable the possibility to simulate the set up daily working scenario within the studied system, this model exercises a special timer consisting of item.lix library blocks and almost without any changes – repeating the same construction based on the “discrete event” paradigm.

Differences in simulation paradigms influence the possible extent of how separate processes are displayed within the system. The development character of these processes depends primarily on the way how the flows are presented (they are shown on the Fig. 3.2). Table 3.10 shows the process of inventory decrease in the first day of system’s operation, when customers receive 54 units of goods. In “continuous” model, only the result of this process can be seen in the form of one stage of diagram, related to the day as a whole. In “discrete event” model, it can be seen how many customers were handled their goods that day, with that: when they were serviced (exact time) and how many units of goods each of them was handled. Each customer has just one stage on the diagram illustrated in Table 3.10. Finally, in “discrete rate” model this process is shown as a segment of inclined line, because the relevant system’s output flow has a constant intensity of 10 units of goods per hour, which defines the slope angle of the segment. The length of the segment depends on the lifetime of the flow, which is defined by the volume of transferred consignment. In this example, time is calculated as follows: 54/10=5.4 hours.

Table 3.10 Methods of inventory dynamics representation with different simulation paradigms

Simulation paradigm	Inventory dynamics on the first day of the process (*Inventory* variable)
Continuous (time unit – 1 day)	

Discrete Event (time unit – 1 minute)	
Discrete Rate (time unit – 1 hour)	

Above-mentioned and some other properties of the designed models are shown in Table 3.11. It is worth noting that the developer makes little effort to program these inventory control strategies. In all designed models, these strategies are encoded in a few lines of a programming code. The majority of time spent when building these models goes for the integration of the material flows structure and gaining required operation results of the studied inventory control system.

Table 3.11 Comparative analysis of developed models properties

Properties of the model	**Simulation paradigm**		
	Continuous	**Discrete Event**	**Discrete Rate**
The level of detail display of real systems properties	low	high	average
The level of abstract display process of real system properties	high	low	average
Characteristics of conceptual model	brief description	detailed and complex declaration	comparative brief description
Variety of library blocks	small (31 block, only 8 of them working with flows)	large (31 block)	average (11 blocks)
The size and complexity of the computer model structure	small and simple structure	large and complex structure	relatively small and simple structure

Flexibility of the computer model structure	low	high	sufficiently high
The precision of simulation results	absolute in relation to the basic conceptual model	absolute in relation to the basic conceptual model	absolute in relation to the basic conceptual model

3.4.2. Conclusions and recommendations for the use of simulation paradigms

By implementation of all three paradigms supported by ExtendSim package, there were built and tested models of relatively simple inventory control system. The experience in development of these and many other models allows making the following conclusions and recommendations:

1. Selection of simulation paradigm relies on the formulation of the objectives for the studied system, and above all, it relies on the indicators of system's performance which are to be shown in model's run results. Every primary (natural), i.e. not yet in terms of money, indicator is related to particular physical processes within the system. In inventory control systems, such processes generally include transportation, waiting processes (reserves, stocking and etc.) and service (processing) of material items that involve cargo (goods), means of transport and natural persons (customers, sometimes staff, too). In order to create an opportunity to estimate the required indicators, the model should display relevant physical processes.
2. Traditionally, the quality of the studied inventory control strategies is estimated by dint of monetary measures that may include either expenditures (e.g. for ordering, transportation and storage of goods) or incomes related to the sale of goods. From the point of view of simulation, such indicators are secondary because the are defined relying on such primary indicators as: number of deliveries for a certain period, volume of these deliveries, dynamics of inventory development, terms of storage, volume of sales and lost demand etc. Calculation of the price or any other economic indicators is usually purely "arithmetical task", provided that data about the norms, prices, tariffs and etc. is available.
3. "Continuous" paradigms may be successfully applied for the development of model if with the help of this model it is supposed to estimate only the principal features of the studied inventory control strategies. In such models only the most essential processes are displayed, able to influence the process of inventory dynamics. Often, such processes are only input and output flows of the corresponding warehouse. Herewith, input flows are determined by the supply strategy, and output flows – by demand from the customers. In "continuous" type

models generally there are no mathematical problems related to the implementation of inventory control strategies or calculation of indicators, but the density and transparency of such models are retained only if they are not overloaded with “minor details” associated with the transportation and storage of goods. Characteristic of this class models is that Δt time step equals to one day. One hour might also perform same role if the ordering decision requires consideration of the current time.

4. “Discrete event” paradigm most frequently finds its application in simulation of processes in systems of manufacturing and logistics. It is expedient to implement this paradigm for the study of inventory control system if in the problem formulation the necessity to display relatively complex logistics processes within the model have been clearly justified, thereto display of processes in multistage supply chains. The highest level of abstraction related to the display methods in models of this-type phenomena allows creating proper models by the hands of specialists not possessing outstanding mathematical thinking. Almost direct representation of items and events in real-time world in “discrete event” models often leads to the substantial increase in models’ size and complexity. Simulation of the studied inventory control strategies occupies very insignificant position in such models due to different focus of attention when performing model’s computer-integration so that to display logistics processes in an adequate way.
5. „Discrete rate“ paradigm in many respects occupies an intermediate position between “continuous” and “discrete event“ paradigms. Flow processes themselves can be displayed as densely and transparently in these models as in the “continuous” models. The advantage of “discrete rate” models is that they can employ fragments built on the “discrete event” principle. Most often such fragments serve to simulate flow control processes displayed by dint of elements working on the “discrete rate” principle. Since specialists have little experience in “discrete rate” models development, some methodological difficulties arise at the stage of choosing the level of detail for displaying real processes and making decisions regarding their division into continuous and discrete ones. Seeing that in “discrete rate” models there should occur many times less events than in “discrete event” similar models, models of such class, also called mesoscopic (Schenk et al., 2009), (Schenk et al., 2010), are rather promising in respect to the inventory control strategy optimization procedures completion, in which thousands of runs can be performed.

6. The decisive factor influencing the selection of simulation paradigm and the success of the whole research using simulation depends on the level of theoretical training and practical skills of the specialist involved. It is enough to say that development of the conceptual model is already a very nontrivial process. When describing a simple inventory control system, 10 points were included in the composition of basic conceptual model (see article 3.3.2). When developing the "discrete event" model, five more articles were added to this description, and five more to the "discrete rate" model (they differ partially). Already at the stage of conceptual model's development, specialist definitely ought to pursue at least one of the paradigms in which one will take advantage of simulation software in order to computer-integrate the paradigm. It is a compulsory requirement that specialist has some good experience with this simulation tool as a condition of his/her admission to the implementation of real projects.

4. PRACTICAL IMPLEMENTATION OF INVENTORY CONTROL MODELS

The main content of the chapter is devoted to the description of simulation models of inventory management systems developed by the author. Most of the models created using different versions of simulation package ExtendSim. It has been reviewed both relatively simple models and models that use complex strategies of inventory management. For most models it is provided a detailed mathematical description. It has been described experiments aimed at optimizing the inventory management systems. Some of these experiments are based on a comparison of the options set by the researcher. Another part of experiments to optimize inventory management systems implemented in the form of automatic search of the best variant using evolutionary algorithm.

4.1. Classical inventory control models

The first model is a model with fixed reorder point and fixed order quantity. This model describes dependency of average expenses for goods holding, ordering and losses from deficit per time unit on two control parameters – the order quantity and reorder point. The description of this model and analytical method of problem solving are examined in the previous works (Kopitovs, 2004a). Problem has been solved using regenerative approach (Ross, 1992).

The second model is a model with fixed time interval between the moments of placing neighbouring orders. In this model, the order quantity is determined as a difference between the fixed stock level and quantity of goods at the ordering moment. The analytical description of the second model has been considered in this thesis (Kopytov, 2006). Note that in the second model the same economic criterion is used – minimum of average total cost in the inventory system.

There are two inventory control models with continuously review inventory position (permanent stock level monitoring). The strategy of each model selection is based on the real conditions of the business. Thus, the first model can be used for the system with arbitrary order placing time; this situation happens in the inventory system using its own means of transportation for order delivery. The second model is suggested for the system with a fixed order placing moment, where the order transportation depends on schedule of transport departure.

The considered models are realized using simulation method. The numerical results of problem solving are obtained in simulation Extend package.

4.1.1. Model with reorder point

Let's consider a single-product stochastic inventory control model under the following conditions. The demand for goods is a Poisson process with intensity λ. In the moment of time, when the stock level falls until certain level *r*, a new order is placed. The quantity *R* is called as reorder point. The order quantity Q is constant. Supposing that $Q \geq R$. The lead time L (time between placing an order and receiving it) has a normal distribution with a mean μ_L and a standard deviation σ_L. There is the possible situation of deficit, when demand $D_L$during lead time L exceeds the value of reorder point *R*. In case of deficit the last cannot be covered by expected order (Kopytov, 2004a).

Denote as *Z* the quantity of goods in stock in the time moment immediately after order receiving. Quantity of goods *Z* can determined as function of demand D_L during lead time *L*:

$$Z = \begin{cases} R + Q - D_L & if\ D_L < R \\ Q & if\ D_L \geq R \end{cases} \tag{4.1}$$

Expression (4.1) is basic. It allows expressing different economical indexes of considered process (Kopytov, 2004b). Let *T* is the duration of a cycle. Length of the cycle consists of two parts: time T_1 between receiving the goods and placing a new order and lead time L, i.e. $T = T_1 + L$ (see Fig. 4.1).

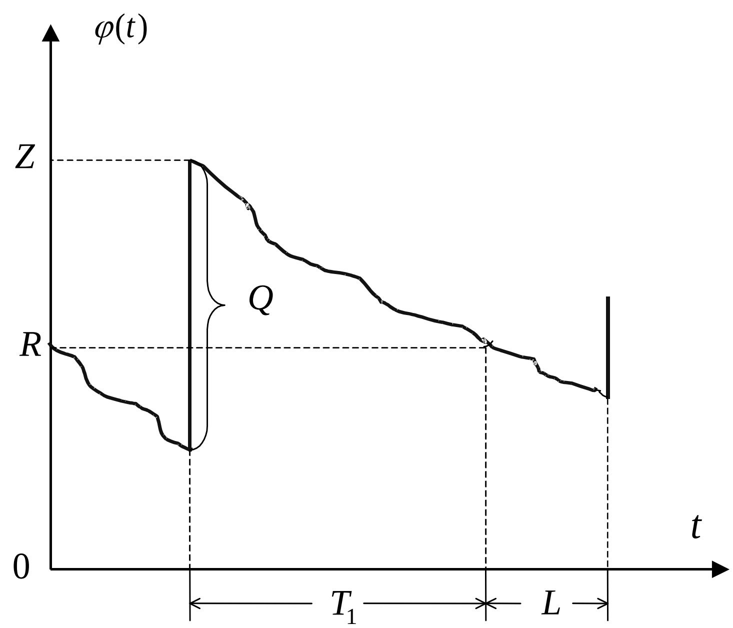


Fig. 4.1 Dynamics of inventory level during one cycle for model 1

Supposing that next parameters of the model are known:

- the ordering cost C_0 is known function of the order quantity Q, i.e. $C_0 = C_0(Q)$;
- the holding cost is proportional to quantity of goods in stock and holding time with coefficient of proportionality C_H;
- the losses from deficit are proportional to quantity of deficit with coefficient of proportionality C_{SH}.

Let denote D_τ as demand for goods within period of time τ. Principal aim of the considered model is to define the optimal values of order quantity Q and reorder point R, which are control parameters of the model. An optimization criterion is the minimum of average total cost per time unit in inventory control system. Denote this average total cost by $E(AC)$ which can be found as average total cost during one cycle divided by average cycle time $E(T)$ (Ross, 1992):

$$E(AC) = \frac{E(TC_H) + E(TC_{SH}) + C_0}{E(T)} \tag{4.2}$$

where $E(TC_H)$ and $E(TC_{SH})$ are average holding and average shortage costs within cycle accordantly. Note that $E(TC_H)$ and $E(TC_{SH})$ depend on control parameters R and Q. Analytical formulas for these economical characteristics are presented in paper (Kopytov, 2004a). For problem solving, we have to minimize criteria (4.2) by R and Q.

Let's consider the single product model 1 with control parameters R and Q. The schema of the simulation task is shown in Fig. 4.2

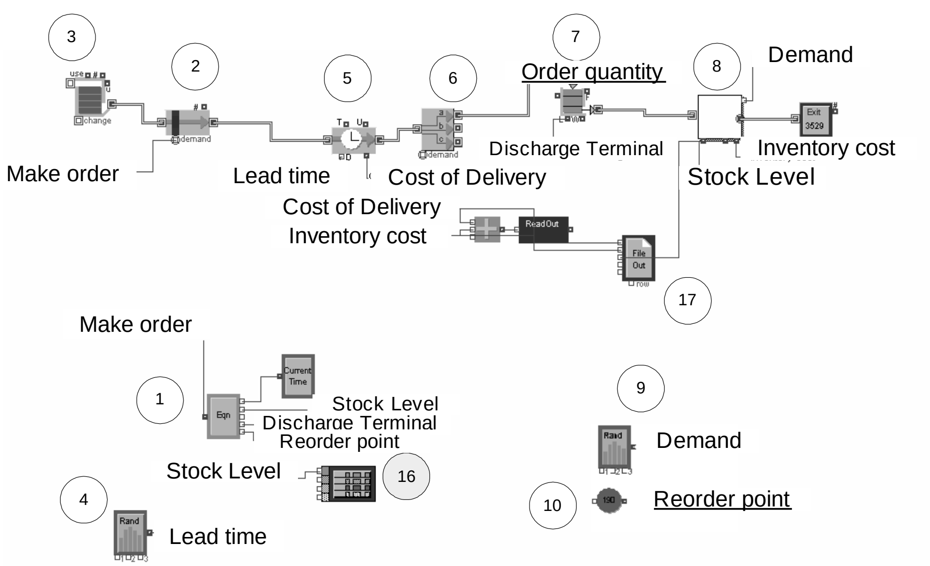


Fig. 4.2 Simulation model overview: inventory control with fixed reorder point and fixed order quantity

Let's consider the main blocks of the simulation model. In the block #1 the decision of a new ordering (*Make Order*) is generated using data about *Reorder point* (block #10) and quantity of goods in stock (*Stock level*). As the result variable *Make Order* takes value 1, it is transmitted to connector of block #2, and a new goods ordering is executed. In block #5 the process of order delivery is simulated. The value of random lead time is generated in block #4 (*Input Random Number*) using parameters μ_L and σ_L of normal distribution. The demand for goods is generated in block #9 as random value with Poisson distribution and known parameter λ. The warehouse is realized in hierarchical block #8, which is illustrated on the scheme on Fig. 4.3. Process of goods realization is simulated in block #11. Block #12 (dummy source of goods) and block #13 (*Set Attribute*) are used for good deficit calculation. The results of simulation are printed out in text file (block #17) and display on the screen (block #16).

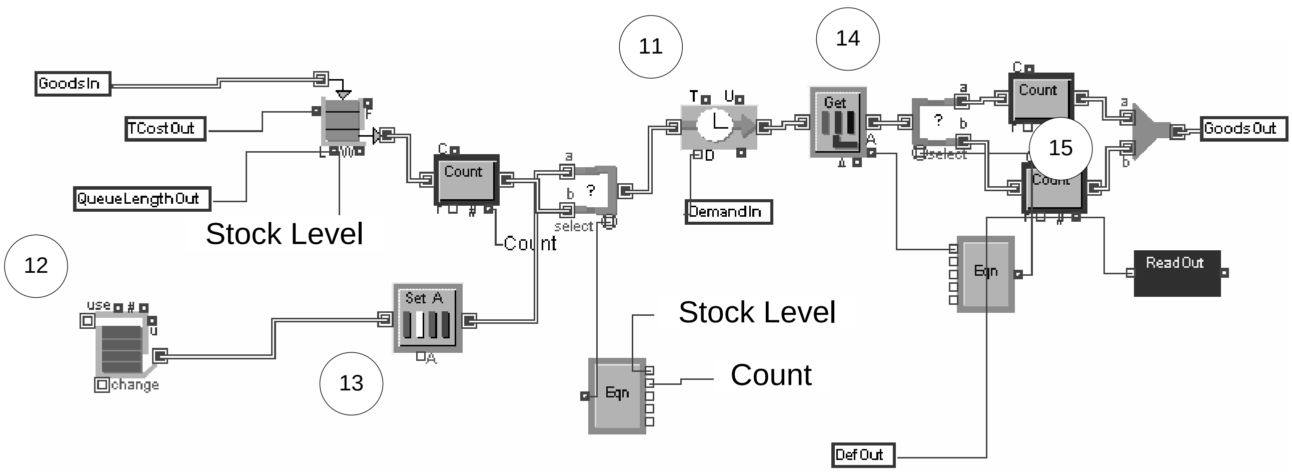


Fig. 4.3 Warehouse simulation model overview

By the use of created simulation model, it is possible to find an optimum solution for inventory control problem with two control parameters – reorder point R and order quantity Q (see example 1) (Muravjovs, 2007a, Muravjovs, 2007b, Muravjovs and 2007c).

Example 1. Let's assume that demand D for goods is a Poisson process with intensity $\lambda = 10$ units per day; lead time L has a normal distribution with a value $\mu_L = 11$ days and a standard deviation $\sigma_L = 3.5$; ordering cost C_0 equals to 200 EUR, holding cost C_H equals to 2 EUR per unit per a year, losses from deficit C_{SH} equals to 8 EUR per a unit; a unit time is 1 year. The period of simulation is one year and number of realization is 100.

The results of simulation are shown in Table 4.1. Note that for given steps of the changing of control parameters, the best results is achieved at the point *Q = 950* and *R = 150* units of goods, where for 100 realizations of average total cost for a period of one year is 1889,34 EUR.

Table 4.1 Average expenses for goods holding, ordering and losses from deficit per year for inventory control system with fixed reorder point and fixed order quantity

	Reorder point, units				
Order quantity, units	*100*	*150*	*200*	*250*	*300*
800	2489,07	1960,80	1987,50	2067,68	2153,34
850	2430,32	1988,34	2026,22	2113,90	2209,30
900	2224,99	2001,77	2051,28	2141,19	2235,84
950	2241,90	**1889,34**	1953,86	2092,33	2236,53
1000	2267,96	1960,65	1993,83	2071,15	2153,34
1050	2387,28	2030,89	2048,83	2135,75	2216,93
1100	2384,31	2064,08	2093,38	2184,38	2268,02
1150	2387,28	2030,89	2048,83	2135,75	2216,93
1200	2211,05	2075,70	2128,13	2225,36	2326,54

4.1.2. Model with fixed time between orders

To put the case that the model has a fixed cycle time *T*, i.e. with fixed time between neighbouring moments of placing the orders. It is a single-product stochastic inventory control model under following conditions. The demand for goods is a Poisson process with intensity λ. The lead time L has a normal distribution with a mean μ_L and a standard deviation σ_L. Supposing that lead time is essentially less, because the cycle time is $\mu_L + 3\sigma_L \ll T$.

There is the possible situation of deficit, when the demand during time between neighbouring moments of orders receiving exceeds the quantity of goods in stock *Z* in the time moment immediately after order receipt. Analogy model 1 in case of deficit the last cannot be covered by expected order.

It can be denoted as *S* the goods quantity, which is needed "ideally" for one period, and it equals to the sum

$$S = \overline{D}_T + S_0 \tag{4.3}$$

where $\overline{D}_T$ is the average demand for cycle time; S_0 is the some safety stock. In the given sentence, *S* perfectly gives the minimum of total expenditure later on for ordering, holding and loses from deficit per unit of time.

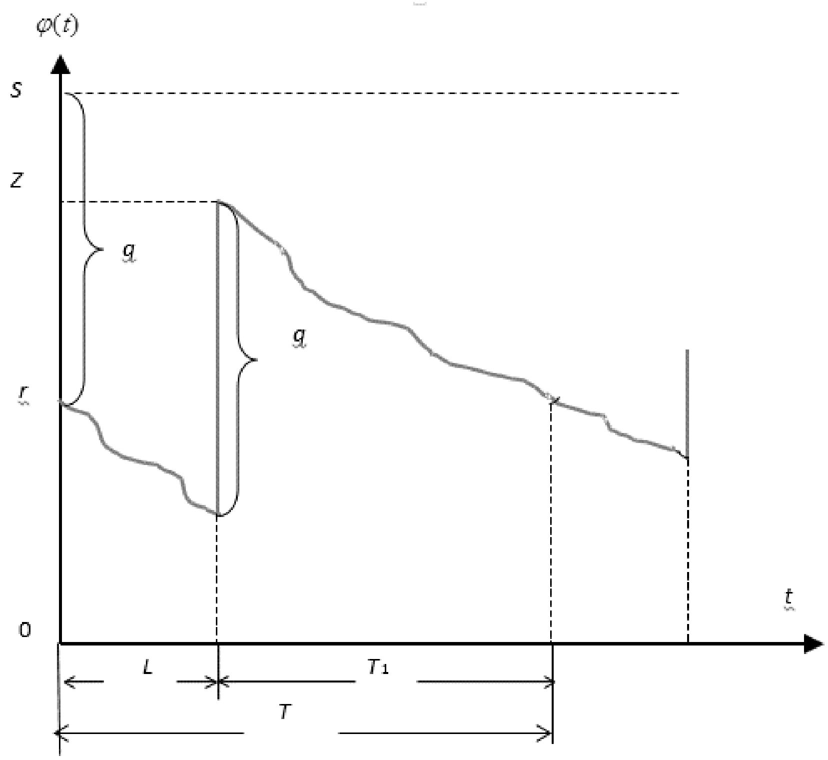


Fig. 4.4 Dynamics of inventory level during a single cycle for model 2

Let's denote the size of the order *q* as difference:

$$q = S - r \tag{4.4}$$

In the suggested model, period of time is *T* and stock level – *S*, and both they are *control parameters.* Supposing that in the moment of time when a new order has to be placed may there may be a situation, when the stock level is so big, that a new ordering doesn't occur. However, for generality of model we will keep the conception of lead time and quantity of goods at the time moment immediately after order receipt in such a case, too. It corresponds to real situation when the customer uses the transport means, which depart at the fixed moments of time not depending on existence of the order and which have the random lead time; for example transportation by trailers, which depart each 1st and 15th day of each month.

In real situation in moment of time *t* the stock level $\varphi(t)$ is equal to *S* only in two cases:

1. $r = S$ and $D_t = 0$, where D_t is the demand for goods during the time t; $0 \leq t \leq T$;
2. $r < S$ and $D_t = 0$, where $L \leq t \leq T$.

Taking into account that in case of deficit it can't be covered by expected order, we can obtain the expression for goods quantity at the moment of time immediately after order receipt

$$Z = \begin{cases} r + q - D_L & if\ D_L < r \\ q & if\ D_L \geq r \end{cases} \tag{4.5}$$

and using (4.4):

$$Z = \begin{cases} S - D_L & if\ D_L < r \\ S - r & if\ D_L \geq r \end{cases} \tag{4.6}$$

The rest *r* at the end of the period and the goods quantity *Z* – at the moment of time immediately after order receipt – takes values from interval [0; *S*]:

- $r = 0$, if in the previous cycle the demand during the time T_1 between the order receiving and placing of the new order is more or equal *Z*;
- *r* = *S*, if in the previous cycle *Z* is equal *S* and there isn't the demand during the time period T_1.
- *Z* = 0, if the rest *r* to the moment of ordering is *S* (i.e. order quantity *q* is 0) and demand D_L during the lead time *L* is more or equal *S*;
- *Z* = *S*, if the rest *r* to the moment of *ordering* is 0 or demand D_L during the lead time *L* is absent.

Finally, average total cost for time unit (criteria of optimization) is expressed by the following formula (Kopytov, 2004d):

$$\boldsymbol{E(AC) = \frac{E(TC_H) + E(TC_{SH}) + C_0}{T}} \tag{4.7}$$

Unlike the model 1 in the considered model expenditures $\boldsymbol{E}(\boldsymbol{TC_H})$ and $\boldsymbol{E}(\boldsymbol{TC_{SH}})$ depend on control parameters *S* and *T*.

Let's consider second strategy of inventory control with fixed period of time *T* between the moments of placing neighbouring orders. Note that in suggested model period of time *T* and required stock level *S* are *control parameters.*

For simulation of inventory control process was created model that is shown in

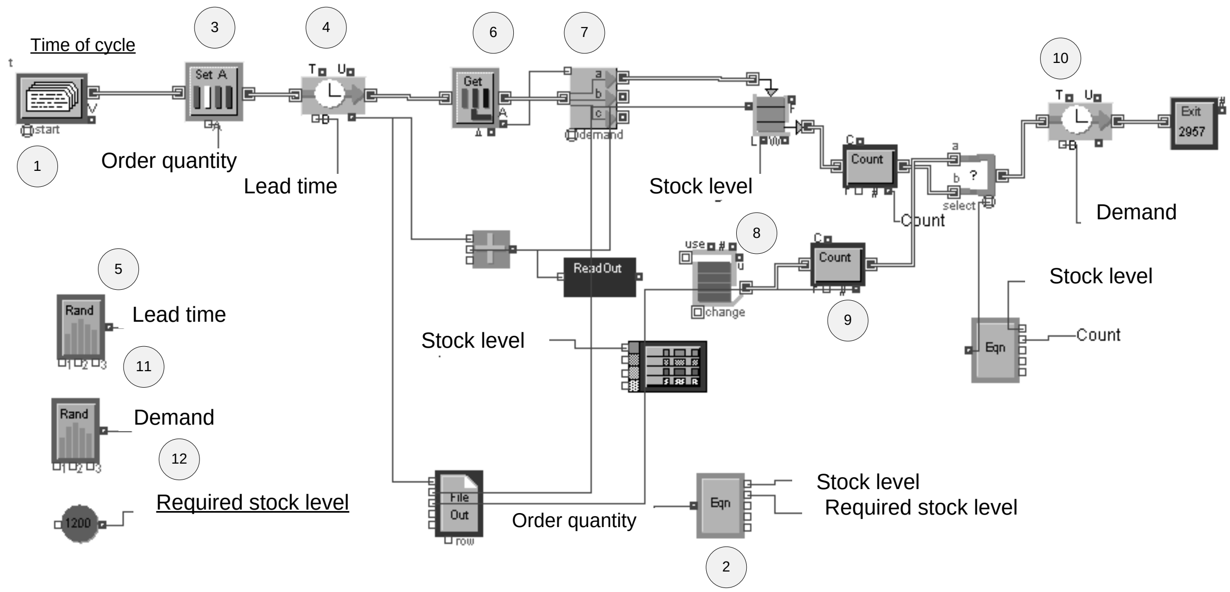


Fig. 4.5Fig. 4.5 Simulation model overview: inventory control with fixed time interval between placing neighbouring orders. Let's consider the main blocks of schema. Block #1 generates the transactions in the fixed moments of time; these transactions are used for simulation of goods ordering during considered time period. Block #2 calculates the *Order quantity* using data about *Stock level* in the moment of ordering and *Required stock level* (quantity of goods which is needed "ideally" for one period); this result is saved in block #3 (*Set Attribute*). Block #4 determines the moment of order delivery using the value of lead time generated in block #5 (*Input Random Number*) as random variable with normal distribution and known parameters. The demand for goods is generated in block #11 as random value with Poisson distribution and known parameter. Process of goods realization is simulated in block #10. Blocks #8 and #9 are used for goods deficit calculation. The results of simulation are saved in the text file and displayed on the screen.

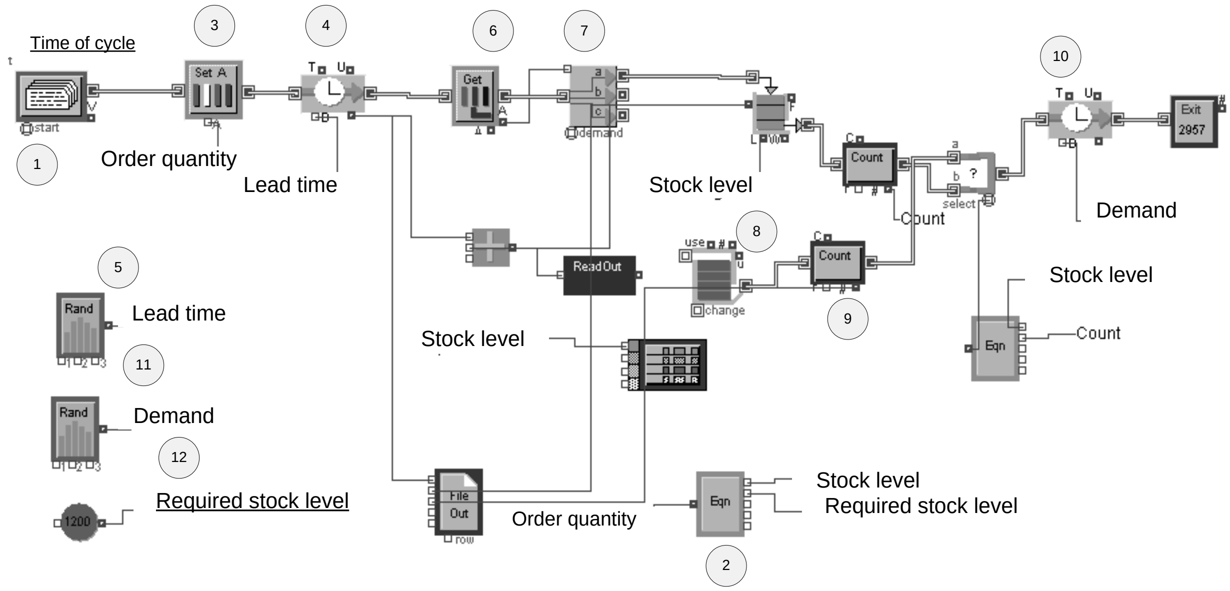


Fig. 4.5 Simulation model overview: inventory control with fixed time interval between placing neighbouring orders

Example 2. (Muravjovs, 2007a, Muravjovs, 2007b, Muravjovs and 2007c) Let's consider another strategy of inventory control according to the model 2, using initial data from example 1. The simulation model is shown in Fig. 4.5. The results of calculation are given in Table 4.2. For the given step changes of control parameters, the best results is achieved at the point S = 900 units of goods and T = 75 days, where for 100 – the implementation total average cost for one year is 1965,9 EUR.

Table 4.2 Average expenses for goods holding, ordering and losses from deficit per year for inventory control system with fixed time interval between placing neighbouring orders

	Time interval between placing neighbouring orders, days						
Level up to order, units	*70*	*75*	*80*	*85*	*90*	*100*	*110*
850	2091,40	2206,92	2826,08	3512,02	3891,42	5213,66	7489,90
900	2108,88	***1965,99***	2287,16	2287,16	3237,74	4365,35	6352,34
950	2203,41	1985,33	2022,51	2341,89	2552,83	3643,92	5308,60
1000	2300,46	2076,96	2044,00	2069,62	2212,28	2945,29	4403,56
1050	2396,41	2179,56	2144,75	2079,61	2075,02	2497,04	3655,52
1100	2497,95	2275,52	2240,75	2177,82	2122,92	2210,92	3035,20

4.2. Simulation of inventory control system for supply chain

It can be considered that a single-product inventory system for the supply chain "producer – wholesaler –customer" has two stages of ordering process (see Fig. 4.6) (Mason, 2009). N customers execute the first stage. In the moment of time when the customer's stock level falls to a certain level – a new order is sent to the wholesaler. The wholesaler executes the second stage. A new order is sent to the producer in the fixed moments of time. Assuming that the wholesaler has his own warehouse.

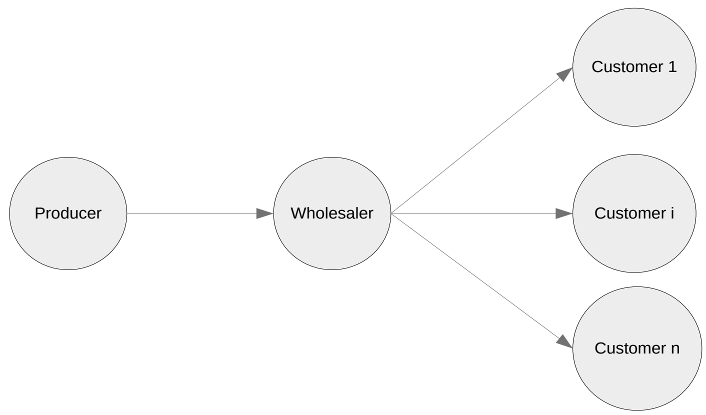


Fig. 4.6 Chain of Product Ordering

The principal aim of the considered problem is to define the exact ordering strategy for *n* customers and the wholesaler to achieve the minimum expenses in inventory control system per time unit (Muravjovs, 2007a, Muravjovs, 2007b, Muravjovs, 2007c and Kopytov, 2004c). Taking into account stochastic character of the inventory control problem, the criterion of optimization is minimum average total expenses E^{total} per time unit, which is calculated by following the formula:

$$\boldsymbol{E^{total}} = \sum_{\boldsymbol{i=1}}^{\boldsymbol{n}} \boldsymbol{E_i^{cust}} + \boldsymbol{E^{wh}} \tag{4.8}$$

where $E_i^{cust}$is *i*-th customer's average total expenses for goods holding, ordering and losses from deficit per time unit, $i = 1,2, \dots n$; E^{wh} is wholesaler's average total expenses for goods holding, ordering and losses from deficit per time unit. Let's consider in detail the stages in the presented chain of product ordering.

First stage of ordering process. The demand for goods D_i of *i*-th customer is Poisson process with intensity λ_i. Time L_i of goods delivery from the wholesaler to *i*-th customer has a normal distribution with parameters μ_i and σ_i. The policy of order forming for *i*-th customer is as follows: a new order is placed at the moment of time, when the stock level falls to a certain level R_i. The order quantity Q_i is constant. It should be kept in mind that $Q_i \geq R_i$ and point R_i and quantity Q_i are control parameters of the first stage model. Dynamics of the inventory level of product for i-th customer during one cycle $T_i = T_i^* + L_i$ (time interval between two neighbouring order deliveries for i-th customer) is shown in Fig. 4.7. There is a possible situation of deficit, when demand $D_i(L_i)$ during lead time L_i exceeds the reorder point R_i. Supposing that in case of deficit the last cannot be covered by expected order.

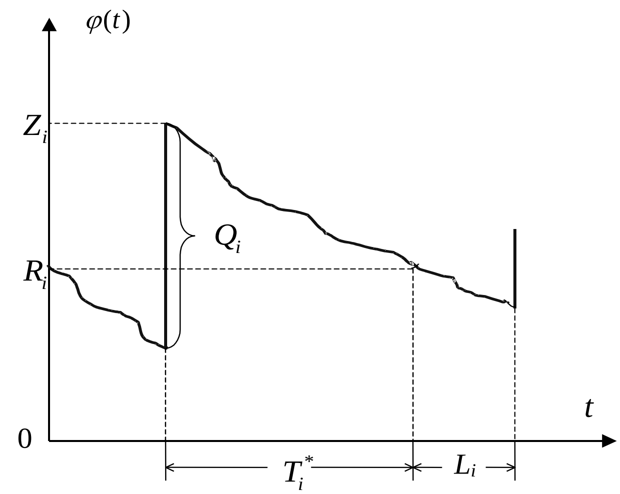


Fig. 4.7 Dynamics of i-th Customer's Inventory Level during One Cycle

Denote as Z_i the quantity of goods in stock in the time moment immediately after order receiving. This random variable Z_i is determined as a function of demand $D_i(L_i)$ during lead time L_i:

$$Z_i = \begin{cases} R_i + Q_i - D_i(L_i), & \text{if} \quad D_i(L_i) < R_i; \\ Q_i, & \text{if} \quad D_i(L_i) \geq R_i. \end{cases} \tag{4.9}$$

Formula (4.9) allows expressing different economical indexes of considered process. Wholesaler has his own warehouse with a definite quantity of goods – q. If customer's order quantity is less or equal than quantity of products in the stock $Q_i \leq q$ the wholesaler performs this order in full volume at once. Otherwise when the order quantity exceeds the stock quantity $Q_i > q$, the customer will receive only a part of goods, which will result in the situation of deficit of $Q_i - q$ product units at the wholesaler's warehouse.

Ordering cost $C_0(Q_i)$ has two components: constant C_1, which includes cost of the order forming and constant part of expenses of order transportation, and variable component $C_2(Q_i)$, which depends on the order quantity Q_i i.e. $C_0(Q_i) = c_1 + c_2(Q_i)$. For all customers the holding cost is proportional to quantity of goods in stock and holding time with coefficient of proportionality C_H; losses from deficit are proportional to quantity of deficit with coefficient of proportionality C_{SH}. For i-th customer the average total cost in inventory system during one cycle $E_i^{cust}(\bar{T}_i)$ is calculated by the following formula:

$$E_i^{cust}(\bar{T}_i) = C_0(Q_i) + E_H(\bar{T}_i) + E_{SH}(\bar{T}_i), \quad i=1,2,\ldots,n \tag{4.10}$$

where $\bar{T}_i$ is average cycle time; $E_H(\bar{T}_i)$ is average holding cost during one cycle; $E_{SH}(\bar{T}_i)$ is average shortage cost during one cycle, and total cost E_i^{cust} per time unit for *i*-th customer can be found as divided by average cycle time $\bar{T}_i$ (Ross, 1992):

$$E_i^{cust} = \frac{E_i^{cust}(\bar{T}_i)}{\bar{T}_i}, \quad i=1,2,\ldots,n \tag{4.11}$$

Note that $E_H(\bar{T}_i)$ and $E_{SH}(\bar{T}_i)$ depend on control parameters R_i and Q_i. Analytical formulas for these economical characteristics are presented in this thesis (Kopytov, 2004c). To solve the problem, criterion

(4.8**)** needs to be minimized by R_i and Q_i.

Second stage of ordering process. Assuming that producer supplies its production to wholesaler according to a fixed schedule. In this case ordering process with constant period of time T between the moments of placing neighbouring wholesaler's orders; and order quantity q is determined as difference between fixed stock level *S* and quantity of goods in the moment of ordering *r* (see Fig. 4.8), i.e. $q = S - r$.

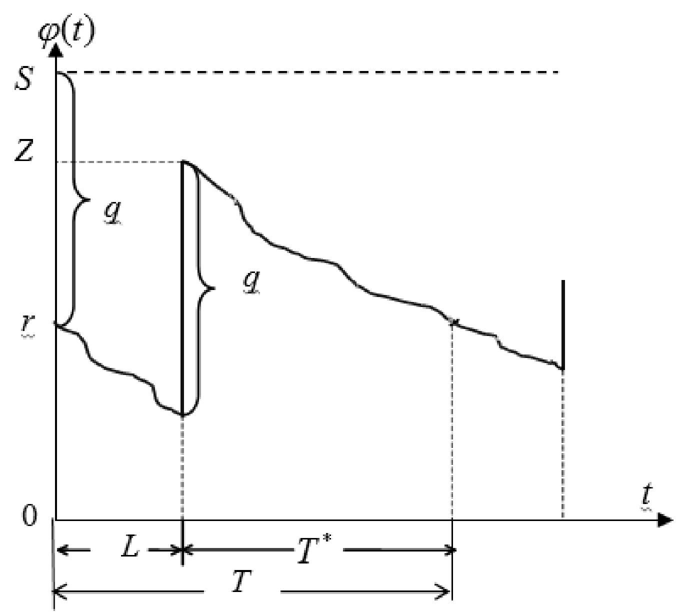


Fig. 4.8 Dynamics of wholesalers' inventory level during one cycle

Let's assume that the lead time L from the producer to the wholesaler has a normal distribution with a mean μ_L and a standard deviation σ_L. Lead time is essentially less as time of the cycle is $\mu_L + \sigma_L \ll T$. Supposingthat during time T the wholesaler has received orders from n customers, these orders can be described by the vector $\{Q_1, Q_2, \dots, Q_n\}$. There is a possible situation of the deficit, when the demand $D(T) = \sum_{i=1}^{n} Q_i$ during time T exceeds the quantity of goods in stock Z in the time moment immediately after order receipt. Similarly to the first stage, in case of deficit the latter cannot be covered by expected order. Denote as S the goods quantity, which is highly necessary for a single period, and it equals to the sum

$$S = \bar{D}(T) + S_0 \tag{4.12}$$

Where $\bar{D}(T)$ is average demand during cycle time; S_0 is some safety stock (emergency stock).

Supposing that "ideally" S gives in future the minimum of total expenditure in inventory control system per unit of time. So, for the second stage in suggested model time period T and stock level S are control parameters. In the moment of time, when a new order has to be placed, may be situation, when the stock level is so big that a new ordering doesn't occur. However, for generality of the model, the concept of lead-time and quantity of goods at the moment immediately after order receiving will be kept in such case too. It corresponds to the real situation, when the wholesaler uses transport means, which depart at the fixed moments of time not depending on existence of the order and which have the random lead-time; for example transportation by trailers which depart the 1st and 15th day of each month. In real situation in the moment of time t, the stock level $\varphi(t)$ are equal to S only in two cases:

1) $r = S$ and $D(t) = 0$, where $D(t)$ is the demand for goods during the time t; $0 \le t \le T$;
2) $r < S$ and $D(t) = 0$, where $L \le t \le T$.

Taking into account that in case of deficit it cannot be covered by expected order, we can obtain the expression for goods quantity at the moment of time immediately after order receiving:

$$Z = \begin{cases} r + q - D(L), & \text{if} \quad D(L) < r; \\ q, & \text{if} \quad D(L) \ge r, \end{cases}$$

Where $D(L)$ is the demand during lead time L and

$$Z = \begin{cases} S - D(L), & \text{if} \quad D(L) < r; \\ S - r, & \text{if} \quad D(L) \geq r. \end{cases}$$

Finally, average total cost for time unit for the wholesaler is expressed by the following formula

$$E^{\mathrm{wh}} = \frac{E_{\mathrm{H}}^{\mathrm{wh}} + E_{\mathrm{SH}}^{\mathrm{wh}} + C_0(q)}{T} \tag{4.13}$$

Unlike stage 1, in the considered stage expenditures $\boldsymbol{E_H^{wh}}$ and $\boldsymbol{E_{SH}^{wh}}$ depend on control parameters S and T.

4.2.1. Simulation model in ExtendSim 8 Environment

Assuming that in considered system we have three customers. The created simulation model for the supply chain “producer – wholesaler – customers” is shown on Fig. 4.9 – Fig. 4.11 and Fig. 4.14 (Muravjovs, 2008a, Muravjovs, 2008b, Muravjovs, 2010a and Muravjovs, 2010b). The main screen of the simulation model is presented in Fig. 4.9. Each zone of the model has a numeric label. In zone 1 an executive block that controls all discreet events in Extend models is placed. Zones 2 and 3 contain blocks which are responsible for modelling result representation on a plotter block placed in zone 2 and in zone 3, total expenses calculation, and data export to Excel spreadsheet. Zone 4 contains a block which is intended for scheduled transact generation; lead time and transport activity for goods transportation to the main store are simulated in blocks placed in zone 5. In the main storehouse zone there are placed: a block for holding activity simulation, a block for order quantity calculation, and an initialization block that performs queue initialization tasks before the model starts. In this situation, all stocks are initialized before starting to represent a typical situation of goods quantity at the warehouse.

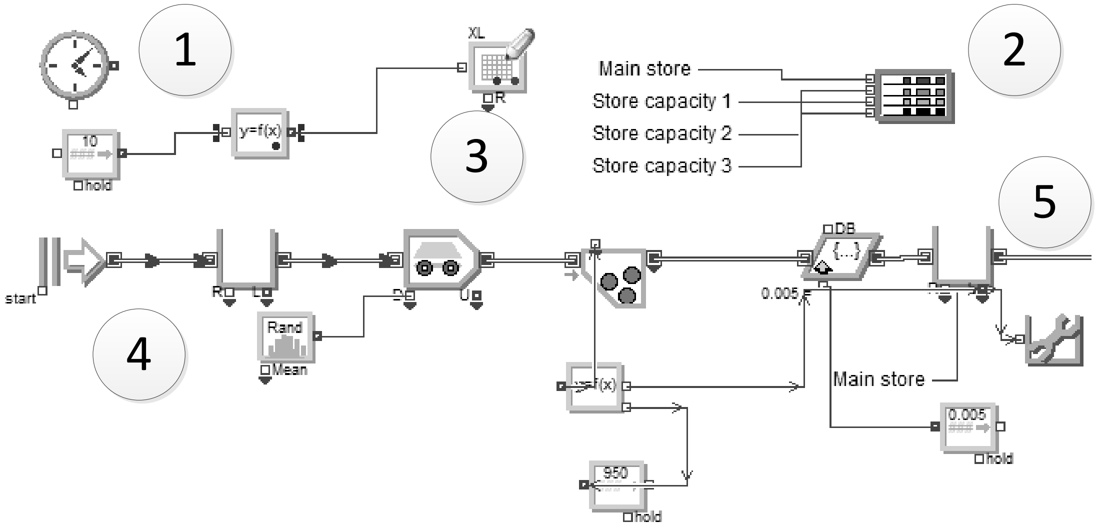


Fig. 4.9 Main Screen of the Simulation Model

After goods delivery to the main warehouse, they are transferred to customers' warehouses according to their orders. The hierarchical blocks shown on Fig. 4.10, performs the "reorder point" goods ordering strategy.

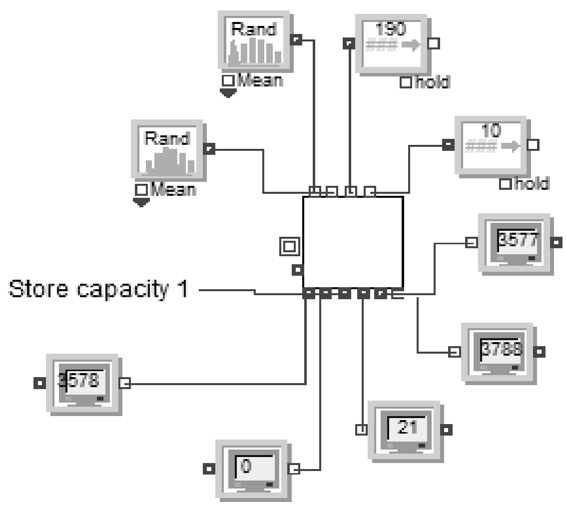


Fig. 4.10 Reorder Point Store

This hierarchical block is made in the way, which allows using it in any necessary Extend model that needs such functionality. In the created model there are three identical reorder point blocks, for three customers stocks modelling respectively. For this reason, all control parameters and results are realized as input and output connectors. The internal parameters for this type of block are: stochastic lead time of goods delivery and demand for goods, shortage, delivery and holding costs, order quantity and reorder point. Specifying these parameters, we can receive appropriate results, such as quantity of sold goods, amount of deficit, total costs that include ordering, holding and shortage costs. These result parameters are used for total cost calculation. Order quantity and reorder point are control parameters and have to be changed during the simulation procedure.

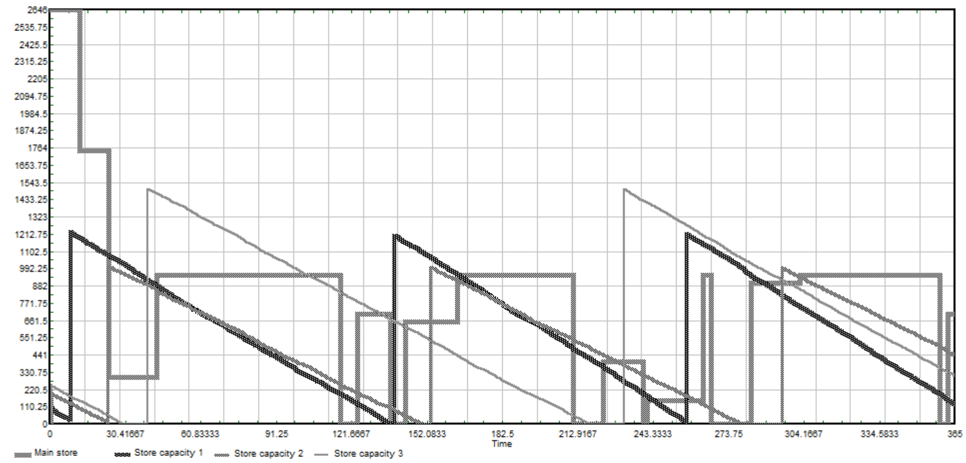


Fig. 4.11 The Example of Simulation of the Inventory Control Process in All Stocks

Using output connectors for goods quantity in stocks and plotter block, ExtendSim builds graphical representation of the dynamics of inventory level in all stocks shown in Fig. 4.11

Fig. 4.12 illustrates internal structure of Reorder point hierarchical block. First block in zone 6 is called Gate, which enables or disables transact entrance to this part of the model. Behaviour of this block is controlled by Equation block that collects information about stock level, reorder point and placed order status.

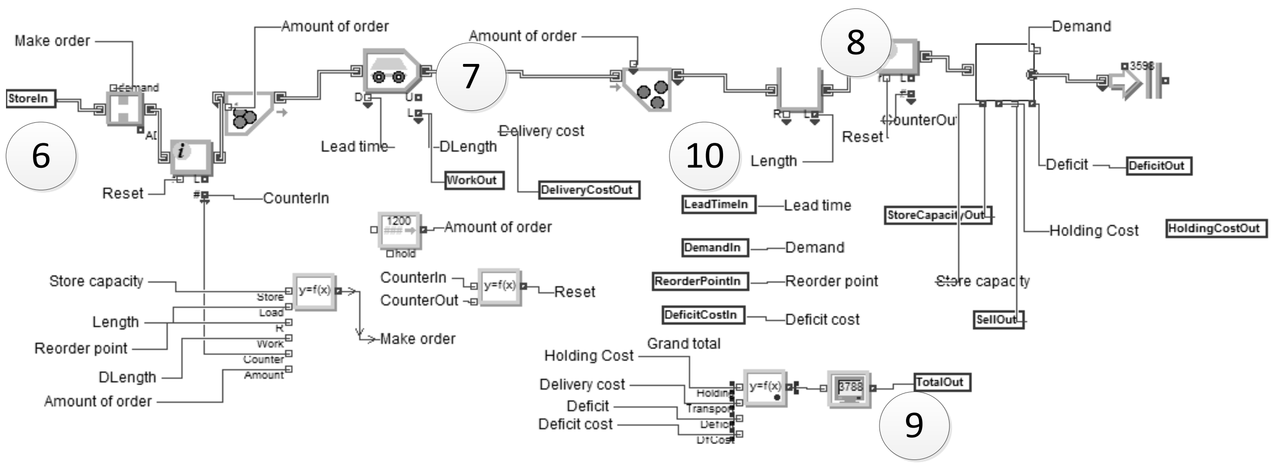


Fig. 4.12 Reorder point hierarchical block

Based on the calculation of these parameters, Equation block sends Boolean value to Gate (0 – close and 1 – open). If transaction is allowed for entrance, than it is passed to activity transport block (zone 7), after appropriate delay to the end store (zone 8). Blocks of zone 9 are used for the calculation of expenditures. Zone 10 is used for internal communication between hierarchical block together with ExtendSim database. Structure of the final hierarchical block is shown in Fig. 4.13

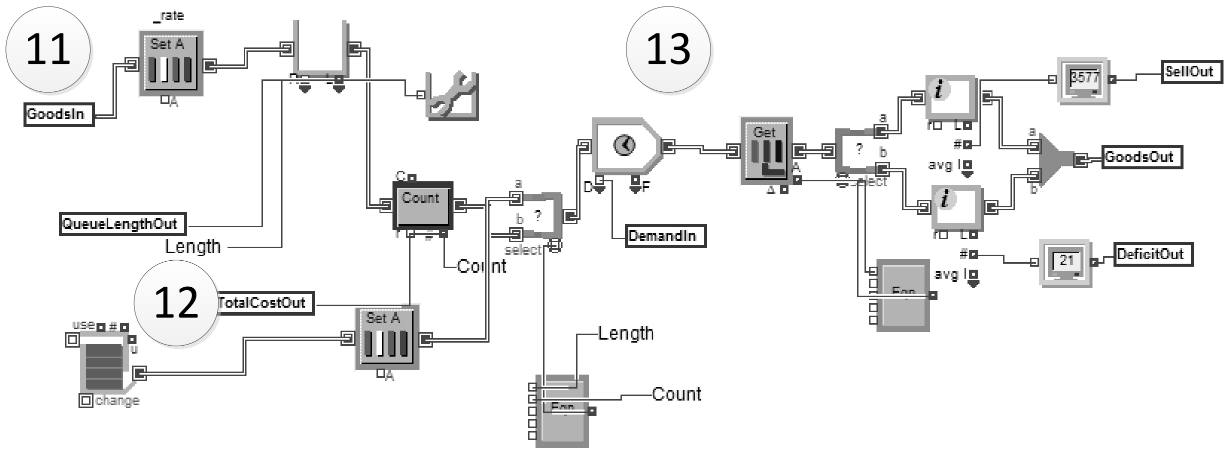


Fig. 4.13 Hierarchical Block Customer Store

In zone 11, transactions are arriving to the warehouse, where they are assigned a holding cost value. Zone 12 is designed for the deficit modelling with dummy supplier and appropriate attribute assigning. Zone 13 represents a market place where goods are sent to customers.

For end users' facilitation – a specialized user interface was designed. Utilizing this interface, user can change control parameters of the model and get final results of simulation. There are several tools for user interface development in ExtendSim. One of them is Notebook window that can be called from any place of the model, and the other one is called a cloning tool that allows clone core control elements from ExtendSim block and place them into Notebook. An example of Notebook's window with initial data and results is shown on Fig. 4.14

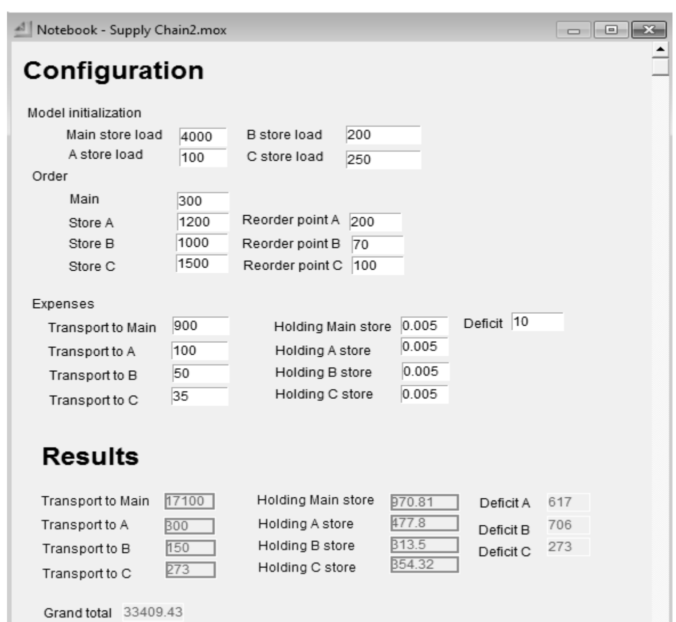


Fig. 4.14 Example of Notebook's Window

4.2.2. Example

There is a two-level inventory control system considered, which includes correspondingly a wholesale warehouse and the warehouses of 3 customers. For delivery of the products, there has been organized a supply chain "producer – wholesaler – customer" with a two-stage ordering process. It is assumed that all the customers are financially independent and organize the whole policy of ordering and holding of the product by themselves; the wholesaler also acts only with the account of the minimization of his costs, losses from the product deficit included.

Table 4.3 Initial Data

Customer, i	Lead time, L_i, days	Demand, λ_i units/day	Order quantity, Q_i, units	Initial stock, Z, units	Ordering cost, C_0, EUR
1	μ_i =11; σ_i =3.5	10	1200	100	100
2	μ_i =12; σ_i =2	8	1000	200	50
3	μ_i =14; σ_i =3.7	8.57	1500	250	35

It is required to find such values of the reorder points R_1, R_2, R_3 with the customers and such value of the desired product stock S with the wholesaler with a minimum sum of the total costs of goods ordering and holding and the losses from the deficit per time unit. The customers' demands for goods D_i, $(i = 1,2,3)$ is represented in Poisson processes with intensity λ_i, and time L_i of goods delivery from wholesaler to i-th customer has a normal distribution, with parameters μ_i and σ_i (see Table 4.3). Ordering costs C_0 (including expenses of order transportation) for each customer are represented in Table 4.3, too.

The producer supplies its production to the wholesaler according fixed schedule, and time period T between the moments of placing neighbouring orders is constant and equals 20 days. The policy of order forming for i-th customer is the following: a new order is placed in the moment of time, when the stock level falls to a certain level R_i. Then goods delivery from producer to wholesaler time L has a normal distribution with a value $\mu_z = 3$ and a standard deviation $\sigma_L = 1$. Ordering cost C_0 (including expenses of order transportation) for wholesaler equals to 900 EUR. For customers and for wholesaler holding cost C_H equals to 0,005 EUR per unit per day, losses from deficit C_{SH} equal to 10

EUR per unit. Initial stock in wholesaler's warehouse is equal 4000 units. Fixed stock level in wholesaler's warehouse S is the control parameter of the model.

The period of simulation is one year and the number of replications is 100. There can be two strategies of optimization. The goal of the first strategy is the total minimization of all expenses for all model participants. The second strategy is the optimization of individual customer and wholesaler activity. In this thesis, we will have a close approach to the first strategy.

After having performed the modelling of the initial variant of the system represented in Table 4.3, we have got the following values of the control parameters: the reorder points with the customers correspondingly: $R_1 = 200$, $R_2 = 70$ and $R_3 = 100$ units, the level of the desired product stock with the wholesaler $S = 1350$ units. In addition, the value of the average total cost per year in the inventory system equals 22621 EUR. The given variant has been taken as the basic one.

Let's perform the optimization of the basic variant of the inventory control system. Note that due to limited volume of the given paper, we will use only one control parameter from each pair: with the customer it will be a reorder point (the second parameter "order quantity" is fixed and determined in Table 4.3), with the wholesaler – it will be a stock level (the interval between orders, as it has been noticed, equals to 20 days). With the account of the above assumptions about the economic independence of a particular customers and wholesaler, it is suggested to use the algorithm of the step-by-step optimization. At each step of the proposed algorithm, the value of the control parameter for the selected structural enterprise is determined (first – customers, then – wholesaler), which, in the considered range of its change, gives the minimal value of the average total cost per year in the inventory system. Due to the illustrative character of the given article, the step of change of the control parameters with the customer is taken as 10 units, and with the wholesaler – 50 units. Let's consider the solution of the task in more detail.

Step 1

Using the data of the basic variant (see Table 4.3), let's perform the simulation of the stock system by changing the value of the reorder point, with Customer 1, in the range from 110 to 230 units with the step of 10, getting for each of the points 100 realizations. The results of the simulation are shown on Fig. 4.15. Note that for the given steps of the control parameter R_1 changing the best result is achieved for reorder point $R_1 = 190$ units, where for 100 realizations average total cost E^{total} for one year period equals 21838 EUR.

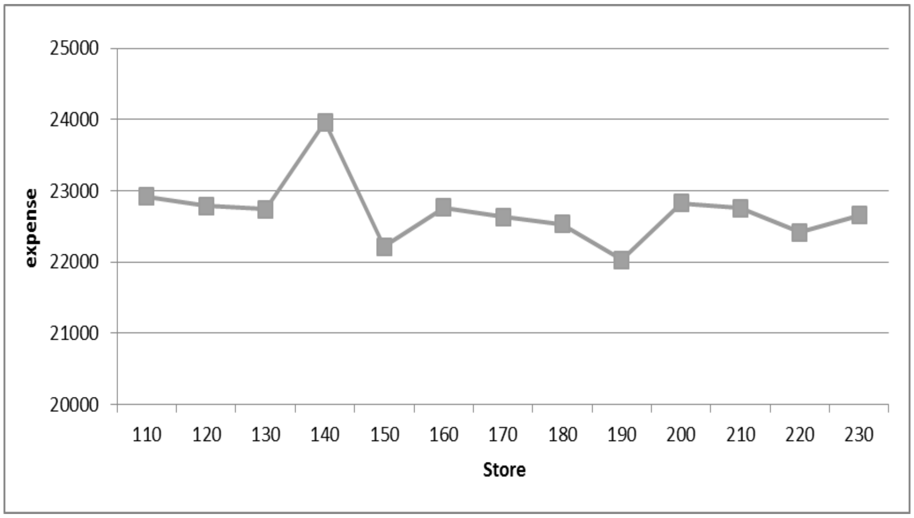


Fig. 4.15 Average Total Expenses per Year in Inventory Control System (Step 1)

Step 2

Using the data received at step 1, let's perform the simulation of the stock system, changing the value of the reorder point, with Customer 2, in the range from 20 to 130 units. The results of the simulation are shown in Fig. 4.16. For the given steps of the control parameter R_2 changing the best result is achieved for reorder point $R_2 = 100$ units, where average total cost E^{total} for one year period equals 21 813 EUR.

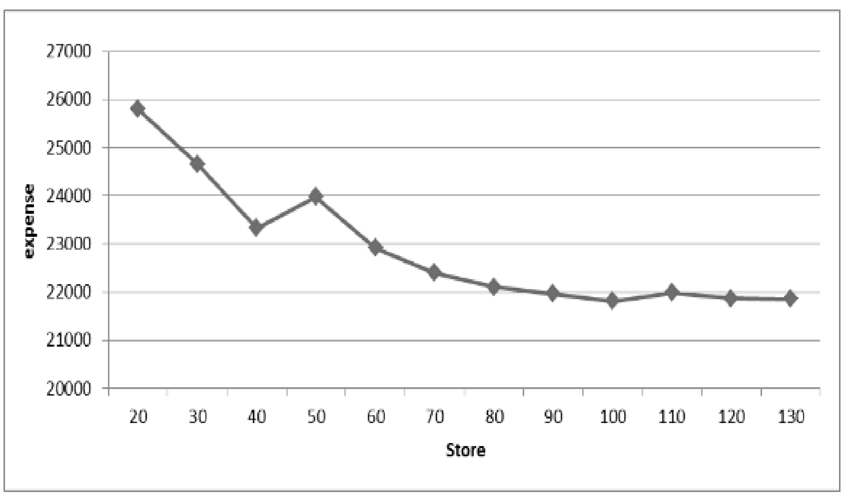


Fig. 4.16 Average Total Expenses per Year in Inventory Control System (Step 2)

Step 3

Using the data received at step 2, lets perform the simulation of the stock system, changing the value of the reorder point, with Customer 3, in the range from 5 to 140 units. The results of the simulation are shown on Fig. 4.17. For the given steps of the control parameter R_3, change of the best result is achieved using reorder point $R_3 = 100$ units, where average total cost for one year period E^{total} is 21635 EUR.

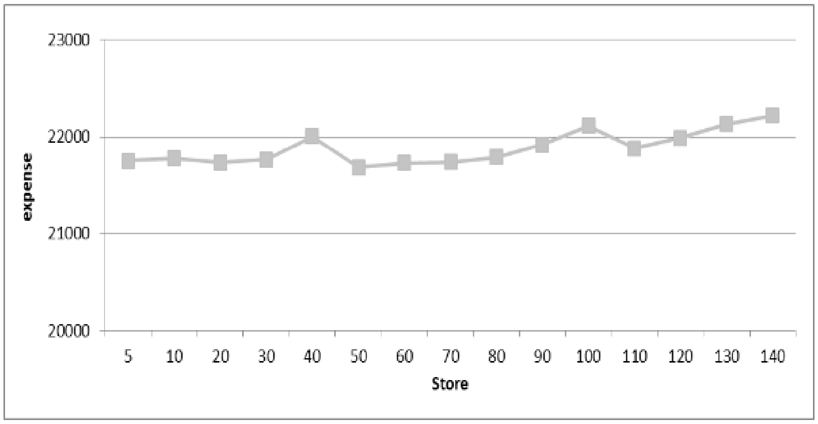


Fig. 4.17 Average Total Expenses per Year in Inventory Control System (Step 3)

Step 4

Using the data received at step 3, lets change the level of the desired stock S with the wholesaler in the range from 900 to 1450 units and perform the simulation for different S values. The results of the simulation are shown on Fig 4.18. Note that for the given steps of the control parameter S changing the best result is achieved for reorder point S = 1000 units, where for 100 realizations average total cost for one year E^{total} period equals 21527 EUR.

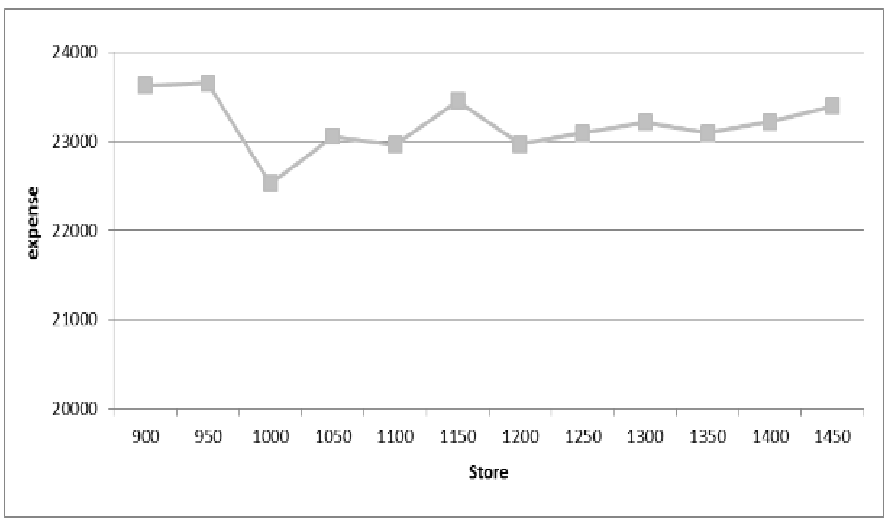


Fig 4.18 Average Total Expenses per Year in Inventory Control System (Step 4)

The results of the considered steps are presented in Table 4.4. Note, that the optimal values of parameters received after each step are underlined. Now it can be noticed that due to the change of values for the selected control parameters, the best variant has been achieved from the considered ones, which provides the reduction of total cost E^{total} in the inventory control system, as compared with the source variant, by 1094 EUR or by 4,84%. It's clear that the given value cannot be seen as the minimum one, since we have changed only one from the each pair of control parameters and used quite a large step of the parameters' change.

Table 4.4 The results of optimization process

Parameters	Values of control parameters				
	Base	Optimization steps in the model			
		Step 1	Step 2	Step 3	Step 4
Reorder point R_1, units	200	<u>190</u>	<u>190</u>	<u>190</u>	<u>190</u>
Reorder point R_2, units	70	70	<u>100</u>	<u>100</u>	<u>100</u>
Reorder point R_3, units	100	100	100	<u>50</u>	<u>50</u>
Desired stock level S, units	1350	1350	1350	1350	<u>1000</u>
Total expenses E^{total}, EUR	22621	21838	21813	21635	<u>**21527**</u>

4.3. Simulation model of current stock of divisible products

In this case the problem of the inventory control system of divisible productions is investigated. In previous examples has been investigated the problem of constructing continuous and unsteady mathematical models to determine the volumes of current stock of divisible productions in one or several interconnected warehouses using apparatus of mathematical physics and continuum principle. Simple models are constructed using the theory of ordinary differential equations. For construction of more complex models, the theory of partial differential equations is applied (Milstein, 1995; Kuznetsov, 2007; Tikhonov, 2004).

It should be noted that the practical implementation of this approach and finding a numerical solution is a rather complicated and time-consuming task. For some proposed models we have found an analytical solution in the closed form, and for some of proposed models the discretization is carried out using stable difference schemes (Guseynov, 2011). In the given paper, we investigate the problem of constructing simulation model for the optimization of current stock of divisible productions. This approach is certainly easier to implement, but it has a lower accuracy of the obtained optimal solutions.

4.3.1. Divisible production model

Considering a stochastic inventory control model for the stock with homogeneous divisible production. The schema of the current stock of divisible production replenishment and distribution is shown in Fig. 4.19 (Muravjovs, 2013).

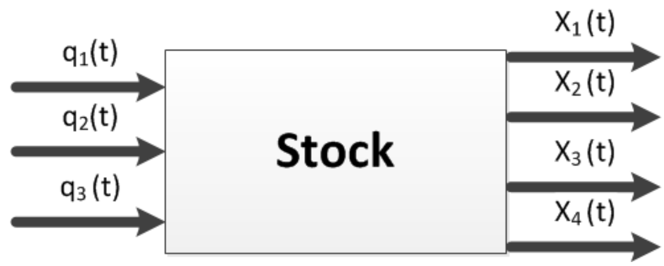


Fig. 4.19 Flows of Production in the Stock

Denote as *z(t)* the quantity of products in stock in the time moment *t*. Describing a continuous replenishment and distribution of the current stock considering the change rate of the current stock volume $\frac{dz(t)}{dt}$ at a given time *t* (Kopytov, 2010).

Let's consider the functions which determine the change rate $\frac{dz(t)}{dt}$:

- function $S_1(t, z(t))$ determines continuous replenishment of the current stock characterized by input flows of production $q_1(t), q_2(t), q_3(t)$;

- function $S_2(t, z(t))$ determines continuous distribution of the current stock characterized by output flows of production $x_1(t), x_2(t), x_3(t), x_4(t)$.

The difference $S_1\big(t, z(t)\big) - S_2(t, z(t))$ is a measure of the change of the current stock volume, i.e.

$$\frac{dz(t)}{dt} = S_1\big(t, z(t)\big) - S_2(t, z(t)).$$

The product replenishment consists of three additive flows (components), namely: from regular replenishment of the stock, which is designated as $q_1(t)$; from irregular replenishment by single orders $q_2(t)$; and from random replenishment $q_3(t)$ (for instance, a random stock replenishment due to an exceptionally high quality of production or an exceptionally low price, or because of an expected sudden deficit of particular products, etc.), which can be described mathematically as a random quantity that designating the total volume of production that has been delivered into a particular warehouse from random and/or non-random sources by the time *t*.

The product distribution consists of four additive flows (components) namely: regular distribution which is denoted as $x_1(t)$; irregular distribution $x_2(t)$; possible losses $x_3(t)$ of divisible productions which take place during holding and distribution processes (for example, for petroleum productions it is evaporation, for grain main reasons of losses are gnawing animals and inundation); and random (rare event) distribution (similar to random replenishment, there can be circumstances due to which random distribution takes place) that can be mathematically presented as a random flow $x_4(t)$ designating the total volume of productions that was taken away from the warehouse by the time *t* due to random circumstances.

Assuming that main parameters of input and output production flows are constant (unchanged) during fixed time span $T = [t_S, t_e]$, where t_S and t_e are day of start and day of the end of the period *T*, respectively. Usually for petroleum and agricultural divisible productions (wheat, rice, meal, etc.) time period *T* is the season period occupying 3 months or 90 days. Lets consider the introduced components in detail.

4.3.2. Product replenishment components

The component $q_1(t)$ can be interpreted as guaranteed replenishment of the current stock of divisible production that takes place regularly in fixed moments of time t_0, $t_0 + \Delta$, $t_0 + 2 \times \Delta, \dots, t_0 + k \times \Delta$ according to a contract during the time period *T* with the constant volume of products $Q_1 = const$. The quantity Q_1 is one of control parameters of the optimization model.

The component $q_2(t)$ obviously depends on random demand for products D_τ during time period τ and also on a certain quantity R_0, which designates the minimal volume of stock in a particular warehouse necessary for administering unregulated stock replenishment on condition that such replenishment is guaranteed. In other words, in the moment of time, when the stock level falls till certain level R_0, a new order is placed. The quantity R_0 is called as reorder point. Demand D_τ has a normal distribution with a mean μ_D and a standard deviation σ_D. In considered task the reorder point is calculated by the following formula:

$$R_0(t) = [\overline{D}(L) + X_1 k(L)]S_0 \tag{4.14}$$

where L is lead time (time between placing an order and receiving it); $\overline{D}(L)$ is average demand for products during lead time L (in considered task lead time L is constant); $k(L)$ is number of cases of regulated (according to contracts) distribution X_1 of products during lead time L, (number

$k(L)$depends on the moment of time *t*, when the order for delivery is placing); S_0 is a safety coefficient which determines certain reserve stock of products, $S_0 \geq 1$.

In case of production deficit the last cannot be covered by expected order. In considered optimization model safety coefficient, S_0 is the second control parameter.

The flow $q_3(t)$ determines the volume of production Q_3 that is delivered into the warehouse by the time *t* due to random (rare event) circumstances from random and/or non-random sources. In considered task, the probability p_3 of occurrence of this event during time unit is known, and it is a quite rare event; for example, for one day assuming that $p_3 = 0.01$.

The vector $Q = \{Q_1(T), Q_2(T), Q_3(T)\}$ determines total volume of products replenishment delivered during time period *T*, where $Q_1(T), Q_2(T), Q_3(T)$ are regular, irregular and random (rare event) replenishments during period *T*.

4.3.3. Products distribution components

The component $x_1\,(t)$ can be interpreted as "strong" (guaranteed) constant distribution of the current stock of divisible productions, i.e. the volume of the current stock is regularly taken away from the warehouse in fixed moments of time $t_1,\ t_1 +\ \Delta_1,\ t_1 + 2\,\Delta_1, \dots, t_1 + k \times\ \Delta_1$ according to a contract during the time period *T* with the constant volume of product x_1.

The component $x_2(t)$depends on random demand for products D_τ during time unit and regular distribution, which determines the stock volume of divisible productions allowing for its unregulated distribution,

The component $x_3(t)$ describes possible losses of the divisible productions in current stock in the processes of storage and distribution. For instance, if we have the oil productions stock, losses will result from the evaporation and/or from the leakage through the reservoirs; if we have the agricultural productions stock (wheat, rice, meal, etc.), there will be unavoidable losses caused by pests, flood, strong winds, etc. Apparently, the value of these losses is a random one.

The flow $x_4(\mathrm{t})$ determines quantity X_4 designates the total volume of productions (unexpected distribution with a large profit) that has been removed from the warehouse by the time t due to random (rare event) circumstances. In considered task we assume that the probability p_4 of occurrence of this event during time unit is known, and it is a quite rare event; we assume that for one day $\mathrm{p}_4 \leq 0.01$.

In the considered problem supposing that the following economic parameters are known:

for *i*-th component of product replenishment ($i = 1,2,3$) the ordering cost of product $C_0^{(i)}(Q_i)$ is a known function of the products quantity Q_i, delivered during time period *T*, and consists of two additive components, namely: constant $C_1^{(i)}$ which includes cost of the order forming and constant part of expenses of products transportation, and variable component $C_2^{(i)}(Q_i)$, which depends on the order quantity Q_i, i.e.

$$C_0^{(i)}(Q_i) = c_1^{(i)} + c_2^{(i)}(Q_i), i = 1,2,3.$$

Considered inventory control system for $i = 1,2,3$ coefficients $c_1^{(i)}$ and $c_2^{(i)}$ are different: $c_1^{(1)} < c_1^{(2)}$; $c_1^{(3)} = 0$; $c_2^{(2)}(1) < c_2^{(1)}(1) < c_2^{(3)}(1)$ where $c_2^{(i)}(1)$ is determined for one unit of delivered production. Therefore: $C_0^{(2)}(1) < C_0^{(1)}(1) < C_0^{(3)}(1)$.

The total ordering cost for time period *T* is determined by the following formula:

$$E_{OD}(T) = C_O^{(1)}(Q_1(T)) + C_O^{(2)}(Q_2(T)) + C_O^{(3)}(Q_3(T)) .$$

The holding cost of the product is proportional to its quantity in the stock and the holding time with the coefficient of proportionality C_H.

The losses from the deficit of the product are proportional to the quantity of its deficit with the coefficients of proportionality C_{SH_j} which are different for each type of product distribution. At the same time losses from the deficit of the product for regular distribution are the largest, but for random (unplanned, rare event) distribution these losses (i.e. lost profit) are the lowest, i.e. $C_{SH_1} > C_{SH_2} > C_{SH_3}$. Losses from damage and loss of product are proportional to the cost of product unit. The total cost *E(T)* in inventory system during the season period *T* is calculated by the following formula:

$$E(T) = E_{OD}(T) + E_H(T) + E_{SH}(T) + E_{CS}(T) \tag{4.15}$$

where $E_{OD}(T)$ is ordering cost; $E_H(T)$ is holding cost; $E_{SH}(T)$ is shortage cost; $E_{CS}(T)$ is losses from damage or loses of products during time period *T*.

Principal aim of the considered task is to define the optimal values of regular order quantity Q_1 and safety coefficient S_0 for irregular replenishment, which are control parameters of the model. Criteria of optimization is minimum of average total cost $\bar{E}(T)$ during time period *T*, which can be calculated by formula (4.15) for average costs and loses $\bar{E}_{OD}(T), \bar{E}_H(T), \bar{E}_{SH}(T)$ and $\bar{E}_{CS}(T)$.

4.3.4. Simulation model in ExtendSim 8 Environment

For this task implementation, continuous simulation approach was chosen. The created model consists of four main parts: “Stock”, “Demand”, “Ordering costs” and “Total costs calculation” that are represented on Fig. 4.20 – Fig. 4.24. The purposes of blocks shown in Fig. 4.20 – Fig. 4.24 are given in captions. Let consider the main sections of the simulation model.

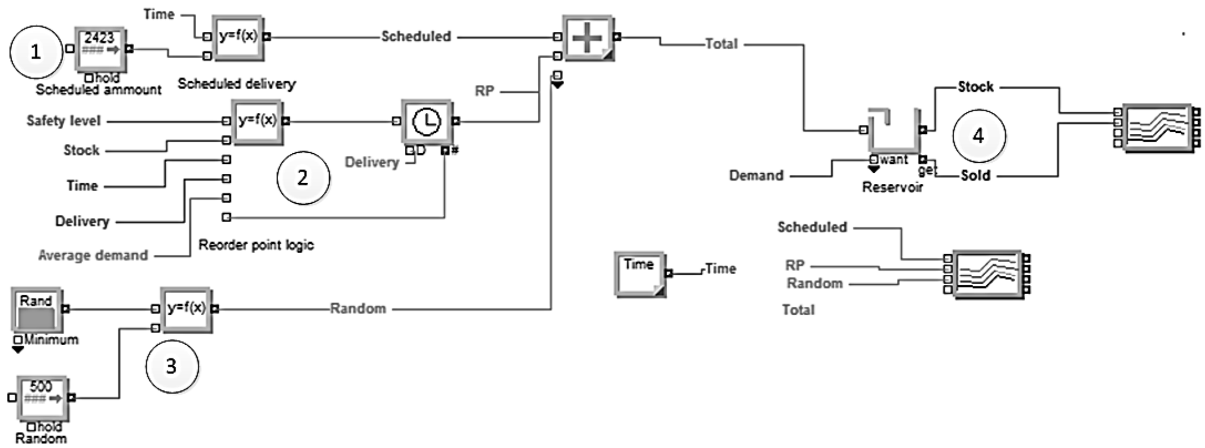


Fig. 4.20 Stock Simulation

Section „Stock" (see Fig. 4.20). In area #1 there are placed blocks that are responsible for scheduled delivery simulation. Area #2 is used for generation emergency delivery orders (irregular replenishment) based on current stock level and time between scheduled orders. Next area #3 generates random deliveries cheap that occurs one out of hundred cases ($p_3 = 0.01$).The stock is realized in area #4.

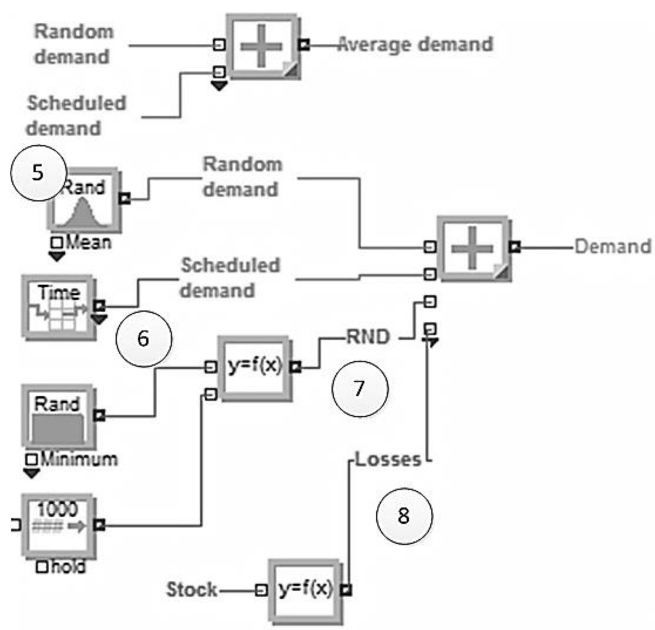


Fig. 4.21 Demand Generation

Section "Demand" (see Fig. 4.21) is created for product distribution simulation and consists of the blocks responsible for demand generation. There are four demand sources: random demand is realized in area #5, scheduled demand (regular distribution) – in area #6, random demand with different distribution – in area #7, and holding – in area #8.

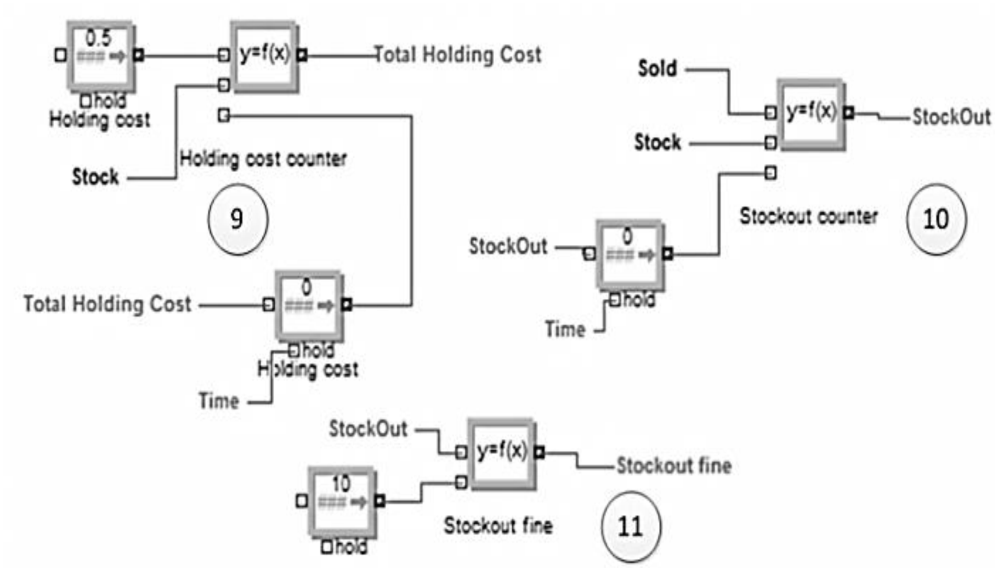


Fig. 4.22 Costs Calculations

Next two sections “Costs” and “Ordering Costs”, shown on Fig. 4.22 and Fig. 4.24 accordingly, include costs calculations blocks, namely: holding, ordering and losses costs for all delivery sources described above.

The total holding cost $E_H(T)$ is calculated in the blocks of areas #9 and #11. Current stock is calculated in blocks of area #10.

Blocks in area #12 are used for order costs calculations from each delivery sources. The total cost *E(T)* in inventory system is calculated in blocks in area #13.

An example of the inventory control process simulation (one realization) is shown in Fig. 4.23. The plot shows the current stock of certain production during period of season *T*.

Using created simulation model we can find the optimal solution for inventory control of stock of divisible production. One of examples is considered in the next section.

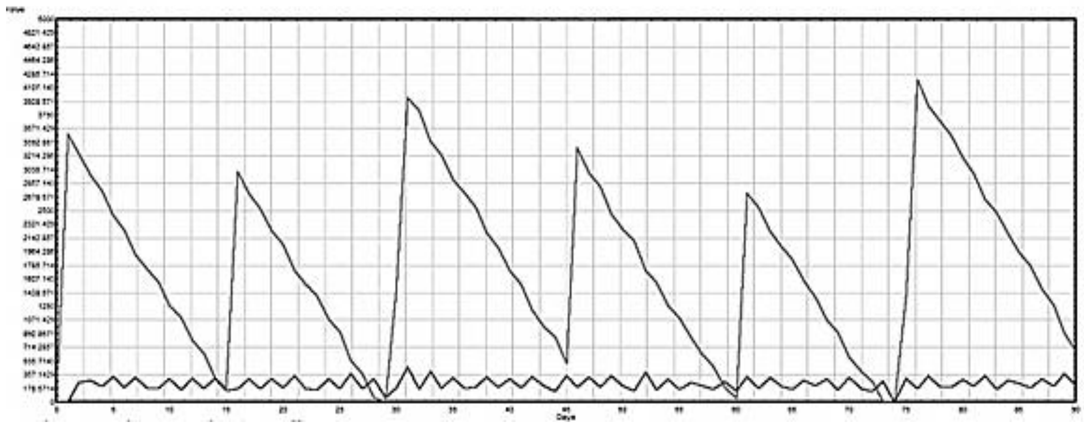


Fig. 4.23 Example of simulation process

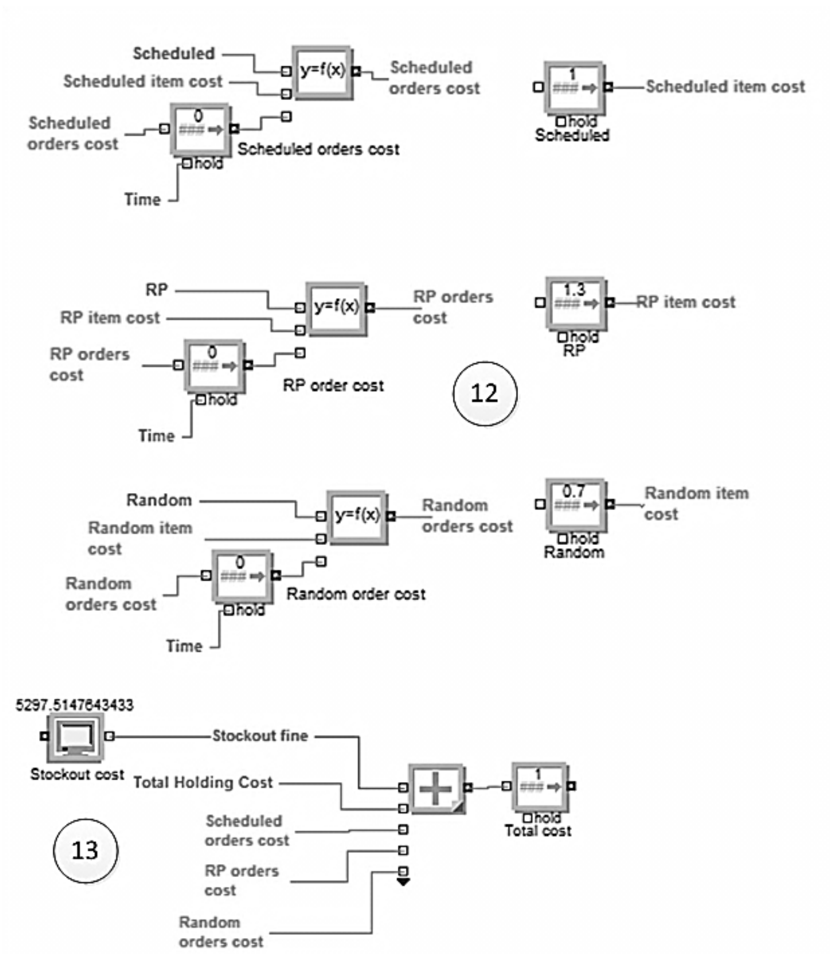


Fig. 4.24 Orderings Costs Calculations

4.3.5. Example and optimization

Let consider a stochastic inventory control model for the stock with homogeneous divisible production shown on Fig. 4.19, Table 4.5 and

Table 4.6 describe main parameters of the products replenishment and distribution.

Table 4.5 Initial Data of Product Replenishment

Source	*Amount*	*Schedule*	*Cost / unit, conventional units (EUR)*
Regular	3000	Bimonthly	1.0
Irregular	According to stock level		1.3
Random	200	Random, p=0.01	0.7

Table 4.6 Initial Data of Product Distribution

Source	*Amount*	*Schedule*
Irregular	Demand D_τ, normal distribution μ_D=170; $\sigma_D = 30$	Daily
Regular	150	Monday, Wednesday, Fryday
Random (rare event)	1000	Random, p=0.01
Holding loses	0.5% of daily stock	Daily

For optimization process amount of regular replenishment Q_1 can be changed from 1000 to 4000 and safety level S_0 from 1.0 to 1.5. The period of simulation is 3 months (one season period) and the number of realization is 100.

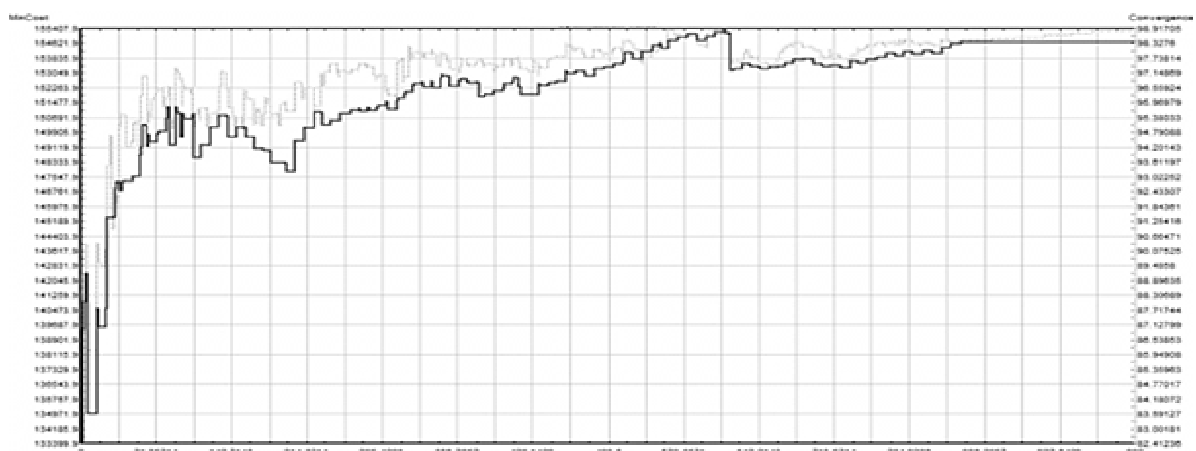


Fig. 4.25 Optimization Process

The optimization model created by ExtendSim optimization tool gives us a flexible solution for optimal result searching. The Fig. 4.25 represents optimization process in ExtendSim environment.

For the given steps of the control parameter Q_1 and S_0 changing, the best result is achieved at point Q_1 = 2435 units and S_0 = 1.34, where for 100 replications the average total cost of the one season period is 154695 EUR It gave us total costs reduction from 163967 EUR (for initial values of control parameters Q_1 = 3000 and S_0 = 1.3) to 154695 EUR

CONCLUSIONS

1. In order to conduct a quantitative research of inventory control systems, there can be implemented two classes of models: analytical (purely mathematical) and simulation (computer) models. Both of them are associated with mathematical models, and each certain model is based on the specific theoretical (conceptual) model.

2. In the class of analytic models, there is a large number of modifications and extensions available for the classical Wilson's model, but inventory control task formulation can always appear and have no existing analytical model, or which, in principle, cannot be even created. In such a case, the only opportunity to study the effectiveness of inventory control strategy lies in the actualization of one of computer simulation types.

3. In the class of computer models there are examples of developments on the basis of spreadsheets, and also with the use of special software related to the simulation software group. The use of discrete time (normally expressed in days) in spreadsheet models and System Dynamics type models due to supposed daily making of decision on the necessity of delivery order placement that happens only once a day. In "discrete event" models there is no such limitation on time representation.

4. The basic principles of process dynamics representation in simulation models are called the simulation paradigms. Nowadays the most recognized are "continuous" and "discrete event" paradigms. "Agent based" paradigm in this sense refers to "discrete event" paradigm, because it is oriented on the same model time calculation mechanism implementation as if in the "discrete event" paradigm. There are some commercial simulation modelling packages that support "continuous" and "discrete event" paradigms. They include, for example, Arena, Automod, AnyLogic and ExtendSim packages. The third paradigm and theoretically the last one of all possible simulation paradigms in material flow systems has recently been defined as "discrete rate", found in ExtendSim package only.

5. A new selection and implementation method of above-mentioned paradigms for the study of inventory control systems has been developed as a result of process properties study within the models on the basis of three paradigms with the use of ExtendSim package. In terms of this method, the following recommendations about the implementation of simulation paradigms in particular have been formulated there.

- "Continuous" paradigms may be successfully applied for the development of model if with the help of this model it is supposed to estimate only the principal features of the studied inventory control strategies. In such models only the most essential processes are displayed, able to influence the process of inventory dynamics. Often, such processes are only input and output flows of the corresponding warehouse.

- “Discrete event” paradigm most frequently finds its application in simulation modelling of processes in systems of manufacturing and logistics. It is expedient to implement this paradigm for the study of inventory control system if in the problem formulation the necessity to display relatively complex logistics processes within the model have been clearly justified, thereto display of processes in multistage supply chains.
- “Discrete rate” paradigm in many respects occupies an intermediate position between “continuous” and “discrete event” paradigms. Since in “discrete rate” models should occur many times less events than in “discrete event” similar models, models of such class are rather promising in respect to the inventory control strategy optimization procedures completion, in which thousands of runs can be performed.

6. The developed method proved worthy as is has been successfully implemented when developing the inventory control systems’ simulation models provided in this thesis.

REFERENCES

(Andraski, 1994) Andraski, J.C. Foundations for Successful Continuous Replenishment Programs. In: The International Journal of Logistics Management, 1994, Vol. 5, Iss. 1, pp. 1-8.

(Angerhofer, 2000) Angerhofer, B.J. and M.C. Angelides. System dynamics modelling in supply chain management: Research review. In: Proceedings of the 2000 Winter Simulation Conference, pp. 342-351.

(Anikina, 1999) Аникина Б.А. Логистика: учебное пособие, М., Инфра-М, 1999.

(Banks et al., 2000) Banks, J., J.S. Carson, II, B.L. Nelson, D.M. Nicol. Discrete-Event System Simulation, Prentice-Hall, Upper Saddle River, NJ, 2000.

(Banks, 2000) Banks, J. Introduction to simulation. In: J.A. Joines, R.R. Barton, K. Kang, & P.A. Fishwick (Eds.), Proceedings of the 2000 Winter Simulation Conference, pp. 510-517.

(Barlas, 1999) Barlas, Y., and A. Aksogan. Product Diversification and Quick Response Order Strategies in Supply Chain Management [web page]. Bogazici University 1997 [cited 27 August 1999]. Available from http://ieiris.cc.boun.edu.tr/faculty/barlas/.

(Baskin, 2005) Баскин А.И., Варданян Г.И. Экономика снабжения предприятия сегодня и завтра. М., Экономика, 2005.

(Borshchev, 2013) Borshchev, A. The Big Book of Simulation Modelling: Multimethod Modelling with Anylogic 6. AnyLogic North America, 2013.

(Chen and Wang, 2010) Chen, Y., Wang, S. Investigating The Impacts of Dynamic Pricing and Price-Sensitive Demand on an Inventory System in the Presence of Supply Disruptions. In: Proceedings of the 2010 Winter Simulation Conference, pp. 1809-1819.

(Chen, 2009) Chen, Y., Li, X. The effect of customer segmentation on an inventory system in the presence of supply disruptions. In: Proceedings of the 2009 Winter Simulation Conference, pp. 2343-2352.

(Chopra et al., 2001) Chopra, S., Meindl, P. Supply Chain Management: Strategy, Planning and Operation. New Jersey: Prentice-Hall Inc., 2001.

(Cooper, 2013) Cooper, K., Sardar, G., Tew, J., Wikum, E. Reducing inventory cost for a medical device manufacturer using simulation. In: Proceedings of the 2013 Winter Simulation Conference, pp. 2109-2115.

(Esnaf, 2000) Esnaf, S. Spreadsheet Simulator for Inventory Management in a Supply Chain. In: Proceedings of the First World Conference on Production and Operations Management, Sevilla, 2000, pp. 1-8.

(Forrester, 1958) Forrester, J.W. Industrial Dynamics: A Major Breakthrough for Decision Makers. In: Harvard Business Review 36 (4), pp. 37-66.

(Forrester, 1961) Forrester, J.W. Industrial Dynamics. Portland (OR): Productivity Press, 1961.

(Franke, 2010) Franke, P. D., Straube, F. (Hrsg.) Vendor-Managed Inventory for High Value Parts. Auflage. Universitätsverlag der TU Berlin, Berlin, 2010.

(Gringlazs, 2005) Gringlazs, L., Kopitovs, J. Krājumu pārvaldības pamati. Rīga, RSEBAA, 2005.

(Guseynov, 2011) Guseynov, Sh.E., Kopytov, E.A., Puzinkevich, E. On continuous models of current stock of divisible productions. In: Dynamical Systems, Differential Equations and Applications Vol. I. American Institute of Mathematical Sciences (AIMS), 2011, pp. 601-613.

(Hadley and Whitin, 1963) Hadley, G., and T.M. Whitin. Analysis of inventory systems. Prentice-Hall, Inc., Englewood Cliffs, N.J., 1963.

(Hajinski, 2010) Гаджинский А.М. Логистика. М., Изд-во «Дашков и Ко», 2010.

(Hajinski, 2011) Гаджинский, А.М. Логистика: учебник для высших и средних специальных учебных заведений. 4-е изд., перераб. и доп., М., Издательско-книготорговый центр "Маркетинг", 2011, с. 232-240.

(He, 2013) He, L., Huang, S., Shen, Z.-J. M. A simulation-based approach to inventory management in batch process with flexible recipes. In: Proceedings of the 2013 Winter Simulation Conference, pp. 2587-2597.

(Hennies, 2014) Hennies, T., Reggelin, T., Tolujew, J., Piccut, P.-A. Mesoscopic supply chain simulation. In: Journal of Computational Science 5(3), 2014, pp. 463-470.

(Hieber, 2001) Hieber, R. Supply Chain Management: A Collaborative Performance Measurement Approach, Zürich: Eidgenössische Techn. Hochsch., 2001.

(Hirano, 2006) Hirano, H., Makota, F. JIT. Is Flow: Practice and Principles of Lean Manufacturing, PCS Press Inc., 2006.

(Hunter, 1992) Hunter N.A., King R.E, Nuttle H.L.M. An Apparel-supply System for QR Retailing. In: Journal of Textile Institute 83(3), pp. 462-471.

(Hwang, 1981) Hwang, C.L., and Yoon, K. Multiple attribute decision making: Methods and applications. Berlin, Heidelberg, New-York: Springer Verlag, 1981.

(Kincade, 1993) Doris H. Kincade. Quick Response Management System for the Apparel Industry: Definition through Technologies. In: Clothing and Textiles Research Journal, September 1995, 13, pp. 245-251.

(Köksalan, 2011) Köksalan, M., Wallenius, J., Zionts, S. (2011). Multiple criteria decision making: from early history to the 21st Century. Singapore: World Scientific, 2011.

(Kopytov, 2004a) Gringlazs, L., E. Kopitovs, F. Tissen. Inventory control model for the typical railways company. In: Reliability and Statistics in Transportation and Communication. Riga, Latvia, 2004, pp. 39-45.

(Kopytov, 2004b) Grīnglazs, L., E. Kopitovs. About a task of optimal inventory control. In: Stability Problems for Stochastic Models. Jūrmala, Latvia, 2004.

(Kopytov, 2004c) Grīnglazs, L., E. Kopitovs. Inventory control model for the chain “Producer–Wholesaler–Customer”. In: Stochastic Models in Reliability, Security and Logistics. Beer Sheva, Israel, 2004.

(Kopytov, 2004d) Kopitovs, E., Tissen F., Greenglaz L. Inventory control model for the typical railways company with constraints. In: Mathematical Modelling and Analysis. Jurmala, Latvia, 2004.

(Kopytov, 2006) Kopitovs E., Greenglaz, L., Tissen, F. Stochastic inventory control model with two stages in ordering process. In: Journal of Business Economics Management, Vol VII, No. 1, 2006, pp. 21-24.

(Kopytov, 2010) Kopytov. E., Guseynov. Sh., Puzinkevich. E., Greenglaz. L. Continuous models of current stock of divisible productions. In: Computer Modelling and New Technologies, Vol. 14, No. 4, 2010, pp. 19-30.

(Krahl, 2007) Krahl, D. ExtendSim 7. In: Proceedings of the 2007 Winter Simulation Conference, pp. 226-232.

(Krahl, 2009) Krahl D. ExtendSim Advanced Technology: Discrete Rate Simulation. In: Proceedings of the 2009 Winter Simulation Conference, pp. 333-338.

(Kuznetsov, 2007) Kuznetsov D.F. Stochastic Differential equations: theory and practice of numerical solutions. Saint Petersburg State Polytechnical University, 2007.

(Law and Kelton, 2004) Лоу А.М., Кельтон В.Д. Имитационное моделирование. Классика CS. 3-е изд., СПб.: Питер; Киев: Издательская группа BHV, 2004.

(Law, 2003) Schriber, T.J., Ståhl, I., Banks, J., Law, A.M., Seila, A.F., & Born, R.G. Simulation Text Books - Old and New (Panel). In: Proceedings of the 2003 Winter Simulation Conference. pp. 238-245.

(Lawrence, 2002) John A. Lawrence, Jr. and Barry A. Pasternack. Applied Management Science: Modelling, Spreadsheet Analysis, and Communication for Decision Making (2nd ed.). California State University at Fullerton. John Wiley & Sons, Inc., 2002.

(Lee et al., 2010) EunSu Lee, Kambiz Farahmand. Simulation of a base stock inventory management system integrated with transportation strategies of a logistic network. In: Proceedings of the 2010 Winter Simulation Conference, pp. 1934-1945.

(Liu, 2013) Qiang Liu, Xinhui Zhang, Yan Liu, Lebin Lin. Spreadsheet Inventory Simulation and Optimization Models and their Application in a National Pharmacy Chain. In: INFORMS Transactions on Education 14(1), 2013, pp. 13-25.

(Mahamani, 2010) Mahamani, A., Rao, K.P. Development of a spreadsheet based vendor managed inventory model for a single echelon supply chain: a case study. In: Serbian Journal of Management 5 (2), 2010, pp. 199-211.

(Mason, 2009) Magableh, G.M., Mason, S.J. An integrated supply chain model with dynamic flow and replenishment requirements. In: Journal of Simulation, 2009, Vol. 3, pp. 84-94.

(Milstein, 1995) Milstein, G.N. Numerical integration of stochastic differential equations. New York-London: Kluwer Academic Publishers, 1995.

(Monk, 2006) Monk, E., Wagner, B. Concepts in Enterprise Resource Planning, 2nd Edition, Thomson Course Technology, 2006.

(Muckstadt, 2010) Muckstadt, J. A., Sapra, A. Principles of Inventory Management. When You Are Down to Four, Order More. Springer Series in Operations Research and Financial Engineering, 2010.

(Muravjovs, 2007a) Eugene Kopytov, Leonid Greenglaz, Aivars Muravjovs, Leonid Burlakov. Investigation of two strategies in inventory control system with random parameters. In: European conference on modelling and simulation. Prague, Czech Republic, 2007, pp. 566-571.

(Muravjovs, 2007b) Eugene Kopytov, Leonid Greenglaz, Aivars Muravjovs. Multiproduct inventory control models with fixed time interval between moments of ordering. In: Reliability and statistics in transportation and communication, Riga, Latvia, 2007, p. 149.

(Muravjovs, 2007c) Eugene Kopytov, Leonid Greenglaz, Aivars Muravjovs, Edvin Puzinkevich. Modelling of two strategies in inventory control system with random lead time and demand. In: Computer modelling and new technologies, Vol. 11, #1, Riga, Latvia, 2007, pp. 21-30.

(Muravjovs, 2008a) Eugene Kopytov, Leonid Greenglaz, Aivars Muravjovs. Modelling of the multiproduct inventory problem. In: International Conference Reliability and Statistics in Transportation and Communication, Riga, Latvia, 2008, pp. 139-145.

(Muravjovs, 2008b) Айвар Муравьев. Моделирование систем управления запасами в среде ExtendSIM. In: Research and technology: step into the future, Vol. 3, #2, Riga, Latvia, 2008, pp. 21-22.

(Muravjovs, 2010a) Eugene Kopytov, Aivars Muravjovs. Supply Chain Simulation in ExtendSim Environment. In: Proceedings of International Conference Reliability and Statistics in Transportation and Communication, Riga, Latvia, 2010, pp. 447-456.

(Muravjovs, 2010b) Eugene Kopytov, Aivars Muravjovs. Supply chain simulation in ExtendSim environment. In: International Conference Reliability and Statistics in Transportation and Communication, Riga, Latvia, 2010, pp. 90-91.

(Muravjovs, 2011) Eugene Kopytov, Aivars Muravjovs. Simulation of inventory control system for supply chain "producer–wholesaler–client" in ExtendSim environment. In: European conference on modelling and simulation. Krakow, Poland, 2011, pp. 580-586.

(Muravjovs, 2012a) Aivars Muravjovs, Eugene Kopytov. Comparative Analysis of Simulation Packages for Inventory Control System Modelling. In: European Conference on Modelling and Simulation, Koblenz, Germany, 2012, pp. 595-601.

(Muravjovs, 2012b) Aivars Muravjovs, Eugene Kopytov. Multiple criteria analysis and choice of simulation tools for inventory control modelling using AHP method. In: International Conference Reliability and Statistics in Transportation and Communication. Riga, Latvia, 2012, pp. 211-220.

(Muravjovs, 2012c) Aivars Muravjovs. Выбор системы моделирования для решения задач управления запасами. In: Research and Technology – Step into Future, Volume 7, Riga, Latvia, 2012, pp. 127-129.

(Muravjovs, 2013) Aivars Muravjovs, Eugene Kopytov. Simulation Model of Current Stock of Divisible Products in ExtendSim Environment. In: European Conference on Modelling and Simulation, Alesund, Norway, 2013, pp. 664-669.

(Nerush, 2001) Nerush Yu.M., Logistika: Uchebnik, Moscow, Publ. House: Yunita, 2001.

(Oberstone, 2007) Joel Oberstone. Teaching Inventory Management Simulation Using E-Learning Software:Blackboard, Elluminate Live!, and Jing. University of San Francisco / School of Business and Management, 2007.

(OR/MS Today, 2009) OR/MS Today 2009 by the Institute for Operations Research and the Management Sciences.

(OR/MS Today, 2013) Simulation Software Survey, OR/MS Today, October 2013http://www.orms-today.org/surveys/Simulation/Simulation.html

(Poles, 2009) Poles, R., Cheong, F. Inventory Control in Closed Loop Supply Chain using System Dynamics. In: 27th International Conference of the System Dynamics Society, 2009.

(Power, 2001) Power, D. What are Analytical Information Systems? In: DSS News, Vol. 2, No. 22, October 21, 2001.

(Ptak, 2011) Ptak, C., Smith, Ch. Orlicky's Material Requirements Planning, 3rd edition, McGraw Hill, New York, 2011.

(Railsback and Grimm, 2011) Railsback, S.F., Grimm, V. Agent-Based and Individual-Based Modelling: A Practical Introduction. Princeton University Press, 2011.

(Ross, 1992) Ross, S. Applied Probability Models with Optimization Applications. Dover Publications, Inc., New York, 1992.

(Rossetti, 2013) Rossetti, M.D., Shbool, M., Varghese, V., Pohl, E. Investigating the effect of demand aggregation on the performance of an (R, Q) inventory control policy. In: Proceedings of the 2013 Winter Simulation Conference, pp. 3318-3329.

(Roy, 1996) Roy, B. Multi-criteria methodology for decision aiding. Dordrecht: Kluwer Academic Publishing, 1996.

(Praude, 2013) Praude, V. Loģistika. Rīga, Burtene, 2013.

(Rud, 2009) Rud, O. Business Intelligence Success Factors: Tools for Aligning Your Business in the Global Economy. Hoboken, N.J: Wiley & Sons, 2009.

(Saaty, 2001) Saaty, T.L. Decision Making for Leaders: The Analytic Hierarchy Process for Decisions in a Complex World, New Edition (Analytic Hierarchy Process Series, Vol. 2), RWS Publications, 2001.

(Samvedi et al., 2011) Avinash Samvedi and Vipul Jain. Studying impact of various inventory policies on a supply chain with intermittent supply disruptions. In: Proceedings of the 2011 Winter Simulation Conference, pp. 1641-1649.

(Sang, 2012) Sang, H., Takakuwa, S. A simulation-based approach for obtaining optimal order quantities of short-expiration date items at a retail store. In: Proceedings of the 2012 Winter Simulation Conference, pp. 1466-1477.

(Savrasov, 2007) Savrasov, M., Toluyew, Y. Application of Mesoscopic Modelling for Queuing Systems Research. In: 7th International Conference, Reliability and Statistics in Transportation and Communication, Riga, 2007, pp. 94-99.

(Savrasov, 2008) Savrasov, M., Tolujew, J. Mesoscopic approach to modeling a traffic system. In: MBITS-08, Transport and Telecommunication Institute, Riga, 2008, pp. 147-151.

(Savrasovs, 2010) Savrasovs, M. The application of a discrete rate approach to traffic flow simulation. In: 10th International Conference, Reliability and Statistics in Transportation and Communication, Riga, 2010, pp. 433-439.

(Savrasovs, 2011) Savrasovs, M. Urban Transport Corridor Mesoscopic Simulation. In: 25th European Conference on Modelling and Simulation (ECMS'2011), pp. 587-593.

(Schenk et al., 2009) Schenk M., Tolujew J., Reggelin T. Mesoscopic Modelling and Simulation of Logistics Networks. In: Preprints of the 13th IFAC Symposium on Information Control Problems in Manufacturing. Moscow, 2009, pp. 586–591.

(Schenk et al., 2010) Schenk M., Tolujew J., Reggelin T. A Mesoscopic Approach to the Simulation of Logistics Systems. In: Advanced Manufacturing and Sustainable Logistics, Springer Berlin Heidelberg, 2010, pp. 15–25.

(Schriber, 2002) Schriber, T., Brunner, D. Inside discrete-event simulation software: How it works and why it matters. In: Proceedings of the 2002 Winter Simulation Conference, pp. 97-107.

(Seila, 2002) Seila, A.F. Spreadsheet Simulation. In: Proceedings of the 2002 Winter Simulation Conference, pp. 17-22.

(Seila, 2003) Seila, A.F., Ceric, V., Tadikamalla, P. Applied Simulation Modelling, Australia: Thomson Learning, 2003.

(Seila, 2006) Seila, A.F. Spreadsheet Simulation. In: Proceedings of the 2006 Winter Simulation Conference, pp. 11-18.

(Sprancmanis, 2011) Sprancmanis, N. Uzņēmējdarbības loģistikas pamati. Rīga, Burtene, 2011.

(Sterligova, 2005a) Стерлигова А.Н. Систематизация элементов моделей управления запасами в звеньях цепей поставок. Логистика и управление цепями поставок, №4, 2005, с. 36-54.

(Sterligova, 2005b) Стерлигова А.Н. Оптимальный размер заказа, или Загадочная формула Вильсона. Логистик&система, №2, 2005, с. 64-69, №3, 2005, с. 62-71.

(Sterman, 1989) Sterman, J.D. Modelling Managerial Behaviour: Misperceptions of Feedback in a Dynamic Decision Making Experiment. In: Management Science, 1989, 35 (3), pp. 321-339.

(Sterman, 2000) Sterman, J. Business Dynamics: Systems Thinking and Modelling for a Complex World. Irwin/McGraw-Hill, 2000.

(Strickland, 2011) Strickland, J. Discrete Event Simulation using ExtendSim 8. Publisher Lulu Inc., 2011.

(Suri, 1998) Suri, R. Quick Response Manufacturing. A Companywide Approach to Reducing Lead Times, Productivity Press, 1998.

(Terlunen, 2014) Terlunen S., Horstkemper D., Hellingrath B. Adaption of the discrete rate-based simulation paradigm for tactical supply chain decisions. In: Proceedings of the 2014 Winter Simulation Conference, pp. 2060–2071.

(Tersine, 1994) Tersine, R. J. Principles of Inventory and Materials Management. 4th ed. New York: North-Holland, 1994.

(Tikhonov, 2004) Tikhonov A.N., A.A. Samarsky. Equations of Mathematical Physics. Moscow, Lomonosov MSU Press, 2004.

(Tolujew, 2008) Tolujew, J., Savrasov, M. Transport System Mesoscopic Model Validation Using Simulation on Microlevel. In: 8th International Conference, Reliability and Statistics in Transportation and Communication, Riga, 2008, pp. 297-304.

(Triantaphyllou, 2000) Triantaphyllou, E. Multi-criteria decision making methods: A comparative study. London: Kluwer Academic Publishers, 2000.

(Verma, 2009) Verma, R., Gupta, A., Singh, K. A critical evaluation and comparison of four manufacturing simulation software. In: Kathmandu university journal of science, Engineering and technology, 2009, Vol. 5, No. 1, pp. 104-120.

(Vlachos, 2007) Vlachos, D., Georgiadis, P., Iakovou, E. A system dynamics model for dynamic capacity planning of remanufacturing in closed-loop supply chains. In: Computers & Operations Research, vol. 34, no. 2, 2007, pp. 367-394.

(Waldner, 1992) Waldner, J.-B. CIM: Principles of Computer Integrated Manufacturing. Chichester: John Wiley & Sons Ltd, 1992.

(Watson, 2007) Watson, H.J., Wixom, B.H. The Current State of Business Intelligence. Computer, vol. 40, no. 9, 2007, pp. 96-99.

(Wu, 2013) Wu, H., Evans, G., Heragu, S. Optimization of production and inventory policies for dishwasher wire rack production through simulation. In: Proceedings of the 2013 Winter Simulation Conference, pp. 2666-2676.

(Zabawa, 2007) Zabawa, J., Mielczarek, B. Tools of Monte Carlo simulation in inventory management problems. In: Proceedings 21st European Conference on Modelling and Simulation ECMS 2007, pp. 56-61.

(Zipkin, 2000) Zipkin, P. H. Foundations of Inventory Management. New York: Irwin/ McGraw-Hill, 2000.

(Zwicker, 1980) Zwicker, E. System Dynamics in Inventory and Production Planning. In: OR Spektrum Vol. 1, 1980, pp. 143-168.

Appendix 1. OR/MS Today, October 2013: Simulation Software Survey

Software	Vendor	Typical Applications of the software	Primary Markets for which the software is applied
aGPSS	aGPSS Simulation System Education	General purpose discrete events simulation of situations with uncertainty, requiring many runs	Education, esp. students of business, logicstics, supply chain systems
Analytica	Lumina Decision Sytems, Inc	Analytics and statistics, data exploration, dynamic systems, Monte Carlo, optimization.	Many, including energy, environment, economics, health, manufacturing, high tech, education, gov, defense
Arena	Rockwell Automation	Arena is typically used to model existing systems and test out proposed changes to those system environments.	Defense, Manufacturing, Supply Chain, Health Care, Oil and Gas, and Academia
Bluesss Simulation System	Stanislaw Raczynski	General purpose, discrete/continuous simulation package	Academic, business, engineering
Clinical Trials Simulator	ProModel Corporatio n	decision support technology that generates realistic data on how patient recruitment will perform	Pharmaceuticals; Life Sciences
CSIM20	Mesquite Software, Inc.	CSIM20 is a library that enables C/C++ programmers to create process-oriented simlation models.	engineers, analysts and programmers with a need for simulation models of large, complex systems
DPL	Syncopation Software	capital investment decision analysis, R&D prioritization, risk analysis, Monte Carlo simulation, valuation	pharmaceutical, oil & gas, energy, new product development, environmental, utilities
Enterprise Dynamics 9	INCONTROL Simulation Solutions	Simulation of large scale industrial, logistics and transportation systems	Manufacturing, Warehousing, Supply Chains, Automated Material Handling, Rail, Ports and Airports
Enterprise Portfolio Simulator (EPS)	ProModel Corporation	Web-based simulation analysis of multiple, simultaneous project/product plans across one or more portfolios of projects	Project & Portfolio Planning;Strategic Resource Capacity Planning;Product Development,R&D;Projec t Selection & Prioritization
ExtendSim AT	Imagine That Inc.	Simplifies the modelling and analysis of complex systems; plus simulates rate-based flows, batch processes, and bulk systems.	High-speed/volume or mixed-mode environments, packaging lines, chemical processes, distribution, system design & reliability.
ExtendSim OR	Imagine That Inc.	Adds message-based DE architecture & capabilities to a powerful	Mfg & business modelling, healthcare, supply chain, transportation, communication, logistics, lean, six sigma, cost analysis

		simulation engine to track & analyze varying entity behaviour.	
ExtendSim Suite	Imagine That Inc.	Professional 3D modelling of continuous, DE, and discrete rate processes. Create impressive presentations for upper management	Traffic and transportation systems, emergency rooms, production lines, customer flow, defence, ports, management presentation
FlexSim	FlexSim Software Products, Inc.	Simulation and modelling of any process, with the purpose of analyzing, understanding, and optimizing that process.	Manufacturing, packaging, warehousing, material handling, supply chain, logistics, healthcare, factory, aerospace, mining.
FlexSim Healthcare	FlexSim Software Products, Inc.	Simulation and modelling of any healthcare process, with the purpose of analyzing, understanding, and optimizing that process.	Healthcare, healthcare systems, architecture.
Fluid Flow Simulator Fluids6	Stanislaw Raczynski	Comutational fluid dynamics	Engineering, research, academic, scientific
ForeTell-DSS	DecisionPath, Inc.	scenario-based "what-if" simulations of critical busines/government decisions	Government, Life Sciences, Financial Services
GoldSim	GoldSim Technology Group	engineering risk analysis, strategic planning, system design and reliability, water resource management, waste management	environmental engineering, mining, water resources, energy, nuclear, waste management
GPSS/H	Wolverine Software	General-Purpose discrete-event simulation applications	Queueing models of modest-to-medium size
Integrated Performance Modelling Environment (IPME)	Alion Science	human performance modelling, workload modelling, trade-off analysis	manufacturing, defence, nuclear, air traffic control, other commercial applications
MedModel Optimization Suite	ProModel Corporatio n	Design, plan, evaluate and improve processes of hospitals, clinics, and other healthcare systems to optmize performance	Hospitals, Clinics, Healthcare Systems, Medical Device Manufacturing and Sales
Micro Saint Sharp	Alion Science and Technology	General purpose-Process improvement/optimization, cost justification, lean implementations, human/system performance.	Healthcare, Human Performance, Supply Chains, Manufacturing, Defence, Marketing, Finance, Energy, Education, Transportation

Oracle Crystal Ball Suite	Oracle Corporation	Spreadsheet-based Monte Carlo simulation, optimization, and time-series forecasting	Business, financial, energy, pharma, environmental, healthcare, defence, manufacturing, education, telecommunication
Patient Flow Simulator	ProModel Corporation	Strategic High Level Patient Flow and Bed Management	Healthcare
Pedestrian Dynamics	INCONTROL Simulation Solutions	Simulation of pedestrians in large infrastructures	Stadiums, Railway Stations, Airports, Vessels, Commercial Infrastructures, Urban Planning
Portfolio Simulator	ProModel Corporation	Simulation and optimization analysis of multiple,simultaneous project/product plans across one or more portfolios of proje	Project & Portfolio Planning;Strategic Resource Capacity Planning;Product Development;R&D;Project Selection & Prioritization
Process Simulator	ProModel Corporation	Lean, SixSigma, value stream mapping, process mapping, flow chart simulation, continuous process improvement	All
Project Simulator	ProModel Corporation	Enables project managers to more accurately predict outcomes of their project plans	Anyone who uses Microsoft Project
ProModel Optimization Suite	ProModel Corporation	Lean, SixSigma, capacity planning, cost analysis, process modelling, cycle time reduction, throughput optimization and more	Manufacturing and logistics, pharmaceutical, defense
Proof Animation - P3D (3D) & P5 (2D)	Wolverine Software	High-end 2D and 3D animation of dioscrete-event simulations	Logistice, transportation, material flow, manufacturing
SAS Simulation Studio	SAS	Discrete event simulation: supply chains, resource management, capacity planning, flow analysis, cost analysis, more.	Manufacturing, banking, pharmaceutical, health care, energy, government agencies, retail, insurance, transportation, etc.
Service Model Optimization Suite	ProModel Corporation	Design, plan, evaluate and improve service industry systems such as Financial Services, Logistics, Business Re-Engineering	Financial Services, Logistics, Transportation, Food & Hotel Services, Entertainment, and Other Service Industries
Simio Design/Team	Simio LLC	Ideal product for professional modellers and researchers. Powerful	Aerospace, Health Care, Logistics, Defense, Mining, Pharma, Foods, Airports, Transportation, Automotive, Electronics

		OO modelling and integrated 3D animation for rapid modelling.	
Simio Enterprise	Simio LLC	Increase your model lifecycle -- single tool builds Design models and extends to Risk-based Planning, and Scheduling.	Aerospace, Health Care, Logistics, Defense, Mining, Pharma, Foods, Airports, Transportation, Automotive, Electronics
Simio Express	Simio LLC	Powerful, fully functional object-based modelling with integrated 3D animation providing both a fast start and rapid modelling.	Aerospace, Health Care, Logistics, Defense, Mining, Pharma, Foods, Airports, Transportation, Automotive, Electronics
SIMPROCESS	CACI	Business Process Improvement, Process Management, Predictive Analytics	Government, Commercial, Education
SIMSCRIPT III	CACI	High-fidelity discrete-event modelling and simulation applications	Military simulations, Air traffics control simulations, war gaming, transportation, networks analysis, logistics
SIMUL8 Professional	SIMUL8 Corporation	Lean, Assembly Line, Strategic planning, Operations, BPMN, Line Balancing, Healthcare Systems, Shared Services, Capacity Plan	Manufacturing, Healthcare, Education, Supply Chain, Logistics, Business, BPMN, Lean, Government, Back Office, Contact centres
SLIM	MJC2	Simulation & modelling of logistics networks and supply chains	Logistics, manufacturing, transport
SLX	Wolverine Software	High-end applications for which off-the-shelf software does not exist.	Logistics, transportation, material-handling, telecommunications
Stat::Fit	Geer Mountain Software Corp.	Statistically fits to your data the most useful analytical distribution and exports into specific forms for simulation.	Simulation and Modelling, Risk Assessment, Reliability, Quality, Engineering and Financial Management
TARGIT Decision Suite 2013	TARGIT	Analytics and Reporting on datasets with fewest possible clicks, integrates with SQL Server, HADOOP, Google BigQuery	Business Intelligence, Analytics and reporting across all industries
Vanguard Business Analytics Suite	Vanguard Software	Strategic Forecasting, Financial Analysis, Cost Modelling, R&D Pipeline Modelling, Portfolio Analysis, Risk Analysis	All
Vanguard System	Vanguard Software	Strategic Forecasting, Financial Analysis, Cost Modelling, R&D	All

		Pipeline Modelling, Portfolio Analysis, Risk Analysis	

Appendix 2. Selection of simulation tools on the basis of AHP method

The existence of a variety of simulation tools makes the issue of choosing the most suitable package for inventory control system modelling rather difficult. Many authors considered the problem of simulation tools, and find that efficiency of evaluation and selection to be the most appropriate solutions. Usually, this problem is formulated as multiple criteria choice problem (for instance, see Law, 2003 & Verma, 2009). It should be noted that in these researches the amount of estimated indicators is significantly different. Therefore, the authors of the paper (Law, 2003) have investigated the effectiveness of 20 discrete event simulation tools using a small number of indicators, but the authors of the work (Verma, 2009) have assessed four software tools, estimating more than 200 parameters. Some of the parameters have been evaluated by expert methods; some parameters were obtained as a result of the experiments.

On the other hand, it should be emphasized that the known studies do not consider the problem of simulation tools efficiency for inventory control modelling, which has a number of specific characteristics. For this reason, the presented research has been executed. Taking in account the specificity of simulation of inventory control models the authors have formed the system of criteria which includes 28 indicators characterizing the efficiency of simulation packages. In the previous articles, (Muravjovs, 2012) were proposed a simple assessment of simulation tools efficiency based on the peer review. In the present paper, one of the most effective methods of multiple criteria choice – the Analytic Hierarchy Process (AHP) method (Saaty, 2001) – is applied. In our opinion, this method is the most suitable for the examined problem solving. It is necessary to mention important advantages of the AHP method: it gives possibility to distribute the criteria by several groups and to evaluate the significance of every group components independently; the computable consistency of the judgments allows controlling the accuracy of estimation; it does not require any special software; the algorithm of AHP operation and the table form of representation of principal and intermediate results give possibility to demonstrate visually the reason for choosing the certain alternative. To illustrate the offered approach, the authors have evaluated two alternatives of simulation tools: packages ExtendSim and AnyLogic. During this research, various inventory control models have been created and realized in ExtendSim 8 and AnyLogic 6.7 environments, and one of these models is presented in the paper.

The procedure of evaluation the effectiveness and choice of simulation tools for inventory control system includes the following stages:

1. Selection of simulation tools (packages) for evaluation;
2. Implementation of various inventory control problems in the selected tools environment;
3. Formation of the system of criteria characterizing the efficiency of simulation tools;
4. Choice of method for simulation tools assessment;
5. Performance of the analysis and choice of simulation tools.

The contents of the separate stages are considered below.

Selection of criteria for inventory control tasks

In the process of criteria formation, the authors have been focused on the specificity of inventory control models simulation. This research offers a system of criteria which includes twenty eight indicators. The criteria are distributed in five groups shown in the Table 5.1. Distributing indicators in the groups allows involving various experts in the assessment process: programmers, graphic interface creators, support team, etc. The hierarchical structure of the criteria is shown on Fig. 5.1. This structure has two levels of the hierarchy (Muravjovs, 2012a, Muravjovs, 2012b, Muravjovs, 2012c).

Table 5.1 Groups of criteria of the effectiveness of inventory control simulation tools

Group Name	Criteria
General	Programming language, Primary domain, Operating system, Data connectivity with other applications, Model packaging, Price for universities, Optimization
Programming aspects	Programming flexibility, Support of programming concepts, Built in functions, Debugging, Code editor, External code connection, Built-in random numbers generators
Visualization	Animation, Logical animation, Payback mode, 3D Animation, Graphic library
Simulation	Model execution speed, Simplification of simulation process, Variety of simulation approaches, Hardware requirements
User support	Documentation, Training courses, Users forum, Knowledge base, Demo models and libraries

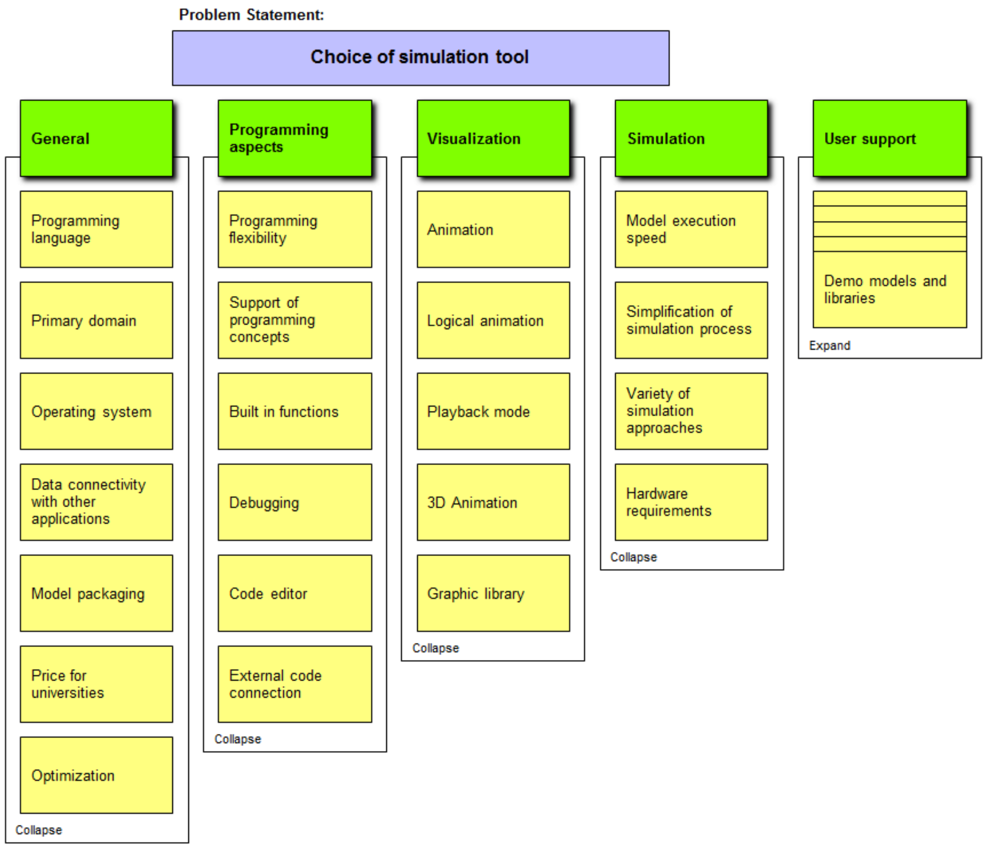


Fig. 5.1 Hierarchy of the criteria for evaluating simulation packages

AHP algorithm

There are currently various methods that have been developed and implemented to analyse and choose from a range of alternatives using multiple-criteria. These methods include multiple criteria decision making (MCDM), multiple criteria decision analysis (MCDA), and multiple attribute decision making (MADM) (Köksalan, 2011). The existence of this variety of methods makes the issue of choosing the most suitable one rather difficult (Triantaphyllou, 2000). The authors have analyzed the possibility of employing various popular MCDM methods (the simple additive weighting (SAW) method (Hwang, 1981), the AHP method and collections of ELECTRE methods (Roy, 1996) to solve the problem of choosing the best simulation tools for inventory control modelling. In our opinion, the AHP method seems to be a more attractive choice since it allows structuring the choice procedure as a hierarchy of several levels. It allows the distribution of the criteria by several groups, and evaluates the significance of each group's components. Consequently, different groups of criteria can be evaluated by different experts. The opportunity of the pairwise comparison of a smaller number of criteria in every group allows the experts to determine better weighted values according to these criteria. The estimation of the significance of the criteria groups can be determined by the experts with greater qualification. The AHP method also allows controlling the consistency of experts' judgments, making it possible to increase the reliability of estimation.

Summary, in the judgment of the authors, AHP method is the most efficient for choosing the optimal simulation tool. The method allows arranging the alternatives of simulation tools by degree of their efficiency and showing their difference in the given set of criteria.

Selection of a simulation modelling system to solve the inventory control problem

To evaluate the efficiency of the selected simulation packages for inventory control tasks, we have investigated various models, among them: single- and multiple-product, with random demand, with random and fixed lead time, with different ordering strategies, with restrictions on storage and financing resources (for instance, see Muravjovs, 2011).

In this paper as example lets consider a single-product stochastic inventory control model under following conditions. The demand for goods D has a normal distribution with known parameters mean μ and standard deviation σ. In the moment of time, when the stock level $\varphi(t)$ falls till certain level R, a new order is placed (see Fig. 5.2). The quantity R is called as reorder point. The order quantity Q is constant. Supposing that $Q \geq R$. The lead time L (time between placing an order and receiving it) is fixed. There is the possible situation of deficit, when demand D_L during lead time L exceeds the value of reorder point R. supposing that in case of deficit the latter cannot be covered by expected order.

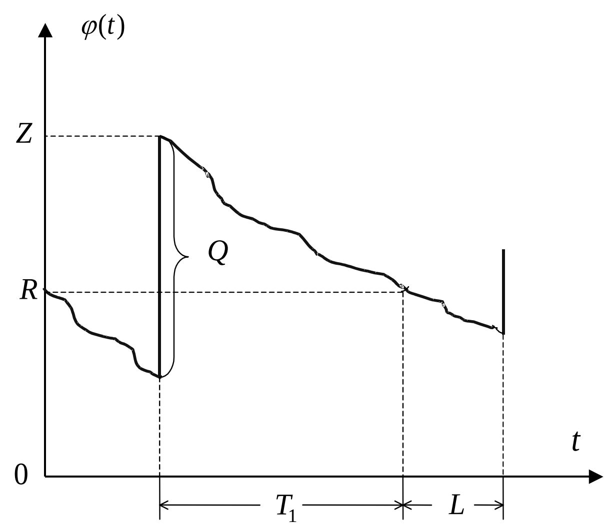


Fig. 5.2 Dynamics of inventory level during one cycle

The quantity of goods in stock in the time moment immediately after order receiving is denoted as Z. We can determine this quantity of goods Z as function of demand D_L during lead time L:

$$Z = \begin{cases} R + Q - D_L, & if\ D_L\ < R \\ Q, & if\ D_L\ < R \end{cases} \tag{5.1}$$

Expression (5.1) is basic. It allows expressing different economical indexes of the considered process.

Let T be the duration of a cycle. Length of the cycle consists of two parts: time T_1 between receiving the goods and placing a new order and lead time L, i.e. $T = T_1 + L$ (seeFig. 5.2 and Fig. 5.2).

Supposing that following parameters of the model are known:

- the ordering cost C_0 is fixed;
- the holding cost is proportional to quantity of goods in stock and holding time with coefficient of proportionality C_H;
- the shortage C_{SH} should not exceed 1,5% of demand.

The principal aim of the considered model is to define the optimal values of order quantity *Q* and reorder point *R*, which are control parameters of the model. A criterion of optimization is the minimum of average total cost in inventory control system per time unit. Denote this average total cost by *E(AC)*, which can be found as average total cost during one cycle divided by average cycle time *E(T)*:

$$E(AC) = \frac{E(TC_H) + E(TC_{SH}) + C_0}{E(T)} \tag{5.2}$$

Where $E(TC_H)$ and $E(TC_{SH})$ are average holding and average shortage costs within cycle accordantly.

Note that costs $E(TC_H)$and $E(TC_{SH})$ depend on control parameters R and *Q*. Analytical formulas for these economic indicators have been presented in the paper (Kopytov 2004). For problem solving we have to minimize criteria (5.2) by R and *Q*.

The realizations of considered model in ExtendSim 8 and AnyLogic 6.7 environments are presented in the next sections. In the examples of simulation presented below we have used the following initial data. The demand for goods *D* has a normal distribution with parameters mean $\mu =$

80 and standard deviation $\sigma = 18$. The lead time L equals 5 days. Starting values for control parameters are: R = 500 and Q = 600.

Simulation model in ExtendSim 8 environment

For solving the problem considered above we have used Discrete Events simulation method realized in the package ExtendSim 8 (Strickland, 2011). In discrete event simulation the operation of a system is represented as a chronological sequence of events. Each event occurs at an instant in time and marks a change of state in the system (Krahl, 2007).

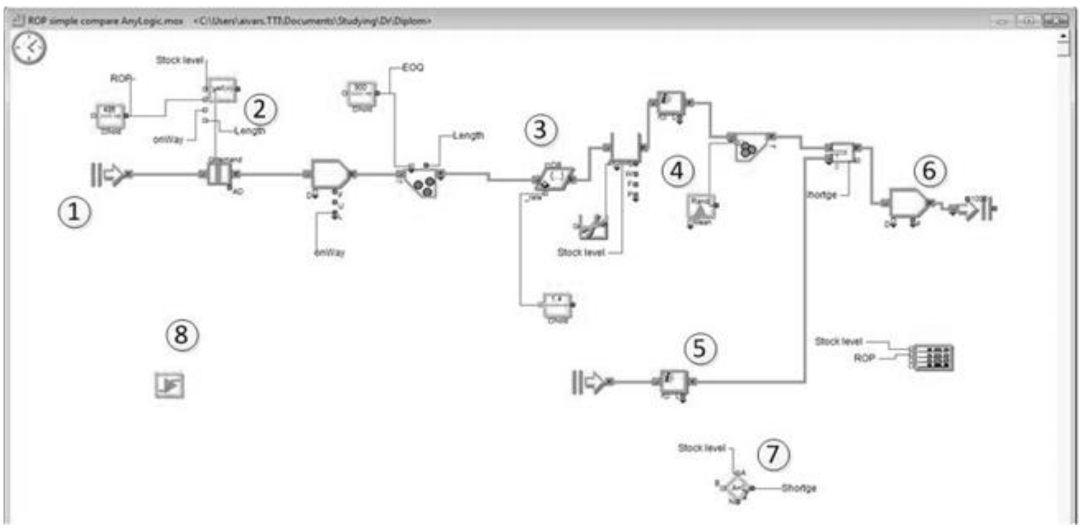


Fig. 5.3 Simulation model

Let’s consider the main parts of the simulation model shown in Fig. 5.3. In the area #1, there are placed executive and generation blocks that control model time and transaction generation in the model. Area #2 is responsible for order making with equation block results in area #7. Next area #3 represents transportation activity. Demand for goods and reductions in stock are simulated in area #4. Shortage occurrences are represented in area #5. Area #6 is the end of the model and is used for transaction termination. An example of inventory control simulation is presented on Fig. 5.4 Example of simulation process in ExtendSim. The plot shows the stock level for 100-day period simulation.

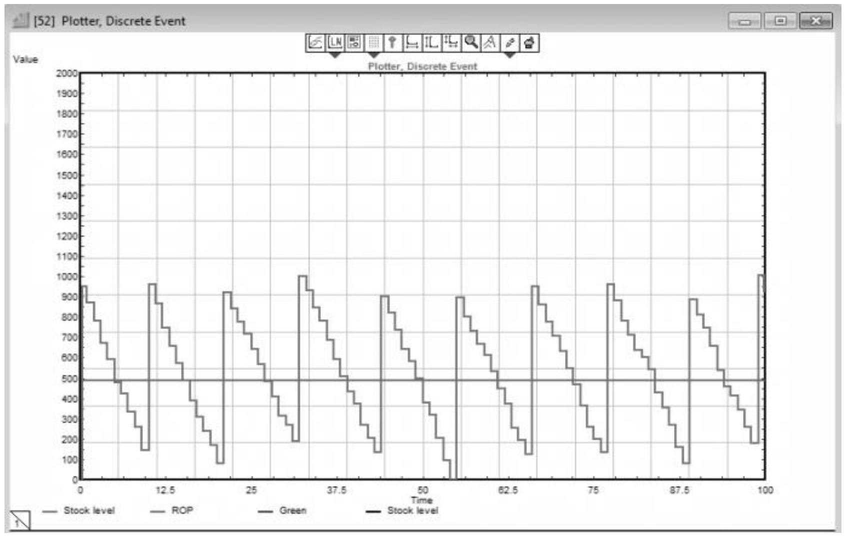


Fig. 5.4 Example of simulation process in ExtendSim

Simulation model in Anylogic 6.7 environment

The model considered above uses discrete event simulation approach. On the other hand, AnyLogic developers promote agent-based modelling (ABM) approach. In ABM, the focus is on the individual agents, their rules, their behaviours, and their interactions with each other and the environment. Collectively agents may exhibit emergent behaviours, such as self-organization. Since agents do not follow a pre-scripted flow (as in Discrete Event) and their structure is not pre-specified at the global/aggregate level (as in System Dynamics), they can exhibit novel or surprising behaviours that were not anticipated during the development. ABM is a great methodology for exploring non-linear dynamic environments. ABM is also well suited for situations with no precedent or where past data or experience does not exist. When combined with data and data analytics, ABM forms one of the most powerful predictive analytics/forecasting methodology. From an architectural viewpoint, a typical AnyLogic agent based model would have at least two active object classes. There would be a main class for a top-level object where agents would be contained and a class for an agent or person. The Person class in most cases would be declared as Agent, which is a special subclass of the ActiveObject class that extends the latter with services useful for agent based modelling. A number of agents would be embedded into the Main object, as a replicated object of type Person. One or more Environment constructs may be defined at the level of Main to specify properties shared by the agents.

The suggested inventory model realization is presented on Fig. 5.5. In this figure, we can see variable and parameter window that also contains agents for distributer, retailer, truck, and events for ordering and transportation tasks. Those agents can be used in models that are more complex with multiple retailers, distribution point and multiproduct ordering.

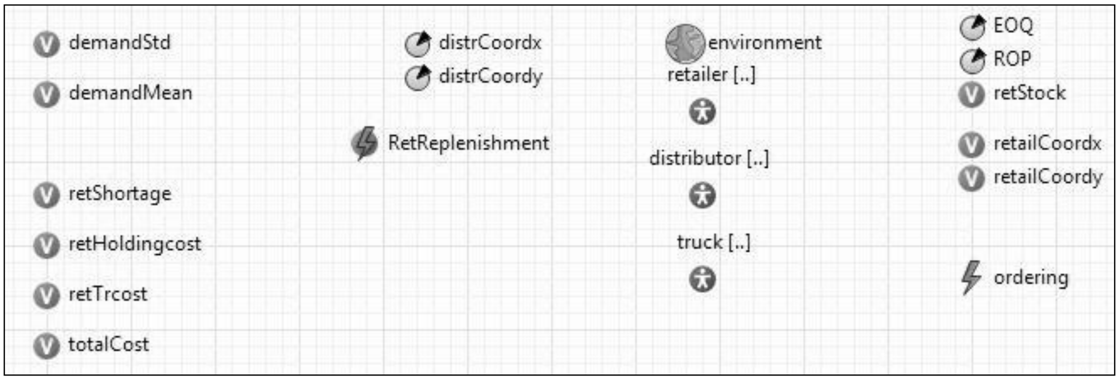


Fig. 5.5 Inventory control model in AnyLogic environment

In AnyLogic 6.2 special graphical tools Action Charts are introduced. The designers of AnyLogic have suggested Action Charts as a simple and commonly accepted language, that makes action/decision logic visual, easy to communicate to other people and easier to develop at the same time. Action Charts consist of nested elements, each corresponding to a Java statement: decision-statement, several kinds of loops, local variable declaration, code section, etc. An action chart is straightforwardly mapped to a Java method and therefore is equally efficient. The developers can choose colours and labels of the action chart boxes to further improve its expressiveness. An example of action chart for inventory control model with reorder point is shown on Fig. 5.6.

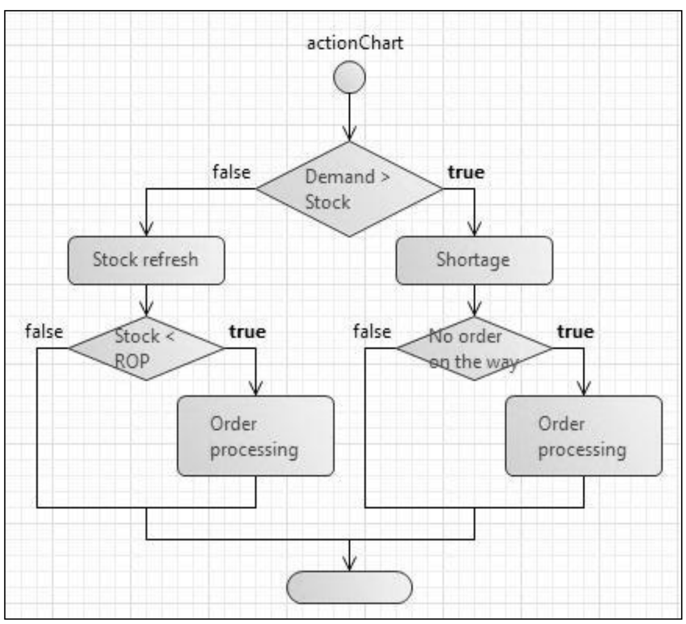


Fig. 5.6 Action chart for inventory control model

The example of simulation process realization in AnyLogic environment is presented on Fig. 5.7 Example of simulation process in AnyLogic. It can be noticed with a naked eye that plots on Fig. 5.4 and Fig. 5.7 are very similar. This is natural, because plots show the simulation results of the same task.

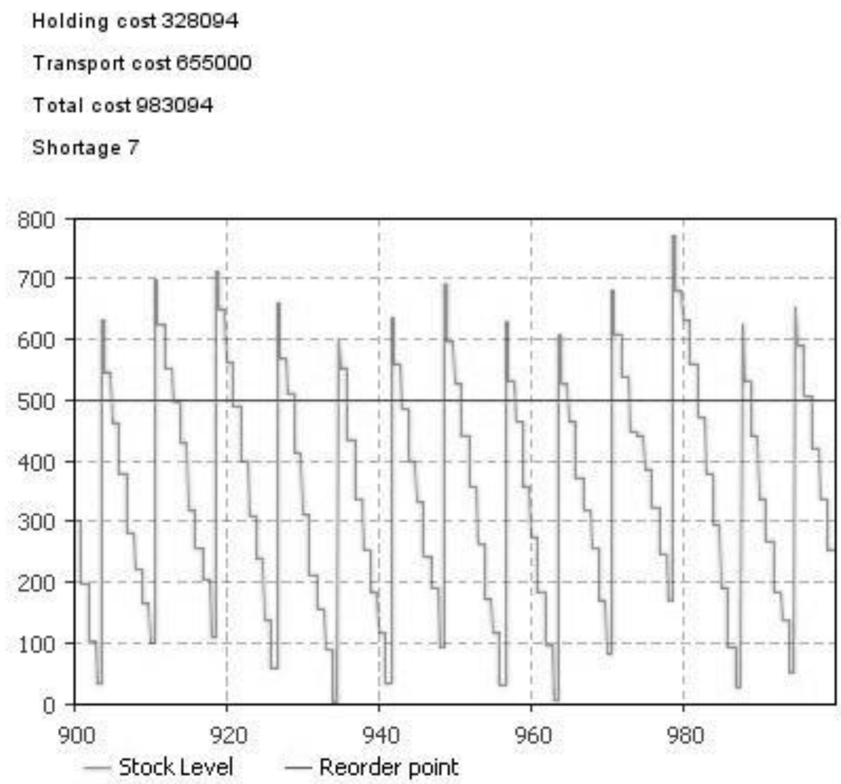


Fig. 5.7 Example of simulation process in AnyLogic

Optimization of inventory control system

As it was mentioned above that control parameters for presented model are: order quantity Q and reorder point R. In the example considered, the optimum search of control parameters is carried out in the range for $400 \leq Q \leq 1200$ and $200 \leq R \leq 800$. Both presented simulation packages have integrated optimization tools, which can be used to find the optimal result.

At first let’s consider optimization process in AnyLogic. To run the optimization process we should manually create and tune an optimization experiment. In tuning process, we need to create user interface, define objective function, optimization parameters and constraints. An example of optimization process in AnyLogic environment is presented on Fig. 5.8.

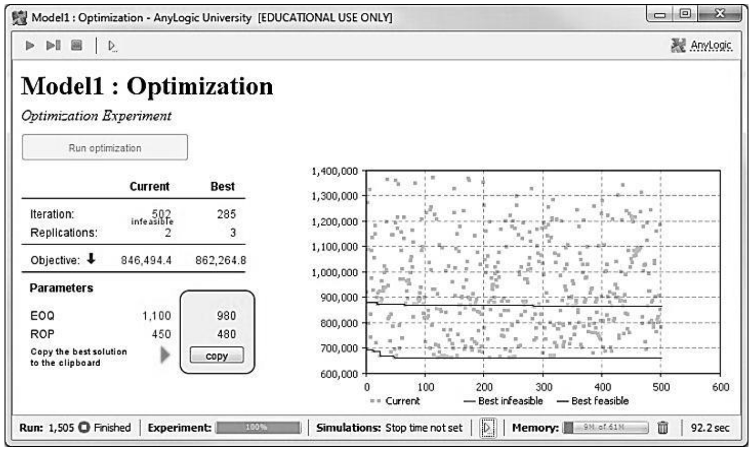


Fig. 5.8 Example of optimization process in AnyLogic

Next, let us look at the same procedure in ExtendSim tool. We can use the same user interface, just putting optimization block into the model window; all other steps are similar to AnyLogic except ExtendSim, where we can use optimization parameters only in constraints. The example of optimization process in ExtendSim environment is shown on Fig. 5.9. In

Table 5.2 the final results of optimization for both simulation tools are shown. Apparently the obtained results are very similar to each other.

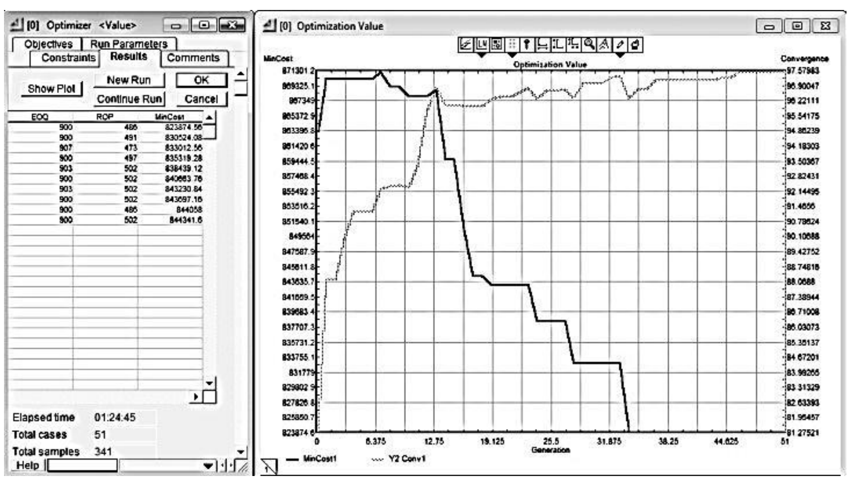


Fig. 5.9 Example of optimization process in ExtendSim

Table 5.2 Results of inventory control system optimization

Parameter	Simulation tools	
	AnyLogic	ExtendSim
Reorder point	480	486
Order quantity	980	900
Total cost	862 264 EUR	823 874 EUR

The author also have investigated more complex models of inventory control, which are not presented in this paper due to size limitation for the article. The simulation processes were analysed by author in the comparative assessment of simulation tools presented below. The system of criteria shown on Fig. 5.1 has been used for comparative assessment of efficiency of ExtendSim 8 and AnyLogic 6.7 packages for inventory control system simulation. Consequently, the different groups of criteria have been evaluated by different qualified experts: programmers have assessed general and programming aspects criteria, experts from the supporting service have evaluated user support criteria, while decision makers have estimated visualization and simulation criteria.

To perform the calculations of criteria, the author have used standard algorithms of the AHP method with the commonly used pairwise comparison scale 1–9, proposed by Saaty. In the case when alternative A1 is compared with alternative A2, this scale has the following values: 1 – if A1 and A2 are equal in importance; 3 – if A1 is weakly more important than A2; 5 – if A1 is strongly more important than A2; 7 – if A1 is very strongly more important than A2; 9 – if A1 is absolutely more important than A2; and 2, 4, 6, and 8 are intermediate values between the two adjacent judgments. The summary data of the pairwise comparisons for the criteria of the first hierarchy level are presented in Table 5.3 . The importance of the criteria is evident from the evaluation of the criteria priority vector. It is easy to notice, that “Programming aspects” criteria with value 0.434783 (or 43.48%) have the highest importance.

Table 5.3 Paired comparisons matrix for criteria (first hierarchy level)

Criteria	General	Programming aspects	Visualization	Simulation	User support	Priority vector
General	1	1/10	1	1/5	1	0.053140
Programming aspects	10	1	5	1	10	0.434783
Visualization	1	1/5	1	1/5	5	0.119163
Simulation	5	1	5	1	10	0.354267
User support	1	1/10	1/5	1/10	1	0.038647

We have calculated the matrices of the evaluations of the priority vector for the suggested simulation tools based on the evaluation of the criteria priority vector of two levels of the hierarchy. Table 4 presents an example of calculating the priorities of the second level criteria “Programming aspects". Similar calculations were made for all other the second level criteria – "General", "Visualization", "Simulation" and "User support". To perform the verifying the correctness of judgments in the criteria evaluation, the consistency ratio has been calculated, and for different groups of criteria it is between 3.7% and 9.22 %. The values of consistency ratio under 10% indicate that experts' judgments are sufficiently consistent. The final matrix of the evaluations of the global priority vector for the suggested simulation tools is shown above in Table 5. This can be used for choosing simulation package in a particular inventory problem solving. The value of the global criteria priority for AnyLogic is 0.5966, and it is significantly higher than the final evaluation of ExtendSim, which criteria priority has the value 0.4034. For groups "General", "Programming aspects" and "User support", the criteria values (priorities) of AnyLogic are greater than these criteria values of ExtendSim.

Table 5.4 Matrix of evaluations of the vector of the criteria priorities in the "Programming aspects" group

Alternatives	Criteria						Priorities in group 'Programming aspects'
	Programming flexibility	Support of programming concepts	Built in functions	Debugging	Code editor	External code connection	
	Numerical value of priority vector						
	0.20134	0.26174	0.22148	0.13758	0.05034	0.12752	
AnyLogic	0.80000	0.88889	0.66667	0.66667	0.25000	0.83333	0.75196
ExtendSim	0.20000	0.11111	0.33333	0.33333	0.75000	0.16667	0.24804

A special note for AnyLogic should be given to the criteria "Programming aspects" with value 0.7520; this criterion is three times higher than ExtendSim efficiency, having 0.2480. ExtandSim package evaluation exceeds AnyLogic package evaluation for group "Visualization" only, where the criterion value of ExtendSim has been estimated at 0.6291, while AnyLogic has been estimated at 0.3709. In the group "Simulation" both packages have practically the same values (0.491 and 0.5086). Consequently, the evaluation results show that the package AnyLogic has the higher value of priority and is recommended as a simulation tool for inventory control tasks. Nevertheless, the package ExtendSim is recommended for an employment in case, when property of visualization is the most important.

Table 5.5 Final evaluating result for simulation tools

Alternatives	Criteria					Global criteria priorities
	General	Programming aspects	Visualization	Simulation	User support	
	Numerical value of priority vector					
	0.0531	0.4348	0.1192	0.3543	0.0386	
AnyLogic	0.5726	0.7520	0.3709	0.4914	0.5406	0.5966
ExtendSim	0.4274	0.2480	0.6291	0.5086	0.4594	0.4034

Comparison of AnyLogic and ExtendSim packages estimation results is presented in the authors' previous paper and in the research under consideration. It demonstrates the coordination of the assessment results for the criteria of the first and of the second hierarchy levels. However, the estimations obtained with employment of simplified expert evaluation method are insignificantly different for AnyLogic and ExtendSim packages, while AHP method shows substantial differences in the efficiency of packages for the criteria of the first and the second hierarchy levels. In the authors' opinion, employment of AHP method allows more objective estimation of the efficiency of implementing the simulation tools in various inventory control tasks. AHP method's important merit lies in the possibility to control the consistency of the experts' evaluations, which facilitates the reliability of estimation.